Multirate Adaptive Robust Control for Discrete-Time Non-Minimum Phase Systems and Application to Linear Motors

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***Abstract*—It is well known that a plant becomes non-minimum phase in discrete-time domain when the relative degree of the original continuous-time plant is greater than 2 even if the plant is minimum-phase in continuous-time domain. Thus, it was difficult to apply the conventional adaptive controllers directly to these systems. In this paper, multirate adaptive robust control (MARC) is proposed for these systems. This scheme is developed by the good combination of perfect tracking control (PTC) with multirate feedforward control which has been proposed by the first author and discontinuous projection based adaptive robust control (ARC) which has been proposed by the second author. Although the original PTC can assure perfect tracking only for nominal plant, the proposed MARC can guarantee: 1) perfect tracking for plant with parametric uncertainty and 2) overall stability even if there exist modeling error and disturbance. The proposed scheme is applied to the high-speed position control of a linear motor, and the advantages are demonstrated through experiments.**

***Index Terms*—Adaptive robust control, linear motor, multirate control, non-minimum phase system.**

I. Introduction

In most studies of the model reference adaptive system (MRAS) or direct self tuning regulator (STR), the plant is assumed to be minimum phase system [1]. These adaptive controllers are usually implemented as digital control system. However, the discrete-time plant $P[z]$ discretized by zeroth-order hold with short sampling period becomes non-minimum phase system when the relative degree of the continuous-time plant $P_c(s)$ is greater than 2 [2]. Even if the relative degree is 2, $P[z]$ has a poorly dumped zero which cannot be canceled by controller to obtain smooth control input [1]. Thus, the MRAS and direct STR could not be utilized directly for these systems. In the indirect STR and the adaptive pole placement, only poles and stable zeros can be allocated and the unstable zeros remains in the reference model, which leads the poor performance of command response.

The unstable-zeros problem of discrete-time plant has been resolved by zero assignment based on multirate control [3]–[5]. This advantage has been applied to adaptive control in [6] and [7]. However, it is shown that those methods sometimes have the disadvantages which are large overshoot and oscillation in the intersample points [6], [8]. The reason of intersample oscillation is that the sign of multirate control input changes positive and negative alternatively during one sampling period. Then, [9] tried to remove this input oscillation as possible.

In this paper, a novel adaptive controller is proposed using multirate feedforward control [10] without zero assignment. The advantage of this method is that the controller can generate smooth control input by introducing the desired trajectory of state variables. Moreover, by combining the proposed adaptive feedforward control, robust feedback control, and discontinuous projection based parameter identification [11], the overall stability can be guaranteed even if the plant has modeling error and disturbance. If the plant has parametric uncertainty only and persistent excitation (PE) condition is satisfied, perfect tracking is achieved at every sampling point of reference trajectory.

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II. Motivation and Formulation

A. Perfect Tracking Control

In many control applications, tracking controllers are often employed in order to let the plant output follow a smoothed desired trajectory. The best tracking controller is perfect tracking controller (PTC), which has zero tracking error [12]. Perfect tracking control can be achieved by using d-step preview action and the feedforward controller $C_1[z]$ which is realized by the inverse of closed-loop system $G_{\mathrm{cl}}[z]$

$$C_1[z] = \frac{1}{z^d G_{\mathrm{cl}}[z]} = \frac{1 - P[z]C_2[z]}{z^d P[z]} \tag{1}$$

$$r[i] = y_d[i+d] \tag{2}$$

where d is the relative degree of $G_{\mathrm{cl}}[z], r[i]$ is the reference input, $y_d[i]$ is the desired trajectory, and $C_2[z]$ is the feedback controller, as shown in Fig. 1. $\mathcal{H}$ and $\mathcal{S}$ represent holder and sampler, respectively.

However, the discrete-time plant $P[z]$ discretized by zero-order hold has unstable zeros or poorly dumped zeros as mentioned above. Thus, $C_1[z]$ becomes unstable because $G_{\mathrm{cl}}[z]$ has unstable zeros. Therefore, in conventional digital control systems utilizing zeroth-order holds, perfect tracking is essentially impossible [12].

