

Nonlinear Adaptive Robust Control Of Electro-Hydraulic Servo Systems With Discontinuous Projections

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Abstract

This paper studies the high performance robust motion control of electro-hydraulic servo-systems. The dynamics of hydraulic systems are highly nonlinear and the system may be subjected to non-smooth and discontinuous nonlinearities due to directional change of valve opening, friction, and valve overlap. Aside from the nonlinear nature of hydraulic dynamics, hydraulic servo systems also have large extent of model uncertainties. To address these challenging issues, the recently proposed adaptive robust control (ARC) is applied and an ARC controller based on discontinuous projection method is constructed. The resulting controller is able to take into account the effect of the parameter variations of inertia load and cylinder and the uncertain nonlinearities such as uncompensated friction forces and external disturbances. Non-differentiability of the inherent nonlinearities associated with hydraulic dynamics is carefully examined and addressing strategies are provided. Compared with previously proposed ARC controller, the ARC controller in the paper has a more robust parameter adaptation process and is more suitable for implementation.

Keywords

Electro-Hydraulic System, Motion Control, Adaptive Control, Robust Control, Servo Control

1 Introduction

Hydraulic systems have been used in industry in a wide number of applications by virtue of their small size-to-power ratios and the ability to apply very large force and torque. However, the dynamics of hydraulic systems are highly nonlinear [1]. Furthermore, the system may be subjected to non-smooth and discontinuous nonlinearities due to control input saturation, directional change of valve opening, friction, and valve overlap. Aside from the nonlinear nature of hydraulic dynamics, hydraulic servosystems also have large extent of model uncertainties. The uncertainties can be classified into two categories: *parametric uncertainties* and *uncertain nonlinearities*. Examples of parametric uncertainties include the large changes in load seen by the system in industrial use and the large variations in the hydraulic parameters (e.g., bulk modulus) due to the change of temperature and component wear [2].

Other general uncertainties, such as the external disturbances, leakage, and friction, cannot be modeled exactly and the nonlinear functions that describe them may not be known. These kinds of uncertainties can be treated as uncertain nonlinearities only. Thus, nonlinear robust control techniques, which can deliver high performance in spite of both parametric uncertainties and uncertain nonlinearities, are essential for successful operations of high-performance hydraulic systems.

In the past, much of the work in the control of hydraulic systems uses linear control theory [3, 4, 5, 7] and feedback linearization techniques [8, 9]. In [10], Alleyne and Hedrick applied the nonlinear adaptive control to the force control of an active suspension driven by a double-rod cylinder. They demonstrated that nonlinear control schemes can achieve a much better performance than conventional linear controllers. They considered the parametric uncertainties of the cylinder only.

Recently, Yao and Tomizuka proposed an adaptive robust control (ARC) approach for high performance robust control of uncertain nonlinear systems in the presence of both parametric uncertainties and uncertain nonlinearities [11, 12, 13, 14]. A general theoretical framework is formalized by Yao in [15]. In [16], the ARC approach was generalized to provide a rigorous theoretic framework for the high performance robust control of a one DOF electro-hydraulic servo-system by taking into account the particular nonlinearities and model uncertainties of the electro-hydraulic servo-systems. A novel strategy was provided to overcome the difficulty in carrying out the backstepping design via ARC Lyapunov function [14] caused by the non-smooth nonlinearities of the hydraulic dynamics.

This paper continues the work done in [16] and will construct a simpler but more robust ARC controller for electro-hydraulic servo-system. Specifically, in [16], smooth projections [13, 14] were used to solve the design conflicts between adaptive control technique and robust control technique, which is technical and may not be ideal for practical implementation. Here, instead of using the smooth projection [16], the widely used discontinuous projection method in adaptive systems [17, 18] will be used to solve the conflicts between the robust control design and adaptive control design. As a result, the resulting controller becomes simpler and the parameter adaptation process is more robust in the presence of uncertain nonlinearities. The discontinuous projection method has been successfully implemented and tested in the motion control of robot

manipulators [11, 12] and the motion control of machine tools [19], in which the design techniques for both systems are essentially for nonlinear systems with "relative degree" of *one*. For nonlinear systems with "relative degree" of more than one, the underlining parameter adaptation laws in the previously proposed ARC controllers [13, 14] and the robust adaptive control designs [20, 21] are based on the tuning function based adaptive backstepping design [22], which needs to incorporate the adaptation law in the design of control functions at each step. As a result, either smooth projections [13, 14] or smooth modifications of adaptation law [20, 11] are necessary since the control functions have to be smooth for backstepping design [22, 23]; either method is technical and may be hard to implement. Only recently, in [15], Yao is able to construct simple ARC controllers for nonlinear systems with "relative degree" of more than one by using discontinuous projection method. However, the scheme in [15] cannot be directly applied to the control of electro-hydraulic servo systems studied here since, as will be shown in the paper, parametric uncertainties will also appear in the input channel of each layer [14, 24]. Therefore, the paper not only constructs a practical ARC controller for electro-hydraulic servo systems but also extends the theoretical results in [15].

2 Problem Formulation and Dynamic Models

The system under consideration is the same as that in [16], which is depicted in Fig. 1. The goal is to have the inertia load to track any specified motion trajectory as closely as possible; examples like a machine tool axis [19].

