PRECISION CONTROL OF LINEAR MOTOR DRIVEN HIGH-SPEED/ACCELERATION ELECTRO-MECHANICAL SYSTEMS

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OUTLINE

- Motivation and Issues
- Problem Formulation
- Direct Adaptive Robust Control (DARC)
- Desired Compensation DARC
- Indirect Adaptive Robust Control (IARC)
- Integrated Direct/Indirect ARC (DIARC)
- Conclusions

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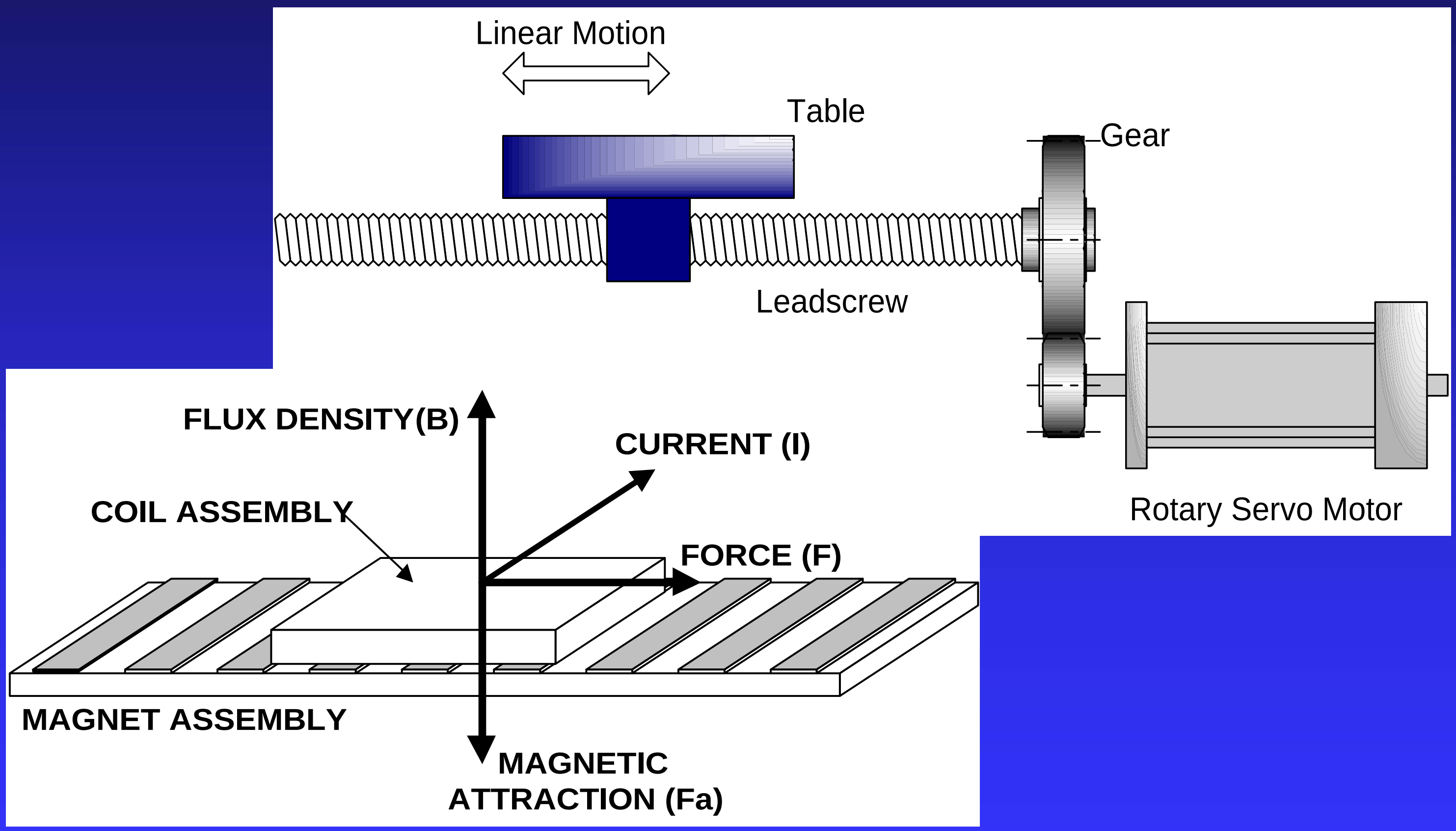
LINEAR MOTOR VS ROTARY MOTOR



LINEAR MOTOR DRIVE SYSTEMS

- Mechanical simplicity (no mechanical transmission mechanisms), higher reliability, and longer lifetime
- No backlash and less friction, resulting in the potential of having high load positioning accuracy
- No mechanical limitations on achievable acceleration and velocity
- Bandwidth is only limited by encoder resolution, measurement noise, calculation time, and frame stiffness

CONTROL ISSUES OF LINEAR MOTOR

- Model Uncertainties

  Parametric uncertainties (e.g., *load inertia*)

  Discontinuous disturbances (e.g., *Coulomb friction*); external disturbances (e.g., *cutting force*)

- Drawback of without mechanical transmissions

  Gear reduction reduces the effect of model uncertainties and external disturbance

- Significant uncertain nonlinearities due to position dependent electro-magnetic *force ripples* (e.g. iron-core linear motors)

- Implementation issues (e.g., *measurement noise*)

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MATHEMATICAL MODEL

- Linear Motor with Negligible Electrical Dynamics

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
M\dot{x}_2 &= u - Bx_2 - F_{fn}(x_2) - F_r(x_1) + F_d(x_1, x_2, t) \\
y &= x_1
\end{aligned}
$$

where

x_1 : position

x_2 : velocity

y : output

F_{fn} : nonlinear friction

F_d : lumped disturbance

M : mass of load/motor

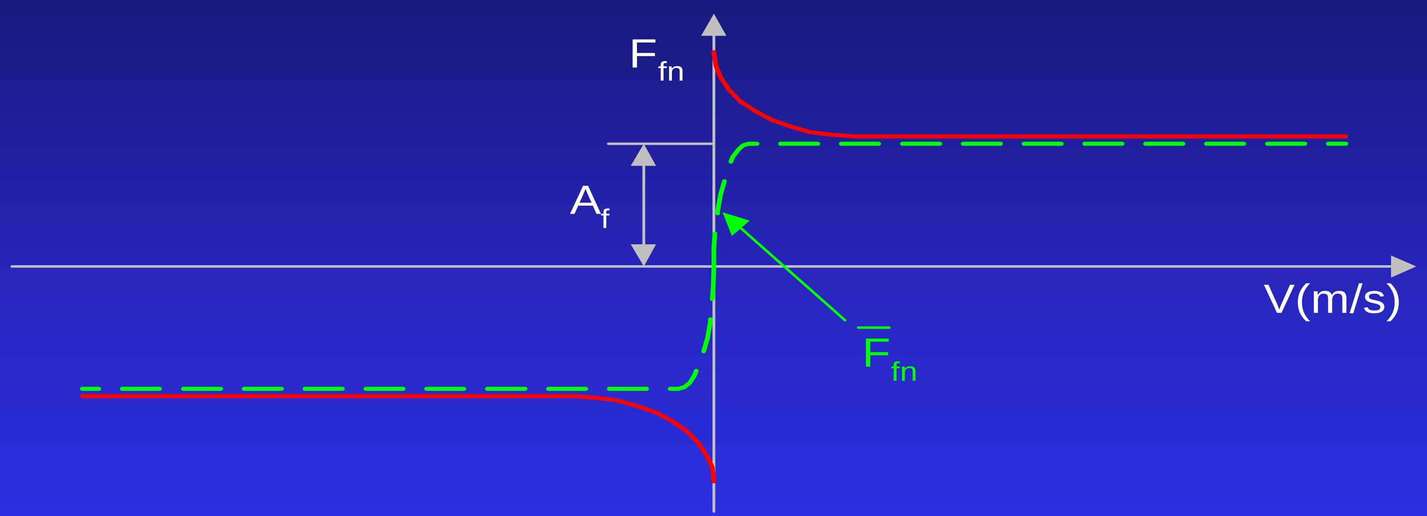
u : input voltage

B : viscous friction const

F_r : force ripple

FRICTION MODELLING

- Coulomb Friction Discontinuous at Zero Velocity



- Continuous Friction Model Approximation

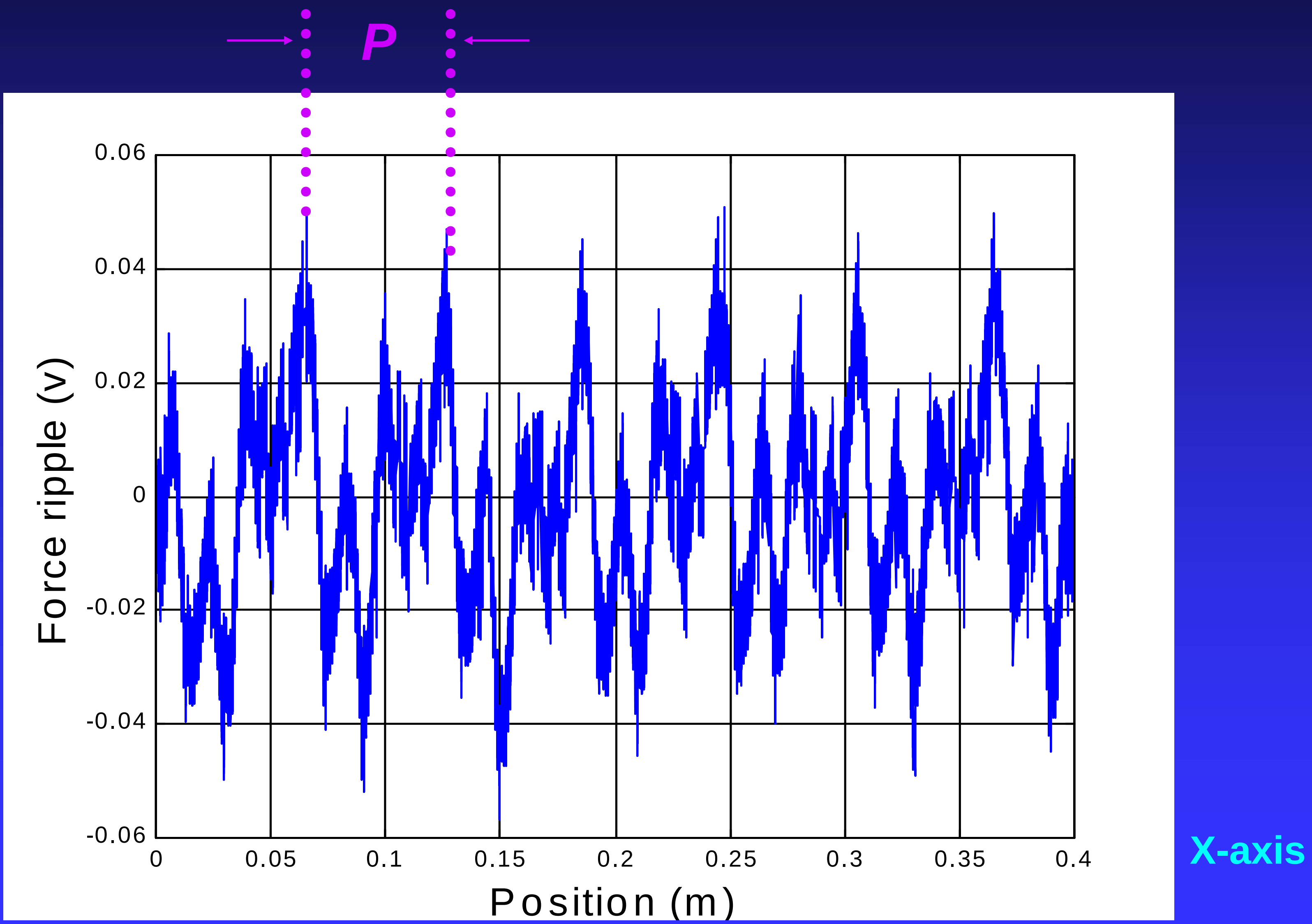
$$\overline{F}_{fn}(x_2) = A_f S_f(x_2)$$

FORCE RIPPLE MODELLING

- ***Cogging force*** is a magnetic force developed due to the attraction between the permanent magnets and the cores of the coil assembly. It depends only on the relative position of the motor coils with respect to the magnets, and is independent of the motor current

- ***Reluctance force*** is developed due to the variation of self inductance of the windings, which causes a position-dependent force in the direction of motion when current flows through the coils

MEASUREMENT OF FORCE RIPPLE



FORCE RIPPLE MODELLING

- The identical and equally spaced permanent magnets of the linear motor at a pitch of *P* :

$$F_r(x_1 + P) = F_r(x_1)$$

- Approximation via first *q* harmonics function of position with period of *P* :

$$\overline{F_r}(x_1) = S_r^T(x_1)\, A_r \approx F_r(x_1)$$

where

$$A_r = [A_{r1s}, A_{r1c}, \cdots, A_{rqs}, A_{rqc}]^T$$

$$S_r(x_1) = [\sin(\frac{2\pi}{P}x_1), \cos(\frac{2\pi}{P}x_1), \cdots, \sin(\frac{2\pi q}{P}x_1), \cos(\frac{2\pi q}{P}x_1)]^T$$

ARC DESIGN MODEL

- Actual System Dynamics

$$M\dot{x}_2 = u - Bx_2 - A_f S_f(x_2) - S_r^T(x_1)A_r + d$$

where

$$d = (\overline{F}_{fn} - F_{fn}) + (\overline{F}_r - F_r) + \Delta$$

- Parametrized Design Model

$$\dot{x}_1 = x_2$$

$$\theta_1 \dot{x}_2 = u - \theta_2 x_2 - \theta_3 S_f - \theta_{4b}^T S_r(x_1) + \theta_5 + \tilde{d}$$

$$y = x_1$$

where

$$\theta_1 = M, \quad \theta_2 = B, \quad \theta_3 = A_f, \quad \theta_{4b} = A_r$$

$$\theta_5 = d_n, \quad \tilde{d} = d - d_n$$

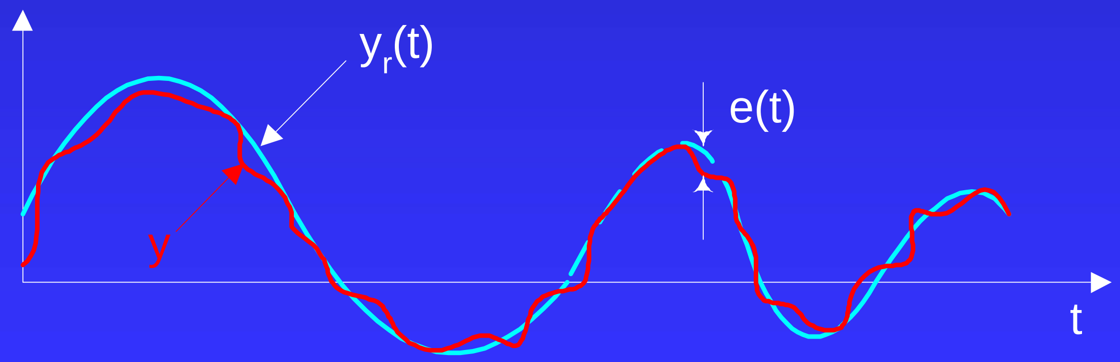
ASSUMPTIONS AND OBJECTIVES

- **Assumption:**

$$\begin{aligned} \theta &\in \Omega_\theta = \{\theta : \quad \theta_{\min} < \theta < \theta_{\max}\} \\ \tilde{d} &\in \Omega_d = \{\tilde{d} : \quad |\tilde{d}| \le \delta_d(x,t)\} \end{aligned}$$

- **Primary Objective:**

Synthesize a control input *u* such that the output *y* track the reference motion trajectory y_r as closely as possible



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DIRECT ADAPTIVE ROBUST CONTROL

- Define a switching-function-like quantity as

$$p = \dot{e} + k_1 e = x_2 - x_{2eq}, \qquad x_{2eq} = \dot{y}_d - k_1 e$$

where

$$e = y - y_d(t)$$

- Error dynamics

$$M\dot{p} = u + \varphi^T \theta + \tilde{d}$$

where

$$\varphi^T = [-\dot{x}_{2eq}, -x_2, -S_f(x_2), -S_r(x_1), 1]$$
$$\dot{x}_{2eq} = \ddot{y}_d - k_1 \dot{e}$$