On the other hand, the first author has developed perfect tracking control method using multirate feedforward control instead of the single-rate zeroth-order hold in [10]. In the perfect tracking control, the tracking error of plant state becomes completely zero at every sampling point of reference input for the nominal

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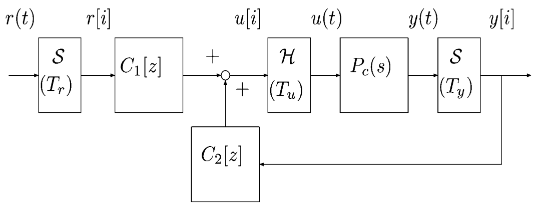

Fig. 1. Two-degree-of-freedom control system.

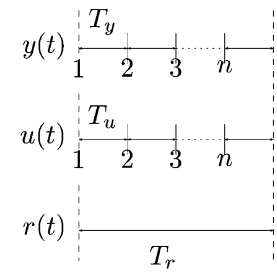

Fig. 2. Multirate feedforward control.

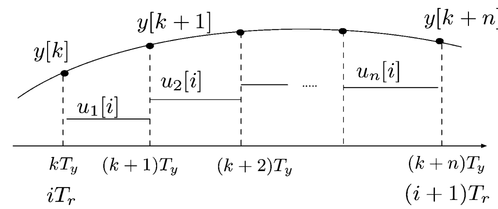

Fig. 3. Multirate hold.

plant.[1] However, perfect tracking was not achieved when the plant had modeling error, although good tracking performance was preserved by combining the perfect tracking feedforward controller with a robust feedback controller. In this paper, the feedforward controller is tuned by the parameters which are obtained through on-line identification in order to keep the tracking performance against the parametric uncertainty of plant. This new scheme is named multirate adaptive robust control (MARC).

A digital tracking control system generally has two samplers for the reference signal $r(t)$ and the output $y(t)$, and one holder on the input $u(t)$, as shown in Fig. 1. Therefore, there exist three time periods T_r, T_y, and T_u, which represent the periods of $r(t), y(t)$, and $u(t)$, respectively [13], [14]. The input period T_u is generally decided by the speed of the actuator, the D/A converter, or the calculations on the CPU. On the other hand, the output period T_y is determined by the speed of the sensor or the A/D converter.

In this paper, multirate adaptive robust controller is proposed in the simplest case of a SISO plant without hardware restrictions on the sampler and holder, that means $T_y = T_u$. Because actual control systems usually have restrictions on T_u and/or T_y, the proposed method can be extended to general systems with these restrictions $(T_y \neq T_u)$ by the formulation of [10]. In the proposed multirate feedforward control, the control input $u(t)$ is changed n times during one sampling period (T_r) of reference input $r(t)$, as shown in Fig. 2, where n is the plant order of (3).

B. Plant Modeling and Discretization

Consider the continuous-time nth-order plant $P_\theta(s)$ described by

$$P_\theta(s) = \frac{y(s)}{u(s)} = \frac{c_m s^m + c_{m-1}s^{m-1} + \cdots + c_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} \quad (3)$$

where $m < n$ and $\boldsymbol{\theta} := [a_{n-1}, \ldots, a_0, c_m, \ldots c_0]^T$ is unknown constant parameter. The following practical assumption is made.