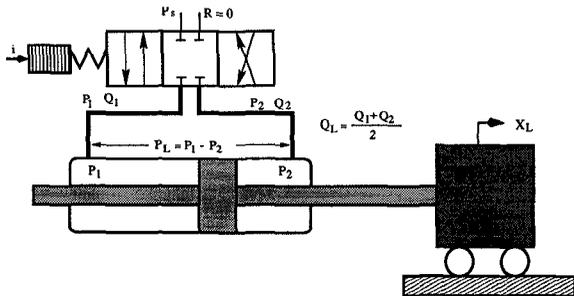


Figure 1: A One DOF Electro-Hydraulic Servo System

The dynamics of the inertia load can be described by

$$m\ddot{x}_L = P_L A - b\dot{x}_L - F_{fc}(\dot{x}_L) + \tilde{f}(t, x_L, \dot{x}_L) \quad (1)$$

where x_L and m represent the displacement and the mass of the load respectively, $P_L = P_1 - P_2$ is the load pressure of the cylinder, A is the ram area of the cylinder, b represents the combined coefficient of the modeled damping and viscous friction forces on the load and the cylinder rod, F_{fc} represents the modeled Coulomb friction force, and $\tilde{f}(t, x_L, \dot{x}_L)$ represents the external disturbances as well as terms like the unmodeled friction forces. Neglecting the effect of external leakage flows in the cylinder and the servovalve, the actuator (or the cylinder) dynamics can be written as [1]

$$\frac{V_i}{4\beta_e} \dot{P}_L = -A\dot{x}_L - C_{tm}P_L + Q_L \quad (2)$$

where V_i is the total volume of the cylinder and the hoses between the cylinder and the servovalve, β_e is the effective bulk modulus, C_{tm} is the coefficient of the total internal leakage of the cylinder due to pressure, and Q_L is the load flow. Q_L is related to the spool valve displacement of the

servovalve, x_v , by [1]

$$Q_L = C_d w x_v \sqrt{\frac{P_s - \text{sgn}(x_v) P_L}{\rho}} \quad (3)$$

where C_d is the discharge coefficient, w is the spool valve area gradient, and P_s is the supply pressure of the fluid.

For simplicity, the same servovalve as in [25] will be used in this study; the spool valve displacement x_v is related to the current input i by a first-order system given by

$$\tau_v \dot{x}_v = -x_v + K_v i \quad (4)$$

where τ_v and K_v are the time constant and gain of the servo-valve respectively.

As seen in the simulation, scaling of state variables is also very important in minimizing the numerical error and facilitating the gain-tuning process. For this purpose, we introduced scaling factors to the load pressure and valve opening as $\bar{P}_L = \frac{P_L}{S_{c3}}$ and $\bar{x}_v = \frac{x_v}{S_{c4}}$, where S_{c3} and S_{c4} are constant scaling factors. Defining the state variables as $x = [x_1, x_2, x_3, x_4]^T \triangleq [x_L, \dot{x}_L, \bar{P}_L, \bar{x}_v]^T$, the entire system, Eqs. (1)-(3) and (4), can be expressed in state space form as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{AS_{c3}}{m} (x_3 - \bar{b}x_2 - \bar{F}_{fc}) + d(t, x_1, x_2) \\ \dot{x}_3 &= \frac{4\beta_e}{V_i} \frac{S_{c4}}{\sqrt{S_{c3}}} \frac{C_d w}{\sqrt{\rho}} [-\bar{A}x_2 - \bar{C}_{tm}x_3 + g_3(x_3, x_4)] \\ \dot{x}_4 &= -\frac{1}{\tau_v} x_4 + \frac{K_v}{\tau_v} u \end{aligned} \quad (5)$$

where $g_3(x_3, x_4) = \sqrt{\bar{P}_s - \text{sgn}(x_4) x_3}$, $\bar{P}_s = \frac{P_s}{S_{c3}}$, $d = \frac{1}{m} \tilde{f}(t, x_1, x_2)$, $\bar{b} = \frac{b}{AS_{c3}}$, $\bar{F}_{fc} = \frac{F_{fc}}{AS_{c3}}$, $\bar{A} = \frac{A}{S_{c4}\sqrt{S_{c3}}} \frac{\sqrt{\rho}}{C_d w}$, $\bar{C}_{tm} = \frac{C_{tm}}{S_{c4}\sqrt{S_{c3}}} \frac{\sqrt{\rho}}{C_d w}$, $\bar{K}_v = \frac{K_v}{S_{c4}}$, and $u = i$ is the control input.

Given the desired motion trajectory $x_{Ld}(t)$, the objective is to synthesize a control input u such that the output $y = x_1$ tracks $x_{Ld}(t)$ as closely as possible in spite of various model uncertainties.

3 ARC of Electro-Hydraulic Servo Systems With Discontinuous Projections

Design Model and Issues

To begin the controller design, practical and reasonable assumptions on the system have to be made. In general, the system is subjected to parametric uncertainties due to the variations of m , b , F_{fc} , β_e , C_{tm} , C_d , ρ , τ and K . For simplicity, in this paper, we only consider the parametric uncertainties of important parameters like m , β_e , and the nominal value of the disturbance d , d_n . Other parametric uncertainties can be dealt with in the same way if necessary. In order to use parameter adaptation to reduce parametric uncertainties to improve performance, it is necessary to linearly parametrize the state space equation (5) in terms of a set of unknown parameters. To this end, define the unknown parameter set $\theta = [\theta_1, \theta_2, \theta_3]^T$ as $\theta_1 = \frac{AS_{c3}}{m}$, $\theta_2 = d_n$, $\theta_3 = \frac{4\beta_e}{V_i} \frac{S_{c4}}{\sqrt{S_{c3}}} \frac{C_d w}{\sqrt{\rho}}$. The state space equation (5) can thus be linearly parametrized in terms of θ as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \theta_1 (x_3 - \bar{b}x_2 - \bar{F}_{fc}(x_2)) + \theta_2 + \bar{d}(t, x_1, x_2) \\ \dot{x}_3 &= \theta_3 [-\bar{A}x_2 - \bar{C}_{tm}x_3 + g_3(x_3, x_4)] \\ \dot{x}_4 &= -\frac{1}{\tau_v} x_4 + \frac{K_v}{\tau_v} u \end{aligned} \quad (6)$$

where $\tilde{d} = d - d_n$. Since the extents of parametric uncertainties and uncertain nonlinearities are known, we make the following practical assumption **A1**: parametric uncertainties and uncertain nonlinearities satisfy $\theta \in \Omega_\theta \triangleq \{\theta : \theta_{min} < \theta < \theta_{max}\}$, $|\tilde{d}| \leq \delta_d$, where $\theta_{min} = [\theta_{1min}, \theta_{2min}, \theta_{3min}]^T$, $\theta_{max} = [\theta_{1max}, \theta_{2max}, \theta_{3max}]^T$, and δ_d are known. In the above, \bullet_i represents the i -th component of the vector \bullet and the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors. Physically $\theta_1 > 0$ and $\theta_3 > 0$. So we assume that $\theta_{1min} > 0$ and $\theta_{3min} > 0$.