DIRECT ADAPTIVE ROBUST CONTROL

- ARC control law

$$u = u_a + u_s, \qquad u_a = -\varphi^T \hat{\theta}$$

$$u_s = u_{s1} + u_{s2}, \qquad u_{s1} = -k_2 p$$

- Error dynamics

$$M\dot{p} + k_2 p = u_{s2} - \varphi^T \tilde{\theta} + \tilde{d}$$

- Choose robust control such that

$$i \qquad p\left(u_{s2} - \varphi^T \tilde{\theta} + \tilde{d}\right) \leq \varepsilon$$

$$ii \qquad p u_{s2} \leq 0$$

where ε is a design parameter

- Example

$$u_{s2} = -\frac{1}{4\varepsilon}(\|\theta_{\max} - \theta_{\min}\| \cdot \|\varphi\| + \delta_d)^2 p$$

THEORETICAL PERFORMANCE OF DARC

If the adaptation function τ is chosen as $\tau = \varphi p$, then the ARC control law guarantees that

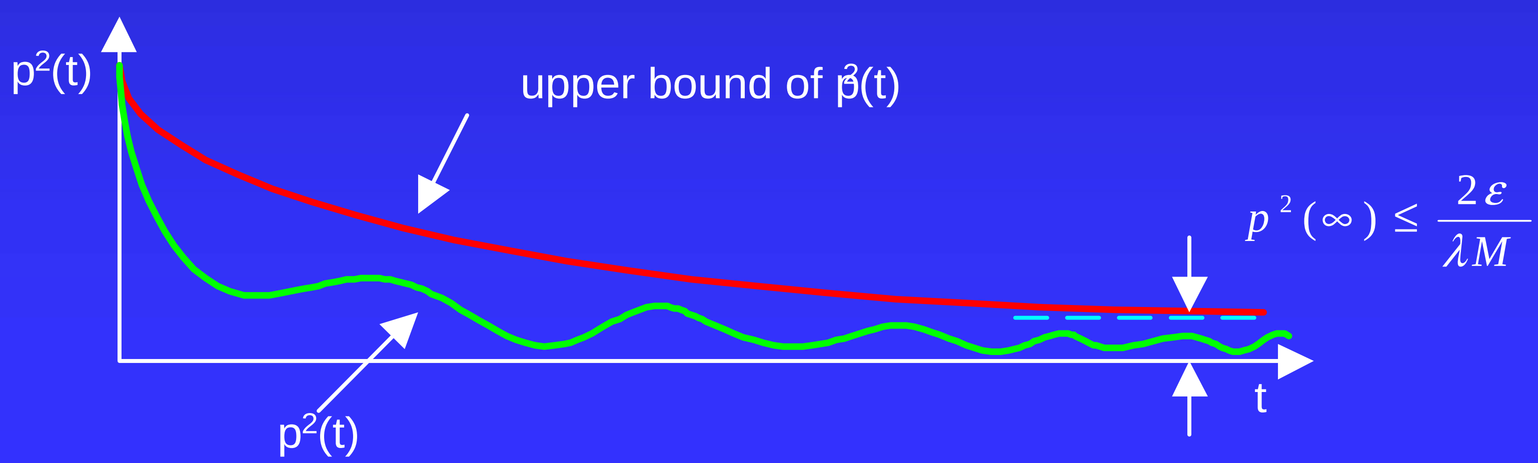
- In general, all signal are bounded. Furthermore, the positive definite function V_s defined by

$$V_s = \frac{1}{2} M p^2$$

is bounded by

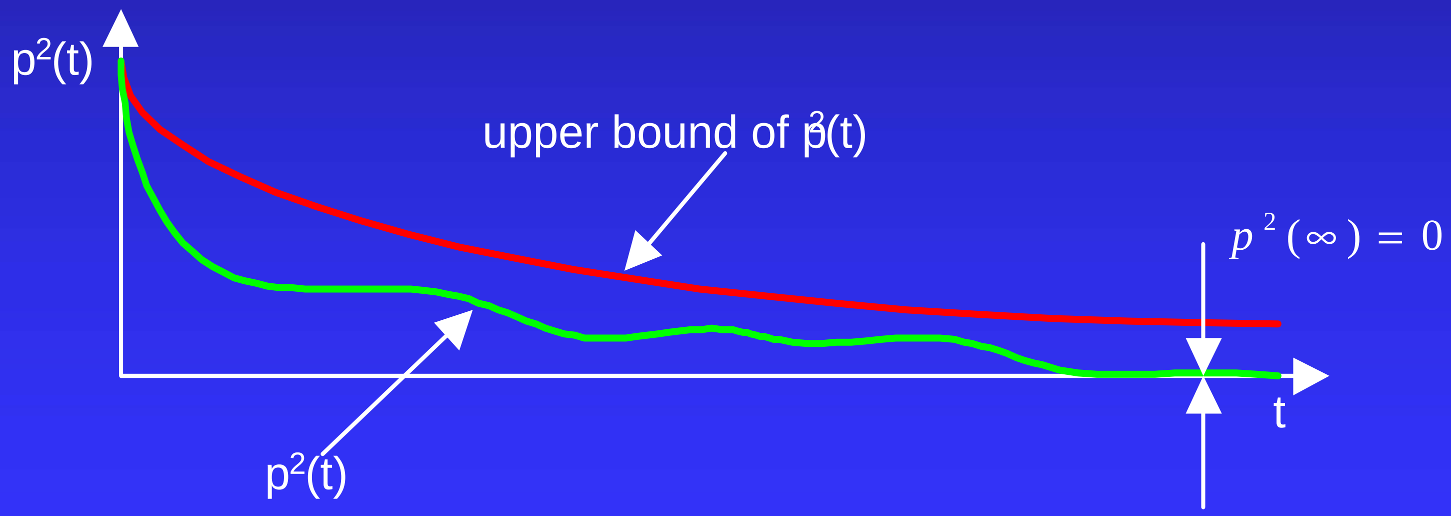
$$V_s \le \exp(-\lambda t) V_s(0) + \frac{\varepsilon}{\lambda}[1 - \exp(-\lambda t)]$$

where $\lambda = 2k_2/\theta_{1\max}$



THEORETICAL PERFORMANCE OF DARC

- In addition, zero final tracking error is achieved in the presence of parametric uncertainties only (i.e., $\tilde{d} = 0, \forall t \geq t_0$)



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IMPLEMENTATION ISSUES OF DARC

- Regressor φ has to be calculated on-line based on the actual measurement of the velocity x_2
  - The effect of velocity measurement noise may be severe
  - Slow adaptation rate has to be used

- Model compensation u_a depends on the actual feedback of the state
  - Creates certain interactions between the model compensation u_a and the robust control u_s
  - Complicates the controller gain tuning process

DESIRED COMPENSATION ARC

- DCARC control law

$$u = u_a + u_s, \qquad u_a = -\varphi_d{}^T \hat{\theta}$$

$$\tau = \varphi_d p$$

where $\varphi_d{}^T = [-\ddot{y}_d, -\dot{y}_d, -S_f(\dot{y}_d), -S_r(y_d), 1]$

- Error dynamics

$$M\dot{p} = u_s - \varphi_d{}^T \tilde{\theta} + \underbrace{(\theta_1 k_1 - \theta_2)\dot{e} + \theta_3[S_f(\dot{y}_d) - S_f(x_2)] + \theta_{4b}^T[S_r(y_d) - S_r(x_1)]}_{addition \quad terms} + \tilde{d}$$

- Notice that (applying Mean Value Theorem)

$$S_f(\dot{y}_d) - S_f(x_2) = g_f(x_2, t)\dot{e}, \quad S_r(x_1) - S_r(y_d) = g_r(x_2, t)e$$

where $g_f(x_2, t)$ and $g_r(x_1, t)$ are certain nonlinear functions

DESIRED COMPENSATION ARC

- Robust control function

$$u_s = u_{s1} + u_{s2}, \qquad u_{s1} = -k_{s1} p$$

where k_{s1} is a nonlinear gain such that the matrix A defined below is p.d.

$$A = \begin{bmatrix} k_{s1} - k_2 - \theta_1 k_1 + \theta_2 + \theta_3 g_f & -\frac{1}{2}(k_1\theta_2 + k_1\theta_3 g_f - \theta_{4b}^T g_r) \\ -\frac{1}{2}(k_1\theta_2 + k_1\theta_3 g_f - \theta_{4b}^T g_r) & \frac{1}{2} M k_1^3 \end{bmatrix}$$

For example

$$k_{s1} \geq k_2 + \theta_1 k_1 - \theta_2 - \theta_3 g_f + \frac{1}{2\theta_1 k_1^3}(\theta_2 k_1 + \theta_3 k_1 g_f + \left|\theta_{4b}^T g_r\right|)^2$$

- Choose u_{s2} such that

$$i \qquad p\left(u_{s2} - \varphi_d{}^T \tilde{\theta} + \tilde{d}\right) \leq \varepsilon$$

$$ii \qquad p u_{s2} \leq 0$$

THEORETICAL PERFORMANCE OF DCARC

If the DCARC law is applied, then

- In general, all signal are bounded. Furthermore, the positive definite function V_s defined by

$$V_s = \frac{1}{2} M p^2 + \frac{1}{2} M k_1^{\,2} e^2$$

is bounded by

$$V_s \le \exp(-\lambda t) V_s(0) + \frac{\varepsilon}{\lambda}[1 - \exp(-\lambda t)]$$

where $\lambda = \min\{2k_2/\theta_{1\max}, k_1\}$

- In addition, zero final tracking error (i.e., asymptotic tracking) is achieved in the presence of parametric uncertainties only

IMPLEMENTATION OF DCARC LAW

- Digital implementation of the adaptation law

$$\hat{\theta}_i[(j+1)\Delta T] = \begin{cases} \theta_{i\min} & \text{if} \quad \hat{\theta}_{\mathrm{i}} = \hat{\theta}_{\mathrm{imax}} \quad \text{and} \quad \bullet > 0 \\ \theta_{i\max} & \text{if} \quad \hat{\theta}_{\mathrm{i}} = \hat{\theta}_{\mathrm{imin}} \quad \text{and} \quad \bullet < 0 \\ \Theta_i & \text{otherwise} \end{cases}$$

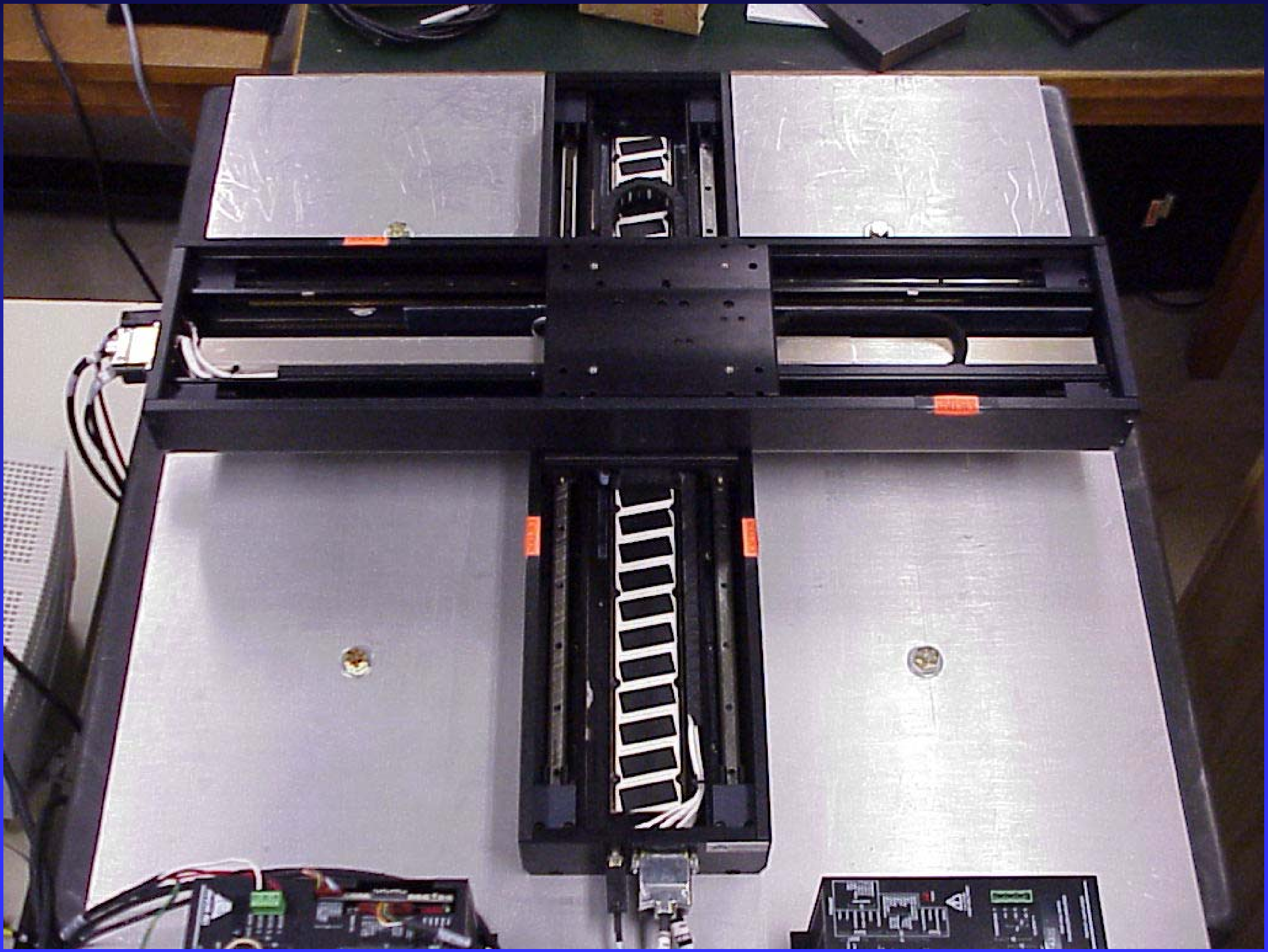
where

$$\Theta_i = \hat{\theta}_i(j\Delta T) + \gamma_i \int_{j\Delta T}^{(j+1)\Delta T} \varphi_{d,i}(\dot{e} + k_1 e)dt$$

