DESIRED COMPENSATION ADAPTIVE ROBUST CONTROL [1]

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Abstract

A desired compensation adaptive robust control (DCARC) approach is proposed for nonlinear systems having both parametric uncertainties and uncertain nonlinearities. DCARC of nonlinear systems transformable to a normal form is first solved. A DCARC backstepping design is then developed to overcome the design difficulties associated with unmatched model uncertainties. The proposed DCARC has the unique feature that the adaptive model compensation part depends on the reference trajectory and parameter estimates only. Such a structure has several implementation advantages. First, the regressor in the model compensation part can be calculated off-line and on-line computation time may be reduced. Second, the interaction between the parameter adaptation and the robust control law is minimized, which may facilitate the controller gain tuning process considerably. Third, the effect of measurement noise is minimized since the regressor does not depend on actual measurements. As a result, a fast adaptation rate may be chosen in implementation to speed up the transient response and to improve overall tracking performance. These claims have been verified in the comparative experimental studies for the control of robot manipulators.

1 Introduction

During the past twenty years, a great deal of effort has been devoted to the control of uncertain nonlinear dynamics. The problem is motivated by the fact that almost every physical system is subjected to certain degrees of model uncertainties. The causes of model uncertainties can be classified into two distinct categories: (i) repeatable or constant unknown quantities such as the unknown physical parameters (e.g., the inertia load of any industrial drive systems), and (ii) nonrepeatable unknown quantities such as external disturbances and imprecise modeling of certain physical terms. Two nonlinear control methods have been popular and well documented: adaptive control [1, 2, 3] for parametric uncertainties, and deterministic robust control (DRC) such as sliding mode control [4, 5, 6, 7, 8, 9] for both parametric uncertainties and uncertain nonlinearities. Recently, as in the robust adaptive control (RAC) of linear systems [10, 11], much of the effort in nonlinear adaptive control area has been devoted to robustifying the adaptive backstepping designs [1] with respect to bounded disturbances and significant progress has been made [12, 13, 14, 15, 16].

In [17, 18, 19, 20], an adaptive robust control (ARC) approach has been proposed for the design of a new class of high-performance robust controllers. The approach effectively combines the design methods of deterministic robust control (DRC) and adaptive control (AC). The resulting ARC controllers achieve the results of both DRC and AC while naturally overcoming the practical limitations associated with each method. The approach was originally developed for the trajectory tracking control of robot manipulators in [17]. In [18], the methodology was extended to a class of single-input single-output (SISO) nonlinear systems with arbitrary known ”relative degrees” in a semi-strict feedback form [13] by combining the backstepping adaptive control [1] with the general deterministic robust control. MIMO nonlinear systems transformable to semi-strict feedback forms were studied in [19]. A general framework was formalized in [20] and a discontinuous projection based ARC was also proposed. Comparative experimental results for the motion control of robot manipulators [21] and the high-speed/high-accuracy trajectory tracking control of machine tools [22] have demonstrated the substantially improved performance of the suggested ARC approach. Other applications include the motion and force control of robot manipulators in various contacting environment [23, 24, 25] and the control of electro-hydraulic systems [26, 27].

The proposed ARC approach was originally motivated by the excellent research done in the conventional robust adaptive control (RAC) area [10, 11, 12, 13]. However, it should be realized that there are some subtle but fundamental differences between the proposed ARC and the conventional RAC [12, 13], even including the recently presented tuning function based RAC approach [14]. First, in terms of fundamental viewpoint, the proposed ARC [17, 18, 19, 20] puts more emphasis on the robust control law design in achieving a guaranteed robust performance. In fact, the parameter adaptation law in ARC can be switched off at any time without affecting global stability and sacrificing the guaranteed transient performance result since the resulting controller becomes a deterministic robust controller. Second, in terms of

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the achievable performance, in the proposed ARC, the upper bound on the absolute value of the tracking error over entire time-history is given and is related to certain controller design parameters in a *known* form, which is more transparent than that in RAC [14, 13]. Finally, in terms of specific approaches used for the controller design and the proof of achievable performance, the proposed ARC uses two Lyapunov functions; one the same as that in DRC [4, 5, 7, 8] and the other the same as that in adaptive control [1], while the robust adaptive control [14, 13] uses the same Lyapunov function as in adaptive control [1] only. Because of these subtle differences, the terminology of "*adaptive robust control*" is used for the proposed combined design method to differentiate the approach from the conventional RAC approach and to reflect the strong emphasis on the robust control law design for robust performance.

For some applications with relatively more transparent dynamics, one may have several options for the design of the robust control law and the parameter adaptation law for the proposed ARC. It is thus important to identify the desirable ARC controller structures so that one can select the most appropriate one for a particular application. One of them is the desired compensation ARC structure–the regressor in the model compensation and adaptation law depends on the reference trajectory only. The desired compensation adaptation law was first proposed by Sadegh and Horowitz [28] for the trajectory tracking control of robot manipulators. The idea was then incorporated in the ARC design in [29], of which the resulting controller has the following desirable features: (a) The regressor can be calculated off-line and thus on-line computation time can be reduced; (b) The interaction between the parameter adaptation and the robust control law is minimized, which leads to an almost total separation of the robust control law design and parameter adaptation design; and (c) The effect of measurement noise is minimized since the regressor does not depend on actual measurements. As a result, a fast adaptation rate may be chosen in the implementation to speed up the transient response and to improve overall tracking performance. These claims have been verified by the comparative experiments on the motion control of robot manipulators [21, 25].

