ADAPTIVE ROBUST CONTROL OF NONLINEAR SYSTEMS: EFFECTIVE USE OF INFORMATION

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Abstract: This paper considers the adaptive robust control of nonlinear systems in the presence of parametric uncertainties and uncertain nonlinearities. The assumption on the allowable uncertain nonlinearities is relaxed to the extent that uncertain nonlinearities are required to be bounded by some known functions multiplied by unknown but bounded time-varying disturbances only. The approach uses robust feedback to attenuate the effect of uncertainties to achieve a prescribed transient performance and final tracking accuracy. The approach also uses parameter adaptation to reduce model uncertainties to achieve asymptotic tracking in the presence of parametric uncertainties without using switching or infinite-gain feedback. Different design methods are examined and compared.

Keywords: Adaptive Control, Nonlinear Systems, Robust Control, Uncertainties

1. INTRODUCTION

Nonlinearities in physical systems appear in various forms, and they may not be exactly known in advance. In many cases, the shapes or functional forms of nonlinear terms are known, but their influence is not known. Such a nonlinear term may be expressed as the product of a known nonlinear function and an unknown parameter. Furthermore, the unknown parameter may be known to stay in a known range. It is usually not possible to make a mathematical model which represents all dynamic characteristics of a real system. When modeling errors must be explicitly considered in the design of controllers, they may be represented by extra nonlinear terms. Such terms are hard to characterize, but it is often valid to assume that they are bounded by certain known nonlinear functions multiplied by unknown but bounded magnitudes. In this paper, we examine how we may effectively utilize prior information on various nonlinearities in the system model in the design of high performance nonlinear controllers.

There are two approaches to the control of terms with uncertainties. The first is to reduce or eliminate uncertainties if possible; this can be achieved through certain learning mechanisms such as parameter adaptation as in conventional adaptive control (Krstic *et al.*, 1992; Sastry and Isidori, 1989; Marino and Tomei, 1993; Pomet and Praly, 1992). By learning from past information, uncertainties can be reduced and thus a better performance can be expected without using infinite gain feedback. However, to be able to learn from the past information, the system must have certain invariant properties, which limits the applicability of this approach. All the adaptive controllers mentioned before deal with the ideal case of parametric uncertainties only. Nonlinearities of the system are assumed known and unknown parameters are assumed to appear linearly with respect to these known nonlinear functions. The second approach utilizes feedback to make the system as robust as possible to withstand the effect of model uncertainties: e.g., the deterministic robust control (DRC) (Corless and Leitmann, 1981) and the sliding mode control (Utkin, 1992). Since no attempt is made to learn from past information, the achievable performance of this approach may be limited. For example, asymptotic tracking may require infinite switching (Utkin, 1992) or infinite gain feedback (Qu *et al.*, 1992), which are not practical.

Recently, the adaptive robust control (ARC) has been proposed in (Yao and Tomizuka, 1994a; 1995; 1996; Yao, 1997) for the design of high-performance robust controllers by combining the two approaches mentioned above. Specifically, ARC uses robust filter structures to attenuate the effect of model uncertainties as much as possible to guarantee transient performance and final tracking accuracy in general; this solves the long-standing practical problems associated with adaptive control (Krstic *et al.*, 1992)– poor transient performance and non-robustness to uncertain nonlinearities (Reed and Ioannou, 1989). It also uses parameter adaptation as in adaptive control to reduce model uncertainties; this overcomes the conservativeness of DRC designs and an improved performance is obtained–asymptotic tracking is achieved in the presence of parametric uncertainties without using infinite switching or infinite-gain feedback as in adaptive control. Comparative experimental results for the motion control of robot manipulators (Yao and Tomizuka, 1994*b*) have demonstrated the effectiveness of the suggested methods. The approach is also applied to the motion control of machine tools (Yao *et al.*, 1997) by incorporating digital feedforward control (Tomizuka, 1987). Experimental results show that the maximum tracking errors during the entire high-speed operations can be reduced to the encoder resolution level.