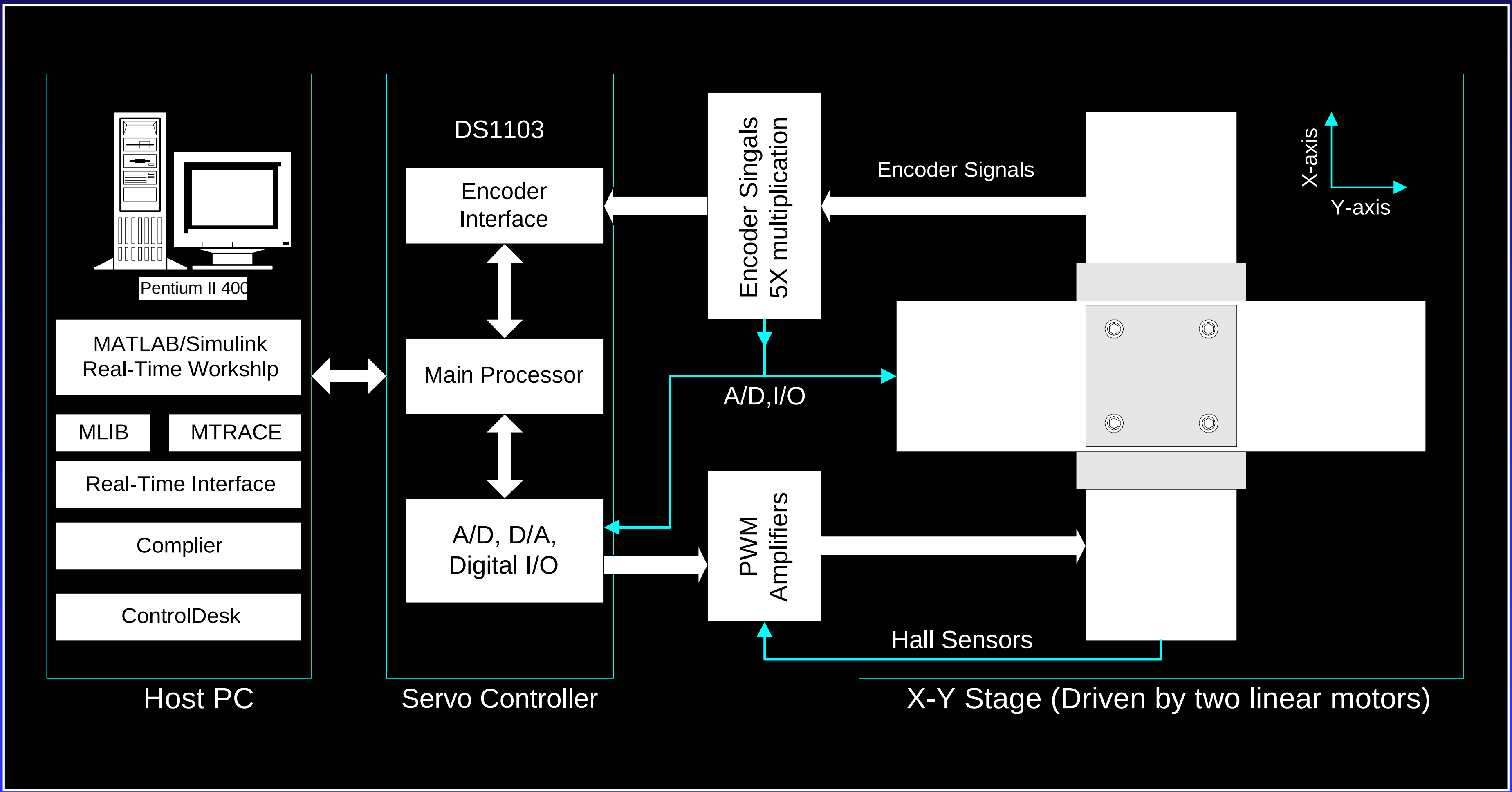
- Velocity-free implementation of adaptation law

$$\Theta_i = \hat{\theta}_i(j\Delta T) + \gamma_i \left(k_1 \int_{j\Delta T}^{(j+1)\Delta T} \varphi_{d,i} e dt + \dot{\varphi}_{d,i} e \Big|_{j\Delta T}^{(j+1)\Delta T} - \int_{j\Delta T}^{(j+1)\Delta T} \dot{\varphi}_{d,i}(t) e dt \right)$$

LINEAR MOTOR POSITIONING STAGE



EXPERIMENTAL SETUP



PERFORMANCE INDEXES

- **Transient Performance:** $$e_M = \max_t \{|e(t)|\}$$
- **Final Tracking Accuracy:** $$e_F = \max_{t \in [T_f - 2, T_f]} \{|e(t)|\}$$
- **Average Tracking Performance:** $$L_2[e] = \sqrt{\frac{1}{T_f} \int_0^{T_f} |e|^2 \, dt}$$
- **Average Control Input:** $$L_2[u] = \sqrt{\frac{1}{T_f} \int_0^{T_f} |u(t)|^2 \, dt}$$
- **Degree of Control Chattering:**

$$c_u = \sum \frac{L_2[\Delta u]}{L_2[u]}, \qquad L_2[\Delta u] = \sqrt{\frac{1}{N} \sum_{j=1}^{N} |u(j\Delta T) - u((j-1)\Delta T)|^2}$$

COMPARATIVE EXPERIMENTS (Y-AXIS)

- PID with model compensation:

$$u = \hat{\theta}_1(0)\ddot{y}_d(t) + \hat{\theta}_2(0)\dot{y}_d(t) + \hat{\theta}_3(0)S_f(\dot{y}) - K_p e - K_i \int e dt - K_d \dot{e}$$

$$S_f(\dot{y}) = \frac{2}{\pi}\arctan(900\dot{y}), \quad K_p = 5.4\times10^3, \quad K_i = 5.4\times10^5, \quad K_d = 18$$

- ARC:

$$k_1 = 400, \quad k_2 = 32, \quad \hat{\theta}(0) = [0.05, 0.25, 0.1, 0]^T$$

$$\Gamma = diag[5, 0, 2, 1000]$$

- DRC:
  - Same controller law with ARC but without parameter adaptation

- DCARC:

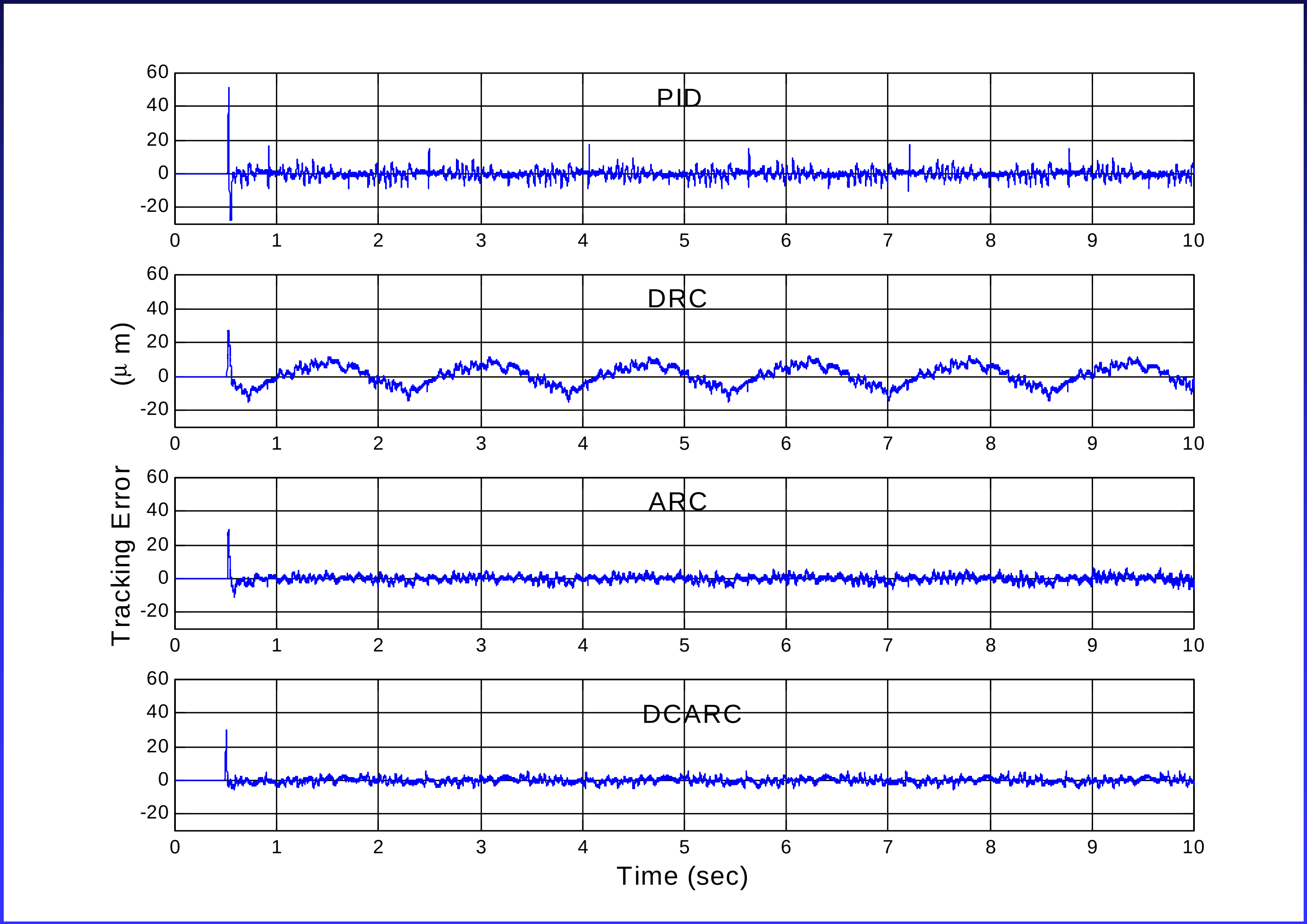
$$k_1 = 400, \quad k_2 = 32, \quad \hat{\theta}(0) = [0.05, 0.25, 0.1, 0]^T$$

$$\Gamma = diag[25, 0, 5, 1000]$$

SINUSOIDAL TRAJECTORY TRACKING

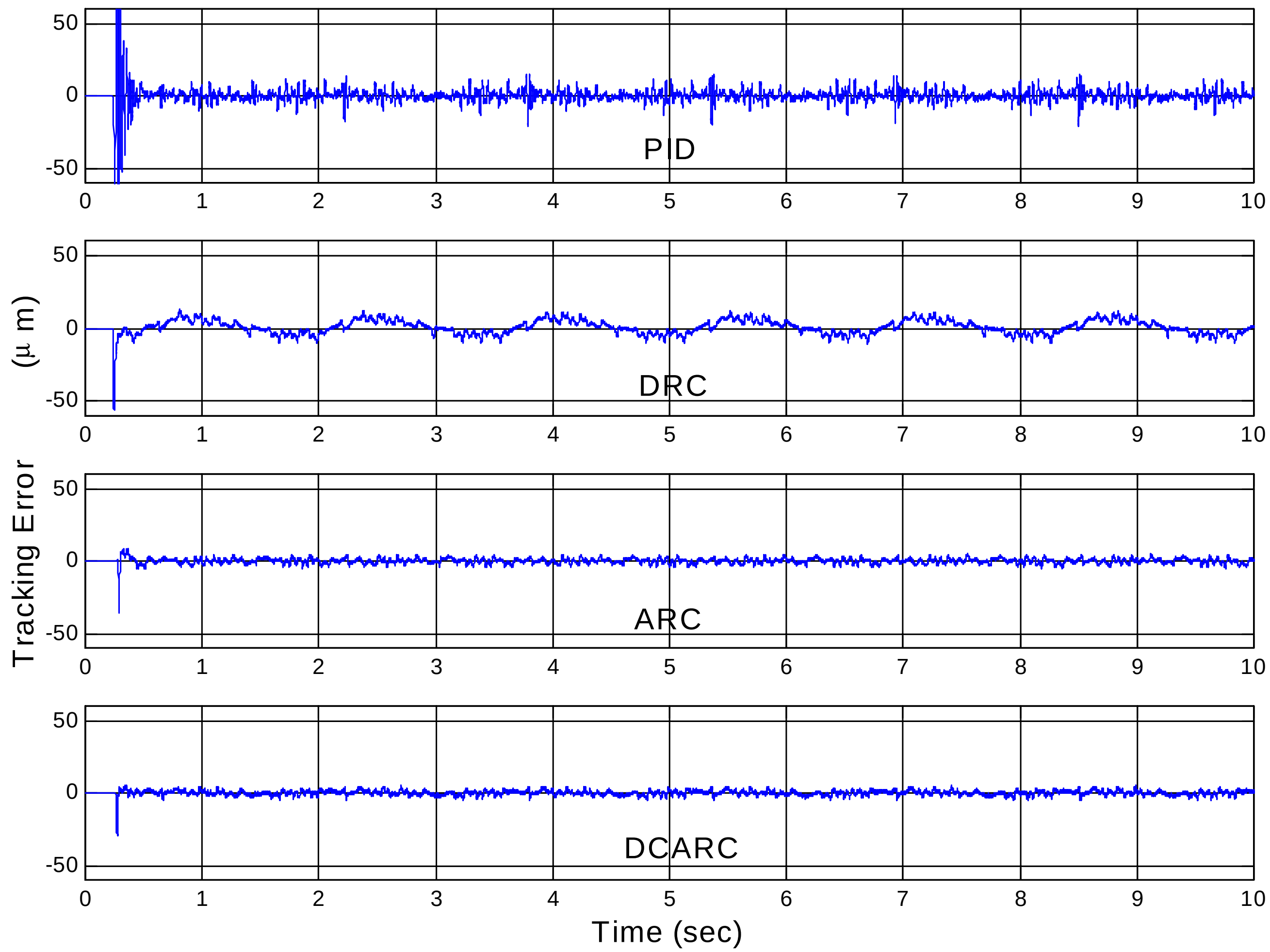
- Set 1:
  - To test the nominal tracking performance, the motors are run without payload
- Set 2:
  - To test the performance robustness of the algorithms to parameter variations, a 20lb payload is mounted on the motor
- Set 3:
  - A large step disturbance (a simulated 0.5V electrical signal) is added at about t=2.5s and removed at t=7.5s to test the performance robustness of each controller to disturbance

TRACKING ERRORS (UNLOADED)



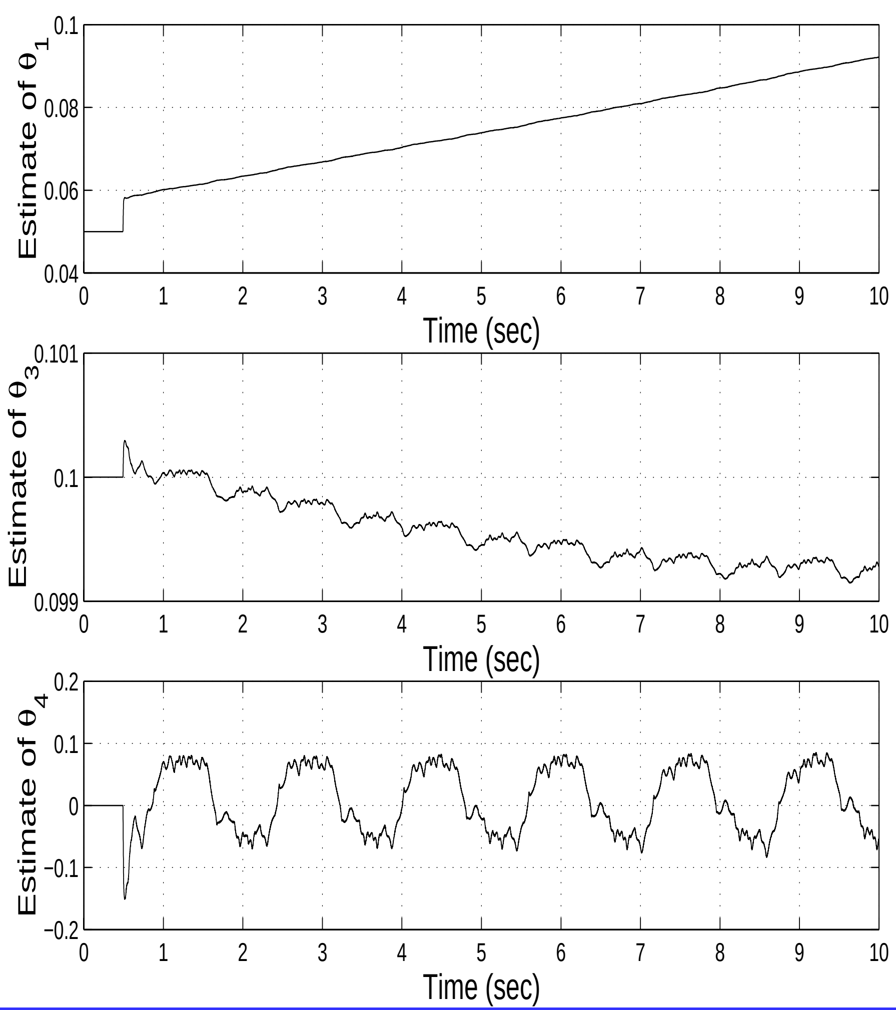
$y_d = 0.05 sin(4t)$

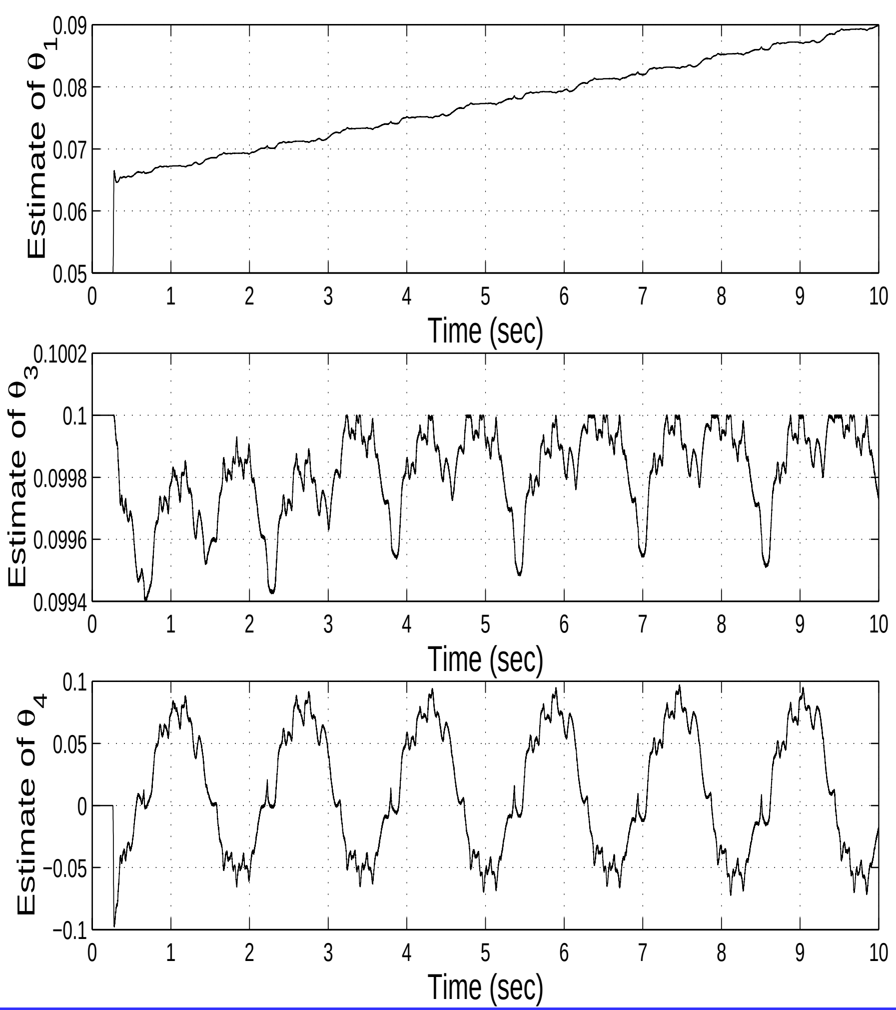
TRACKING ERRORS (LOADED)