This paper continues the work done in [29] and formalizes the desired compensation ARC (DCARC) design by generalizing the approach to a more general class of nonlinear systems. In particular, desired compensation ARC controllers will be constructed for a class of nonlinear systems transformable to a normal form. A DCARC backstepping design is then developed to overcome the design difficulties associated with unmatched model uncertainties to enlarge the applicable nonlinear systems.

This paper is organized as follows. Section II uses a simple first-order uncertain nonlinear system to illustrate the discontinuous projection based ARC design in [17, 20]. Section III presents the proposed DCARC design for the same first-order system to illustrate the uniqueness and advantages of the new ARC design. Section IV considers DCARC of nonlinear systems in a normal form. Section V talks about DCARC backstepping design and section VI concludes the paper.

2 Discontinuous Projection Based ARC

In this section, tracking control of a simple first-order system will be used to illustrate the discontinuous projection based ARC designs presented in [17, 20]. The system is described by

$$\dot{x} = f(x,t) + u, \qquad f = \varphi^T(x)\theta + \Delta(x,t) \tag{1}$$

where $x, u \in R$, and f is an unknown nonlinear function. In general, f can be approximated by a group of known basis functions $\varphi(x) \in R^p$ with unknown weights $\theta \in R^p$, and the approximation error is denoted by the unknown nonlinear function $\Delta(x,t)$. The objective is to let x track its desired trajectory $x_d(t)$ as closely as possible. The following reasonable and practical assumption is made, which is satisfied by most applications [17, 21, 22]:

A1 . The extent of parametric uncertainties and uncertain nonlinearities is known, i.e.,

$$\begin{array}{rcl} \theta & \in & \Omega_\theta \stackrel{\Delta}{=} \{\theta : \ \theta_{min} < \theta < \theta_{max} \} \\ \Delta & \in & \Omega_\Delta \stackrel{\Delta}{=} \{\Delta : \ \|\Delta(x,t)\| \leq \delta(x,t) \ \} \end{array} \tag{2}$$

where $\theta_{min}, \ \theta_{max}$ and $\delta(x,t)$ are known. $\diamondsuit$

Throughout the paper, it is implicitly assumed that if a function depends on t explicitly (e.g., $\delta(x,t)$), then, the function and all its partial derivatives are bounded with respect to time t. The following notations are used: $\bullet_i$ represents the i-th component of the vector $\bullet$ and the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors.

Under Assumption A1, the discontinuous projection based ARC design [17, 20] can be applied to solve the robust tracking control problem for (1). Specifically, the parameter estimate $\hat{\theta}$ is updated through a parameter adaptation law having the form given by

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\tau) \tag{3}$$

where Γ is any symmetric positive definite (s.p.d.) adaptation rate matrix, τ is an adaptation function to be specified later, and the projection mapping $Proj_{\hat{\theta}}(\bullet)$ is defined by [30, 31] (for simplicity, assume that Γ is a diagonal matrix in the following)

$$Proj_{\hat{\theta}}(\bullet) = \begin{cases} 0 & if \quad \begin{cases} \hat{\theta}_i = \hat{\theta}_{imax} & and \quad \bullet > 0 \\ \hat{\theta}_i = \hat{\theta}_{imin} & and \quad \bullet < 0 \end{cases} \\ \bullet & \quad otherwise \end{cases} \tag{4}$$

It can be shown [30, 31, 17] that the projection mapping has the following nice properties

$$\begin{array}{ll} \text{P1} & \hat{\theta} \in \bar{\Omega}_{\theta} = \{\hat{\theta}: \ \theta_{min} \le \hat{\theta} \le \theta_{max}\} \\ \text{P2} & \tilde{\theta}^T(\Gamma^{-1} Proj_{\hat{\theta}}(\Gamma \bullet) - \bullet) \le 0, \quad \forall \bullet \end{array} \tag{5}$$

The ARC control law consists of two parts given by

$$\begin{array}{ll} u = u_f + u_s, & u_f = \dot{x}_d(t) - \varphi^T \hat{\theta} \\ & u_s = u_{s1} + u_{s2}, \quad u_{s1} = -kz \end{array} \tag{6}$$

where $z = x - x_d$ is the tracking error. In (6), u_f is the adjustable model compensation needed for achieving perfect tracking, and u_s is the robust control law consisting of two parts: u_{s1} is used to stabilize the nominal system, which is a simple proportional feedback in this case; and u_{s2} is a robust feedback used to attenuate the effect of model uncertainties, which is required to satisfy the following two constraints

$$\begin{array}{ll} \text{i} & z[-\varphi^T \tilde{\theta} + \Delta(x,t) + u_{s2}] \le \varepsilon \\ \text{ii} & z u_{s2} \le 0 \end{array} \tag{7}$$

where ε is a positive design parameter representing the attenuation level of the model uncertainties. In (7), condition i is used to represent the fact that u_{s2} is synthesized to dominate the model uncertainties coming from both the parametric uncertainties and uncertain nonlinearities to achieve a guaranteed level of attenuation ε, and the passive-like constraint ii is imposed to make sure that introducing u_{s2} does not interfere with the nominal identification process of parameter adaptation. The specific forms of u_{s2} satisfying constraints like (7) can be found in ARC designs in [18, 19, 20].