In the above ARC controllers, uncertain nonlinearities are assumed to be bounded by some known functions. In this paper, this assumption will be relaxed to the extent that uncertain nonlinearities are only assumed to be bounded by some known functions multiplied by some unknown but bounded time-varying disturbances. This formulation of uncertain nonlinearities was used in (Freeman and Kokotovic, 1995) where a robust integral control was developed by employing smooth modifications of parameter adaptation law similar to those in (Yao and Tomizuka, 1994*a*). However, transient performance of the robust integral control (Freeman and Kokotovic, 1995) was not guaranteed as contrast to the ARC methods. The paper will compare different methods in solving the conflicts between robust control design and adaptive control. Finally, as in (Yao and Tomizuka, 1996), unknown parameters are also allowed to enter control channels and the last layer's state equations do not have to be completely linearly parametrized. The solvable form can thus be applied to the control of mechanical systems such as robot manipulators.

2. ARC OF A FIRST-ORDER SYSTEM

In this section, robust tracking control of a simple first-order system will be used to illustrate the problem formulation and the general philosophy of the proposed ARC. The system is described by

$$\dot{x} = f(x,t) + u \ , \qquad f = \varphi^T(x,t)\theta + \Delta(x,u,t) \quad (1)$$

where $x, u \in R$, and $\theta \in R^p$. In (1), the unknown nonlinear function f is characterized by two parts: the first part represents the approximation of f by a group of known basis functions $\varphi = [\varphi_1, \dots, \varphi_p]^T$ multiplied by unknown but constant parameters $\theta = [\theta_1, \dots, \theta_p]^T$, and the second part Δ represents the residual or other uncertain nonlinearities. In (Yao and Tomizuka, 1996), $\Delta(x,t)$ is assumed to be bounded by a known function. Here, this assumption will be relaxed and the following assumptions are made

Assumption 1. The unknown parameters lie in a known bounded region Ω_θ and the unknown nonlinear function Δ is bounded by a known function $\delta(x,t)$ multiplied by an unknown but bounded time-varying disturbance $d(t)$, i.e.,

$$\begin{array}{l} \theta \ \in \ \Omega_\theta \stackrel{\Delta}{=} \{\theta : \ \ \theta_{min} < \theta < \theta_{max} \ \} \\ \Delta \ \in \ \Omega_\Delta \stackrel{\Delta}{=} \{\Delta : \ \ |\Delta(x,\theta,u,t)| \le \delta(x,t) d(t) \ \} \end{array} \quad (2)$$

where $\theta_{min} = [\theta_{1min}, \dots, \theta_{pmin}]^T$, $\theta_{max} = [\theta_{1max}, \dots, \theta_{pmax}]^T$, and $\delta(x,t)$ are known. In general, $\bullet_i$ represents the i-th component of the vector $\bullet$ and the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors. $\diamond$

The objective is to let x track its desired trajectory $x_d(t)$. To achieve this objective, available tools for the robust control of nonlinear systems with uncertainties–adaptive control and deterministic robust control (DRC)–will be reviewed first, from which one can gain insights about the development and the advantages of the proposed ARC.

2.1. Adaptive Control

In the absence of uncertain nonlinearities, i.e., $\Delta = 0$, adaptive control (Krstic *et al.*, 1992) can be applied to solve the problem. A typical adaptive control law would be

$$\begin{array}{l} u_a = u_f + u_p \\ u_f = \dot{x}_d(t) - \varphi^T \hat{\theta}(t), \quad u_p = -ke \end{array} \quad (3)$$

where $e = x - x_d$ is the tracking error and $\hat{\theta}(t)$ is the estimate of θ. In (3), u_f is used to provide correct model compensation for perfect tracking and u_p is used to stabilize the nominal system. Substituting (3) into (1), the resulting error dynamics is

$$\dot{e} + ke = -\varphi^T \tilde{\theta} + \Delta \quad (4)$$

where $\tilde{\theta}{=}\hat{\theta} - \theta$ represents the parameter estimation error, i.e., parametric uncertainties. The essence of adaptive control is that, *when* $\Delta = 0$, by learning from the past information through the parameter adaptation mechanism

$$\dot{\hat{\theta}} = \Gamma \varphi e, \quad \Gamma = diag\{\gamma_1, \dots, \gamma_p\}, \ \ \gamma_I > 0 \quad (5)$$

the effect of parametric uncertainties can be eliminated; this is seen from the fact that $\varphi^T \tilde{\theta} \longrightarrow 0$ as $t \longrightarrow \infty$, which can be proved by the use of a quadratic positive definite (p.d.) function $V_a = \frac{1}{2}e^2 + \frac{1}{2}\tilde{\theta}^T \Gamma \ \tilde{\theta}$. As a result, asymptotic tracking or zero final tracking error is obtained for any feedback gain k since the right hand side of (4) converges to zero.