$y_d = 0.05 sin(4t)$ 20*lb* Load

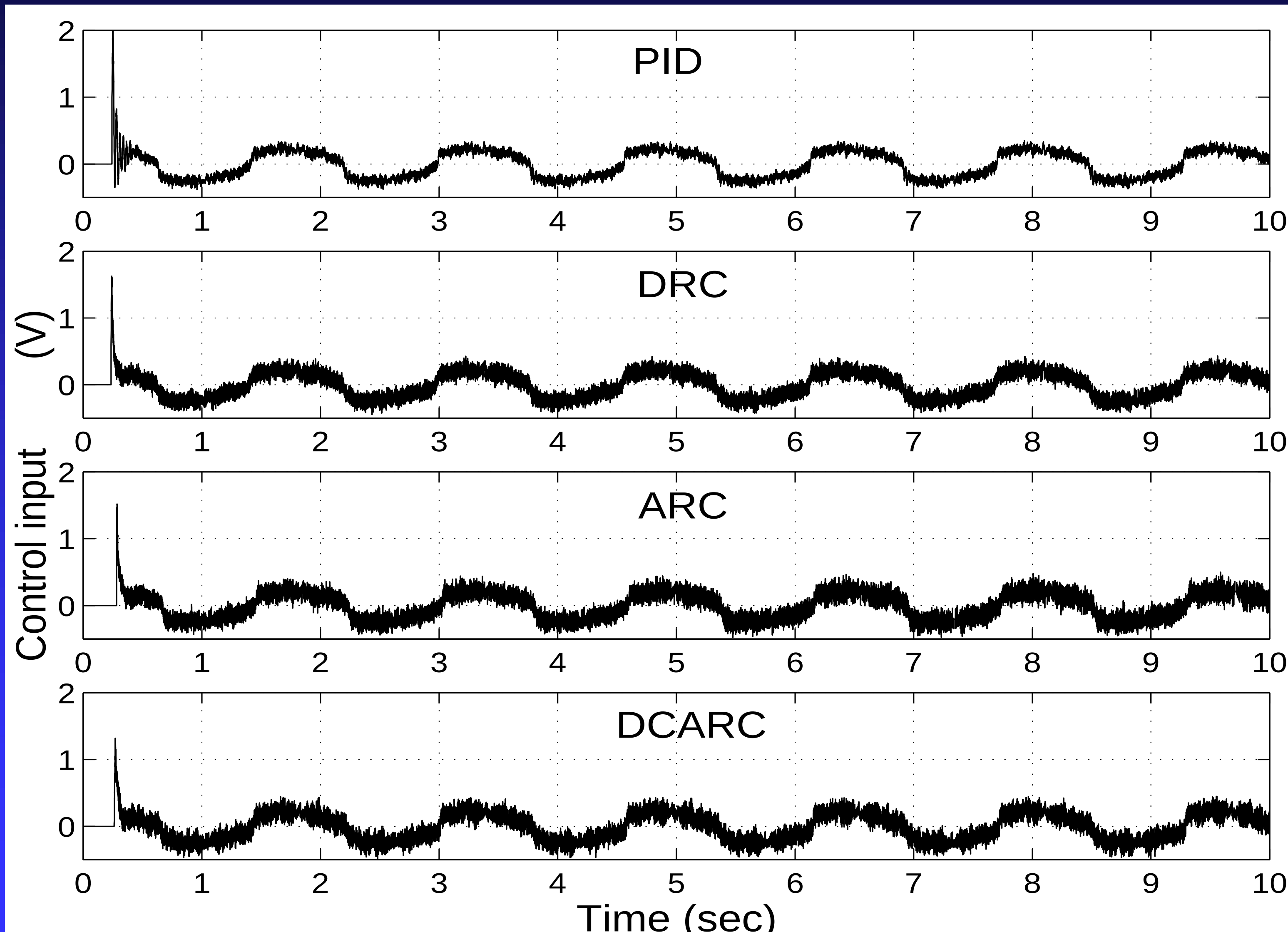
PARAMETER ESTIMATES (LOADED)



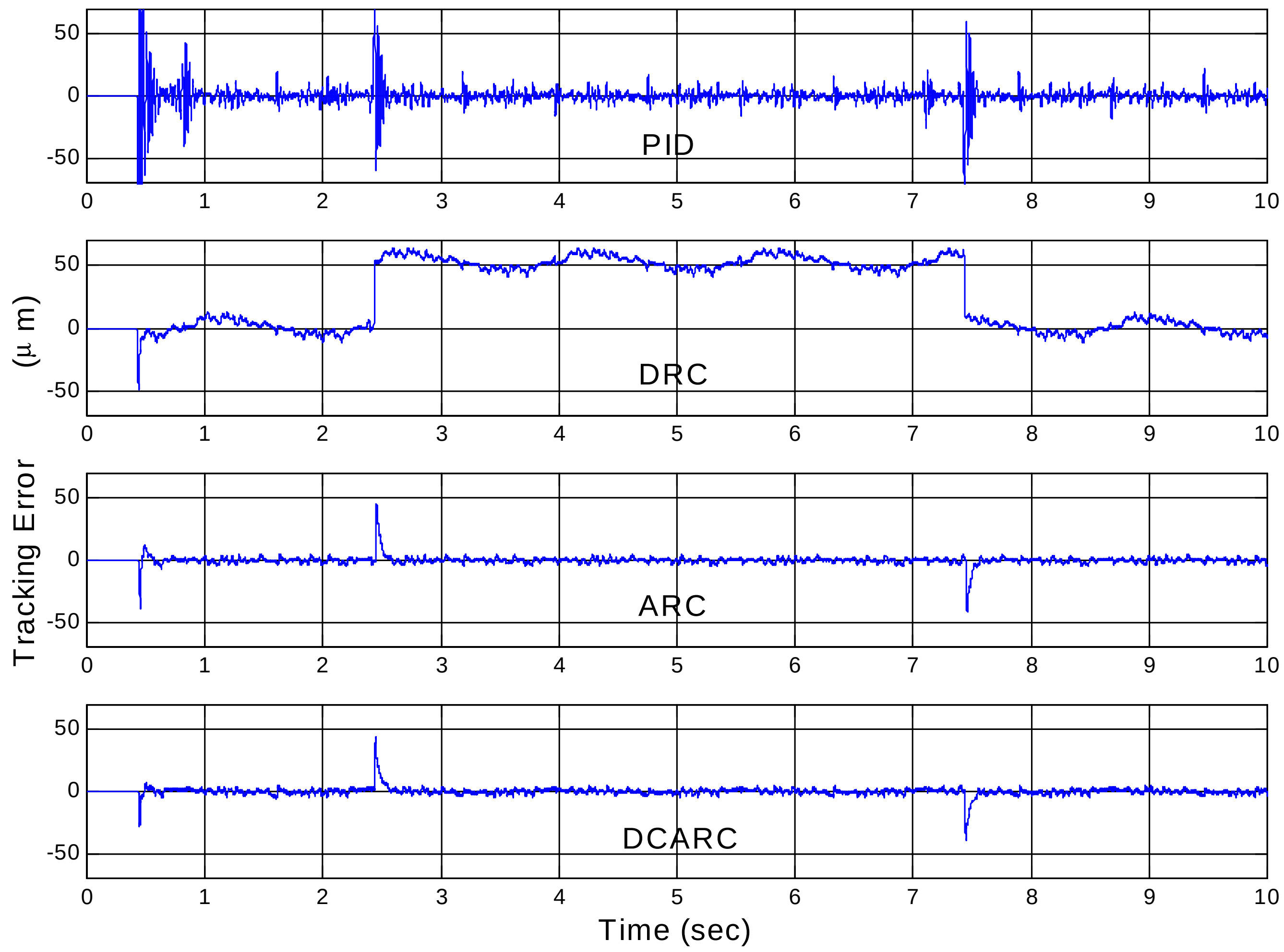


ARC $y_d = 0.05 sin(4t)$ **20*lb* Load** **DCARC**

CONTROL INPUTS (LOADED)

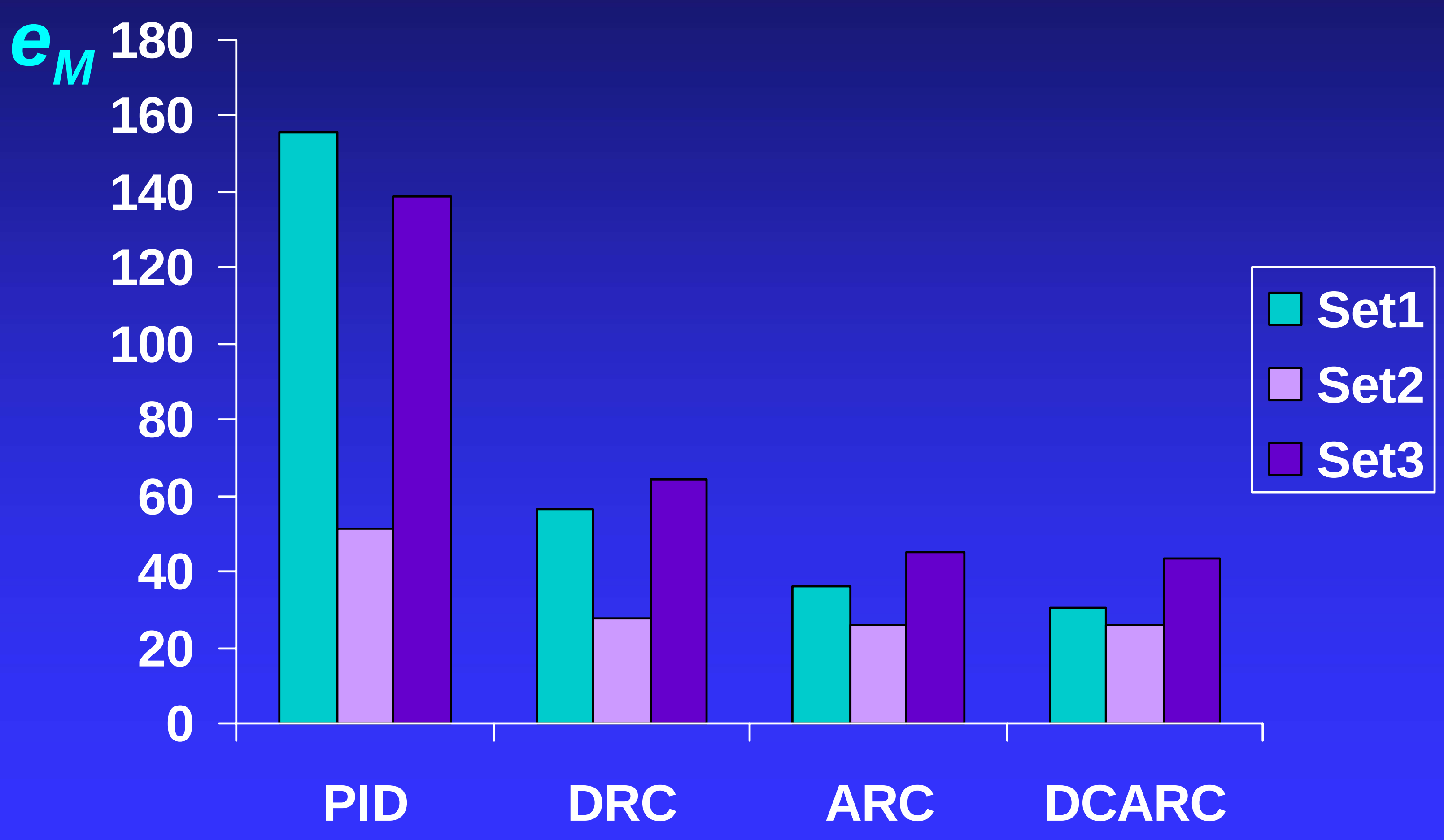


TRACKING ERRORS (DISTURBANCE)

