Adaptive Robust Control of Mechanical Systems with Nonlinear Dynamic Friction Compensation [1]

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Abstract

In this paper, an adaptive robust control (ARC) scheme based friction compensation strategy is presented for a class of mechanical systems in the presence of dynamic friction effects. In contrast to existing deterministic robust control (DRC) and adaptive control (AC) schemes, the proposed ARC scheme utilizes both the structural information of the dynamic friction model and the available prior information on a physical system such as the bounds of parameter variations and unmeasurable friction state for high performance. In particular, the physical bound on the unmeasurable dynamic friction state is fully exploited to construct certain projection type modifications to the dynamic friction state observers. By doing so, a controlled dynamic friction state estimation process is achieved even in the presence of disturbances. The resulting controller achieves a guaranteed transient performance and final tracking accuracy. Furthermore, in the absence of uncertain nonlinearities, asymptotic output tracking is achieved.

1 Introduction

Friction has been shown to be a dynamic nonlinear effect which is detrimental to the performance of a number of mechanical systems. Most of the dynamic behaviors of friction, such as the presliding displacement, the friction lag and the Stribeck effect to name a few, all occur in the so-called *low-velocity* and the *presliding* regions. To capture these effects, several researchers have proposed various dynamic friction models [1, 2]. The idea behind obtaining these highly accurate and sophisticated models is to be able to predict the friction more accurately so that friction compensation can be done more effectively. However, no matter how accurate these mathematical models may be, it is usually difficulty to capture the nonlinear features of friction exactly, since almost every physical system is subjected to certain degrees of model uncertainties.

To account for the model uncertainties, two approaches, deterministic robust control (DRC) [3, 4] and adaptive control (AC) [5, 6, 7], may apply. In general, DRC designs can achieve a guaranteed transient performance and final tracking accuracy. However, since no attempt is made to learn from past behavior to reduce the effect of parametric and dynamic uncertainties, the designs are conservative and may involve switching or infinite gain feedback [3, 4] for asymptotic tracking. In contrast to DRC designs, adaptive controllers are able to achieve asymptotic tracking in the absence of uncertain nonlinearities without resorting to infinite gain feedback. In [5], Canudas de Wit *et al.* illustrated how the control structure proposed in [2] could be modified to adapt for selected unknown friction parameters. In [6], an adaptive controller utilizing an *observer/filter* structure, was proposed to handle non-uniform variations in the friction force. Similar result was achieved in [7], in which a *dual-observer* structure was utilized to estimate different nonlinear effects of the unmeasurable friction state. However, these AC designs suffer from two main drawbacks - unknown transient performance and possible non-robustness to disturbances [8].

In this paper, a dynamic friction compensation strategy is proposed by utilizing the idea of adaptive robust control (ARC) [9, 10, 11]. By exploiting available prior information on physical systems such as the bounds of parameter variations and the unmeasurable friction state as much as possible, the proposed scheme effectively combines the design methods of DRC and AC while naturally overcoming their practical limitations. Specifically, based on the available bounds on the unmeasurable friction state, the widely used discontinuous projection mapping is utilized to modify the observer design proposed in [7], which guarantees that the friction state estimates belong to a known bounded region all the time no matter if the system is subjected to disturbances or not and what type of adaptation law is used. As a result, the possible destabilizing effect of friction state estimation errors can be dealt with via certain simple robust feedback effectively for a better performance. The resulting controller achieves a guaranteed transient performance and final tracking accuracy. Furthermore, in the absence of uncertain nonlinearities, asymptotic output tracking is achieved.