2.2. Deterministic Robust Control

Adaptive control has two main drawbacks-unknown transient performance and non-robustness. The system may have large tracking errors during the initial transient period or have a sluggish response. In the presence of even small disturbances Δ, the closed-loop system may be unstable (Reed and Ioannou, 1989) since parameter adaptation may learn in a wrong way. In contrast, instead of using parameter adaptation, deterministic robust control (DRC) uses robust feedback to attenuate the effect of model uncertainties. A DRC law is given by

$$u = u_a + u_s \quad (6)$$

where u_a is given by (3) but with a fixed $\hat{\theta} = \hat{\theta}(0)$. Substituting (6) into (1), the resulting error dynamics is

$$\dot{e} + ke = -\varphi^T \tilde{\theta}_0 + \Delta + u_s \quad (7)$$

where $\tilde{\theta}_0 = \hat{\theta}(0) - \theta$. The essence of DRC is that the added robust control term u_s can be synthesized to dominate the model uncertainties coming from both parametric uncertainties and uncertain nonlinearities and to attenuate their effects. This requirement can be represented by the following constraint: choosing u_s such that

i. $$e[-\varphi^T(x,t)\tilde{\theta}_0 + \Delta(x,t) + u_s] \le \varepsilon_0 + \varepsilon_1 d^2 \quad (8)$$

where ε_0 and ε_1 are two design parameters which may be arbitrarily small. From (7), the derivative of $V_s = 1/2e^2$ is

$$\begin{aligned}\dot{V}_s &= -ke^2 + e[-\varphi^T\tilde{\theta} + \Delta + u_s] \\ &\le -2kV + \varepsilon_0 + \varepsilon_1 d^2\end{aligned} \quad (9)$$

From (9),

$$\begin{aligned}|e|^2 \le\ & exp(-2kt)|e(0)|^2 \\ &+ \frac{\varepsilon_0 + \varepsilon_1\|d\|_\infty^2}{2k}[1 - exp(-2kt)]\end{aligned} \quad (10)$$

where $\|d\|_\infty$ stands for the infinity norm of $d(t)$. Thus the tracking error exponentially decays and is ultimately bounded. The exponentially converging rate $2k$ and the size of the final tracking error ($|e(\infty)| \le \sqrt{\frac{\varepsilon_0+\varepsilon_1\|d\|_\infty^2}{2k}}$) can be freely adjusted by the design parameters ε_0, ε_1, and k. In other words, transient performance and final tracking accuracy are guaranteed.

Remark 1. One example of a smooth u_s satisfying (8) is given by

$$u_s = -h\ tanh\left(\frac{0.2785he}{\varepsilon_0}\right) - \frac{1}{4\varepsilon_1}\delta^2(x,t)e \quad (11)$$

where $h(x,t)$ is any smooth function satisfying that $h \ge \|\theta_{max} - \theta_{min}\|\|\varphi(x,t)\|$. Since $\theta \in \Omega_\theta$ and $\theta(0) \in \Omega_\theta$, it is obvious that $h \ge |\varphi^T\tilde{\theta}_0|$. Thus, using the same technique as in (Yao and Tomizuka, 1995), it can be shown that

$$e\left[-\varphi\tilde{\theta}_0 - h\ tanh\left(\frac{0.2785he}{\varepsilon_0}\right)\right] \le \varepsilon_0 \quad (12)$$

Furthermore, by using the completion of square,

$$\begin{aligned}e\left[\Delta - \frac{1}{4\varepsilon_1}\delta^2 e\right] &\le |e|\delta\ d - \frac{1}{4\varepsilon_1}\delta^2 e^2 \\ &= -(\frac{1}{2\sqrt{\varepsilon_1}}\delta|e| - \sqrt{\varepsilon_1}d)^2 + \varepsilon_1 d^2 \le \varepsilon_1 d^2\end{aligned} \quad (13)$$