At this stage, it is ready to see that the main difficulties in controlling (6) are: (a) the system has unmatched model uncertainties; this can be overcome by employing backstepping design as done in the following; (b) the nonlinear static flow gain g_3 is *non-smooth* since it is a function of $sgn(x_4)$; this prohibits the direct application of the general results in [14] to obtain an ARC controller.

Theoretically, all above difficulties have been solved in our recent study [16]. However, in [16], smooth projections [13, 14] were used to solve the design conflicts between adaptive control technique and robust control technique, which is technical and may not be ideal for practical implementation. Here, instead of using the smooth projection [16], the widely used discontinuous projection method in adaptive systems [17, 18] will be used to solve the conflicts between the robust control design and adaptive control design.

Notations and Discontinuous Projection

Let $\hat{\theta}$ denote the estimate of θ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta} = \hat{\theta} - \theta$). A simple discontinuous projection can be defined [17, 18] as

$$Proj_{\hat{\theta}}(\bullet) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{imax} \text{ and } \bullet > 0 \\ 0 & \text{if } \hat{\theta}_i = \theta_{imin} \text{ and } \bullet < 0 \\ \bullet & \text{otherwise} \end{cases} \quad (7)$$

By using an adaptation law given by

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\tau) \quad (8)$$

where $\Gamma > 0$ is a diagonal matrix and τ is an adaptation function to be synthesized later. It can be shown [11] that for any adaptation function τ , the projection mapping used in (8) guarantees that **(P1)**. $\hat{\theta} \in \bar{\Omega}_\theta \triangleq \{\hat{\theta} : \theta_{min} \leq \hat{\theta} \leq \theta_{max}\}$, and **(P2)**. $\tilde{\theta}^T(\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\tau) - \tau) \leq 0, \forall \tau$.

For simplicity, let \hat{x}_2 and \hat{x}_3 represent the calculable part of the \dot{x}_2 and \dot{x}_3 respectively, which are given by $\hat{x}_2 = \hat{\theta}_1(x_3 - \bar{b}x_2 - \bar{F}_{fc}(x_2)) + \hat{\theta}_2$, and $\hat{x}_3 = \hat{\theta}_3[-\bar{A}x_2 - \bar{C}_{tm}x_3 + g_3(x_3, x_4)x_4]$.

Controller Design

The design parallels the recursive backstepping design procedure via ARC Lyapunov functions in [15, 14] as follows.

Step 1

Since the first equation of (6) does not have any uncertainties, an ARC Lyapunov function can thus be constructed for the first two equations of (6) directly. Define a switching-function-like quantity as $z_2 = x_2 - x_{2eq}$ where $x_{2eq} = \dot{x}_{1d} - k_p e_1$, $e_1 = x_1 - x_{1d}(t)$, and $x_{1d}(t)$ is the desired trajectory to be tracked by x_1 . Since $G_s(s) = \frac{e_1(s)}{z_2(s)} = \frac{1}{s+k_p}$ is a stable transfer function, making e_1 small or converging

to zero is equivalent to making z_2 small or converging to zero. So the rest of the design is to make z_2 as small as possible with a guaranteed transient performance. Differentiating z_2 and noting (6)

$$\dot{z}_2 = \theta_1(x_3 - \bar{b}x_2 - \bar{F}_{fc}) + \theta_2 + \tilde{d} - \dot{x}_{2eq} \quad (9)$$

In (9), if we treat x_3 as the input, we can synthesize a virtual control law α_2 for x_3 such that z_2 is as small as possible. Since (9) has both parametric uncertainties θ_1 and θ_2 and uncertain nonlinearity \tilde{d} , the ARC approach proposed in [15] will be generalized to accomplish the objective. The generalization comes from the fact that parametric uncertainties appear in the input channel of (9) while the system studied in [15] assumes no parametric uncertainties in the input channel of each step.

The control function α_2 consists of two parts given by

$$\alpha_2 = \alpha_{2a} + \alpha_{2s}, \quad \alpha_{2a} = \bar{b}x_2 + \bar{F}_{fc} + \frac{1}{\hat{\theta}_1}(\dot{x}_{2eq} - \hat{\theta}_2) \quad (10)$$

in which α_{2s} is a robust control law and α_{2a} functions as an adjustable model compensation used to reduce model uncertainties through on-line parameter adaptation given by (8). If x_3 were the actual control input, then τ in (8) would be $\tau_2 = w_2\phi_2 z_2$ where $\phi_2 \triangleq [\alpha_{2a} - \bar{b}x_2 - \bar{F}_{fc}, 1, 0]^T$. In the tuning function based backstepping adaptive control [22], one of the key points is to incorporate the adaptation function τ (or tuning function) in the construction of control functions to compensate for the possible destabilizing effect of the time-varying adaptation law. Here, due to the use of discontinuous projection (7), the adaptation law (8) is discontinuous and thus *cannot* be used in the control law design at each step; backstepping design needs the control function synthesized at each step to be sufficiently smooth in order to obtain its partial derivatives. To compensate for this loss of information, the robust control law has to be strengthened; the robust control function α_{2s} consists of two terms given by

$$\alpha_{2s} = \alpha_{2s1} + \alpha_{2s2}, \quad \alpha_{2s1} = -k_{2s1}z_2, \quad k_{2s1} = \frac{1}{\theta_{1min}}(k_2 + \|C_{\phi_2}\Gamma\phi_2\|^2) \quad (11)$$

where $k_2 > 0$, C_{ϕ_2} is a positive definite constant diagonal matrix to be specified later and α_{2s2} is a robust control function designed as follows. Let $z_3 = x_3 - \alpha_2$ denote the input discrepancy. Substituting (10) and (11) into (9),

$$\dot{z}_2 = \theta_1 z_3 - \theta_1 k_{2s1} z_2 + \theta_1 \alpha_{2s2} - \tilde{\theta}^T \phi_2 + \tilde{d} \quad (12)$$