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2 Problem Formulation

The mathematical model for a mechanical system in the presence of friction is assumed to be of the form

$$\dot{x}_1 = x_2, \tag{1}$$
$$m\dot{x}_2 = u - F + \Delta(x,z,,u,t), \tag{2}$$

where $x = [x_1 \;\; x_2]^T$ represents the state vector consisting of the position and velocity, m denotes the unknown inertia, F is the friction, Δ represents the lumped unknown nonlinear functions such as disturbances and modeling errors, and $u(t)$ is the control input. Using the dynamic friction model proposed in [2], the dynamic friction force F is given by

$$F = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 x_2, \tag{3}$$
$$\dot{z} = x_2 - \frac{|x_2|}{g(x_2)} z, \tag{4}$$

where z represents the unmeasurable internal friction state, and σ_0, σ_1, σ_2 are unknown friction force parameters. The function $g(x_2)$ is used to describe the Stribeck effect [2]:

$$g(x_2) = F_C + (F_S - F_C)e^{-(x_2/v_s)^2}, \tag{5}$$

where F_C and F_S represent the levels of the normalized Coulomb friction and stiction force respectively, and v_s is the Stribeck velocity. This model considers the dynamic effects of the friction as arising out of the deflection of bristles used to model the asperities between two contacting surfaces. The friction state essentially captures the average deflection of these bristles. Understandably, the state $z(t)$ does not proffer itself for direct measurement. It is shown in [2] that the model has the following **finite bristle deflection** property:

P1 If $|z(0)| \le F_S$, then $|z(t)| \le F_S, \; \forall t \ge 0$.

This property is physically intuitive and will be extensively used in the subsequent ARC controller designs. Substituting the friction dynamics (3) and (4) into (2), we rewrite the equation of motion as

$$m\dot{x}_2 = u - \sigma_0 z + \sigma_1 \frac{|x_2|}{g(x_2)} z - (\sigma_1 + \sigma_2)x_2 + \Delta. \tag{6}$$

If we define an unknown parameter set $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T$, where $\theta_1 = m$, $\theta_2 = \sigma_0$, $\theta_3 = \sigma_1$ and $\theta_4 = \sigma_1 + \sigma_2$, (6) can be rewritten as

$$\theta_1 \dot{x}_2 = u - \theta_2 z + \theta_3 \frac{|x_2|}{g(x_2)} z - \theta_4 x_2 + \Delta. \tag{7}$$

In the paper, the following notations are used: $\bullet_{\min}$ for the minimum value of $\bullet$, $\bullet_{\max}$ for the maximum value of $\bullet$, the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors, and $\hat{\bullet}$ denotes the estimate of $\bullet$. The following practical assumptions are made:

Assumption 1 *The extent of parametric uncertainties is known, and the uncertain nonlinearity Δ is bounded by a known function $\delta(x,t)$ multiplied by an unknown but bounded time-varying disturbance $d(t)$, i.e.,*

$$\theta \in \Omega_\theta \stackrel{\Delta}{=} \{\theta : \;\; 0 < \theta_{\min} < \theta < \theta_{\max}\}, \tag{8}$$
$$\Delta \in \Omega_\Delta \stackrel{\Delta}{=} \{\Delta : \;\; |\Delta(x,z,u,t)| \le \delta(x,t)d(t)\}, \tag{9}$$

where $\theta_{\min}$ and $\theta_{\max}$ are known. ◇

The control objective is to synthesize a control input u such that x_1 tracks a desired position trajectory $x_d(t)$ that is assumed to be second-order differentiable. To achieve this objective, available tools for control of such a mechanical system - AC and DRC - will be applied first, from which one can gain insights about the development and the advantages of the proposed ARC.