(12) and (13) show that u_s given by (11) satisfies (8). Another example of a smooth u_s is given by

$$u_s = -\frac{1}{4}\left[\frac{1}{\varepsilon_0}h^2 + \frac{1}{\varepsilon_1}\delta^2\right]e \quad (14)$$

which can be proved by using the completion of square as in (13). The main difference between (11) and (14) is that when $|e|$ is large, the gain in u_s given by (11) is in the order of h while that of (14) is in the order of h^2, which may require a larger control effort. However, (14) has a simpler expression and needs less computation time. ◇

2.3. Adaptive Robust Control

It seems that DRC presented above has solved the robust control problem since theoretically both transient error and final tracking error can be made as small as possible by decreasing ε_0 and ε_1. However, this will increase feedback gains, which subsequently raises the bandwidth of the resulting closed-loop system. Due to neglected factors such as sampling time and high-frequency dynamics, every physical system has a finite bandwidth. Thus, the accuracy that DRC can achieve in practice is limited as will be later in the simulation results. This necessitates the need to introduce learning mechanisms such as parameter adaptation as in adaptive control so that the model uncertainties can be reduced and performance will be improved. The combination of adaptive control and DRC is the adaptive robust control (ARC). The idea is natural and it seems that all one has to do is to update the parameter estimate used in DRC law (6) by the same adaptation law (5) as in adaptive control. Unfortunately, this simple solution does not work since some serious conflicts exist between the two design techniques. Namely, the parameter estimates provided by (5) may go unbounded when the plant has uncertain nonlinearities and thus may destabilize the system. In contrast, since a fixed $\hat{\theta}(0)$ is used in DRC design, the model uncertainties are thus bounded and the robust control term u_s can be found to guarantee stability and performance. Therefore, one of the major difficulties in combining these two design techniques is the ability to getting rid of the destabilizing effect of parameter adaptation while retaining its nominal learning capability. Intelligent utilization of available prior information such as the bounds of parametric uncertainties is the key to achieving this goal. Two major methods exist and are explained in the following.

2.3.A. Smooth Projection Method

The use of the smooth projection in the design of ARC is detailed in (Yao and Tomizuka, 1995). The smooth projection mapping $\pi(\bullet)$ ensures that the projection of the parameter estimate, $\hat{\theta}_\pi = \pi(\hat{\theta})$, belongs to a *known* bounded region all the time, i.e., $\hat{\theta}_\pi \in \Omega_{\hat{\theta}} = \{\bullet :\ \theta_{min} - \varepsilon_\theta \le \bullet \le \theta_{max} + \varepsilon_\theta\}$ where ε_θ is a vector of known positive design parameters which can be arbitrarily small. This ensures that a robust control term similar to (8) can be found to guarantee transient performance and final tracking accuracy. The projection also guarantees that when the parameter estimates are within the actual range Ω_θ, then, $\hat{\theta}_\pi = \hat{\theta}$, i.e., no modification is made so that the use of projection does not interfere with the nominal identification capability of parameter adaptation law. Along this line, the proposed ARC law has a similar structure as the DRC law (6) but with the fixed $\hat{\theta}(0)$ replaced by $\hat{\theta}_\pi$. The resulting ARC law is

$$u = u_a + u_s, \qquad u_a = \dot{x}_d - \varphi^T\hat{\theta}_\pi(t) - ke \quad (15)$$

where $\hat{\theta}$ is up-dated by (5) as in adaptive control. Since $\hat{\theta}_\pi \in \Omega_{\hat{\theta}}$ and $\Omega_{\hat{\theta}}$ is a known bounded region, the robust control term u_s in (15) can thus be determined in the same way as in DRC design so that a similar constraint as (8) is satisfied, i.e.,

i. $$e[-\varphi^T(x,t)\tilde{\theta}_\pi + \Delta(x,t) + u_s] \le \varepsilon_0 + \varepsilon_1 d^2 \quad (16)$$

where $\tilde{\theta}_\pi = \hat{\theta}_\pi - \theta$ is the projected estimation error. To ensure that the robust control term u_s does not interfere with the nominal identification process of parameter adaptation, the following passivity-like requirement is also imposed:

$$\text{ii.} \qquad eu_s \leq 0 \tag{17}$$

Example of u_s satisfying (16) and (17) is given by (11) or (14) with the bounding function h redefined to be $h = \|\theta_{max} - \theta_{min} + \varepsilon_\theta\| \|\varphi\|$.