Theorem 1 *[20, 17] If the adaptation function in (3) is chosen as*

$$\tau = \varphi(x) z \tag{8}$$

then, the ARC law (6) with the parameter adaptation law (8) guarantees that

A . *In general, all signals are bounded and the tracking error is bounded by*

$$|z|^2 \le exp(-2kt)|z(0)|^2 + \frac{\varepsilon}{k}[1 - exp(-2kt)] \tag{9}$$

i.e., the tracking error exponentially decays to a ball. The exponential converging rate $2k$ and the size of the final tracking error ($|z(\infty)| \le \sqrt{\frac{\varepsilon}{k}}$) can be freely adjusted by the controller parameters ε and k in a known form.

B . *If after a finite time, there exist parametric uncertainties only (i.e., $\Delta(x,t) = 0$, $\forall t \ge t_0$), then, in addition to the results in A, zero final tracking error is achieved, i.e, $z \longrightarrow 0$ as $t \longrightarrow \infty$.* △

3 Desired Compensation ARC (DCARC)

In the ARC design presented in section II, the regressor $\varphi(x)$ in the model compensation u_f in (6) and the parameter adaptation function (8) depends on the state x. Such an adaptation structure may have several potential implementation problems. Firstly, the regressor $\varphi(x)$ has to be calculated online based on the actual measurement of the state x. Thus, the effect of measurement noise may be severe, and a slow adaptation rate may have to be used, which in turn reduces the effect of parameter adaptation. Secondly, despite that the intention of introducing u_f is for model compensation, because of $\varphi(x)$, u_f depends on the actual feedback of the state also. Although theoretically the effect of this added implicit feedback loop has been considered in the robust control law design as seen from condition i of (7), practically, there still exists certain interactions between the model compensation u_f and the robust control u_s. This may complicate the controller gain tuning process in implementation. In the following, the idea of desired compensation adaptation law introduced in [28] will be combined with the proposed ARC design to obtain a DCARC controller structure to solve these practical problems.

For simplicity, denote the desired regressor as $\varphi_d(t) = \varphi(x_d(t))$. Let the regressor error be $\tilde{\varphi} = \varphi(x) - \varphi_d$. Noting that θ is unknown but bounded as assumed in (2), there exists a known function $\delta_\phi(x,t)$ such that

$$|\tilde{\varphi}^T \theta| = |\varphi(x)^T \theta - \varphi(x_d)^T \theta| \le \delta_\phi(x,t)|z| \tag{10}$$

The proposed desired compensation ARC law and the adaptation function have the same forms as (6) and (8) respectively but with the desired regressor $\varphi_d(t)$ and a strengthened robust control u_s, which are given by

$$\begin{array}{ll} u = u_f + u_s, & u_f = \dot{x}_d(t) - \varphi_d^T(t) \hat{\theta} \\ & u_s = u_{s1} + u_{s2}, \quad u_{s1} = -k_{s1} z \\ \tau = \varphi_d(t) z & \end{array} \tag{11}$$

where k_{s1} can be any nonlinear gain satisfying

$$k_{s1} \ge k + \delta_\phi(x,t) \tag{12}$$

and u_{s2} is required to satisfy constraints similar to (7) with the constraint i modified to

$$\text{i.} \qquad z[-\varphi_d^T \tilde{\theta} + \Delta(x,t) + u_{s2}] \le \varepsilon \tag{13}$$

Remark 1 *Examples of smooth u_{s2} satisfying (13) can be found in the following way. Let $h(x,t)$ be any smooth function satisfying*

$$h \ge ||\theta_M|| ||\varphi_d(t)|| + \delta(x,t) \tag{14}$$

where $\theta_M = \theta_{max} - \theta_{min}$. Then, using the same technique as in [18], it can be shown that

$$u_{s2} = -h \, tanh\left(\frac{0.2785 hz}{\varepsilon}\right) \tag{15}$$

satisfies (13) and condition ii of (7). Another simple choice would be [19, 25]

$$u_{s2} = -\frac{1}{4\varepsilon} h^2 z \tag{16}$$

Other smooth or continuous examples of u_{s2} can be found in [19, 25]. ◇

Theorem 2 *If the DCARC law (11) is applied, the same results as stated in Theorem 1 are achieved.* △