Assumption 1: The extent of unknown parameters is known, i.e.,

$$\theta_j \in (\theta_{j\,\min}, \theta_{j\,\max}) \quad (4)$$

where $\theta_{j\,\min}$ and $\theta_{j\,\max}$ are known for all $j = 1, \ldots, p, p := n + m + 1$. □

The controllable canonical form of (3) is given by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{b}_c u(t), \quad y(t) = \boldsymbol{c}_c\boldsymbol{x}(t) \quad (5)$$

where $\boldsymbol{x} := [x, x^{(1)}, \ldots, x^{(n-1)}]^T, x^{(i)} := d^i x(t)/dt^i$, and

$$\boldsymbol{A}_c := \begin{bmatrix} 0 & 1 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix}, \quad \boldsymbol{b}_c := \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (6)$$

$$\boldsymbol{c}_c := [c_0, \ldots, c_m, 0 \ldots 0]. \quad (7)$$

Note that the coefficients include the unknown constants. The discrete-time plant $P_\theta[z_s]$ discretized by the short sampling period T_y $(=T_u)$ becomes

$$\boldsymbol{x}[k+1] = \boldsymbol{A}_s\boldsymbol{x}[k] + \boldsymbol{b}_s u[k] \quad (8)$$

$$y[k] = \boldsymbol{c}_s\boldsymbol{x}[k] \quad (9)$$

where $\boldsymbol{x}[k] = \boldsymbol{x}(kT_y), z_s := e^{sT_y}$, and

$$\boldsymbol{A}_s := e^{\boldsymbol{A}_c T_y}, \quad \boldsymbol{b}_s := \int_0^{T_y} e^{\boldsymbol{A}_c \tau}\boldsymbol{b}_c \, d\tau, \quad \boldsymbol{c}_s := \boldsymbol{c}_c. \quad (10)$$

In Fig. 3, the state transition state-space model of multirate plant $\boldsymbol{P}_\theta[z]$ from $t = iT_r = kT_y$ to $t = (i+1)T_r = (k+n)T_y$ can be represented by

$$\boldsymbol{x}[i+1] = \boldsymbol{A}\boldsymbol{x}[i] + \boldsymbol{B}\boldsymbol{u}[i] \quad (11)$$

$$\boldsymbol{y}[i] = \boldsymbol{C}\boldsymbol{x}[i] + \boldsymbol{D}\boldsymbol{u}[i] \quad (12)$$

where $\boldsymbol{x}[i] = \boldsymbol{x}(iT_r), z := e^{sT_r}$ and multirate input and output vectors $\boldsymbol{u}, \boldsymbol{y}$ are defined as[2]

$$\begin{aligned}\boldsymbol{u}[i] &:= [u_1[i], \ldots, u_n[i]]^T \\ &= [u(kT_y), \ldots, u((k+n-1)T_y)]^T \end{aligned} \quad (13)$$

[1] The phrase "perfect tracking control" is originally defined in [12], which means the plant output tracks the desired trajectory with zero tracking error at every sampling point. Note that perfect tracking was impossible by conventional single-rate controller even in the ideal situation without disturbance and modeling error.

[2] In this paper, the time index k and operator z_s represent the shorter period $T_y = T_u$, and i and z represent the longer period T_r. The operations of (13) and (14) are called "discrete-time lifting" in advanced sampled-data control theory [15].

$$\boldsymbol{y}[i] := [y_1[i], \ldots, y_n[i]]^T = [y(kT_y), \ldots, y((k+n-1)T_y)]^T. \tag{14}$$

The coefficient matrices are given by (15) at the bottom of the page.

III. Multirate Adaptive Robust Control

A. Design of Feedback Controller

Before the proposed feedforward controller is designed, the feedback controller $C_2[z_s]$ is determined. Here, the following assumption is made.

Assumption 2: The feedback controller $C_2[z_s]$ stabilizes the plant (3), i.e., $(1 + P_\theta[z_s]C_2[z_s])^{-1}$ is a stable transfer function. □

Due to the recent advancement of robust control theory, it became possible to design a stabilizing controller for a plant with uncertainty. Thus, this assumption may not be so difficult. The only problem of the fixed robust controller is that the performance of closed-loop system becomes too conservative when the plant variation is too large. The proposed controller can recover the tracking performance by adaptive feedforward control.