3 Adaptive Control

In the absence of uncertain nonlinearities, i.e., $\Delta = 0$, adaptive control [7] can be generalized to solve the problem. Define a switching-function-like quantity as

$$e_2 = \dot{e}_1 + ke_1 = x_2 - x_{2eq}, \quad x_{2eq} \stackrel{\Delta}{=} \dot{x}_d - ke_1, \tag{10}$$

where $e_1 = x_1 - x_d$ is the position tracking error, and k is any positive feedback gain. If e_2 is small or converges to zero exponentially, then e_1 will be small or converge to zero exponentially since $G(s) = \frac{e_1(s)}{e_2(s)} = \frac{1}{s+k}$ is a stable transfer function. So the rest of the design is to make e_2 as small as possible. The structure of (7) motivates us to design the adaptive control law as follows:

$$u = u_a + u_{s1}, \tag{11}$$
$$u_a = \hat{\theta}_2 \hat{z}_0 - \hat{\theta}_3 \frac{|x_2|}{g(x_2)} \hat{z}_1 + \hat{\theta}_4 x_2 + \hat{\theta}_1 \dot{x}_{2eq}, \tag{12}$$
$$u_{s1} = -k_s e_2, \tag{13}$$

where k_s is a positive design constant, $\hat{z}_0$ and $\hat{z}_1$ are estimates of the unmeasurable friction state z. After taking the time derivative of (10), substituting the expressions given by (7) and (11), and then simplifying the resulting expression, we obtain

$$m\dot{e}_2 = -k_s e_2 - \varphi^T \tilde{\theta} + \theta_2 \tilde{z}_0 - \theta_3 \frac{|x_2|}{g(x_2)} \tilde{z}_1 + \Delta, \tag{14}$$

where $\varphi^T = [-\dot{x}_{2eq}, -\hat{z}_0, \frac{|x_2|}{g(x_2)}\hat{z}_1, -x_2]$, $\tilde{z}_0 = \hat{z}_0 - z$ and $\tilde{z}_1 = \hat{z}_1 - z$ represent the estimation errors of the unmeasurable friction state, and $\tilde{\theta} = \hat{\theta} - \theta$ represents parameter estimation error. When $\Delta = 0$, the effect of dynamic uncertainties can be canceled by using the following observers [7] :

$$\dot{\hat{z}}_0 = x_2 - \frac{|x_2|}{g(x_2)} \hat{z}_0 - \gamma_0 e_2, \tag{15}$$
$$\dot{\hat{z}}_1 = x_2 - \frac{|x_2|}{g(x_2)} \hat{z}_1 + \gamma_1 \frac{|x_2|}{g(x_2)} e_2, \tag{16}$$

where γ_0 and γ_0 are two positive design constants. The effect of parametric uncertainties, on the other hand, can be eliminated by using the following parameter adaptation mechanisms

$$\dot{\hat{\theta}} = \Gamma \varphi e_2, \quad \Gamma > 0. \tag{17}$$

The closed-loop stability of the adaptive controller/observer can be proved via a Lyapunov-type argument. Specifically, define the following nonnegative function

$$V_a = \frac{1}{2} m e_2^2 + \frac{1}{2\gamma_0}\theta_2 \tilde{z}_0^2 + \frac{1}{2\gamma_1}\theta_3 \tilde{z}_1^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}. \tag{18}$$

By using similar techniques as in [7], it can be proved that all the signals are bounded and asymptotic position tracking is achieved.

4 Deterministic Robust Control

Adaptive control has two main drawbacks - unknown transient performance and possible non-robustness to disturbances. The system may have large tracking errors during the initial transient period or have a sluggish response. In the presence of uncertain nonlinearity Δ, the closed-loop system may be unstable [8], since parameter adaptation may learn in a wrong way or the observer states may go unbounded. In contrast, instead of using parameter adaptation and friction observers, deterministic robust control (DRC) only uses robust feedback to attenuate the effect of various model uncertainties for a guaranteed performance as done in the following. Consider the following DRC law

$$u = u_a + u_{s1} + u_{s2}, \tag{19}$$

where u_a is given by (12) but with fixed state estimate $\hat{z}_0 = \hat{z}_1 = \hat{z}(0)$ and parameter estimate $\hat{\theta} = \hat{\theta}(0)$, u_{s1} is given in (13) and u_{s2} is a robust control term to be synthesized later. Substituting (19) into (7), we obtain the following error dynamics