Substituting (15) into (1), the error equation is

$$\dot{e} + ke = -\varphi^T \tilde{\theta}_\pi + \Delta(x,t) + u_s \tag{18}$$

Noting (16) and (18) and following the same proof as in DRC, it is easy to show that (9) and (10) are still valid, which means that *the same performance as in DRC can be obtained.* Furthermore, by using a non-quadratic p.d. function $V_{an} = V_s + V_\theta$ where V_θ is defined by

$$V_\theta = \sum_{i=1}^{p} \frac{1}{\gamma_i} \int_0^{\tilde{\theta}_i} (\pi_i(\mu + \theta_i) - \theta_i) d\mu \tag{19}$$

noting (17) and following the same derivation as in (Yao and Tomizuka, 1995), it can be proved that *asymptotic tracking is achieved in the presence of parametric uncertainties for any feedback gain k and design parameters ε_0 and ε_1*. This also shows that the proposed ARC has the same kind of identification property as adaptive control.

2.3.B. Discontinuous Projection Method

The main advantage of using smooth projection is that the resulting control function such as (15) is smooth; the smoothness of the resulting control function is necessary for the design of ARC controllers for a general nonlinear system with "relative degree" larger than one where backstepping design procedure (Krstic *et al.*, 1992) has to be employed in constructing the actual control law. Because of this consideration, it has been used in (Yao and Tomizuka, 1995; Yao and Tomizuka, 1996) in formulating the ARC for a general nonlinear system. However, this method suffers from the drawback that the internal parameter $\hat{\theta}$ may still go unbounded although it does not affect the stability of the actual system when the system has large uncertain nonlinearities. As an alternative, the second method is to modify the adaptation law (5) to make it robust to uncertain nonlinearities so that $\hat{\theta}_\pi$ in (15) can be replaced by $\hat{\theta}$ directly without using the smooth projection. The modification can adopt either the variation of σ-modification, which is a smooth modification, or the discontinuous projection method. The variations of σ-modification method used in (Yao and Tomizuka, 1994*a*; Freeman *et al.*, 1996) suffer from the problem that, in general, the bound on the estimated parameter $\hat{\theta}$ can not be known in advance. Thus, the robust control term cannot be determined from (16) for a predetermined ε_0. As a result, transient performance cannot be prespecified. In contrast, the discontinuous projection method used in (Yao and Tomizuka, 1994*a*) guarantees that $\hat{\theta}$ stays in a known bounded region all the time. Thus, it does the same job as the smooth projection but does not have the problem associated with the smooth projection mentioned before. The resulting adaptation law for $\hat{\theta}$ is

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma \varphi e) \tag{20}$$

where the projection mapping $Proj_{\hat{\theta}}(\bullet)$ is defined by (Sastry and Bodson, 1989; Goodwin and Mayne, 1989)

$$Proj_{\hat{\theta}}(\bullet) = \begin{cases} 0 \quad if & \begin{cases} \hat{\theta}_i = \hat{\theta}_{imax} \text{ and } \bullet > 0 \\ \hat{\theta}_i = \hat{\theta}_{imin} \text{ and } \bullet < 0 \end{cases} \\ \bullet & otherwise \end{cases} \tag{21}$$

It can be shown (Sastry and Bodson, 1989; Goodwin and Mayne, 1989; Yao and Tomizuka, 1994*a*) that the discontinuous projection mapping has the following nice properties

$$\begin{array}{ll} \text{P1} & \hat{\theta} \in \bar{\Omega}_\theta = \{\hat{\theta} : \theta_{min} \leq \hat{\theta} \leq \theta_{max}\} \\ \text{P2} & \tilde{\theta}^T (\Gamma^{-1} Proj_{\hat{\theta}}(\Gamma \bullet) - \bullet) \leq 0, \quad \forall \bullet \end{array} \tag{22}$$

Noting P1 of (22), u_s in (15) can be determined in the same way as before so that a condition similar to (16) (replacing $\tilde{\theta}_\pi$ by $\tilde{\theta}$) is satisfied to achieve a guaranteed transient performance and final tracking accuracy. Finally, P2 of (22) will ensure that asymptotic tracking as in adaptive control will not be lost. The details can be worked out as in (Yao and Tomizuka, 1994*a*).