The robust control function α_{2s2} is chosen to satisfy the following conditions

$$\begin{aligned} \text{condition i} & \quad z_2[\theta_1 \alpha_{2s2} - \tilde{\theta}^T \phi_2 + \tilde{d}] \leq \varepsilon_2 \\ \text{condition ii} & \quad z_2 \theta_1 \alpha_{2s2} \leq 0 \end{aligned} \quad (13)$$

where ε_2 is a design parameter which can be arbitrarily small. Essentially, condition i of (13) shows that α_{2s2} is synthesized to dominate the model uncertainties coming from both parametric uncertainties $\tilde{\theta}$ and uncertain nonlinearities \tilde{d} , and condition ii is to make sure that α_{2s2} is dissipating in nature so that it does not interfere with the functionality of the adaptive control part α_{2a} . How to choose α_{2s2} to satisfy constraints like (13) can be found in [13, 14, 15].

For the positive semi-definite (p.s.d.) function V_2 defined by $V_2 = \frac{1}{2}w_2 z_2^2$, where $w_2 > 0$ is a weighting factor, from(12), its time derivative is

$$\dot{V}_2 = w_2 \theta_1 z_2 z_3 + w_2 z_2 (\theta_1 \alpha_{2s2} - \bar{\theta}^T \phi_2 + \bar{d}) - w_2 \theta_1 k_{2s1} z_2^2 \quad (14)$$

Step 2

If we neglect the effect of the directional change of the valve opening (i.e., $\text{sgn}(x_4)$) as in previous studies [10], g_3 in the third equation of (6) would be a function of x_3 only. In that case, Step 2 would be to synthesize a control function α_3 for the virtual control x_4 such that x_3 tracks the desired control function α_2 synthesized in Step 1 with a guaranteed transient performance. Since most operations (especially the position or force regulation at the end of an operation) do involve the directional change of the valve opening, the effect of the discontinuous sign function $\text{sgn}(x_4)$ will be carefully treated. Here, instead of defining x_4 as the virtual control for the third equation of (6), we define actual load flow rate $Q_L = g_3(x_3, x_4)x_4$ as the virtual control, which makes physical sense since physically it is the flow rate that regulates the pressure inside the cylinder. Thus in this step, we will synthesize a control function α_3 for Q_L such that x_3 tracks the desired control function α_2 synthesized in Step 1 with a guaranteed transient performance.

Let $z_4 = Q_L - \alpha_3$, from (6),

$$\dot{z}_3 = \theta_3 z_4 + \theta_3 [-\bar{A}x_2 - \bar{C}_{tm}x_3 + \alpha_3] - \dot{\alpha}_{2c} - \dot{\alpha}_{2u} \quad (15)$$

where

$$\begin{aligned} \dot{\alpha}_{2c} &= \frac{\partial \alpha_2}{\partial x_1} x_2 + \frac{\partial \alpha_2}{\partial x_2} \dot{x}_2 + \frac{\partial \alpha_2}{\partial t} \\ \dot{\alpha}_{2u} &= \frac{\partial \alpha_2}{\partial x_2} [-(x_3 - \bar{b}x_2 - \bar{F}_{fc})\bar{\theta}_1 - \bar{\theta}_2 + \bar{d}] + \frac{\partial \alpha_2}{\partial \theta} \dot{\theta} \end{aligned} \quad (16)$$

In (15), $\dot{\alpha}_{2c}$ is the calculable part of $\dot{\alpha}_2$ and can be used in the design of control functions but $\dot{\alpha}_{2u}$ cannot due to various uncertainties. Therefore, $\dot{\alpha}_{2u}$ has to be dealt with in this step design. Consider the augmented p.s.d. function V_3 given by $V_3 = V_2 + \frac{1}{2}w_3 z_3^2$, where $w_3 > 0$. Using similar ideas and techniques as in Step 1, the control function α_3 are obtained as

$$\begin{aligned} \alpha_3 &= \alpha_{3a} + \alpha_{3s} \\ \alpha_{3a} &= -\frac{1}{\theta_3} \left[\frac{w_2}{w_3} z_2 \hat{\theta}_1 + \hat{\theta}_3 (-\bar{A}x_2 - \bar{C}_{tm}x_3) - \dot{\alpha}_{2c} \right] \\ \alpha_{3s} &= \alpha_{3s1} + \alpha_{3s2}, \quad \alpha_{3s1} = -k_{3s1} z_3 \end{aligned} \quad (17)$$

where

$$\begin{aligned} k_{3s1} &= \frac{1}{\theta_{3min}} (k_3 + \|C_{\theta_3} \left(\frac{\partial \alpha_2}{\partial \theta} \right)^T\|^2 + \|C_{\phi_3} \Gamma \phi_3\|^2) \\ \phi_3 &= \begin{bmatrix} \frac{w_2}{w_3} z_2 - \frac{\partial \alpha_2}{\partial x_2} (x_3 - \bar{b}x_2 - \bar{F}_{fc}) \\ -\frac{\partial \alpha_2}{\partial x_2} \\ -\bar{A}x_2 - \bar{C}_{tm}x_3 + \alpha_{3a} \end{bmatrix} \end{aligned} \quad (18)$$

in which k_3 is a positive constant, C_{θ_3} , and C_{ϕ_3} are positive definite constant diagonal matrices, and α_{3s2} is a robust control function satisfying the following two conditions

$$\begin{aligned} \text{condition i} & \quad z_3 [\theta_3 \alpha_{3s2} - \bar{\theta}_1^T \phi_3 - \frac{\partial \alpha_2}{\partial x_2} \bar{d}] \leq \varepsilon_3 \\ \text{condition ii} & \quad z_3 \theta_3 \alpha_{3s2} \leq 0 \end{aligned} \quad (19)$$

where ε_3 is a design parameter.