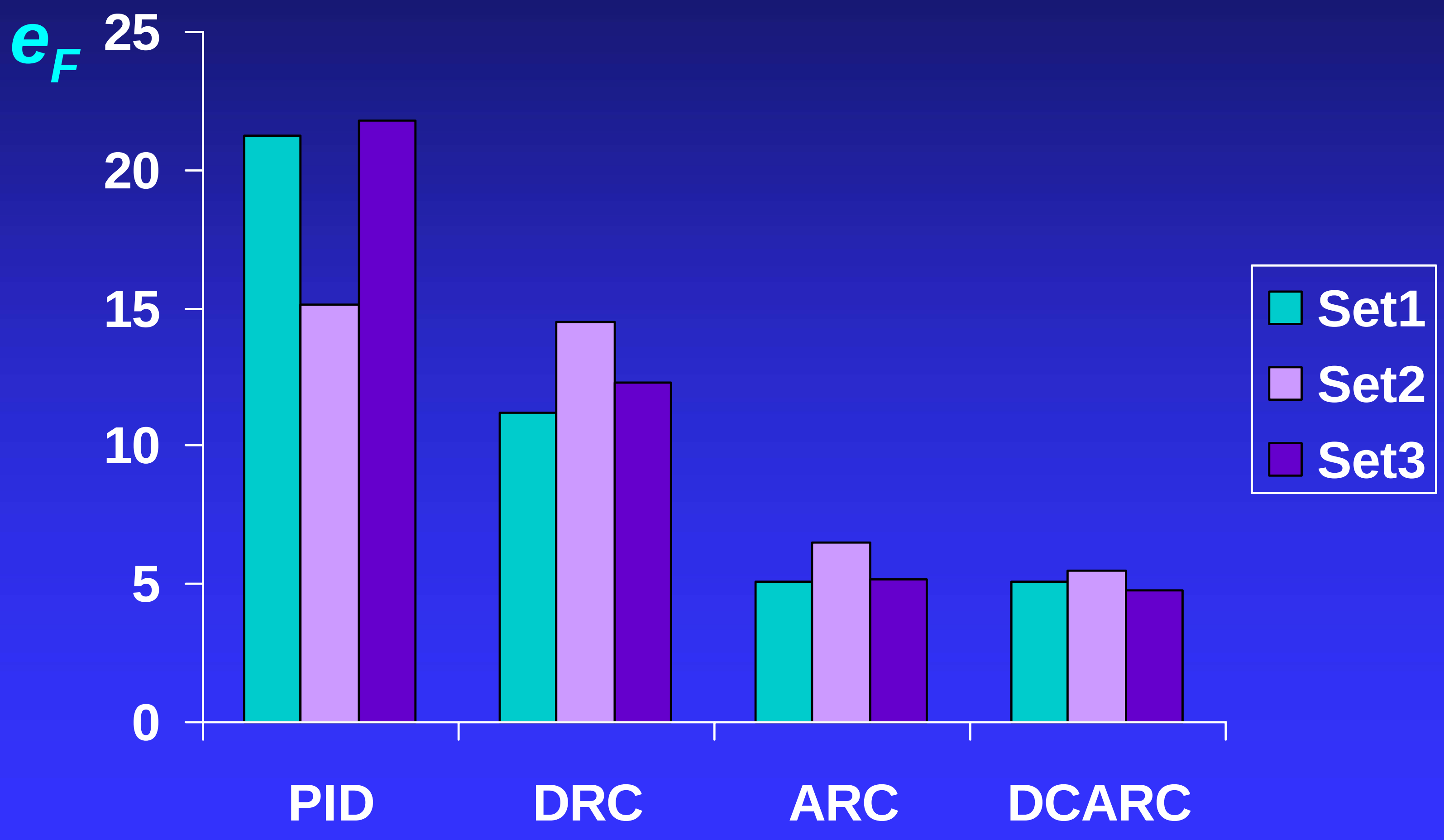


$y_d = 0.05\sin(4t)$ 20*lb* Load Step disturbance between 2.4 and 7.4 sec

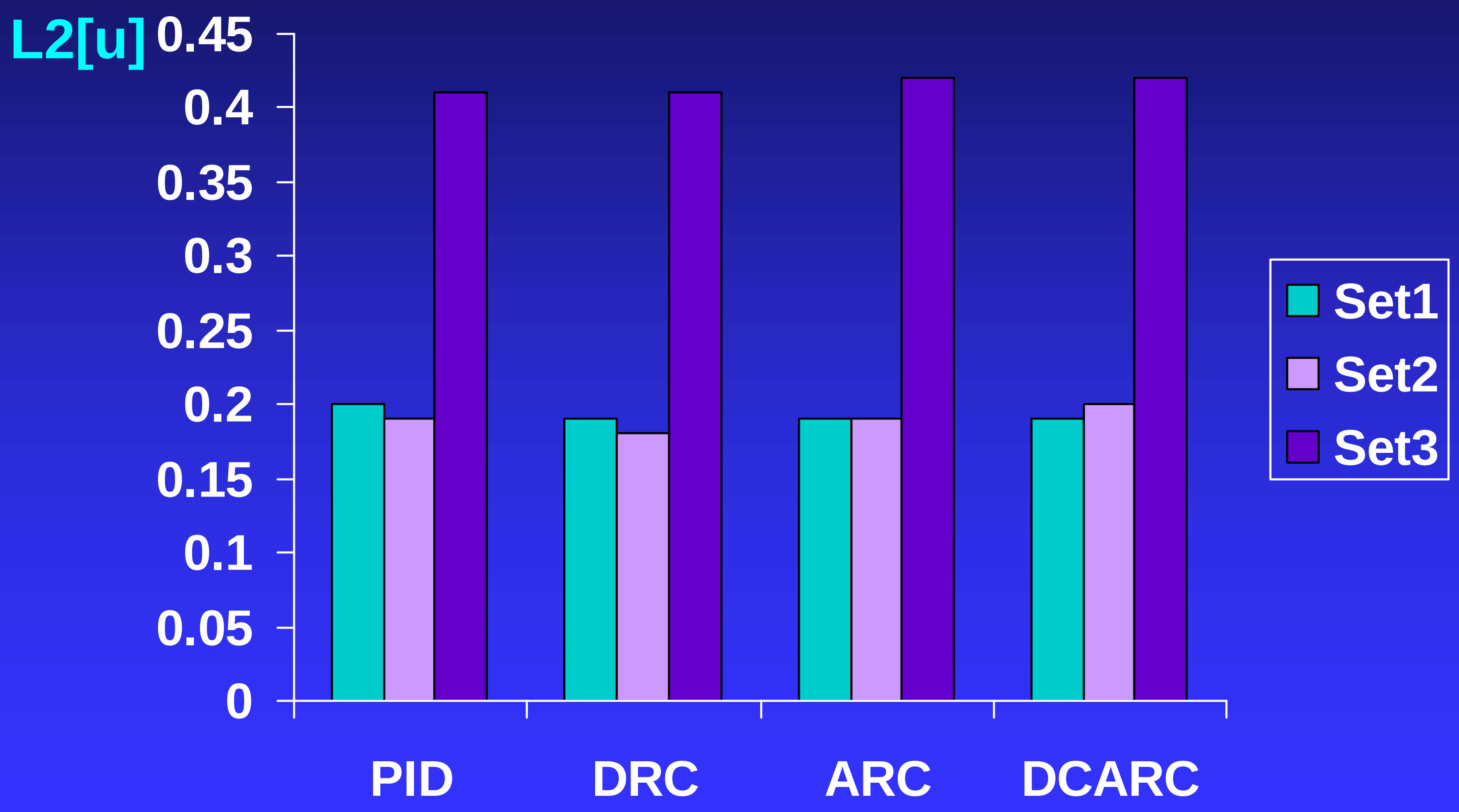
TRANSIENT PERFORMANCE



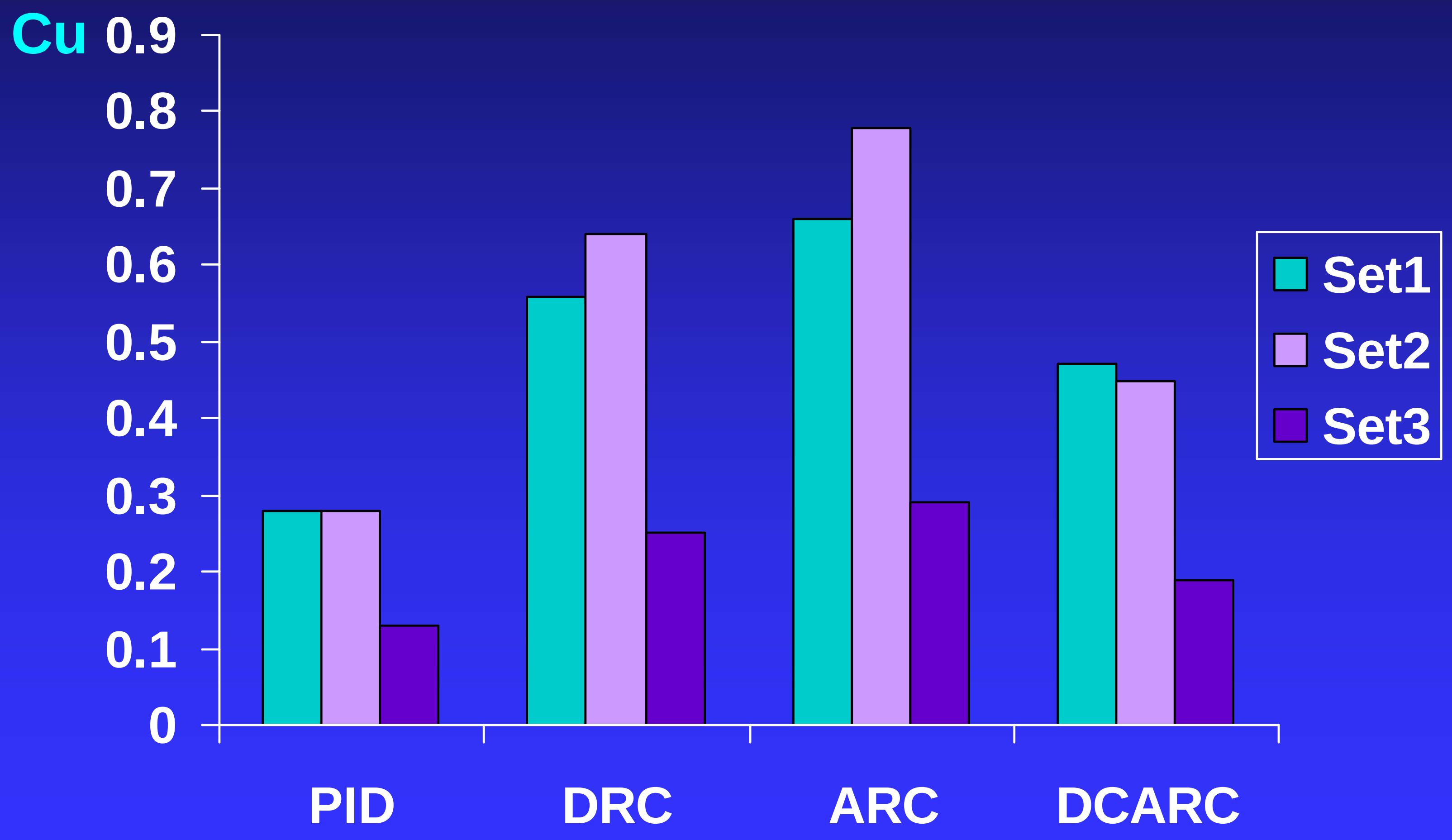
FINAL TRACKING ACCURACY



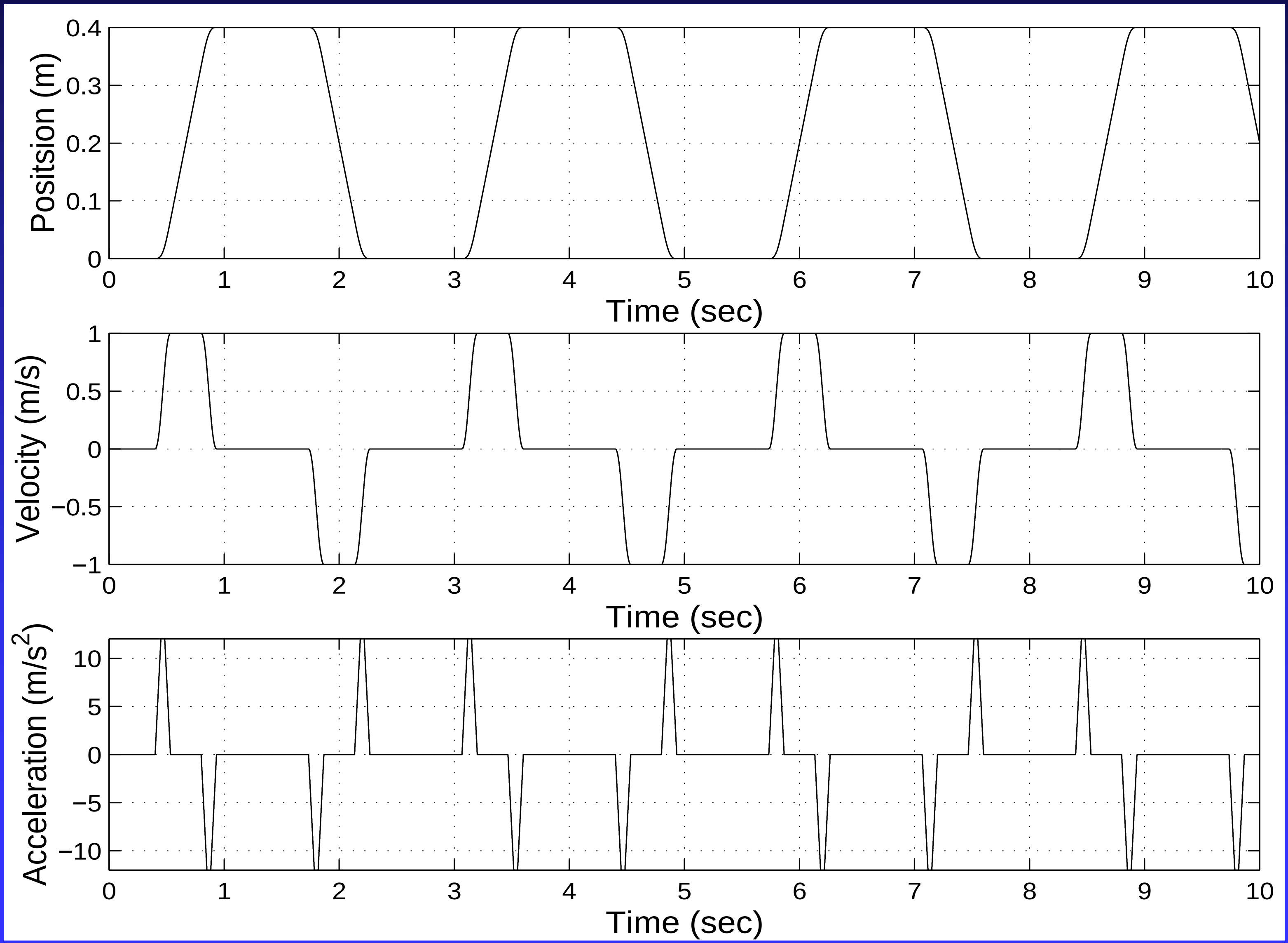
CONTROL EFFORT



CONTROL INPUT CHATTERING

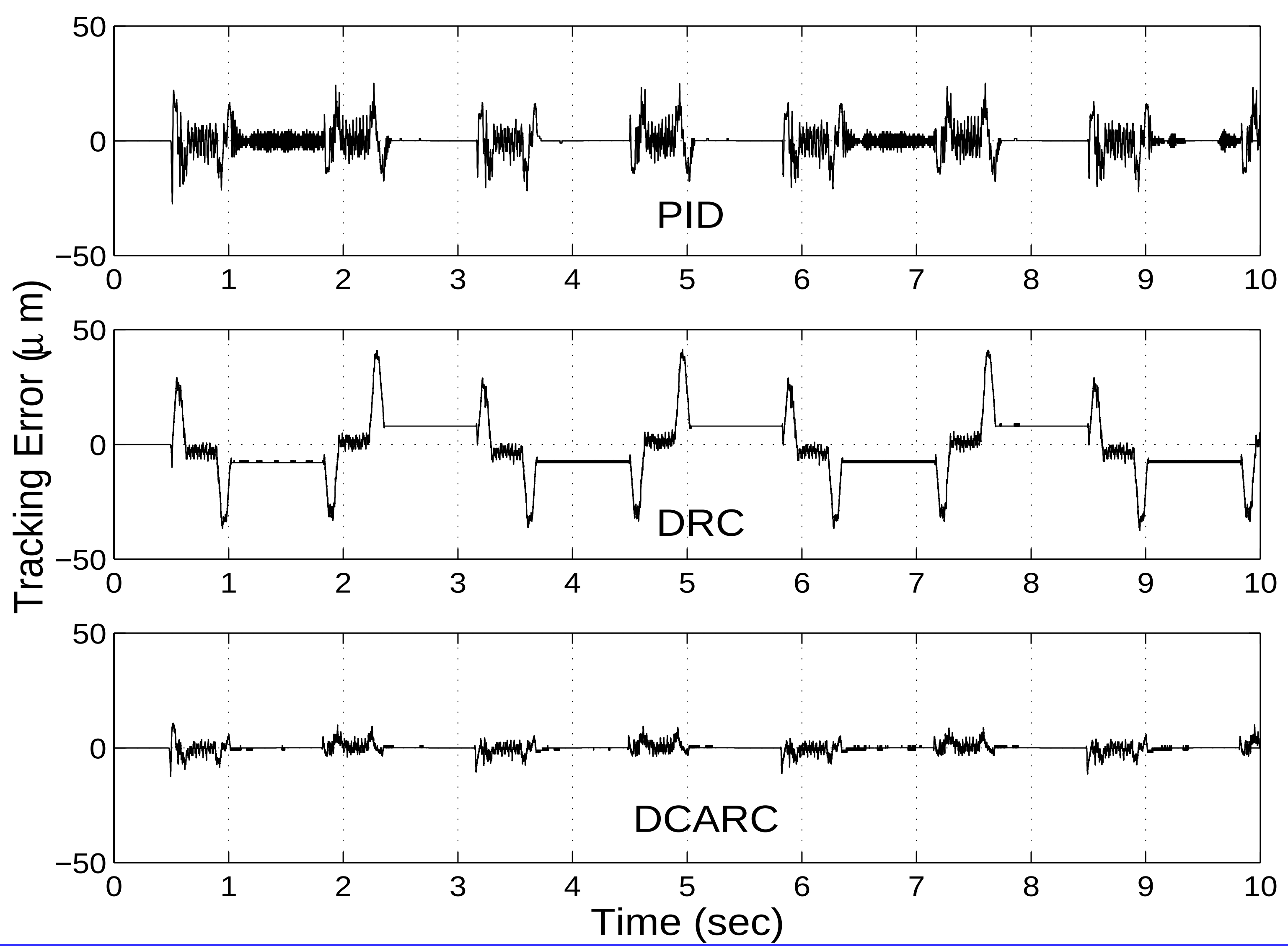


POINT-TO-POINTMOTION TRAJECTORY

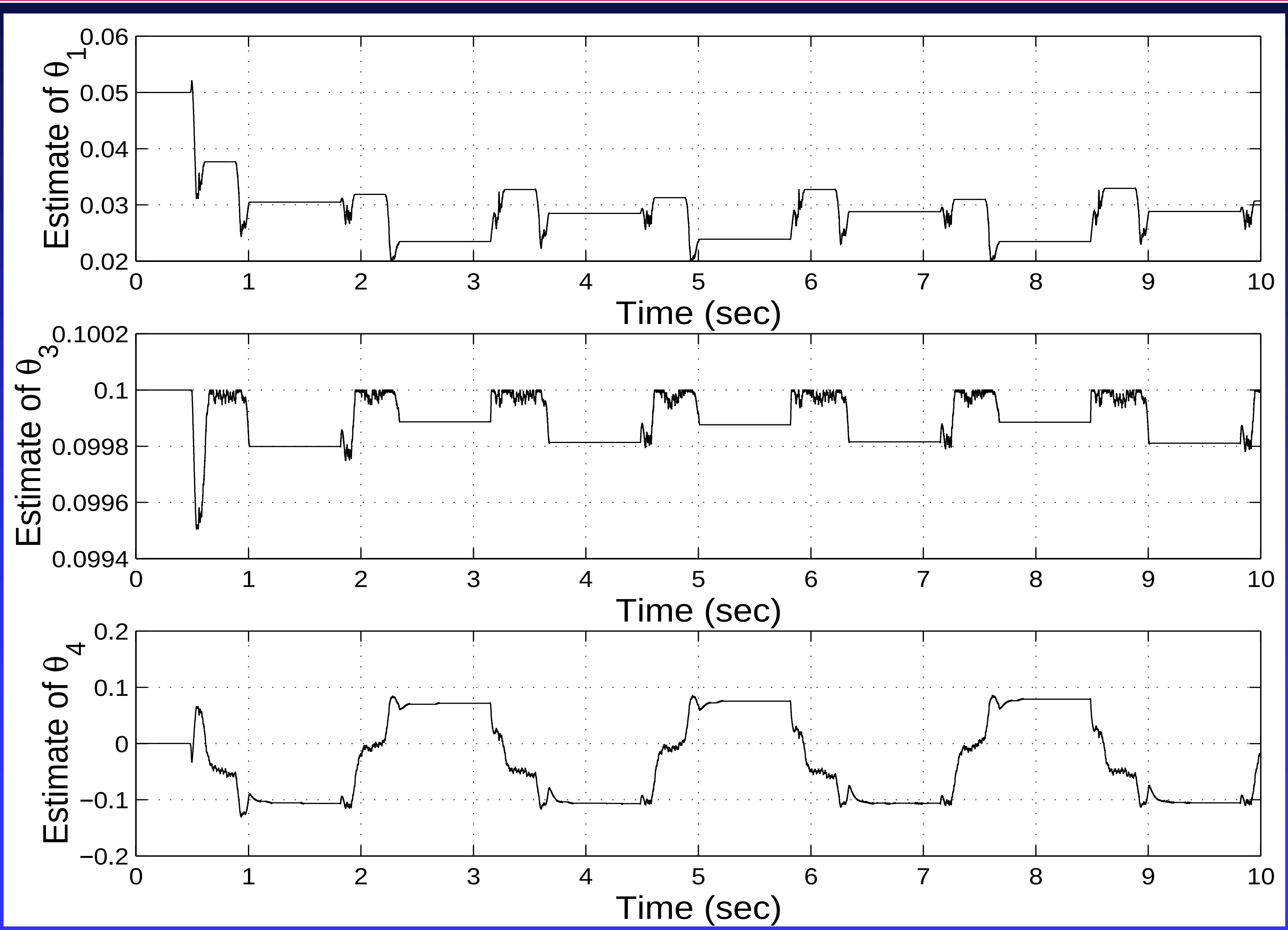


UNLOADED EXPERIMENTS

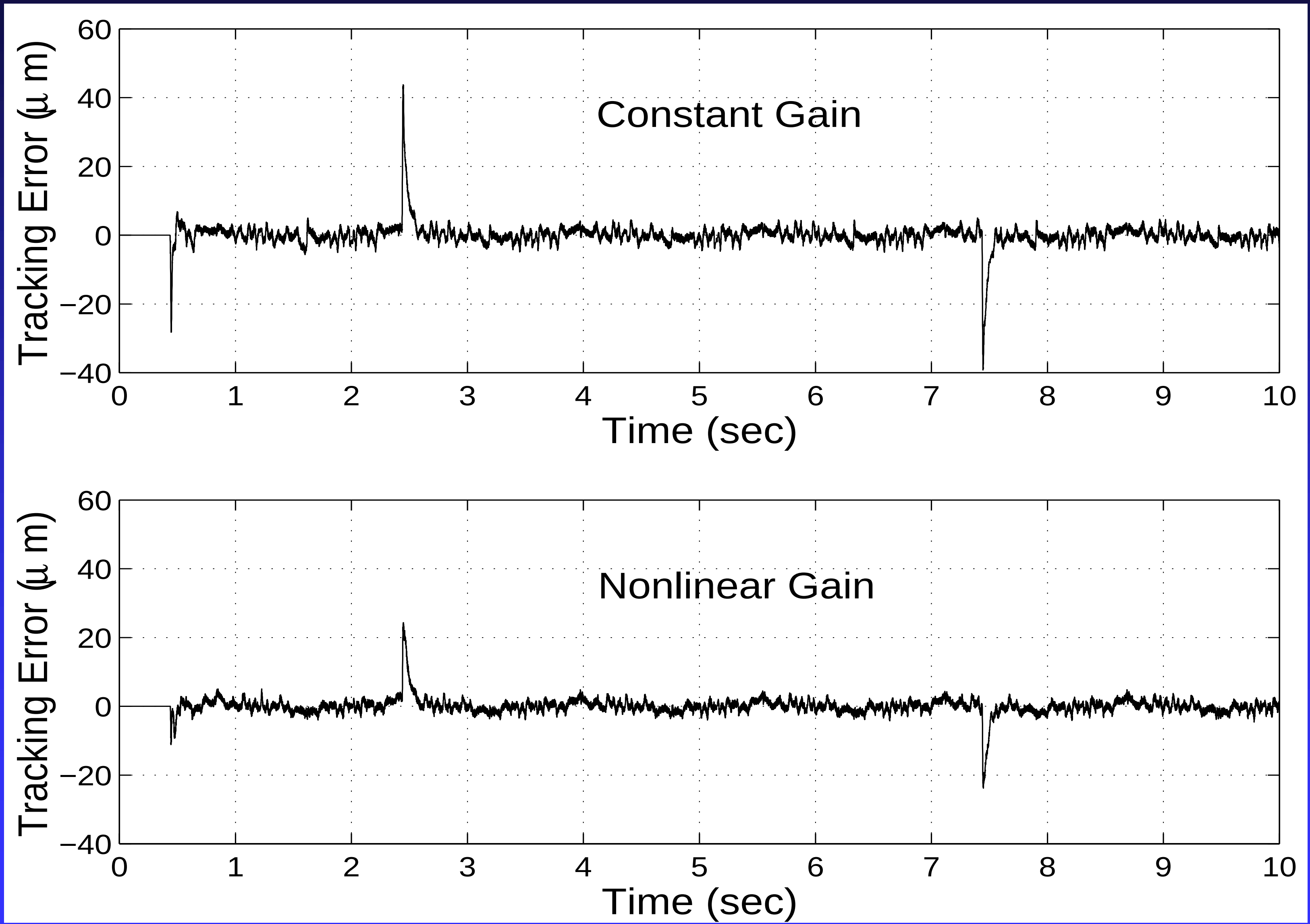
POINT-POINT MOTION TRACKING ERRORS



DARC PARAMETER ESTIMATES

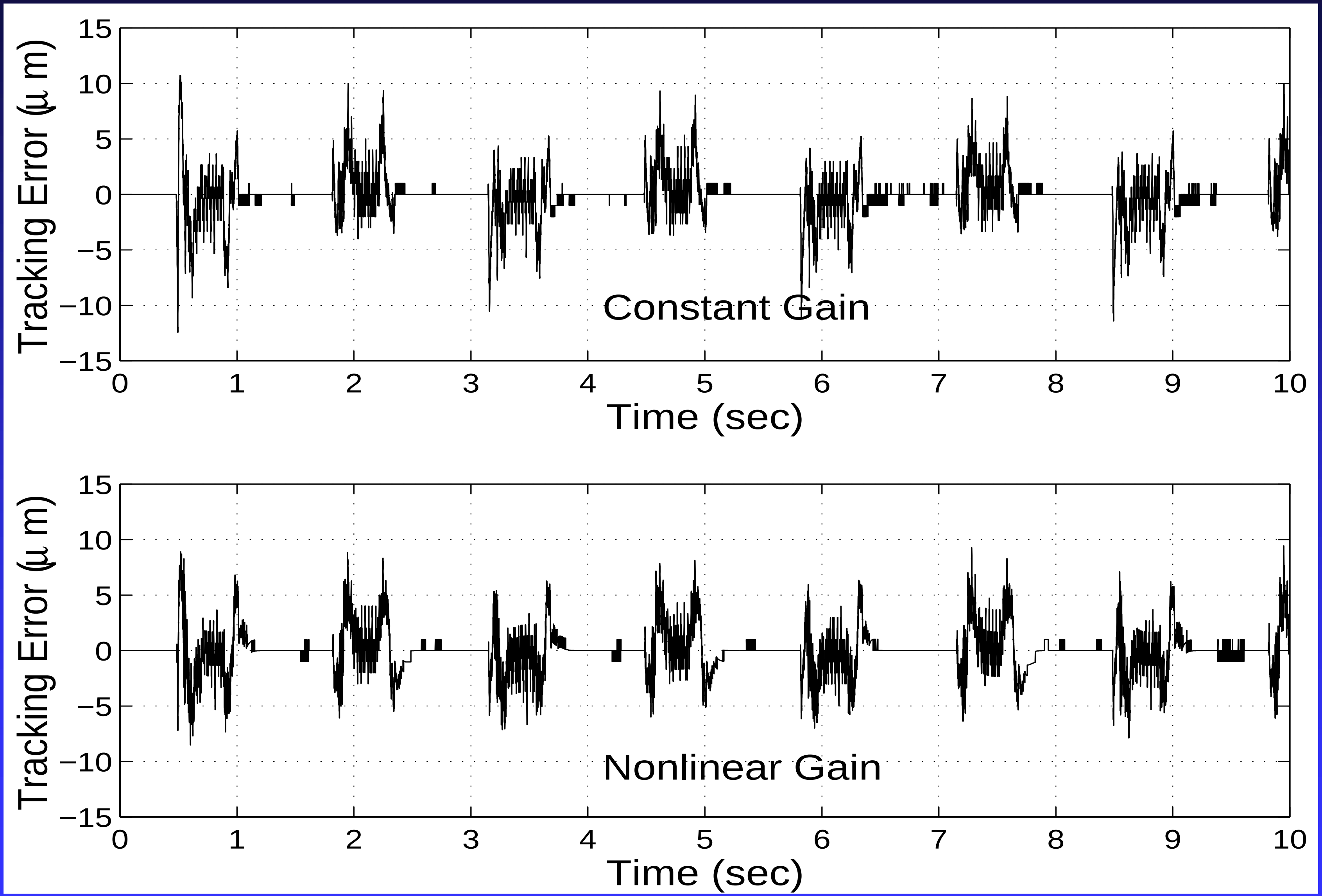


NONLINEAR VS CONSTANT GAIN



y_d =0.05*sin*(4t) 20*lb* Load Step disturbance between 2.4 and 7.4 sec

NONLINEAR VS CONSTANT GAIN



Point to Point Motion: **acceleration 12m/sec^2** **velocity 1 m/sec**

LOADED EXPERIMENTS

TRACKING ERRORS WITH LOAD

SUMMARY OF DARC DESIGNS

- **Unique Features of Direct Nonlinear Adaptive Robust Control Strategy:**
  - Nonlinear *physical model* based; easy to incorporate physical intuition into the controller design stage.