Remark 2. The above discontinuous projection based ARC design is simple and has been successfully implemented and tested for the motion control of robot manipulators (Yao and Tomizuka, 1994*b*) and machine tools (Yao *et al.*, 1997). However, it cannot be straightforwardly generalized to nonlinear systems with a "relative degree" higher than one since smoothness of the resulting control function is lost. This technical difficulty has recently been solved by Yao in (Yao, 1997) by strengthening the underline robust control design. ◇

2.4. Simulation Results

To illustrate the above designs, simulation results are shown for a plant described by (1), in which $p = 1$, $\varphi = sin(\pi x)$, and $\theta = 18$. The initial estimate is set at $\hat{\theta}(0) = 2$. The desired trajectory is a sinusoidal curve given by $x_d = 0.5(1 - cos(1.4\pi t))$. The region and the function in (2) are $\Omega_\theta = (0, 20)$ and $\delta = 1$. Sampling time used in the simulation study is $1ms$. The following four controllers are compared:

- ARC(d) : the proposed ARC law (15) with the discontinuous projection (20) and u_s determined from (14). The controller parameters are $k = 10$, $\varepsilon_0 = 0.3$, $\varepsilon_1 = 0.001$, and $\Gamma = 4000$.
- ARC(s) : the proposed ARC law (15) with a smooth projection. The smooth projection mapping $\pi(\bullet)$ is defined in the same way as in the simulation in (Yao and Tomizuka, 1995) with $\varepsilon_\theta = 1$. The remaining controller parameters are the same as in ARC(d).
- DRC : Deterministic Robust Control–the same control law as in ARC(d) but without parameter adaptation.
- AC : Adaptive Control law (3) with (5).

To test nominal performances, simulations are run for the parametric uncertainties only (i.e., $\Delta = 0$).

Tracking errors are shown in Fig. 1, in which the ARC(d)'s and ARC(s)'s errors are indistinguishable. It is seen from the figure that the four controllers all have very good tracking ability. Compared to the non-convergent tracking error of DRC, the final tracking errors of ARCs, and AC are almost zero because the parameter estimates approach to the true value for ARCs and AC. However, AC has the worst transient response among the four controllers.

To test performance robustness, a square wave disturbance $\Delta = (-1)^{round(t)}$ is added to the system. The tracking errors are shown in Fig. 2. Again, the ARC(d)'s and ARC(s)'s errors are indistinguishable. It is seen that the proposed ARCs still achieve the best tracking performance and AC has a very large tracking error. Although not shown, control inputs of the four controllers are almost the same. These results illustrate the effectiveness of the proposed ARC.

Finally, a large disturbance $\Delta = 5$ is added from $t = 0$ to $t = 2s$. The tracking errors are shown in Fig. 3 (AC's tracking error is very large and thus not shown). Again, both ARC(d) and ARC(s) outperform their robust or adaptive counterparts. Although there is not much difference between ARC(d) and ARC(s) for $t \le 2$, the tracking error of ARC(d) converges to zero much more quickly than that of ARC(s) after the disturbance is removed. This is because the parameter estimate in ARC(s) becomes very large when the disturbance is added as seen in Fig.4. These results illustrate that the ARC law based on discontinuous projection method has a more stable parameter estimation process and thus, in some cases, achieves a better tracking performance than the ARC law based on smooth projection.

3. ARC OF MULTIVARIABLE NONLINEAR SYSTEMS

The ARC design presented in section II can be extended to MIMO nonlinear systems transformable to the following MIMO semi-strict feedback form:

$$\begin{aligned}
\dot{x}_i &= f_i^0(\bar{\chi}_i,t) + F_i(\bar{\chi}_i,t)\theta \\
&\quad + B_i(\bar{\chi}_i,\theta,t)\bar{x}_{i+1,m_i} + D_i\Delta_i \\
\dot{\eta}_i &= \Phi_i^0(\bar{\chi}_i,t) + \Phi_i^1(\bar{\chi}_i,t)\theta, \quad 1 \le i \le r-1 \\
\dot{x}_r &= M^{-1}(\bar{\chi}_{r-1},\beta,t)[f_r^0 + F_r\theta \\
&\quad + F_\beta\beta + B_r u + D_r\Delta_r] \\
\dot{\eta}_r &= \Phi_r(\chi,\theta,\beta,t) \\
y &= [y_{1b}^T,\ldots,y_{rb}^T]^T, \qquad y_{ib} = N_i^T x_i
\end{aligned} \tag{23}$$