$$m\dot{e}_2 = -k_s e_2 - \varphi_0^T \tilde{\theta}_0 + \theta_2 \tilde{z} - \theta_3 \frac{|x_2|}{g(x_2)} \tilde{z} + \Delta + u_{s2}, \tag{20}$$

where $\varphi_0^T = [-\dot{x}_{2eq}, -\hat{z}(0), \frac{|x_2|}{g(x_2)}\hat{z}(0), -x_2]$, $\tilde{z} = \hat{z}(0) - z$ and $\tilde{\theta}_0 = \hat{\theta}(0) - \theta$. Noting Assumption 1 and Property P1, a robust control function u_{s2} can be synthesized such that the following condition is satisfied:

$$e_2[-\varphi_0^T \tilde{\theta}_0 + \theta_2 \tilde{z} - \theta_3 \frac{|x_2|}{g(x_2)} \tilde{z} + \Delta + u_{s2}] \le \varepsilon_0 + \varepsilon_1 d^2, \tag{21}$$

where ε_0 and ε_1 are two design parameters which may be arbitrarily small. From (20) and (21), the derivative of a positive semi-definite function $V_s = \frac{1}{2} m e_2^2$ satisfies

$$\begin{aligned} \dot{V}_s &= -k_s e_2^2 + e_2[-\varphi_0^T \tilde{\theta}_0 + \theta_2 \tilde{z} - \theta_3 \tfrac{|x_2|}{g(x_2)} \tilde{z} + \Delta + u_{s2}] \\ &\le -\lambda_V V_s + \varepsilon_0 + \varepsilon_1 d^2, \end{aligned} \tag{22}$$

where $\lambda_V = 2k_s/\theta_{1\max}$. From (22), it follows that

$$V_s(t) \le \exp(-\lambda_V t) V_s(0) + \frac{\varepsilon_0 + \varepsilon_1 \|d\|_\infty^2}{\lambda_V}[1 - \exp(-\lambda_V t)], \tag{23}$$

where $\|d\|_\infty$ stands for the infinity norm of $d(t)$. The exponentially converging rate λ_V and the final tracking accuracy index ($|V_s(\infty)| \le \frac{\varepsilon_0 + \varepsilon_1 \|d\|_\infty^2}{\lambda_V}$) can be freely adjusted by the design parameters ε_0, ε_1 and k_s in a *known* form. In other words, transient performance and final tracking accuracy are guaranteed. One example u_{s2} satisfying (21) can be found in the following way. Let h_0, h_1 and h_2 be any smooth bounding functions satisfying the following conditions,

$$h_0 \ \ge \ \|\varphi_0\| \|\theta_{\max} - \theta_{\min}\|, \tag{24}$$

$$h_1 \ \ge \ \theta_{2\max}(z_{\max} - z_{\min}), \tag{25}$$

$$h_2 \ \ge \ \theta_{3\max} \frac{|x_2|}{g(x_2)} (z_{\max} - z_{\min}), \tag{26}$$

where $z_{\max} = F_S$ and $z_{\min} = -F_S$ in viewing Property P1. Then, u_{s2} can be chosen as

$$u_{s2} = -\frac{1}{4}(\frac{3}{\varepsilon_0} h_0^2 + \frac{3}{\varepsilon_0} h_1^2 + \frac{3}{\varepsilon_0} h_2^2 + \frac{1}{\varepsilon_1}\delta^2) e_2. \tag{27}$$

5 Adaptive Robust Control

In the following, we will extend the discontinuous projection based ARC design [10] and propose a dynamic friction compensation scheme for a class of second order mechanical systems as outlined in Section 2.