  - Address *nonlinearities* associated with linear motor dynamics directly.
  - Address *parameter variations* due to change of load, nominal value of disturbances, …
  - Effectively handle the effect of *hard-to-model terms* (e.g., uncompensated friction)
  - Achieve *high performance* through the use of learning techniques such as on-line parameter estimation.

SUMMARY OF DARC DESIGNS

- **Comparative Experimental Results**
  - Illustrate the above claims
  - The proposed schemes outperform a PID controller with model compensation significantly in terms of output tracking accuracy
  - Tracking errors for high-speed/high-acceleration movements are very small, even during transient period. Final tracking errors are mostly within measurement resolution level of $1\mu m$

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COMPARATIVE EXPERIMENTS

- Off-line estimates (no load):

$$\theta_1 = 0.027, \quad \theta_2 = 0.273, \quad \theta_3 = 0.09$$

- Off-line estimates (with 20lb load):

$$\theta_1 = 0.1 \ (V / m / s^2)$$

- Bounds

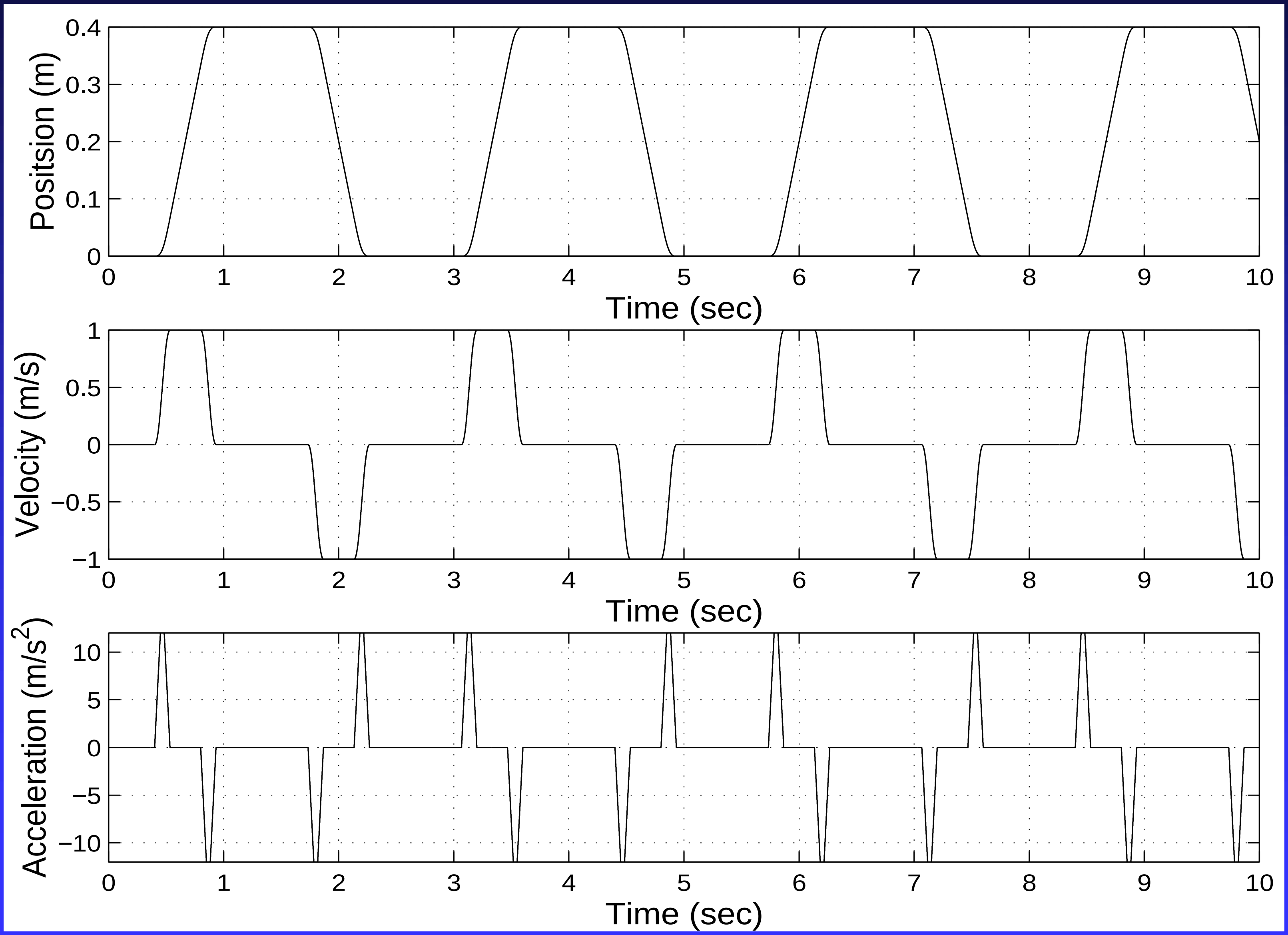
$$\theta_{\min} = [0.02,\ 0.22,\ 0.02,\ -1]^T$$