In [16], it is proven that the feedback characteristics such as disturbance rejection performance and stability robustness cannot be improved by the multirate feedback control. This is true only when there is no hardware restriction in the sampling scheme, that is $T_y = T_u$. Thus, it is not necessary to design a multirate feedback controller here. A single-rate feedback controller $C_2[z_s]$ is adequate in the simple case of $T_y = T_u$. However, when the system has sampling restriction ($T_y \neq T_u$), multirate feedback controller should be designed to obtain higher feedback performance [13], [14].

B. Design of MARC

In this section, the multirate adaptive robust controller is proposed as a feedforward controller based on perfect tracking control [10] and on-line robust adaptation [11]. From (11), the transfer functions from $\boldsymbol{x}[i+1]$ to $\boldsymbol{u}[i]$ and $\boldsymbol{y}[i]$ are described as

$$\boldsymbol{u}[i] = \boldsymbol{B}^{-1}(\boldsymbol{x}[i+1] - \boldsymbol{A}\boldsymbol{x}[i]) = \boldsymbol{B}^{-1}(\boldsymbol{I} - z^{-1}\boldsymbol{A})\,\boldsymbol{x}[i+1] \tag{16}$$

$$\boldsymbol{y}[i] = z^{-1}\boldsymbol{C}\,\boldsymbol{x}[i+1] + \boldsymbol{D}\,\boldsymbol{u}[i]. \tag{17}$$

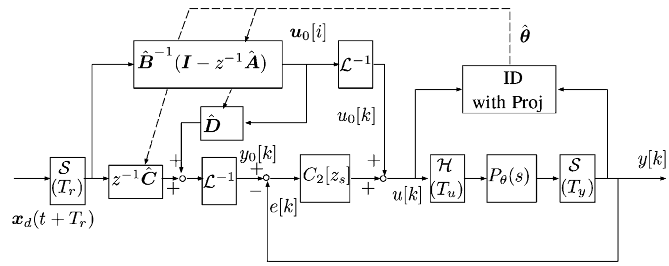

Fig. 4. Multirate adaptive robust controller (MARC). ($T_y = T_u = T_r/n$.)

As shown in Fig. 4, the feedforward input $\boldsymbol{u}_0$ is generated with estimated parameter $\hat{\boldsymbol{\theta}}$ as

$$\boldsymbol{u}_0[i] = \hat{\boldsymbol{B}}^{-1}(\boldsymbol{I} - z^{-1}\hat{\boldsymbol{A}})\,\boldsymbol{x}_d[i+1] \tag{18}$$

where $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ can be calculated by (15) with $\hat{\boldsymbol{\theta}}[i]$.

In Fig. 4, $\boldsymbol{x}_d(t+T_r)$ is the previewed desired trajectory of plant state. It is assumed that the trajectory $\boldsymbol{x}_d(t) = [x_d, x_d^{(1)}, \ldots, x_d^{(n-1)}]^T$ is bounded. $\mathcal{L}$ is the discrete-time lifting operator [15] which is defined in (13) and (14). The function of $\mathcal{H}\mathcal{L}^{-1}$ is also represented in Fig. 3.

The block "ID" is the proposed robust identification scheme based on recursive least-squares (RLS) algorithm with discontinuous projection and conditional updating. The continuous-time version was originally proposed in [11]. The discrete-time algorithm is represented as

$$\hat{\boldsymbol{\theta}}[k] = \hat{\boldsymbol{\theta}}[k-1] + \mathrm{Proj}_{\hat{\theta}}\{\boldsymbol{K}[k](y[k] - \boldsymbol{\varphi}[k]^T\hat{\boldsymbol{\theta}}[k-1])\} \tag{19}$$

$$\boldsymbol{K}[k] = \frac{\boldsymbol{P}[k-1]\boldsymbol{\varphi}[k]}{\lambda + \boldsymbol{\varphi}[k]^T\boldsymbol{P}[k-1]\boldsymbol{\varphi}[k]} \tag{20}$$