From (15) and (17), it can be verified that

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 |_{\alpha_2} + w_3 \theta_3 z_3 z_4 + w_3 z_3 (\theta_3 \alpha_{3s2} - \bar{\theta}^T \phi_3 \\ & \quad - \frac{\partial \alpha_2}{\partial x_2} \bar{d}) - w_3 \theta_3 k_{3s1} z_3^2 - w_3 z_3 \left(\frac{\partial \alpha_2}{\partial \theta} \dot{\theta} \right) \end{aligned} \quad (20)$$

where $\dot{V}_2 |_{\alpha_2}$ denotes \dot{V}_2 under the condition that $x_3 = \alpha_2$.

Step 3

Noting the last equation of (6), Step 3 is to synthesize an actual control law for u such that Q_L tracks the desired

control function α_3 synthesized in Step 2 with a guaranteed transient performance. This can be done by the same backstepping design as in Step 2 except that here Q_L is not differentiable at $x_4 = 0$ since it contains $\text{sgn}(x_4)$. Fortunately, since the actual control input u can have finite jumps and is the control law to be synthesized in this step, we can proceed the design as follows by noting that Q_L is differentiable anywhere except at the singular point of $x_4 = 0$ and is *continuous everywhere*. By the definition of Q_L and g_3 , it can be checked out that the derivative of Q_L is given by $\dot{Q}_L = \frac{\partial g_3}{\partial x_3} \dot{x}_3 x_4 + g_3(x_3, x_4) \dot{x}_4$, $\forall x_4 \neq 0$, where $\frac{\partial g_3}{\partial x_3} = \frac{\text{sgn}(x_4)}{2\sqrt{P_2 - \text{sgn}(x_4)P_3}}$. Consider the augmented p.s.d. function V_4 given by $V_4 = V_3 + \frac{1}{2}w_4 z_4^2$, noting (6) and (20),

$$\dot{V}_4 = \dot{V}_3 |_{\alpha_3} + w_4 z_4 \left(\frac{K_v}{\tau_v} g_3 u + \alpha_{4e} - \bar{\theta}^T \phi_4 - \frac{\partial \alpha_3}{\partial x_2} \bar{d} - \frac{\partial \alpha_3}{\partial \theta} \dot{\theta} \right) \quad (21)$$

where

$$\begin{aligned} \dot{\alpha}_{3c} &= \frac{\partial \alpha_3}{\partial x_1} x_2 + \frac{\partial \alpha_3}{\partial x_2} \dot{x}_2 + \frac{\partial \alpha_3}{\partial x_3} \dot{x}_3 + \frac{\partial \alpha_3}{\partial t} \\ \alpha_{4e} &= \hat{\theta}_3 \frac{w_3}{w_4} z_3 - \frac{1}{\tau_v} g_3 x_4 + \frac{\partial g_3}{\partial x_3} \dot{x}_3 x_4 - \dot{\alpha}_{3c} \end{aligned} \quad (22)$$

and $\phi_4 = \left[-\frac{\partial \alpha_3}{\partial x_2} (x_3 - \bar{b}x_2 - \bar{F}_{fc}), -\frac{\partial \alpha_3}{\partial x_2}, \frac{w_3}{w_4} z_3 + \left(\frac{\partial g_3}{\partial x_3} x_4 - \frac{\partial \alpha_3}{\partial x_3} \right) (-\bar{A}x_2 - \bar{C}_{tm}x_3 + Q_L) \right]^T$. Similar to (17), the control law consists of two parts given by

$$\begin{aligned} u &= u_a + u_s, \quad u_a = -\frac{\tau_v}{K_v g_3} \alpha_{4e} \\ u_s &= u_{s1} + u_{s2}, \quad u_{s1} = -k_{4s1} z_4, \\ k_{4s1} &= \frac{\tau_v}{g_3 K_v} (k_4 + \|C_{\theta_4} \left(\frac{\partial \alpha_3}{\partial \theta} \right)^T\|^2 + \|C_{\phi_4} \Gamma \phi_4\|^2) \end{aligned} \quad (23)$$

where C_{θ_4} , and C_{ϕ_4} are constant positive definite diagonal matrixes, and u_{s2} is a robust control function satisfying the following two conditions

$$\begin{aligned} \text{condition i} & \quad z_4 \left[\frac{K_v}{\tau_v} g_3 u_{s2} - \bar{\theta}^T \phi_4 - \frac{\partial \alpha_3}{\partial x_2} \bar{d} \right] \leq \varepsilon_4 \\ \text{condition ii} & \quad z_4 \frac{g_3 K_v}{\tau_v} u_{s2} \leq 0 \end{aligned} \quad (24)$$

in which ε_4 is a design parameter. Substituting (23) into (21) and noting (14) and (20)

$$\begin{aligned} \dot{V}_4 &= w_2 z_2 (\theta_1 \alpha_{2s2} - \bar{\theta}^T \phi_2 + \bar{d}) - w_2 \theta_1 k_{2s1} z_2^2 \\ & \quad + w_3 z_3 (\theta_3 \alpha_{3s2} - \bar{\theta}^T \phi_3 - \frac{\partial \alpha_2}{\partial x_2} \bar{d}) - w_3 \theta_3 k_{3s1} z_3^2 \\ & \quad + w_4 z_4 \left(\frac{g_3 K_v}{\tau_v} u_{s2} - \bar{\theta}^T \phi_4 - \frac{\partial \alpha_3}{\partial x_2} \bar{d} \right) - w_4 k_{4s1} z_4^2 \\ & \quad - (w_3 z_3 \frac{\partial \alpha_2}{\partial \theta} + w_4 z_4 \frac{\partial \alpha_3}{\partial \theta}) \dot{\theta} \end{aligned} \quad (25)$$