5.1 Discontinuous Projection Mapping

By exploiting prior information on the physical systems such as the bounds of parameter variations and internal friction state, a discontinuous projection based ARC design is constructed to solve the robust tracking control problem. Specifically, the parameter estimate $\hat{\theta}$ is updated through a parameter adaptation law having the form given by

$$\dot{\hat{\theta}} = \mathrm{Proj}_{\hat{\theta}}(\Gamma\tau), \quad \tau = \varphi e_2, \tag{28}$$

where $\mathrm{Proj}_{\hat{\theta}}(\Gamma\tau) = [\mathrm{Proj}_{\hat{\theta}_1}((\Gamma\tau)_1), \cdots, \mathrm{Proj}_{\hat{\theta}_4}((\Gamma\tau)_4)]^T$, and $(\Gamma\tau)_i$ represents the i-th component of $\Gamma\tau$. The unmeasurable friction state z is estimated by the following robust observers with projection type modifications respectively

$$\dot{\hat{z}}_0 \ = \ \mathrm{Proj}_{\hat{z}_0}\{x_2 - \frac{|x_2|}{g(x_2)} \hat{z}_0 - \gamma_0 e_2\}, \tag{29}$$

$$\dot{\hat{z}}_1 \ = \ \mathrm{Proj}_{\hat{z}_1}\{x_2 - \frac{|x_2|}{g(x_2)} \hat{z}_1 + \gamma_1 \frac{|x_2|}{g(x_2)} e_2\}. \tag{30}$$

The projection mapping $\mathrm{Proj}_{\hat{\zeta}}(\bullet)$ is defined by [9, 12]

$$\mathrm{Proj}_{\hat{\zeta}}(\bullet) = \begin{cases} 0 & \text{if } \hat{\zeta} = \zeta_{\max} \text{ and } \bullet > 0 \\ 0 & \text{if } \hat{\zeta} = \zeta_{\min} \text{ and } \bullet < 0 \\ \bullet & \text{otherwise} \end{cases} \tag{31}$$

where ζ is a symbol that can be replaced by $\theta_1 \sim \theta_4$, z_0 or z_1. Using similar arguments as in [9, 12], we can show that the above projection mappings have the following properties

$$\textbf{P2} \qquad \hat{\zeta} \in \Omega_\zeta = \{\hat{\zeta} : \ \zeta_{\min} \le \hat{\zeta} \le \zeta_{\max}\} \tag{32}$$

$$\textbf{P3} \qquad \tilde{\theta}^T(\Gamma^{-1}\mathrm{Proj}_{\hat{\theta}}(\Gamma\tau) - \tau) \le 0 \tag{33}$$

$$\textbf{P4} \qquad \tilde{z}_0\{\dot{\hat{z}}_0 - (x_2 - \frac{|x_2|}{g(x_2)} \hat{z}_0 - \gamma_0 e_2)\} \le 0 \tag{34}$$

$$\textbf{P5} \qquad \tilde{z}_1\{\dot{\hat{z}}_1 - (x_2 - \frac{|x_2|}{g(x_2)} \hat{z}_1 + \gamma_1 \frac{|x_2|}{g(x_2)} e_2)\} \le 0 \tag{35}$$

5.2 ARC Controller Design

The proposed ARC law has the same structure as the DRC law (19)

$$u = u_a + u_{s1} + u_{s2}, \tag{36}$$

where u_a is given by (12) but with $\hat{\theta}$ updated by (28), and $\hat{z}_0$ and $\hat{z}_1$ updated by (29) and (30) respectively. u_{s1} is given by (13), and u_{s2} is synthesized in a similar way as in DRC design as follows. Substituting the ARC control law into (7), we obtain the following error dynamics

$$m\dot{e}_2 = -k_s e_2 - \varphi^T \tilde{\theta} + \theta_2 \tilde{z}_0 - \theta_3 \frac{|x_2|}{g(x_2)} \tilde{z}_1 + \Delta + u_{s2}. \tag{37}$$