Remark 2 *The DCARC law (11) has the following advantages: (i) Since the regressor φ_d depends on the reference trajectory only, it is bounded and can be calculated off-line to save on-line computation time if needed; (ii) Due to the use of projection mapping in (3), $\hat{\theta}$ is bounded as shown by P1 of (5). Thus the model compensation u_f in (11) is bounded no matter what type of adaptation law is going to be used. This implies that u_f does not affect the system stability at all and the robust control function u_s can be synthesized totally independent from the design of parameter adaptation law for stability; (iii) Gain tuning process becomes simpler since some of the bounds like the first term in the right hand side of (14) can be estimated off-line; and (iv) the effect of measurement noise is reduced.* ♢

Proof of Theorem 2: Substituting (11) into (1), the error equation is

$$\dot{z} + k_{s1}z = \tilde{\varphi}^T\theta - \varphi_d^T\tilde{\theta} + \Delta(x,t) + u_{s2} \tag{17}$$

where $\tilde{\theta}$=$\hat{\theta} - \theta$ is the parameter estimation error. Noting (10) and (12), the derivative of a positive definite function $V_s = \frac{1}{2}z^2$ is given by

$$\begin{aligned} \dot{V}_s &\leq -k_{s1}z^2 + |z||\tilde{\varphi}^T\theta| + z[-\varphi_d^T\tilde{\theta} + \Delta + u_{s2}] \\ &\leq -kz^2 + z[-\varphi_d^T\tilde{\theta} + \Delta + u_{s2}] \end{aligned} \tag{18}$$

Thus, from (13),

$$\dot{V}_s \leq -kz^2 + \varepsilon \leq -2kV_s + \varepsilon \tag{19}$$

which leads to (9) and proves the results in A of Theorem 1.

Now consider the situation in B of Theorem 1, i.e., $\Delta = 0$, $t \geq t_0$. Choose a p.d. function V_a as

$$V_a = V_s + \tfrac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{20}$$

Noticing (18), (11), condition ii of (7), and P2 of (5),

$$\begin{aligned} \dot{V}_a = \dot{V}_s + \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} &\leq -kz^2 + \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \Gamma\tau) \\ &\leq -kz^2 + \tilde{\theta}^T(\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\tau) - \tau) \leq -kz^2 \end{aligned} \tag{21}$$

Therefore, $z \in L_2$. It is easy to check that $\dot{z}$ is bounded. So, z is uniformly continuous. By Barbalat's lemma, $z \longrightarrow 0$ as $t \longrightarrow \infty$, which proves B of Theorem 1. □

4 DCARC of Systems In A Normal Form

In this section, DCARC of SISO nonlinear systems transformable to the following controllable canonical form will be solved. The system under consideration is described by

$$\begin{aligned} &\dot{x}_i = x_{i+1}, \quad i \leq n-1 \\ &\dot{x}_n = \varphi^T(x)\theta + \Delta(x,t) + u \\ &y = x_1 \end{aligned} \tag{22}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$ is the state, y is the output, and θ and $\Delta(x,t)$ are assumed to satisfy (2) as in previous sections. The objective is to design a bounded control law for the input u such that all signals are bounded and the output y tracks the desired output trajectory $y_d(t)$ as closely as possible. As such, if perfect tracking were achieved, the desired state would be $x_d(t) = [y_d(t), \dot{y}_d(t), \ldots, y_d^{(n-1)}]^T \in R^n$, which is known in advance. Thus, we can define the desired regressor $\varphi_d = \varphi(x_d(t))$ and the regressor error $\tilde{\varphi}$ as in section III. Define the state tracking error as $e = x - x_d \in R^n$. Similar to (10), there exists a known vector function $\delta_\phi(x,t) \in R^n$ such that

$$|\tilde{\varphi}^T\theta| = |\varphi(x)^T\theta - \varphi(x_d)^T\theta| \leq \delta_\phi(x,t)^T|e| \tag{23}$$

The system (22) has a relative degree of n and is in the semi-strict feedback form studied in [18]. Thus, in principle, the backstepping designs may be applied to construct intermediate control functions for the first $n-1$ equations (i.e., state equations for $\bar{x}_{n-1} = [x_1, \ldots, x_{n-1}]^T$). However, since the system (22) has matched model uncertainties only. A simple sliding-mode-like technique can be used to construct a control function for the first $n-1$ equations directly, which is adopted in the paper. Furthermore, a dynamic sliding mode can be employed to enhance the dynamic response of the system as in the control of robot manipulators [17, 21, 29]. The design proceeds as follows.

Let a dynamic compensator be

$$\begin{aligned} &\dot{x}_c = A_c x_c + B_c e_1, \qquad x_c \in R^{n_c}, \quad B_c \in R^{n_c \times 1} \\ &y_c = C_c x_c, \qquad\qquad\quad y_c \in R \end{aligned} \tag{24}$$

where (A_c, B_c, C_c) is controllable and observable and e_1 is the first element of e or the actual output tracking error. For simplicity, denote $\bar{e}_{n-1}$ as the first $n-1$ elements of e. Noting (22), $\dot{\bar{e}}_{n-1} = [e_2, \ldots, e_n]^T$, which is known. Define a switching-function-like term as