Theorem 1 Let the parameter estimates be updated by the adaptation law (8) in which τ is chosen as

$$\tau = \sum_{j=2}^4 w_j z_j \phi_j \quad (26)$$

If controller parameters $C_{\theta_j} = \text{diag}\{c_{\theta_{jl}}, l = 1, \dots, 3\}$, $j = 3, 4$ and $C_{\phi_k} = \text{diag}\{c_{\phi_{kl}}, k = 2, 3, 4$ in (11), (17), and (23) are chosen such that $c_{\phi_{kl}}^2 \geq \frac{3w_k}{4} \left(\frac{w_3}{c_{\theta_{3l}}^2} + \frac{w_4}{c_{\theta_{4l}}^2} \right)$, $\forall k, l$, then, the control law (23) with the adaptation law (8) guarantees that

A. In general, output tracking error e_1 and $z = [z_2, z_3, z_4]^T$ are bounded. Furthermore, V_4 is bounded above by

$$V_4(t) \leq \exp(-\lambda_V t) V_4(0) + \frac{\varepsilon_V}{\lambda_V} [1 - \exp(-\lambda_V t)] \quad (27)$$

where $\lambda_V = 2\min\{k_2, k_3, k_4\}$ and $\varepsilon_V = w_2 \varepsilon_2 + w_3 \varepsilon_3 + w_4 \varepsilon_4$.

B. If after a finite time t_0 , $\dot{\bar{d}} = 0$, i.e., in the presence of parametric uncertainties only, then, in addition to results in A, asymptotic output tracking (or zero final tracking error) is also achieved. \triangle

Proof. If C_{ϕ_j} and C_{θ_j} satisfy the condition stated in the theorem, using the same technique as in [15], it can be proved that

$$\left| \sum_{j=3}^4 w_j z_j \frac{\partial \alpha_{j-1}}{\partial \theta} \hat{\theta} \right| \leq \sum_{j=3}^4 w_j \|C_{\theta_j} \left(\frac{\partial \alpha_{j-1}}{\partial \theta} \right)^T\|^2 z_j^2 + \sum_{k=2}^4 w_k \|C_{\phi_k} \Gamma \phi_k\|^2 z_k^2 \quad (28)$$

Thus, noting the formula for k_{2s1} , k_{3s1} , and k_{4s1} , (25) becomes

$$\begin{aligned} \dot{V}_4 \leq & -\sum_{j=2}^4 w_j k_j z_j^2 + w_2 z_2 (\theta_1 \alpha_{2s2} - \tilde{\theta}^T \phi_2 + \tilde{d}) \\ & + w_3 z_3 (\theta_3 \alpha_{3s2} - \tilde{\theta}^T \phi_3 - \frac{\partial \alpha_2}{\partial x_2} \tilde{d}) \\ & + w_4 z_4 \left(\frac{4\beta_e}{\tau_v} u_{s2} - \tilde{\theta}^T \phi_4 - \frac{\partial \alpha_3}{\partial x_2} \tilde{d} \right) \end{aligned} \quad (29)$$

From the condition i of (13), (19), and (24), we have

$$\dot{V}_4 \leq -\sum_{j=2}^4 (w_j k_j z_j^2 + w_j \varepsilon_j) \leq -2\lambda_V V_4 + \varepsilon_v \quad (30)$$

which leads to (27) and A of Theorem 1.

When $\tilde{d} = 0$, noting conditions ii of (13), (19), and (24), (29) becomes

$$\dot{V}_4 \leq -\sum_{j=2}^4 [w_j k_j z_j^2 + w_j z_j \phi_j^T \tilde{\theta}] \leq -2\lambda_V V_4 - \tau^T \tilde{\theta} \quad (31)$$

B of Theorem 1 can thus be proved by using the same technique as in [15]. \square

Trajectory Initialization and Generation

It is seen from (27) that transient tracking error is affected by the initial value $V_4(0)$, which may depend on the controller parameters also. To further reduce transient tracking error, the desired trajectory initialization can be used as in [15]. Namely, instead of simply letting the desired trajectory used for the controller be the actual desired trajectory or position (i.e., $x_{1d}(t) = x_{Ld}(t)$), $x_{1d}(t)$ can be generated using a any stable 4-th order filter with four initials $x_{1d}(0), \dots, x_{1d}^{(3)}(0)$ chosen as

$$\begin{aligned} x_{1d}(0) &= x_1(0), \quad \dot{x}_{1d}(0) = x_2(0), \quad \ddot{x}_{1d}(0) = \hat{x}_2(0) \\ x_{1d}^{(3)}(0) &= \hat{\theta}_1(0) \left[\hat{x}_3(0) - \left(b + \frac{\partial F_{fc}}{\partial x_2}(0) \right) \hat{x}_2(0) \right] \end{aligned} \quad (32)$$

Such a trajectory initialization guarantees that $e_1(0) = 0, z_i(0) = 0, i = 2, 3, 4$ and $V_4(0) = 0$. Transient tracking error is thus minimized. The trajectory initialization is independent from the choice of controller parameters k and ε and can be performed off-line once the initial state of the system is determined.