As in (21), u_{s2} is synthesized to satisfy the following constraint,

$$\text{i} \quad e_2[-\varphi^T \tilde{\theta} + \theta_2 \tilde{z}_0 - \theta_3 \frac{|x_2|}{g(x_2)} \tilde{z}_1 + \Delta + u_{s2}] \le \varepsilon_0 + \varepsilon_1 d^2. \tag{38}$$

Such a robust control term u_{s2} always exists since both parameter and state estimates belong to some known bounded regions by Property P2. To ensure that the robust control term u_{s2} does not interfere with the nominal estimation process of parameter adaptation and friction state observer, the following passivity-like requirement is also imposed:

$$\text{ii} \qquad e_2 u_{s2} \leq 0. \tag{39}$$

An example of u_{s2} satisfying (38) and (39) is given by (27) with h_0 redefined as $h_0 \geq \|\varphi\|\|\theta_{\max} - \theta_{\min}\|$.

Theorem 1 *With the ARC control law (36), parameter adaptation law (28), and friction state observers (29) and (30), the following results hold:*

A. *In general, all signals are bounded. Furthermore, V_s is bounded above by*

$$V_s(t) \leq \exp(-\lambda_V t)V_s(0) + \frac{\varepsilon_0 + \varepsilon_1 \|d\|_\infty^2}{\lambda_V}[1 - \exp(-\lambda_V t)]. \tag{40}$$

B. *If after a finite time t_f, $\Delta = 0$, i.e., in the presence of parametric uncertainties and dynamic friction only, then, in addition to result A, asymptotic position tracking (or zero final tracking error) is achieved.* △

Proof: Consider the same Lyapunov function candidate as that in DRC design $V_s = \frac{1}{2}me_2^2$. With (37) and (38), we obtain

$$\dot{V}_s \leq -\lambda_V V_s + \varepsilon_0 + \varepsilon_1 d^2, \tag{41}$$

which leads to (40). Thus $e_2(t)$ is bounded. Since $e_2(t)$ is related to $e_1(t)$ via an exponentially stable transfer function $G(s)$, $e_1(t)$ is bounded. Since $x_d(t)$ is assumed to be a bounded signal with bounded derivatives up to the second order, noting (10), it follows that x_{2eq} is bounded. Since $e_1 = x_1 - x_d$ and $e_2 = x_2 - x_{2eq}$, we see that the state x is bounded. From Property P2, the boundedness of $\hat{\theta}$, $\hat{z}_0$ and $\hat{z}_1$ is apparent. The control input u is thus bounded. This proves A of Theorem 1.

Now consider the situation in B of Theorem 1, i.e., $\Delta = 0$. Choose the same Lyapunov function candidate V_a (18) as that in AC design. Its derivative along the trajectory in (37) is

$$\begin{aligned} \dot{V}_a \ = \ & -k_s e_2^2 + e_2[-\varphi^T\tilde{\theta} + \theta_2\tilde{z}_0 - \theta_3\tfrac{|x_2|}{g(x_2)}\tilde{z}_1 + u_{s2}] \\ & + \tfrac{\theta_2}{\gamma_0}\tilde{z}_0(\dot{\hat{z}}_0 - \dot{z}) + \tfrac{\theta_3}{\gamma_1}\tilde{z}_1(\dot{\hat{z}}_1 - \dot{z}) + \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}}. \end{aligned} \tag{42}$$

Substituting (4), (28), (29) and (30) into (42), and noting (33), (34), (35) and (39), we obtain