$$\begin{aligned} \xi &= L_\xi^T e + y_c = \bar{L}_{\xi n-1}^T \bar{e}_{n-1} + e_n + y_c \\ &= l_{\xi 1}e_1 + \ldots + l_{\xi n-1}e_1^{(n-2)} + e_1^{(n-1)} + y_c \end{aligned} \tag{25}$$

where $L_\xi = [\bar{L}_{\xi n-1}^T, 1]^T$, $\bar{L}_{\xi n-1} = [l_{\xi 1}, \ldots, l_{\xi n-1}]^T$ is a constant vector to be chosen later. In frequency domain, from (25) and (24), $e_1(s)$ is related to $\xi(s)$ by

$$e_1(s) = G_\xi(s)\xi(s), \quad G_\xi(s) = \frac{1}{s^{n-1} + l_{\xi n-1}s^{n-2} + \ldots + l_{\xi 1} + G_c(s)} \tag{26}$$

where $G_c(s) = C_c(sI_{n_c} - A_c)^{-1}B_c$. It is clear that poles of $G_\xi(s)$ can be arbitrarily assigned by suitably choosing dynamic compensator transfer function $G_c(s)$ and the constant vector L_ξ; $G_\xi(s)$ should be chosen such that the resulting dynamic sliding mode $\{\xi = 0\}$ (i.e., free response of the transfer function $G_\xi(s)$) possesses fast enough exponentially converging rate and the effect of non-zero ξ on e_1 is attenuated to a certain degree. In addition, the initial value $x_c(0)$ of the dynamic compensator (24) can be chosen to satisfy

$$C_c x_c(0) = -L_\xi^T e(0) \tag{27}$$

Then, $\xi(0) = 0$ and transient tracking error may be reduced.

Noting (25) and (24), the state space representation of (26) is obtained as

$$\dot{x}_\xi = A_\xi x_\xi + B_\xi \xi, \quad y_\xi = C_\xi x_\xi \tag{28}$$

where $x_\xi = [x_c^T\ ,\ \bar{e}_{n-1}^T]^T \in R^{n_c+n-1}$ and

$$A_\xi = \begin{bmatrix} A_c & B_c & 0 \\ 0 & 0 & I_{n-2} \\ -C_c & -\bar{L}_{\xi n-1}^T & \end{bmatrix}, \quad B_\xi = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C_\xi = [0,1,0] \tag{29}$$

Since $G_\xi(s)$ is chosen to be stable, there exists an s.p.d. solution P_ξ for any s.p.d. matrix Q_ξ for the following Lyapunov equation,

$$A_\xi^T P_\xi + P_\xi A_\xi = -Q_\xi \tag{30}$$

Furthermore, the exponentially converging rate $\frac{\lambda_{min}(Q_\xi)}{\lambda_{max}(P_\xi)}$ can be any desired value by assigning the poles of A_ξ to the far left plane and suitably choosing Q_ξ.

Define the transformed state error vector as $x_e = [x_\xi^T, \xi]^T = [x_c^T, \bar{e}_{n-1}^T, \xi]^T \in R^{n_c+n}$. The original state error vector e is related to x_e by

$$e = C_e x_e, \quad C_e = \begin{bmatrix} 0 & I_{n-1} & 0 \\ -C_c & -\bar{L}_{\xi n-1}^T & 1 \end{bmatrix} \tag{31}$$

Noting (23), there exist known nonlinear functions $\delta_{x_\xi}(x_e,t)$ and $\delta_\xi(x_e,t)$ such that

$$|\tilde{\varphi}^T\theta| \le \delta_{x_\xi}(x_e,t)\|x_\xi\| + \delta_\xi(x_e,t)|\xi| \tag{32}$$

The proposed DCARC law $\alpha(x_e,\hat{\theta},t)$ for (22) and the associated adaptation function $\tau_\alpha(x_e,t)$ have similar forms as (11) and are given by

$$\begin{aligned} &\alpha = \alpha_f(\hat{\theta},t) + \alpha_s(x_e,t), \quad \alpha_s = \alpha_{s1} + \alpha_{s2}, \\ &\quad \alpha_f = y_d^{(n)}(t) - \varphi_d^T(t)\hat{\theta} \\ &\quad \alpha_{s1} = -k_{x_e}(x_e,t)x_e \\ &\qquad = -k_{s1}\xi - C_c(A_c x_c + B_c e_1) - \bar{L}_{\xi n-1}^T \bar{e}_{n-1} - B_\xi^T P_\xi x_\xi \\ &\tau_\alpha = \varphi_d(t)\xi \end{aligned} \tag{33}$$

In (33), $k_{s1}(x_e,t)$ is any nonlinear gain satisfying

$$k_{s1} \ge k + \delta_\xi + \tfrac{1}{2k_Q}\delta_{x_\xi}^2 \tag{34}$$

where k_Q is any gain less than $\lambda_{min}(Q_\xi)$, and α_{s2} is required to satisfy constraints similar to (7)

$$\begin{aligned} &\text{i.} \qquad \xi[-\varphi_d^T\tilde{\theta} + \Delta + \alpha_{s2}] \le \varepsilon \\ &\text{ii.} \qquad \xi\alpha_{s2} \le 0 \end{aligned} \tag{35}$$

Specific form of α_{s2} can be obtained using the techniques in Remark 1. For example, corresponding to (16),

$$\alpha_{s2} = -\tfrac{1}{4\varepsilon}h^2\xi \tag{36}$$

where h is any function satisfying $h \ge \|\phi_d\|\|\theta_M\| + \delta$.