$$\theta_{\max} = [0.12,\ 0.35,\ 0.2,\ \ 1]^T$$

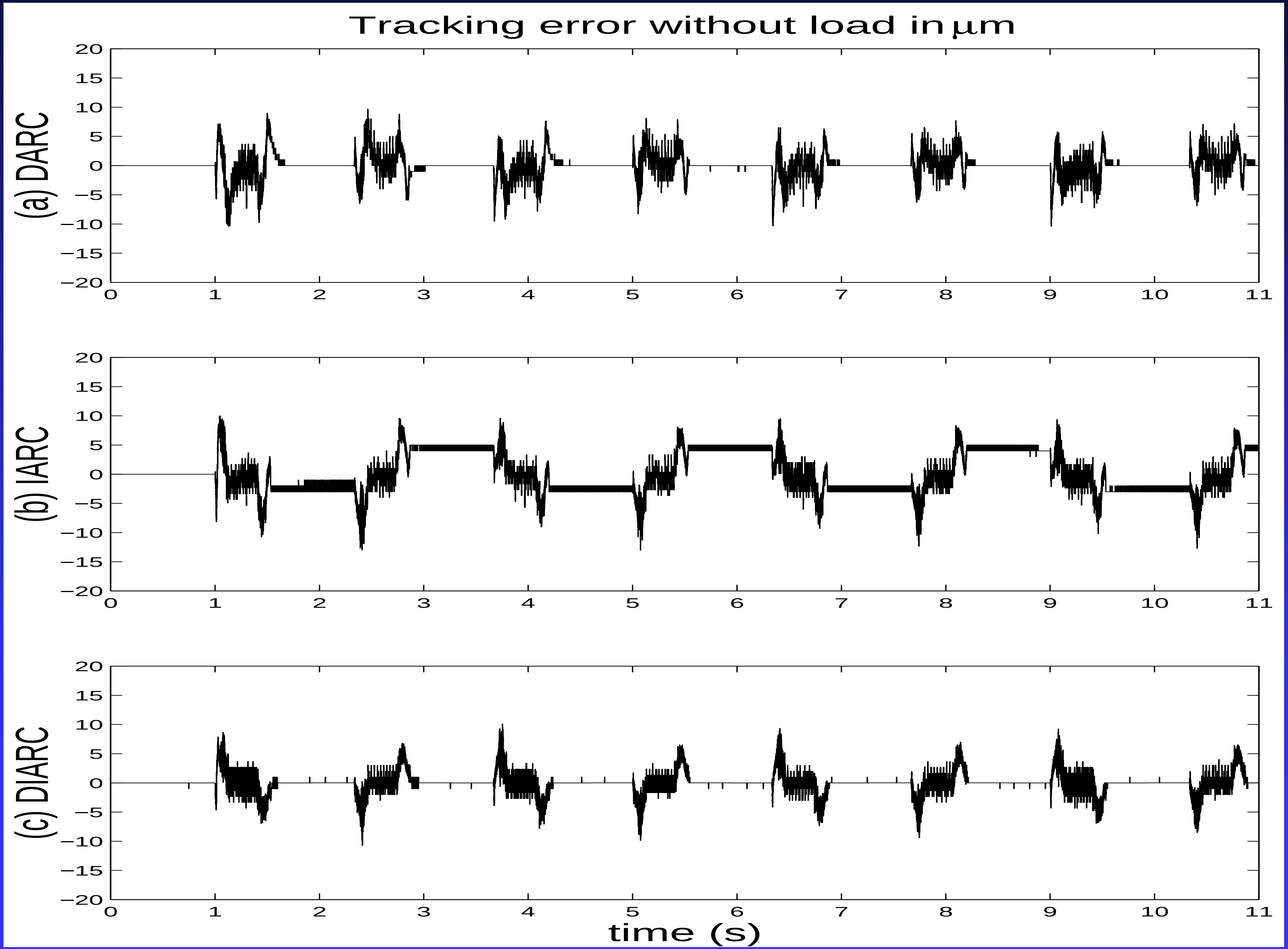
- Initial parameter estimates

$$\hat{\theta}_0 = [0.05,\ 0.24,\ 0.05,\ 0]^T$$

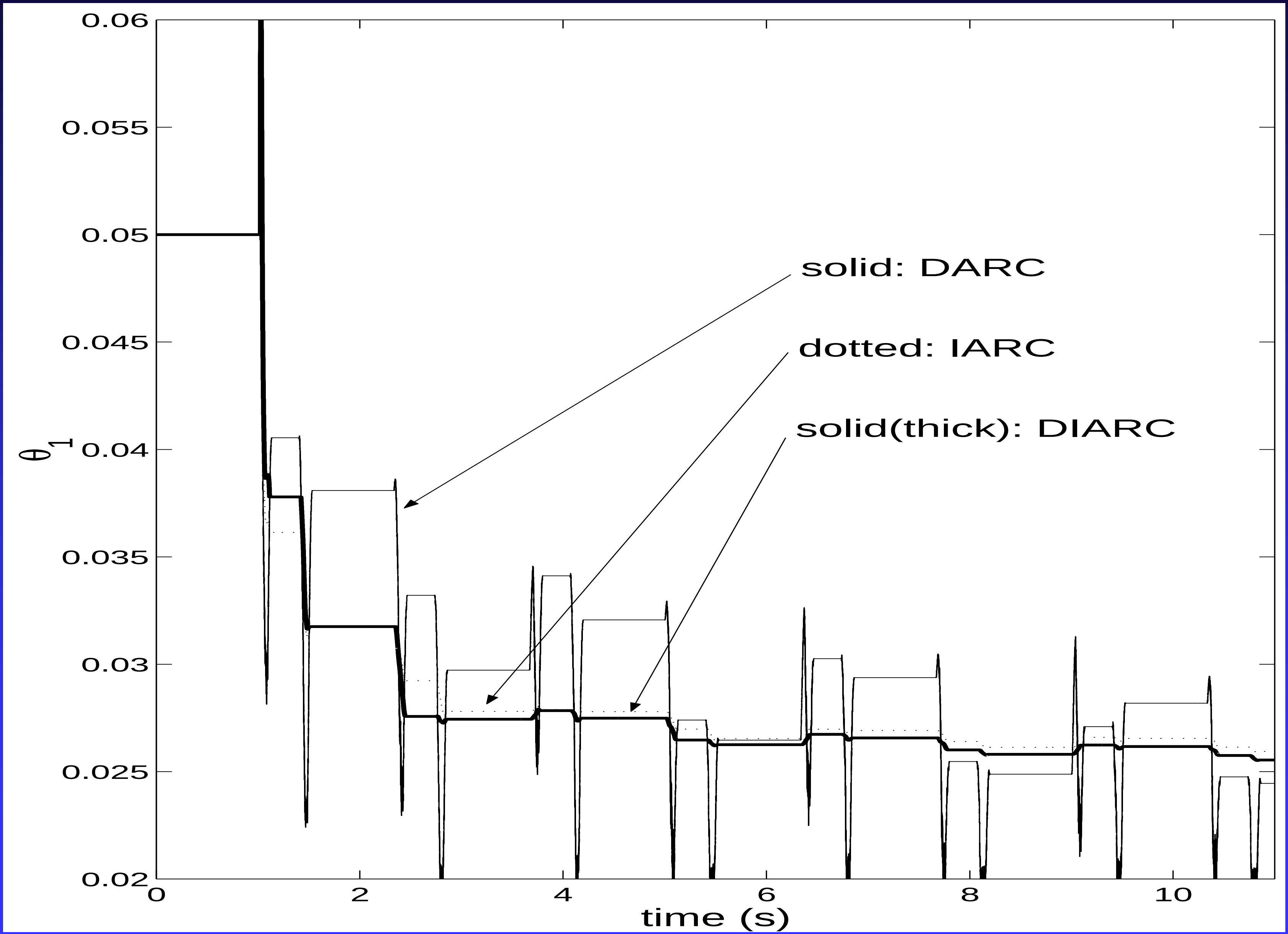
POINT-TO-POINT MOTION TRAJECTORY



TRACKING ERRORS (NO LOAD)



PARAMETER ESTIMATES (NO LOAD)



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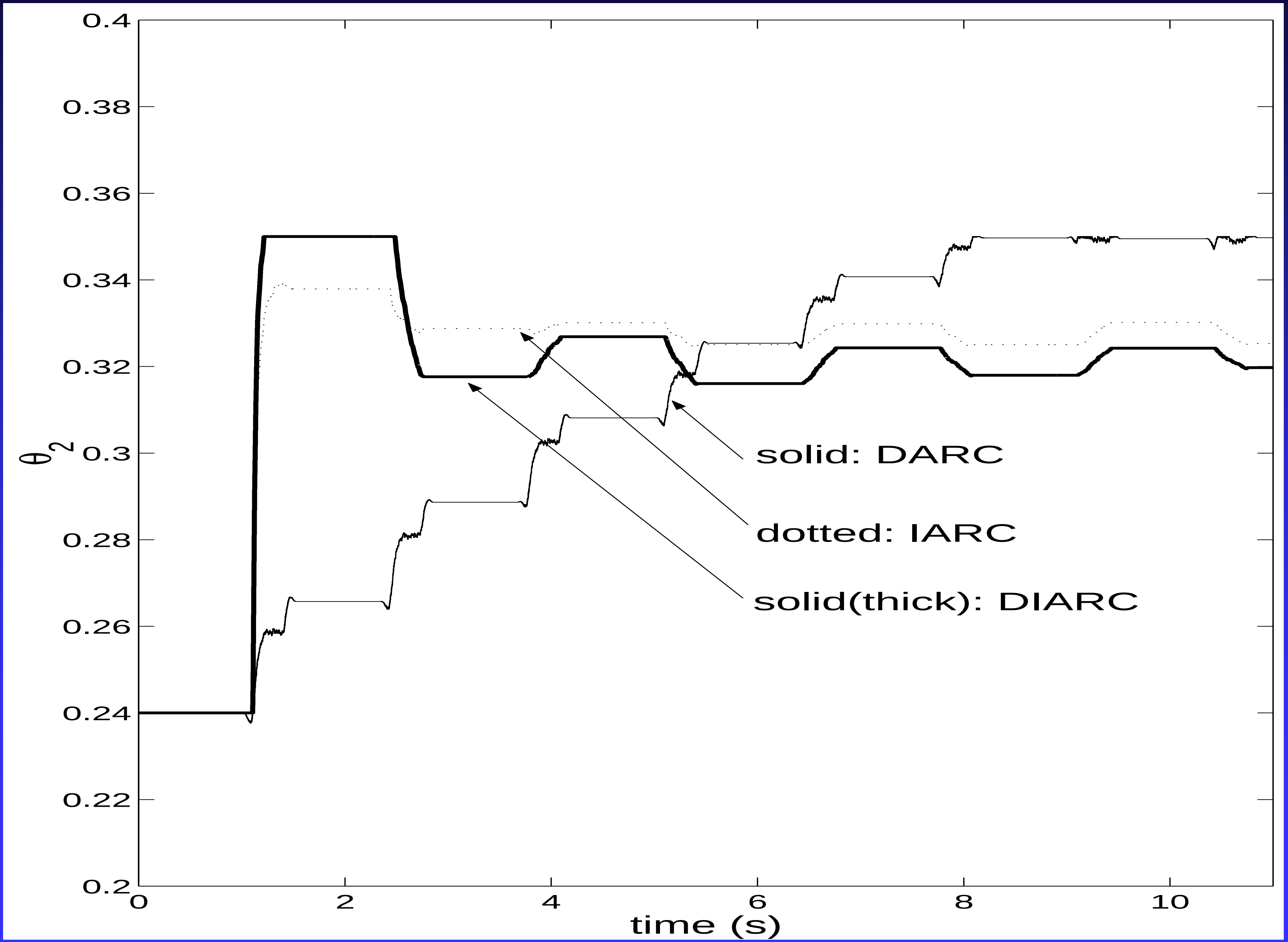
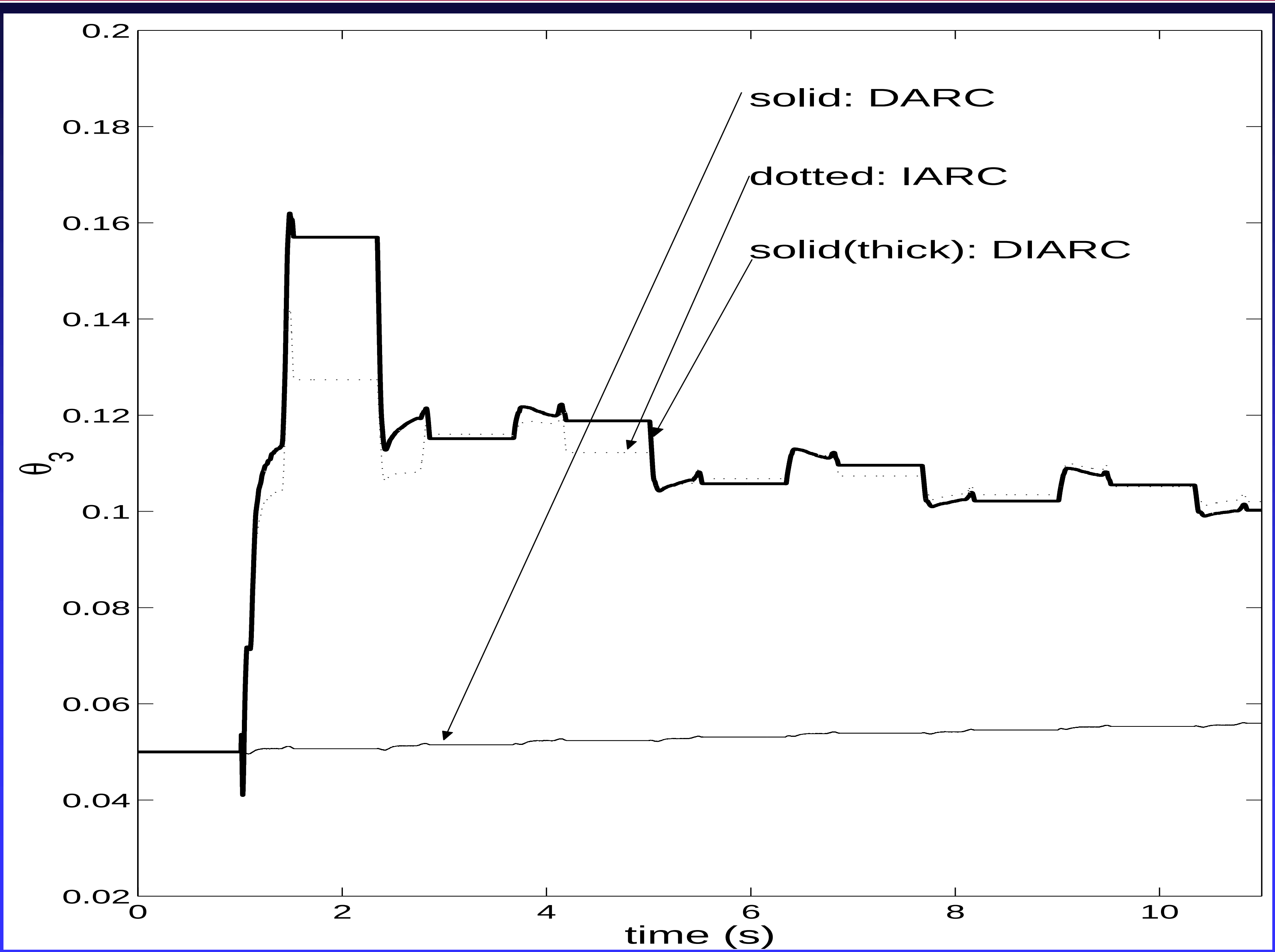
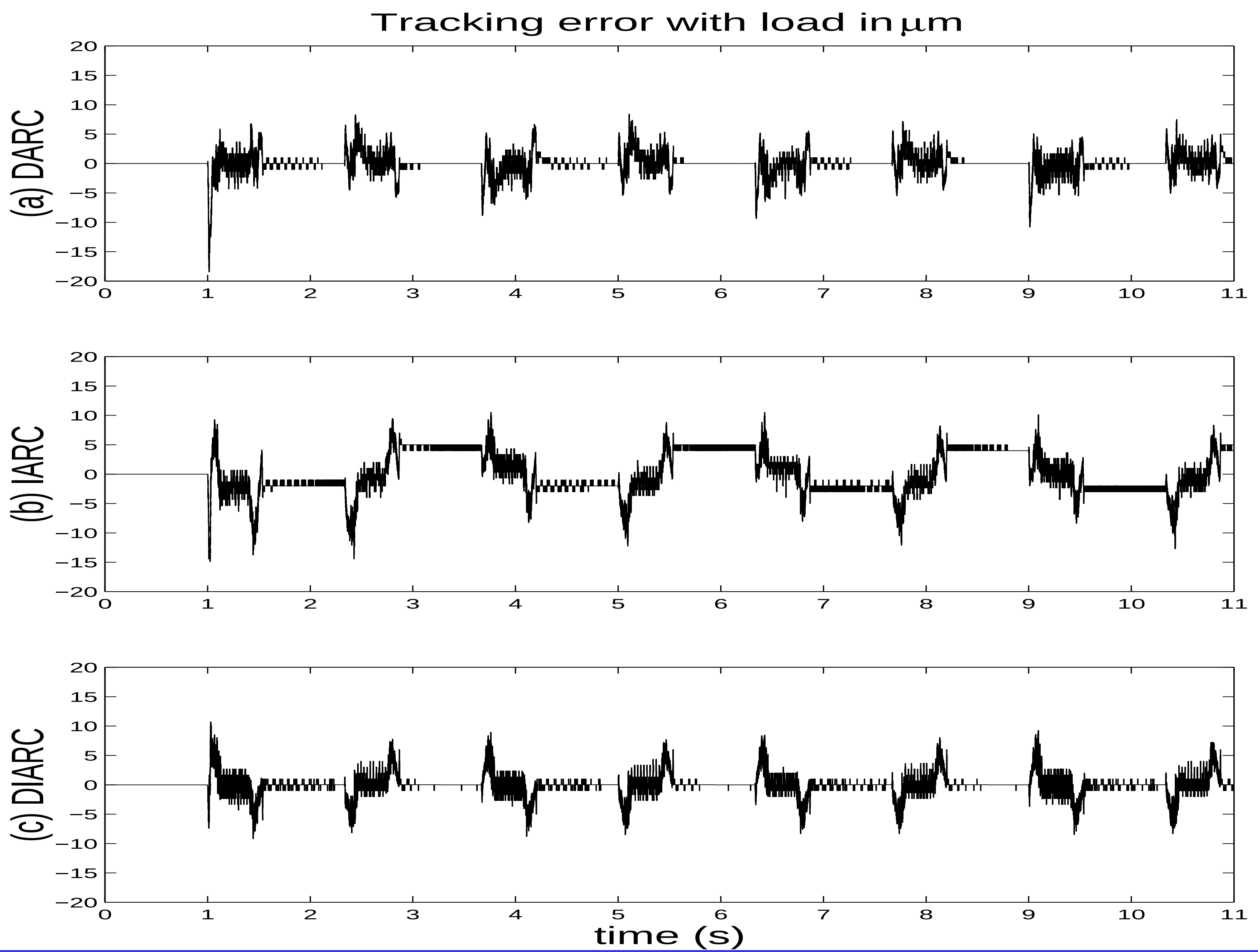