$$\begin{aligned} \dot{V}_a \leq & -k_s e_2^2 + \theta_2\tilde{z}_0(\tfrac{\dot{\hat{z}}_0}{\gamma_0} + e_2) - \tfrac{\theta_2}{\gamma_0}\tilde{z}_0\dot{z} + \theta_3\tilde{z}_1(\tfrac{\dot{\hat{z}}_1}{\gamma_1} - \tfrac{|x_2|}{g(x_2)}e_2) \\ & - \tfrac{\theta_3}{\gamma_1}\tilde{z}_1\dot{z} + \tilde{\theta}^T(\Gamma^{-1}\dot{\hat{\theta}} - \tau) \\ = & -k_s e_2^2 + \tfrac{\theta_2}{\gamma_0}\tilde{z}_0[\dot{\hat{z}}_0 - (x_2 - \tfrac{|x_2|}{g(x_2)}\hat{z}_0 - \gamma_0 e_2)] - \tfrac{\theta_2}{\gamma_0}\tilde{z}_0(\dot{z} - x_2 \\ & + \tfrac{|x_2|}{g(x_2)}\hat{z}_0) + \tfrac{\theta_3}{\gamma_1}\tilde{z}_1[\dot{\hat{z}}_1 - (x_2 - \tfrac{|x_2|}{g(x_2)}\hat{z}_1 + \gamma_1\tfrac{|x_2|}{g(x_2)}e_2)] \\ & - \tfrac{\theta_3}{\gamma_1}\tilde{z}_1(\dot{z} - x_2 + \tfrac{|x_2|}{g(x_2)}\hat{z}_1) + \tilde{\theta}^T(\Gamma^{-1}\dot{\hat{\theta}} - \tau) \\ \leq & -k_s e_2^2 - \tfrac{\theta_2}{\gamma_0}\tfrac{|x_2|}{g(x_2)}\tilde{z}_0^2 - \tfrac{\theta_3}{\gamma_1}\tfrac{|x_2|}{g(x_2)}\tilde{z}_1^2 \end{aligned} \tag{43}$$

This shows that $e_2 \in L_2 \bigcap L_\infty$. It is easy to check that $\dot{e}_2 \in L_\infty$. So, $e_2(t)$ is uniformly continuous. By Barbalat's lemma, $e_2 \longrightarrow 0$ as $t \longrightarrow \infty$. □

6 Comparative Simulation Results

Comparative simulation results are obtained for the model described by (1)∼(5), in which the numerical values of the parameters are taken from reference [6]

$$m = 0.125kg\text{-}m^2, \ \ \sigma_0 = 12Nm/rad, \ \ \sigma_1 = 0.1Nm\text{-}s/rad,$$
$$\sigma_2 = 13.2Nm\text{-}s/rad, \ \ F_c = 3.24, \ \ F_s = 8.45, \ \ v_s = 3.0.$$

For comparison purpose, the inertia m is assumed to be known as in [7]. Thus there are only three parameters to be adapted. The bounds describing the uncertain ranges are given by $\theta_{\min} = [0.125, 0, 0, 0]^T$, $\theta_{\max} = [0.125, 20, 0.3, 2]^T$, $z_{\min} = -8.45$, $z_{\max} = 8.45$ and $\delta = 1$. Three different controllers synthesized early are compared:

- **ARC**: the controller proposed in section 5.
- **AC**: the controller presented in section 3. For comparison purpose, initial conditions and common design parameters of ARC and AC are kept the same as follows:

$$\begin{aligned} & z(0) = \hat{z}_0(0) = \hat{z}_1(0) = x_1(0) = x_2(0) = 0, \\ & \hat{\theta}(0) = [0.125, 6, 0.05, 0.71]^T, \ k = 5, \ k_s = 5, \\ & \Gamma = \mathbf{diag}[0, 2000, 100, 50], \ \gamma_0 = \gamma_1 = 1, \\ & \varepsilon_1 = \varepsilon_2 = 4 \times 10^3. \end{aligned} \tag{44}$$

- **DRC**: the controller presented in section 4. Initial conditions and design parameters remain unchanged except $\Gamma = 0$ and $k = k_s = 25$.