Theorem 3 *If the DCARC law (33) is applied, i.e., $u = \alpha$ with $\hat{\theta}$ updated by (3) and $\tau = \tau_\alpha$, then,*

A . *In general, all signals are bounded. Furthermore, the positive definite function V_s defined by*

$$V_s = \tfrac{1}{2}x_\xi^T P_\xi x_\xi + \tfrac{1}{2}\xi^2 \tag{37}$$

is bounded above by

$$V_s \le exp(-\lambda_V t)V_s(0) + \tfrac{\varepsilon}{\lambda_V}[1 - exp(-\lambda_V t)] \tag{38}$$

where $\lambda_V = min\{\frac{\lambda_{min}(Q_\xi)-k_Q}{\lambda_{max}(P_\xi)},\ 2k\}$.

B . *If after a finite time, $\Delta(x,t) = 0$, $\forall t \ge t_0$, then, in addition to the results in A, zero final output tracking error is achieved, i.e, $e \longrightarrow 0$ and $x_e \longrightarrow 0$ as $t \longrightarrow \infty$.* △

Remark 3 *The DCARC law (33) has the structure that the model compensation α_f depends on the reference trajectory and parameter estimate only, and the robust control term α_s dose not depend on the parameter estimate. In addition, $\alpha_s \longrightarrow 0$ as $\|x_e\| \longrightarrow 0$. It thus has all the nice properties stated in Remark 2.* ♢

Proof of Theorem 3: In order for the results in this section to be used conveniently in the DCARC backstepping design in the next section, formulas are derived for the general case that u might be different from the control function α first. Denote the input discrepancy as $z_u = u - \alpha$, from (22), (25), and (33), it can be checked out that

$$\dot{\xi} = z_u - k_{s1}\xi - B_\xi^T P_\xi x_\xi + \tilde{\varphi}^T\theta - \varphi_d^T\tilde{\theta} + \Delta + \alpha_{s2} \tag{39}$$

Noting (28), (30), (39), and (32), the derivative of V_s given by (37) is

$$\begin{aligned} \dot{V}_s &= -\tfrac{1}{2}x_\xi^T Q_\xi x_\xi - k_{s1}\xi^2 + \xi\tilde{\varphi}^T\theta + \xi[-\varphi_d^T\tilde{\theta} + \Delta + \alpha_{s2}] + \xi z_u \\ &\le -\tfrac{1}{2}(\lambda_{min}(Q_\xi) - k_Q)\|x_\xi\|^2 - k\xi^2 + \xi[-\varphi_d^T\tilde{\theta} + \Delta + \alpha_{s2}] + \xi z_u \end{aligned} \tag{40}$$

in which $\xi\tilde{\varphi}^T\theta \le \delta_{x_\xi}\|x_\xi\||\xi| + \delta_\xi\xi^2 \le \frac{1}{2}k_Q\|x_\xi\|^2 + (\frac{1}{2k_Q}\delta_{x_\xi}^2 + \delta_\xi)\xi^2$ has been used. Thus, if $u = \alpha$ or $z_u = 0$, noting i of (35),

$$\dot{V}_s \le -\lambda_V V_s + \varepsilon \tag{41}$$

which leads to (38) and proves the results in A of Theorem 3. Noting ii of (35), B of Theorem 3 can be proved by using a positive definite function V_a of the form (20) and the same techniques as in (21). □

5 DCARC Backstepping Design

In this section, a DCARC backstepping design will be presented to overcome the design difficulties associated with higher "relative degrees" and *unmatached* model uncertainties. To keep the development concise, the system under consideration is obtained by augmenting the system (22) through a general first-order nonlinear input dynamics, which is described by

$$\begin{aligned} &\dot{x}_i = x_{i+1}, \quad i \le n-1 \\ &\dot{x}_n = \varphi^T(x)\theta + \Delta(x,t) + u \\ &\dot{u} = \varphi_u^T(x,u)\theta + \Delta_u(x,u,t) + v \\ &y = x_1 \end{aligned} \tag{42}$$

where v is the new input of the system and u becomes a measurable state variable. Similar to (2), the unknown nonlinear function Δ_u is assumed to be bounded by

$$|\Delta_u| \leq \delta_u(x,u,t) \quad (43)$$

The goal is the same as in section IV, i.e., want $y \longrightarrow y_d$.