Essentially, the form (23) is an inter-connection of r subsystems. The i-th subsystem is a MIMO nonlinear system with the state vector $\chi_i = [x_i^T,\ \eta_i^T]^T$, $x_i \in R^{n_i}$, $\eta_i \in R^{m_i}$, the input vector $v_i \in R^{m_i}$ and the output vector x_i. The first $(r-1)$ subsystems are described by the first two equations of (23), where $\bar{\chi}_i = [\chi_1^T,\ \ldots,\chi_i^T]^T$, $\chi = \bar{\chi}_r$. The vectors or matrices, f_i^0, F_i, B_i, D_i, Φ_i^0, and Φ_i^1, are known functions of their variables, which include $\bar{\chi}_{i-1}$, the states of all its previous subsystems. Δ_i represents the vector of uncertainties. The r-th subsystem is described by the last two equations of (23), which has the same structure as the i-th subsystem except that its state dynamics cannot be linearly parametrized due to the appearance of M^{-1} as explained in (Yao and Tomizuka, 1996). θ and β in (23) are sets of unknown parameters which satisfy conditions similar to (2). The r subsystems are connected in the following way. Assuming that $0 = m_0 \le m_1 \le m_2 \le \ldots \le m_r = m$, the first m_i outputs of the $i+1$-th subsystem are connected to the inputs of the i-th subsystem, i.e., $v_i = \bar{x}_{i+1,m_i} = U_{i+1}x_{i+1}$ where $U_i = [I_{m_{i-1}} \quad 0] \in R^{m_{i-1}\times m_i}$. The remaining outputs of the $i+1$-th subsystem, $y_{i+1b} = N_{i+1}^T x_{i+1}$ where $N_i = [0 \quad I_{m_i - m_{i-1}}]^T \in R^{m_i \times (m_i - m_{i-1})}$, become the $i+1$-th block of the system outputs. The inputs of the r-th subsystem are the actual inputs u.

The same assumptions as in (Yao and Tomizuka, 1996) are made for the form except that the assumptions on the uncertain nonlinearities are relaxed as in section II: $(\mathbf{A1})$. *$\forall i$, B_i is nonsingular. In addition, $\forall i \le r-1$, $B_i = B_i^0(\bar{\chi}_i,t) + B_i^1(\bar{\chi}_i,\theta,t)$ where B_i^1 is linear w.r.t. θ, and $B_r = B_r^0(\chi,t) + B_{r\theta}(\chi,\theta,t) + B_{r\beta}(\chi,\beta,t)$, where $B_{r\theta}$ and $B_{r\beta}$ are linear w.r.t. θ and β respectively.* $(\mathbf{A2})$. *M is a s.p.d. matrix and there exist positive scalars k_m and k_M such that $k_m I_m \le M(\bar{\chi}_{r-1},\beta,t) \le k_M I_m$. In addition, $M = M_0(\bar{\chi}_{r-1},t) + M_\beta(\bar{\chi}_{r-1},\beta,t)$ in which M_β is linear w.r.t. β.* $(\mathbf{A3})$. *The η_i- subsystem is BIBS stable w.r.t. the input $(\bar{\chi}_{i-1},x_i)$.* $(\mathbf{A4})$. *$\forall i$, there exists known functions $\delta_i(\bar{\chi}_i,t)$ such that $\|\Delta_i(\chi,\theta,t)\| \le \delta_i(\bar{\chi}_i,t)\ d_i(t)$ where d_i are unknown but bounded disturbances.*

The detailed design procedure is omitted here due to space limit. Basically, the same design procedure as in (Yao and Tomizuka, 1996) can be used except that at each step, the robust control term is determined from a condition similar to (16) to accommodate the relaxed assumption on uncertain nonlinearities. The design involves techniques such as backstepping via ARC Lyapunov functions for unmatched uncertainties, trajectory initialization for a guaranteed transient performance, and linear reparametrization of the derivative of ARC Lyapunov function for parameter adaptation.