$$\boldsymbol{P}[k] = (\boldsymbol{I} - \boldsymbol{K}[k]\boldsymbol{\varphi}[k]^T)\boldsymbol{P}[k-1]/\lambda \tag{21}$$

$$\mathrm{Proj}_{\hat{\theta}_j}(\bullet_j) := \begin{cases} 0, & \text{if } \hat{\theta}_j \geq \theta_{j\,\max}\ \&\ \bullet_j > 0 \\ 0, & \text{if } \hat{\theta}_j \leq \theta_{j\,\min}\ \&\ \bullet_j < 0 \\ \bullet_j, & \text{otherwise} \end{cases} \tag{22}$$

where λ is the forgetting factor. When the persistent excitation of regressor $\boldsymbol{\varphi}[k]$ is not satisfied, $\hat{\boldsymbol{\theta}}[k]$ and $\boldsymbol{P}[k]$ are not updated, i.e., $\hat{\boldsymbol{\theta}}[k] = \hat{\boldsymbol{\theta}}[k-1]$ and $\boldsymbol{P}[k] = \boldsymbol{P}[k-1]$. Several practical methods to check the PE condition are proposed in [1] and [11]. From (22), this scheme can guarantee that $\hat{\boldsymbol{\theta}}[k]$ is bounded [11]. The output of the identified plant model can be calculated by

$$\boldsymbol{y}_0[i] = z^{-1}\hat{\boldsymbol{C}}\boldsymbol{x}_d[i+1] + \hat{\boldsymbol{D}}\boldsymbol{u}_0[i] \tag{23}$$

where $\hat{\boldsymbol{C}}$ and $\hat{\boldsymbol{D}}$ is calculated by (15) with $\hat{\boldsymbol{\theta}}[i]$. When the tracking error $e[k]$ is caused by disturbance or identification

$$\left[\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{B} \\ \hline \boldsymbol{C} & \boldsymbol{D} \end{array}\right] := \left[\begin{array}{c|ccccc} \boldsymbol{A}_s^n & \boldsymbol{A}_s^{n-1}\boldsymbol{b}_s & \boldsymbol{A}_s^{n-2}\boldsymbol{b}_s & \cdots & \boldsymbol{A}_s\boldsymbol{b}_s & \boldsymbol{b}_s \\ \hline \boldsymbol{c}_s & 0 & 0 & \cdots & 0 & 0 \\ \boldsymbol{c}_s\boldsymbol{A}_s & \boldsymbol{c}_s\boldsymbol{b}_s & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & & \vdots & \\ \boldsymbol{c}_s\boldsymbol{A}_s^{n-1} & \boldsymbol{c}_s\boldsymbol{A}_s^{n-2}\boldsymbol{b}_s & \boldsymbol{c}_s\boldsymbol{A}_s^{n-3}\boldsymbol{b}_s & \cdots & \boldsymbol{c}_s\boldsymbol{b}_s & 0 \end{array}\right]. \tag{15}$$

error, it can be attenuated by the robust feedback controller $C_2[z_s]$ by applying

$$u[k] = u_0[k] + u_2[k], \quad u_2[k] = C_2[z_s](y_0[k] - y[k]). \quad (24)$$

Theorem 1: If the PE condition of $\boldsymbol{\varphi}[k]$ and zero initial conditions at $t_0 = i_0 T_r$ are satisfied, then perfect tracking is achieved, i.e., $\boldsymbol{x}[i] = \boldsymbol{x}_d[i]$ for $\forall i \geq i_0$. □