Like the system (22), the desired values for the first n state variables for perfect tracking is known in advance and given by $x_d(t) = [y_d, \ldots, y_d^{(n-1)}]^T$. However, the desired value for the state variable u for perfect tracking is unknown because of the appearance of the unmatched model uncertainties in the second equation of (42). To by-pass this technical difficulty, the best estimate of its desired value, $\alpha_f(\hat{\theta},t)$ defined in (33), is used, i.e., let u_d be $u_d = \alpha_f(\hat{\theta},t)$. The desired value of the function $\varphi_u(x,u)$ can thus be calculated as $\varphi_{ud} = \varphi_u(x_d(t), u_d(\hat{\theta},t))$. Similar to (23) and (32), there exist known functions $\delta_{\varphi_u 1}$ and $\delta_{\varphi_u 2}$ such that

$$|\tilde{\varphi}_u^T\theta| = |\varphi_u(x,u)^T\theta - \varphi_{ud}(x_d,u_d)^T\theta| \leq \delta_{\varphi_u 1}\|x_e\| + \delta_{\varphi_u 2}|u-u_d| \quad (44)$$

Noting that $|u-u_d| = |z_u + \alpha_s| \leq |z_u| + |\alpha_s|$ and α_s given by (33) has the property that $\frac{\alpha_s}{\|x_e\|}$ is finite when $\|x_e\| \longrightarrow 0$, from (44), there exist known functions $\delta_{\varphi_u 3}$ and $\delta_{\varphi_u 4}$ such that

$$|\tilde{\varphi}_u^T\theta| \leq \delta_{\varphi_u 3}\|x_e\| + \delta_{\varphi_u 4}|z_u| \quad (45)$$

As shown in Theorem 3, if $z_u = 0$, output tracking would be achieved. Thus, the backstepping design in this section is essentially to synthesize a DCARC law for the actual input v such that z_u converges to a small value with a guaranteed transient performance as follows. The proposed DCARC backstepping law has the following structural form

$$\begin{aligned} v &= v_f(\hat{\theta},t) + v_s(x_e,z_u,\hat{\theta},t), \quad v_s = v_{s1} + v_{s2} + v_{s3}, \\ v_f &= \frac{\partial\alpha_f(\hat{\theta},t)}{\partial t} - \varphi_{ud}^T(\hat{\theta},t)\hat{\theta}, \\ v_{s1} &= -k_{su}z_u - \xi + \frac{\partial\alpha_s}{\partial t} + \frac{\partial\alpha_s}{\partial x_\xi}(A_\xi x_\xi + B_\xi \xi) \\ &\quad + \frac{\partial\alpha_s}{\partial\xi}(z_u - k_{s1}\xi - B_\xi^T P_\xi x_\xi + \alpha_{s2}) \end{aligned} \quad (46)$$

where v_f is the model compensation depending on the reference trajectory and the parameter estimate only, v_{s1} has the same function as that in (33), v_{s2} is synthesized in the following to attenuate the effect of model uncertainties, and v_{s3} is an additional robust control action term synthesized in the following to handle the effect of time-varying parameter estimate.

Noting the particular form (33), from (42), (39), and (46), it can be checked out that

$$\begin{aligned} \dot{z}_u &= \dot{u} - \dot{\alpha} = \varphi_u^T\theta + \Delta_u + v - [\frac{\partial\alpha_f}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\alpha_f}{\partial t} + \frac{\partial\alpha_s}{\partial x_\xi}\dot{x}_\xi + \frac{\partial\alpha_s}{\partial\xi}\dot{\xi} + \frac{\partial\alpha_s}{\partial t}] \\ &= -k_{su}z_u - \xi + \tilde{\varphi}_u^T\theta - \frac{\partial\alpha_s}{\partial\xi}\tilde{\varphi}^T\theta + v_{s2} - \phi_u^T\tilde{\theta} + \Delta_u - \frac{\partial\alpha_s}{\partial\xi}\Delta \\ &\quad + v_{s3} - \frac{\partial\alpha_f}{\partial\hat{\theta}}\dot{\hat{\theta}} \end{aligned} \quad (47)$$

where

$$\phi_u = \varphi_{ud}(\hat{\theta},t) - \frac{\partial\alpha_s}{\partial\xi}\varphi_d(t) \quad (48)$$

Define an augmented p.d. function as

$$V_{st} = V_s + \frac{1}{2}z_u^2 \quad (49)$$

where V_s is defined by (37). Noting (32) and (45), from (40) and (47), it is straightforward to show that

$$\begin{aligned} \dot{V}_{st} \leq & -\frac{1}{2}(\lambda_{min}(Q_\xi) - k_Q)\|x_\xi\|^2 - k\xi^2 + \xi[-\varphi_d^T\tilde{\theta} + \Delta + \alpha_{s2}] \\ & -(k_{su} - \delta_{\varphi_u 4})z_u^2 + [\delta_{\varphi_u 3}\|x_e\| + |\frac{\partial\alpha_s}{\partial\xi}|(\delta_{x_\xi}\|x_\xi\| + \delta_\xi|\xi|)]|z_u| \\ & + z_u[v_{s2} - \phi_u^T\tilde{\theta} + \Delta_u - \frac{\partial\alpha_s}{\partial\xi}\Delta] + z_u[v_{s3} - \frac{\partial\alpha_f}{\partial\hat{\theta}}\dot{\hat{\theta}}] \end{aligned} \quad (50)$$

v_{s2} is now chosen to satisfy conditions similar to (35)

$$\begin{array}{ll} \text{i.} & z_u[v_{s2} - \phi_u^T\tilde{\theta} + \Delta_u - \frac{\partial\alpha_s}{\partial\xi}\Delta] \leq \varepsilon_u \\ \text{ii.} & z_u v_{s2} \leq 0 \end{array} \quad (51)$$