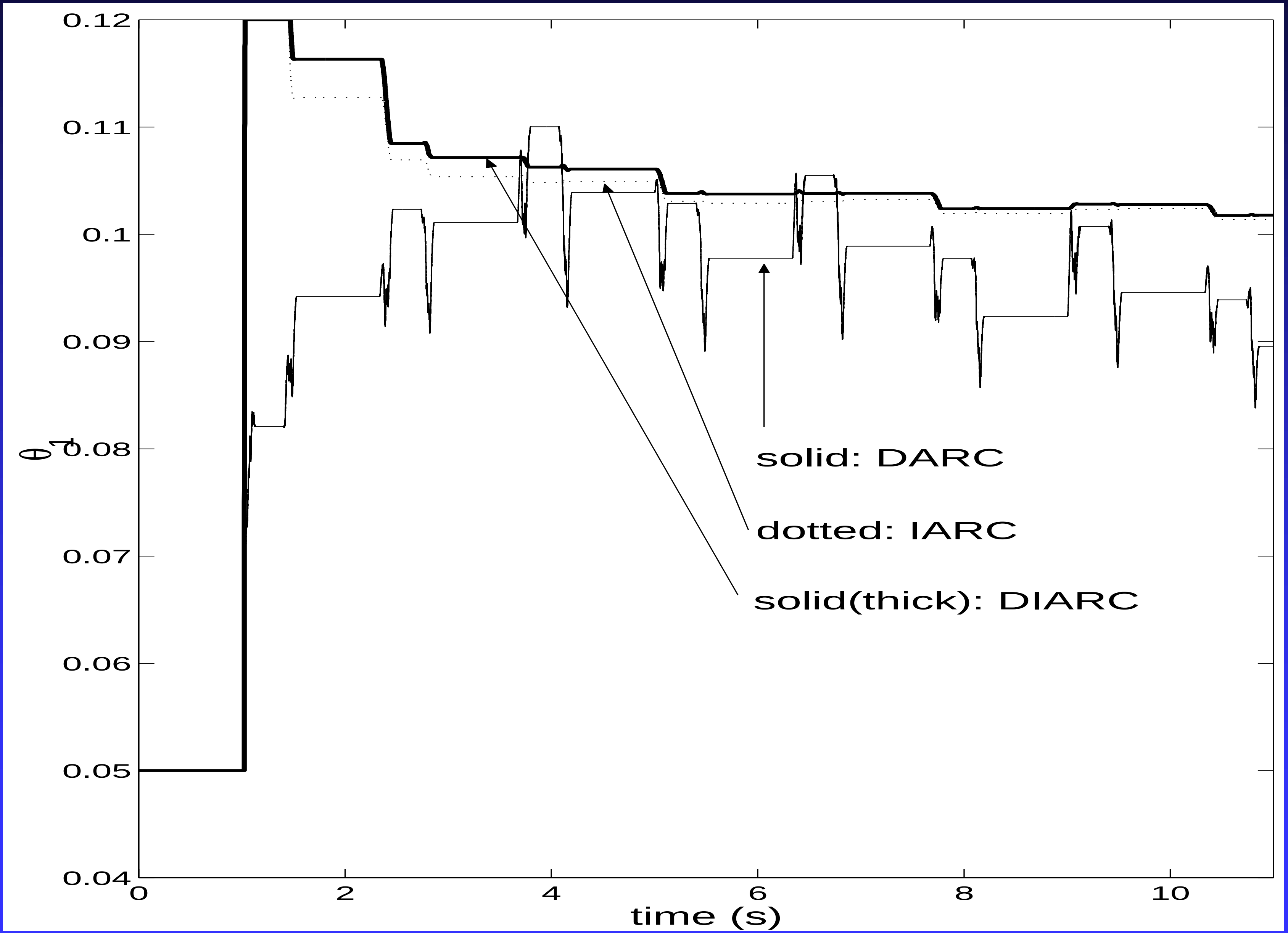
PARAMETER ESTIMATES (NO LOAD)



TRACKING ERRORS (LOADED)



PARAMETER ESTIMATES (LOADED)



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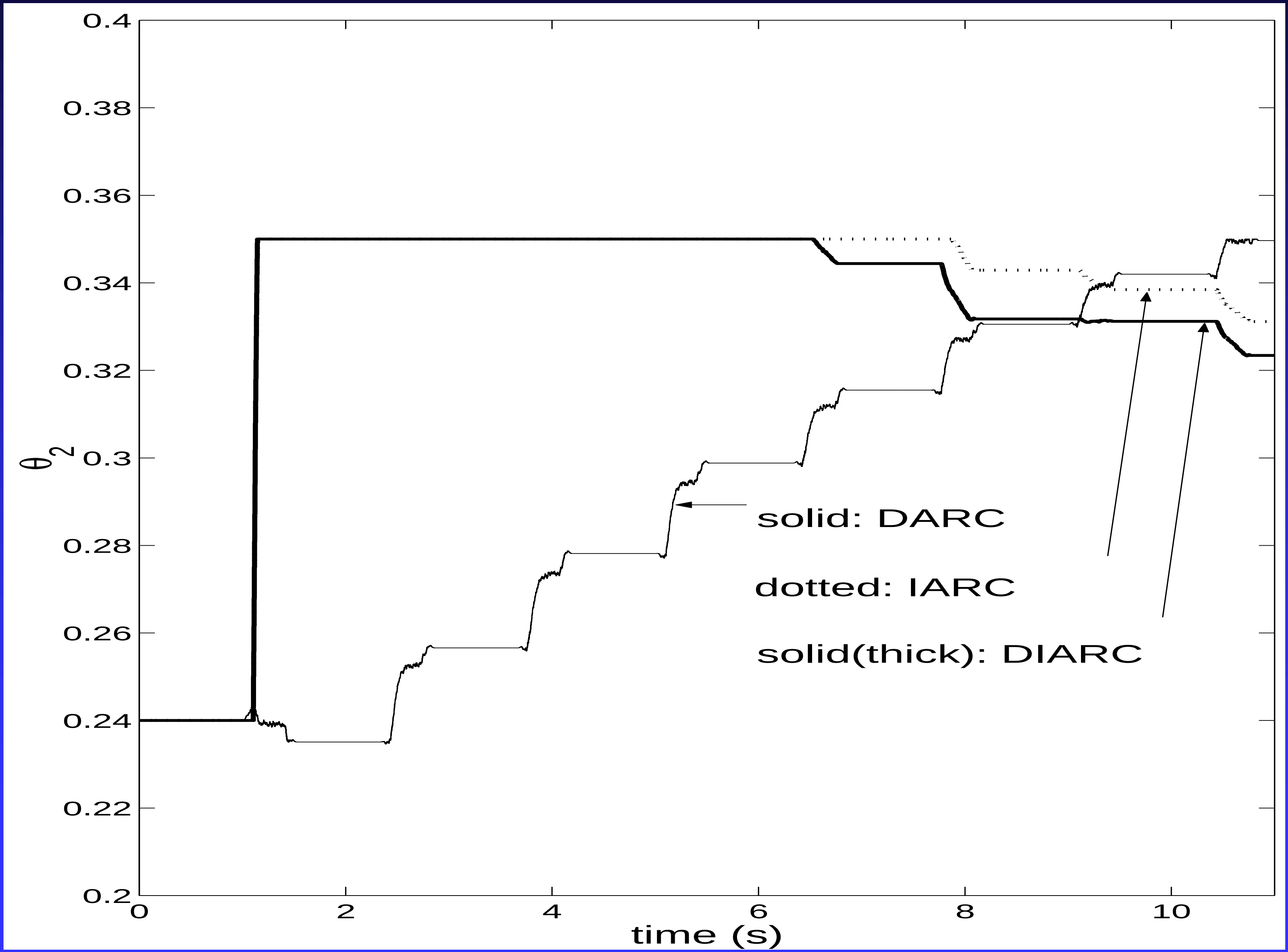

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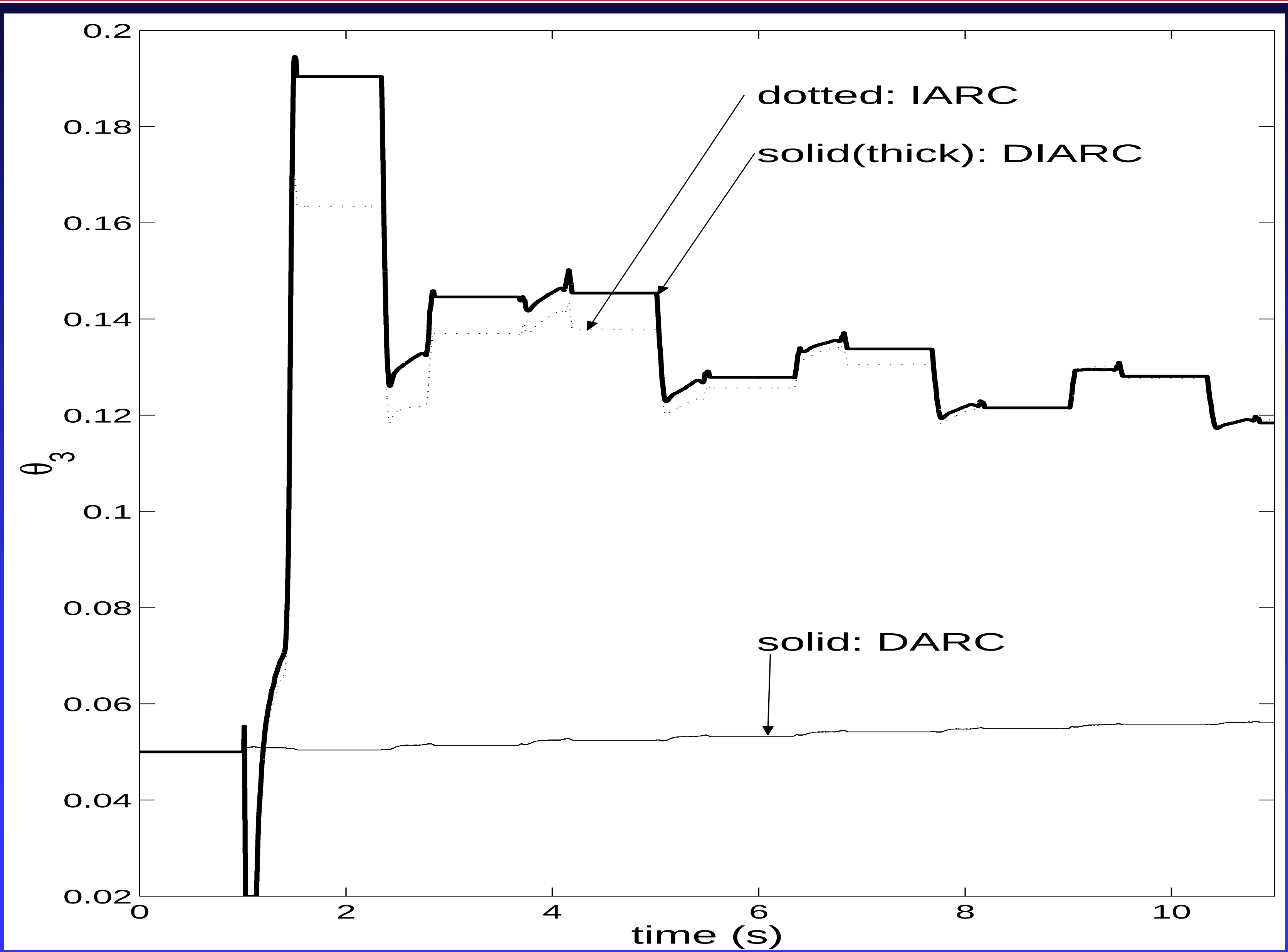

CONCLUSIONS

- **Unique Features of Integrated Direct/Indirect Adaptive Robust Control Strategy:**
  - **Nonlinear *physical model* based; easy to incorporate physical intuition into the controller design stage and estimator designs.**
  - **Effectively deal with *nonlinearities, parameter variations*, and *hard-to-model terms* through both non-linear high-gain feedback and fast adaptation**
  - **Separate estimation process from control law design to achieve *accurate parameter estimates***
  - **Achieve *best tracking performance* through the use of both estimation based parameter adaptation and the fast dynamic compensation based parameter adaptation.**

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Questions ?

More Information can be downloaded from:

http://widget.ecn.purdue.edu/~byao

Thank You !