The semi-strict feedback form includes some important applications such as the control of robot manipulators in various operations from trajectory tracking (Yao and Tomizuka, 1994*b*) and motion and force control in contact with a surface to coordinated control of multiple robot manipulators grasping a common object (Yao and Tomizuka, 1996).

4. CONCLUSIONS

Two methods were presented for the design of adaptive robust controllers (ARC) for nonlinear systems. The design utilized prior information such as shapes of nonlinearities, parameter ranges, and uncertainty bounds so that the combination of adaptive control (AC) and deterministic robust control (DRC) are mathematically sound. The resulting ARCs exhibited the advantages of both AC and DRC. The first method utilized the smooth projection of parameter estimates in the adaptive robust control law (ARC(s)). The second utilized the discontinuous projection in the parameter adaptation law (ARC(d)). Simulation results showed that ARCs performed significantly better than AC and DRC. Between the two ARC designs, ARC(d) performed better and has a more stable parameter estimation process than ARC(s).

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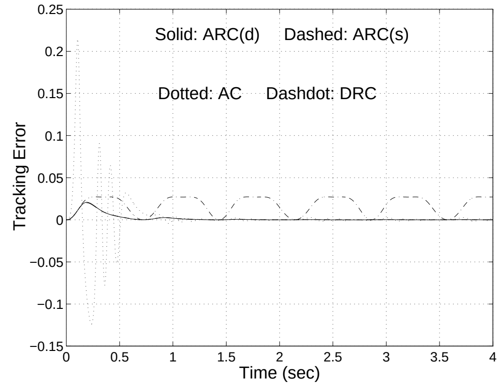

Fig. 1. Tracking errors in the presence of parametric uncertainty

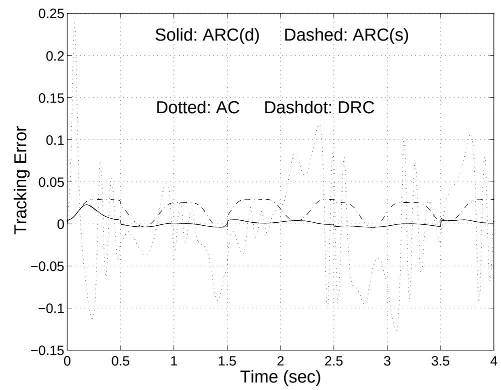

Fig. 2. Tracking errors in the presence of parametric uncertainty and a small disturbance

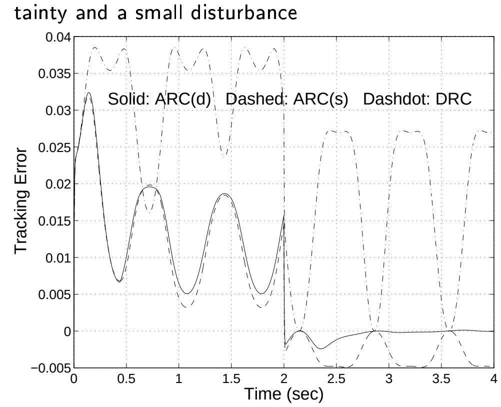

Fig. 3. Tracking errors in the presence of parametric uncertainty and a large disturbance

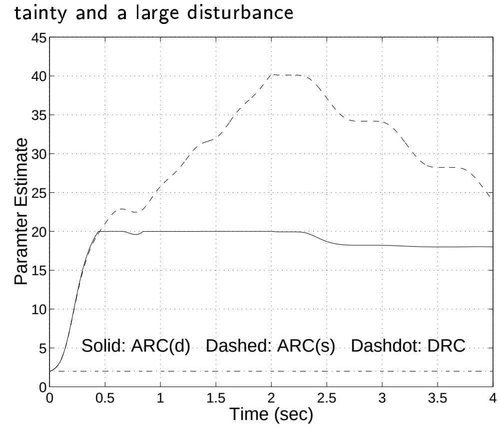

Fig. 4. Parameter estimates in the presence of parametric uncertainty and a large disturbance