Specific form of v_{s2} can be obtained using the techniques in Remark 1. For example, corresponding to (16),

$$v_{s2} = -\frac{1}{4\varepsilon_u}h_u^2 z_u \quad (52)$$

where h_u is any function satisfying $h_u \geq \|\phi_u\|\|\theta_M\| + \delta_u + |\frac{\partial\alpha_s}{\partial\xi}|\delta$. Let the adaptation function be

$$\tau = \tau_a + \phi_u z_u = \varphi_d\xi + \phi_u z_u \quad (53)$$

v_{s3} can now be chosen as

$$v_{s3} = \frac{\partial\alpha_f}{\partial\hat{\theta}}\dot{\hat{\theta}} = \frac{\partial\alpha_f}{\partial\hat{\theta}}Proj_{\hat{\theta}}(\Gamma\tau) \quad (54)$$

to cancel the effect of the time-varying parameter estimate as seen from (50). Note that v_{s3} given by (54) may experience possible finite jumps since the projection mapping is discontinuous at certain boundary points. If this poses a problem, then, instead of the direct cancellation by (54), the technique in [20] can be used to construct a smooth v_{s3} to dominate the effect of the time-varying parameter estimate. The details are quite tedious and omitted here.

Theorem 4 *Consider the DCARC law (46) and the adaptation function (53) for the system (42). Choose the controller gain k_{su} in (46) large enough such that*

$$k_{su} \geq d_1 + \delta_{\varphi_u 4} + \frac{1}{2d_2}\left(\delta_{\varphi_u 3} + |\frac{\partial\alpha_s}{\partial\xi}|\delta_{x_\xi}\right)^2 + \frac{1}{4d_3}\left(\delta_{\varphi_u 3} + |\frac{\partial\alpha_s}{\partial\xi}|\delta_\xi\right)^2 \quad (55)$$

where d_1 is any positive scalar, d_2 and d_3 are any positive nonlinear gains satisfying $d_2 < \lambda_{min}(Q_\xi) - k_Q$ and $d_3 < k$ respectively. Then,

A *In general, all signals are bounded. Furthermore, the p.d. function V_{st} defined by (49) is bounded above by*

$$V_{st} \leq exp(-\lambda_V t)V_{st}(0) + \frac{\varepsilon_V}{\lambda_V}[1 - exp(-\lambda_V t)] \quad (56)$$

where $\lambda_V = min\{\frac{\lambda_{min}(Q_\xi) - k_Q - d_2}{\lambda_{max}(P_\xi)},\ 2(k-d_3),\ 2d_1\}$, and $\varepsilon_V = \varepsilon + \varepsilon_u$.

B *If after a finite time, $\Delta = 0$ and $\Delta_u = 0$, then, in addition to the results in A, zero final output tracking error is achieved, i.e, $e \longrightarrow 0$ as $t \longrightarrow \infty$.* △

Proof of Theorem 4: If (55) is satisfied, by using the completion of square and noting (54), from (50), it is easy to show that

$$\begin{aligned} \dot{V}_{st} \leq & -\frac{1}{2}(\lambda_{min}(Q_\xi) - k_Q - d_2)\|x_\xi\|^2 - (k - d_3)\xi^2 - d_1 z_u^2 \\ & + \xi[-\varphi_d^T\tilde{\theta} + \Delta + \alpha_{s2}] + z_u[v_{s2} - \phi_u^T\tilde{\theta} + \Delta_u - \frac{\partial\alpha_s}{\partial\xi}\Delta] \end{aligned} \quad (57)$$

Noting i of (35) and (51),

$$\dot{V}_{st} \leq -\lambda_V V_{st} + \varepsilon_V \tag{58}$$

which leads to A of Theorem 4.

When $\Delta = 0$ and $\Delta_u = 0$, noting (53) and ii of (35) and (51),

$$\dot{V}_{st} \leq -\lambda_V V_{st} - \tau^T \tilde{\theta} \tag{59}$$

Thus, B of Theorem 4 can be proved by using a positive definite function $V_{at} = V_{st} + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$ and the same techniques as in (21). □

6 Conclusions

A general framework of the proposed desired compensation adaptive robust control (DCARC) has been presented for nonlinear systems having both parametric uncertainties and uncertain nonlinearities. The resulting DCARC controllers have several implementation advantages such as reduced on-line computation time, an almost total separation of robust control design and parameter adaptation to facilitate gain tuning process, the reduced effect of measurement noise, and a faster adaptation rate for a better tracking performance in implementation. The controllers achieve a guaranteed transient performance and a prescribed final tracking accuracy in the presence of both parametric uncertainties and uncertain nonlinearities while achieving asymptotic output tracking in the presence of parametric uncertainties without using an infinite fast switching control law or an infinite-gain feedback.

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