The desired position trajectory is selected to be [6]

$$x_d = 1.0\tan^{-1}(4\sin(0.5t)(1 - \exp(-0.01t^3))) \text{ rads}. \tag{45}$$

The following two typical cases are considered: **1.** No disturbance (i.e., $\Delta = 0$); **2.** A random disturbance signal with an amplitude of 6.25(Nm) is added to the system at $t = 0$ sec and remove at $t = 20$ sec. Simulation results are given in Fig.1∼Fig.5. Overall, AC has the worst transient performance, and DRC has a large non-converging final tracking error; ARC has a more stable parameter estimation process and state estimation process than that of AC. It is seen that ARC achieves a much smaller tracking error during the period when disturbance occurs. In addition, ARC recovers to its nominal performance much faster after the disturbance is removed.

7 Conclusion

In this paper, a discontinuous projection based ARC scheme is presented for a class of mechanical systems in the presence of dynamic friction effects. Like other existing adaptive dynamic friction compensation (AC) schemes, the proposed ARC scheme utilizes the structural information of the friction dynamics to construct nonlinear observers to recover the unmeasurable friction state. In addition, by utilizing available prior information on the system such as the bounds of parameter variations and unmeasurable friction state, the proposed scheme uses projection mappings for a stable parameter and friction state estimation process in the presence of uncertain nonlinearities, and uses certain robust feedback to attenuate the effect of both estimation errors and uncertain nonlinearities for a guaranteed robust performance in general. As a result, the presented design enjoys the benefits of both adaptive dynamic friction compensation and fast robust feedback while naturally overcoming their practical limitations.

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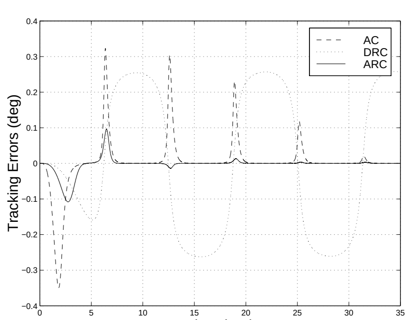

Figure 1: Tracking errors (without disturbance)

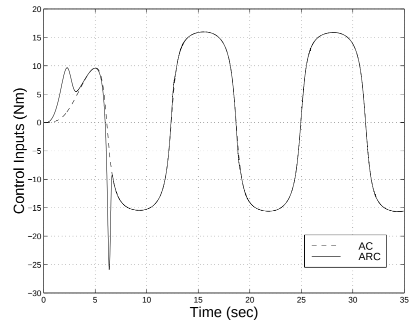

Figure 2: Control inputs (without disturbance)

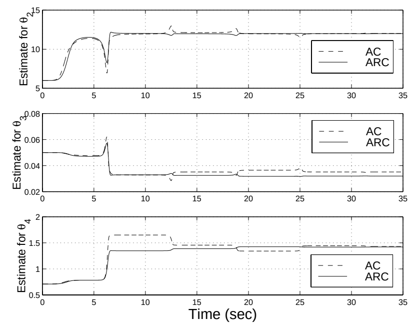

Figure 3: Parameter estimates (without disturbance)

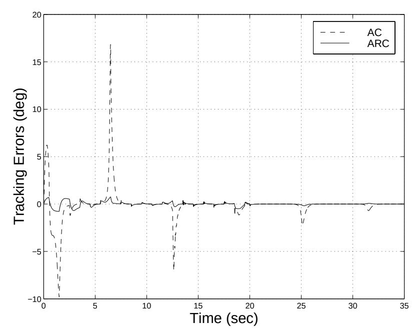

Figure 4: Tracking errors (with disturbance)

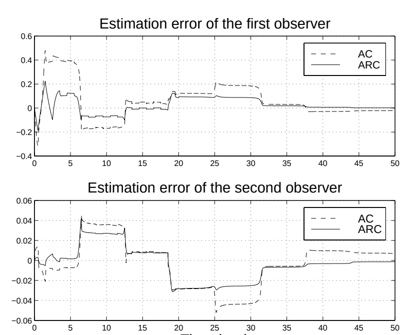

Figure 5: Friction state estimation errors (with disturbance)