Remark 1 In the above design, the intermediate control functions α_i given by (10) and (17) have to be differentiable. In return, the Coulomb friction compensation term $F_{fc}(x_2)$ used in them has to be a differential function of x_2 . This requirement can be easily accommodated in the proposed ARC framework since $F_{fc}(x_2)$ can be chosen as any differentiable function which approximates the actual discontinuous Coulomb friction (e.g., replacing $\text{sgn}(x_2)$ in the conventional Coulomb friction modeling by the smooth $\tanh(x_2)$). The approximation error can be lumped into the uncertain nonlinearity term \tilde{d} . \triangle

4 Simulation Results

To illustrate the above design, simulation results are obtained for a hydraulic cylinder having the following parameters: $m = 100\text{kg}$, $A = 3.35 \times 10^{-4} \text{m}^2$, $\frac{4\beta_e}{V_i} = 4.52 \times 10^{13} \frac{\text{N}}{\text{m}^5}$, $C_{tm} = 2.21 \times 10^{-14} \frac{\text{m}^5}{\text{N}}$, $b = 20 \frac{\text{N}}{\text{s}}$ and $\delta_d = 2$. The scaling factors used are $S_{c3} = 5.97 \times 10^5$ and $S_{c4} = 4.99 \times 10^{-7}$. The desired trajectory is a sinusoidal curve given by $x_d =$

$0.01 \sin \frac{\pi}{2} t$. Sampling time is 1ms . The initial estimate of θ is $\hat{\theta} = [2, 0, 1]$.

The following three controllers are compared: ARC(d), ARC(s), and DRC. ARC(d) represents the discontinuous projection based ARC law presented in this paper. The controller parameters are: $k_p = k_2 = k_3 = k_4 = 220$, $w_i = 1, i = 2, 3, 4$, $C_{\phi_2} = \text{diag}[166, 22.4, 1000]$, $C_{\phi_3} = C_{\phi_4} = \text{diag}[166, 22.4, 86.6]$, $C_{\theta_3} = C_{\theta_4} = \text{diag}[0.01, 3.16 \times 10^{-2}, 0.01]$, $C_{\theta_4} = \text{diag}[3.16 \times 10^{-3}, 3.16 \times 10^{-2}, 7.07 \times 10^{-3}]$, $\varepsilon_2 = 10$, $\varepsilon_3 = 1 \times 10^6$, $\varepsilon_4 = 1 \times 10^{10}$, and $\Gamma = \text{diag}[5 \times 10^{-8}, 3.5 \times 10^{-7}, 2.9 \times 10^{-6}]$. ARC(s) represents the smooth projection based ARC law proposed in [16]. The same smooth projection as in [13] is used where $\varepsilon_\theta = 0.001$, and the remaining controller parameters are the same as in ARC(d). DRC represents deterministic robust control law, which is obtained by using the same control law as in ARC(d) but without parameter adaptation.

Simulation is first run for parametric uncertainties only (i.e., $\tilde{d} = 0$). Tracking errors are shown in Fig. 2. It is seen that the tracking errors of both ARC(d) and ARC(s) converge to zero quickly as in contrast to the non-zero tracking error of DRC; this verifies the effectiveness of introducing parameter adaptation. ARC(d) also has the smallest transient tracking error.

To test the performance robustness of the proposed schemes, a large constant disturbance with an amplitude of 2 is added to the system during the period of $0 < t < 1\text{sec}$. Tracking errors are shown in Fig.3. As seen, the tracking errors of ARC(d) and ARC(s) converge to zero after the disturbance is removed at $t = 1\text{sec}$. However, ARC(d) has a much shorter recover period and a smaller transient tracking error. This is due to the fact that the parameter estimates in the discontinuous projection based ARC stay in the known bounded range as in contrast to the large wrong parameter estimates of ARC(s) during the first 1 sec when the system is subjected to the large disturbance, which are shown in Fig.4. This verifies that the discontinuous projection based ARC has a more robust parameter adaptation process in general. In return, a better performance is achieved.

5 Conclusions

In this paper, instead of using smooth projection, an ARC controller based on discontinuous projection method is constructed for the high performance robust motion control of a typical one DOF electro-hydraulic servosystem. The controller takes into account the particular nonlinearities associated with hydraulic dynamics and allows parametric uncertainties such as variations of inertia load and hydraulic parameters as well as uncertain nonlinearities coming from external disturbances and uncompensated friction forces. Simulation results show that the proposed scheme has a more robust parameter adaptation process and achieves a better tracking performance in general.

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References

- [1] H. E. Merritt, *Hydraulic control systems*. New York: Wiley, 1967.
- [2] J. Whatton, *Fluid power systems*. Prentice Hall, 1989.

[3] T. C. Tsao and M. Tomizuka, "Robust adaptive and repetitive digital control and application to hydraulic servo for noncircular machining," *ASME J. Dynamic Systems, Measurement, and Control*, vol. 116, pp. 24–32, 1994.

[4] P. M. FitzSimons and J. J. Palazzolo, "Part i: Modeling of a one-degree-of-freedom active hydraulic mount; part ii: Control," *ASME J. Dynamic Systems, Measurement, and Control*, vol. 118, no. 4, pp. 439–448, 1996.

[5] A. R. Plummer and N. D. Vaughan, "Robust adaptive control for hydraulic servosystems," *ASME J. Dynamic System, Measurement and Control*, vol. 118, pp. 237–244, 1996.

[6] C. E. Jeronymo and T. Muto, "Application of unified predictive control (upc) for an electro-hydraulic servo system," *JSME Int. J., Series C*, vol. 38, no. 4, pp. 727–734, 1995.

[7] J. E. Bobrow and K. Lum, "Adaptive, high bandwidth control of a hydraulic actuator," *ASME J. Dynamic Systems, Measurement, and Control*, vol. 118, pp. 714–720, 1996.

[8] R. Vossoughi and M. Donath, "Dynamic feedback linearization for electro-hydraulically actuated control systems," *ASME J. Dynamic Systems, Measurement, and Control*, vol. 117, no. 4, pp. 468–477, 1995.

[9] L. D. Re and A. Isidori, "Performance enhancement of nonlinear drives by feedback linearization of linear-bilinear cascade models," *IEEE Trans. on Control Systems Technology*, vol. 3, no. 3, pp. 299–308, 1995.

[10] A. Alleyne and J. K. Hedrick, "Nonlinear adaptive control of active suspension," *IEEE Trans. on Control Systems Technology*, vol. 3, no. 1, pp. 94–101, 1995.

[11] B. Yao and M. Tomizuka, "Smooth robust adaptive sliding mode control of robot manipulators with guaranteed transient performance," in *Proc. of American Control Conference*, pp. 1176–1180, 1994. The full paper appeared in *ASME Journal of Dynamic Systems, Measurement and Control*, Vol. 118, No.4, pp764-775, 1996.

[12] B. Yao and M. Tomizuka, "Comparative experiments of robust and adaptive control with new robust adaptive controllers for robot manipulators," in *Proc. of IEEE Conf. on Decision and Control*, pp. 1290–1295, 1994.

[13] B. Yao and M. Tomizuka, "Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form," *Automatica*, vol. 33, no. 5, pp. 893–900, 1997. (Part of the paper appeared in *Proc. of 1995 American Control Conference*, pp2500-2505).

[14] B. Yao and M. Tomizuka, "Adaptive robust control of MIMO nonlinear systems," 1997. Submitted to *Automatica* (revised in 1998). Parts of the paper were presented in the *IEEE Conf. on Decision and Control*, pp2346-2351, 1995, and the *IFAC World Congress*, Vol. F, pp335-340, 1996.

[15] B. Yao, "High performance adaptive robust control of nonlinear systems: a general framework and new schemes," in *Proc. of IEEE Conference on Decision and Control*, pp. 2489–2494, 1997.

[16] B. Yao, G. T. C. Chiu, and J. T. Reedy, "Nonlinear adaptive robust control of one-dof electro-hydraulic servo systems," in *ASME International Mechanical Engineering Congress and Exposition (IMECE'97), FPST-Vol.4*, (Dallas), pp. 191–197, 1997.

[17] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*. Englewood Cliffs, NJ 07632, USA: Prentice Hall, Inc., 1989.

[18] G. C. Goodwin and D. Q. Mayne, "A parameter estimation perspective of continuous time model reference adaptive control," *Automatica*, vol. 23, no. 1, pp. 57–70, 1989.

[19] B. Yao, M. Al-Majed, and M. Tomizuka, "High performance robust motion control of machine tools: An adaptive robust control approach and comparative experiments," *IEEE/ASME Trans. on Mechatronics*, vol. 2, no. 2, pp. 63–76, 1997. (Part of the paper also appeared in *Proc. of 1997 American Control Conference*).

[20] R. A. Freeman, M. Krstic, and P. V. Kokotovic, "Robustness of adaptive nonlinear control to bounded uncertainties," in *IFAC World Congress, Vol. F*, pp. 329–334, 1996.

[21] M. M. Polycarpou and P. A. Ioannou, "A robust adaptive nonlinear control design," in *Proc. of American Control Conference*, pp. 1365–1369, 1993.

[22] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and adaptive control design*. New York: Wiley, 1995.

[23] W. Lin, "Global robust stabilization of minimum-phase nonlinear systems," *Automatica*, vol. 33, no. 3, pp. 453–462, 1997.

[24] Z. P. Jiang and L. Praly, "A self-tuning robust nonlinear controller," in *13th IFAC World Congress, Vol. K*, pp. 169–174, 1996.

[25] A. Alleyne, "Nonlinear force control of an electro-hydraulic actuator," in *Proc. of JAPAN/USA Symposium on Flexible Automation*, (Boston), pp. 193–200, 1996.

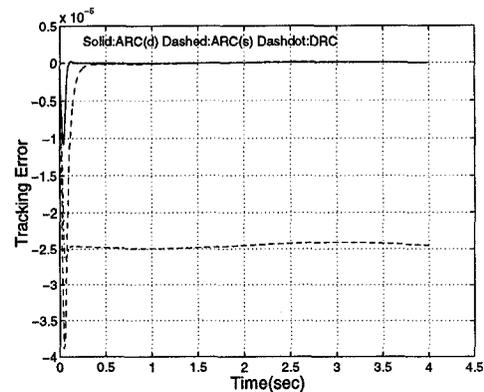


Figure 2: Tracking errors in the presence of parametric uncertainty

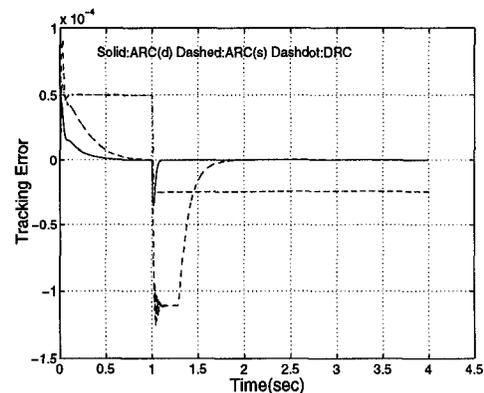


Figure 3: Tracking errors in the presence of parametric uncertainty and large disturbances

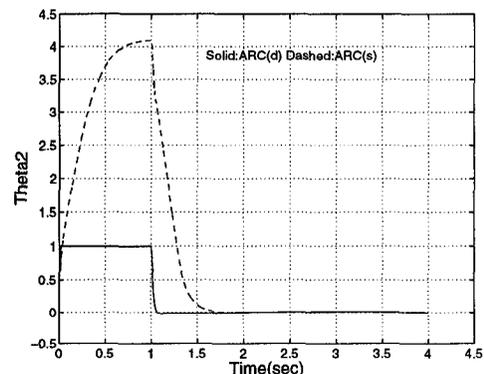


Figure 4: Parameter estimates in the presence of large disturbances