Here, the zero initial conditions are defined as $\boldsymbol{x}[i_0] = \boldsymbol{x}_d[i_0], \boldsymbol{u}_2[i_0] = \mathbf{0}$, where $\boldsymbol{u}_2[i]$ is the lifting of $u_2[k]$. The proof is shown in the Appendix. This theorem assumes that the plant has parametric uncertainty only, which has not disturbance and modeling error. If persistent excitation (PE) condition of the regressor is satisfied and disturbance is sufficient small, the parameter estimation error generally goes to zero ($\hat{\boldsymbol{\theta}} \to \boldsymbol{\theta}_0$) [1], where $\boldsymbol{\theta}_0$ is the true value. Moreover, the zero initial conditions at t_0 will be satisfied by $C_2[z_s]$ if the desired trajectory has a sufficient long period $t_1 \leq t \leq t_0$ when $x_d(t) = 0$. Then, the plant state $\boldsymbol{x}[i]$ completely tracks the desired trajectory $\boldsymbol{x}_d[i]$ at every sampling period T_r after the instance of t_0. Note that perfect tracking is enabled by the proposed new scheme of multirate feedforward control, while it was impossible by conventional single-rate controllers even if $\hat{\boldsymbol{\theta}} \to \boldsymbol{\theta}_0$ and the zero initial conditions are satisfied.

C. Robustness Against Modeling Error and Disturbance

In this section, the robustness of MARC is considered against disturbance d, measurement noise ξ, and modeling error $\Delta(s)$. As shown in Fig. 6, the actual plant is given by

$$y = P(s)(u + d), \quad P(s) := P_\theta(s) + \Delta(s). \quad (25)$$

The measured output y_m includes the noise as $y_m = y + \xi$. In this case, the estimated parameter does not converge to its true value in general. In this analysis, $\hat{\boldsymbol{\theta}}$ is assumed to converge to a constant value $\hat{\boldsymbol{\theta}}_\infty \neq \boldsymbol{\theta}_0$. This assumption could be achieved by the discontinuous projection and conditional updating [11]. The discrete-time transfer functions of identified plant $P_{\hat{\theta}_\infty}(s)$ and real plant $P(s)$ are denoted as $\hat{P}[z_s]$ and $P[z_s]$ with shorter period T_y, respectively.

In order to analyze the robustness of proposed MARC scheme in shorter T_y domain, the transfer function from $u_0[k]$ to $y[k]$ of Fig. 4 is represented as in Fig. 5. Here, $y_0[k] = \hat{P}[z_s]u_0[k]$ is obtained from (18) and (23). The time origin $k = 0$ is defined at a point after $\hat{\boldsymbol{\theta}}$ converged to $\hat{\boldsymbol{\theta}}_\infty$. Fig. 5 can be converted to Fig. 6 equivalently [17], which shows

$$y[k] = \hat{P}[z_s]u_0[k] + \delta[k] \quad (26)$$

$$\delta[k] = -\frac{\hat{P}[z_s] - P[z_s]}{P[z_s]}\hat{S}[z_s]y[k] - T[z_s]\xi[k] + \hat{P}[z_s]S[z_s]d[k] \quad (27)$$

where

$$S[z_s] = (1 + P[z_s]C_2[z_s])^{-1} \quad (28)$$

$$T[z_s] = 1 - S[z_s] \quad (29)$$

$$\hat{S}[z_s] = (1 + \hat{P}[z_s]C_2[z_s])^{-1}. \quad (30)$$

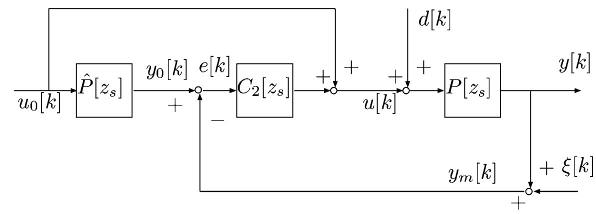

Fig. 5. Single-rate system with T_y from $u_0[k]$ to $y[k]$.

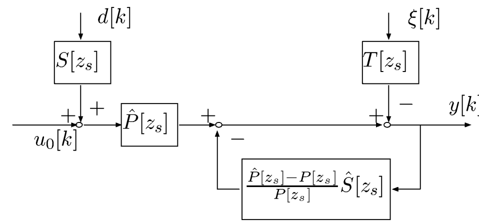

Fig. 6. Equivalent system with Fig. 5.

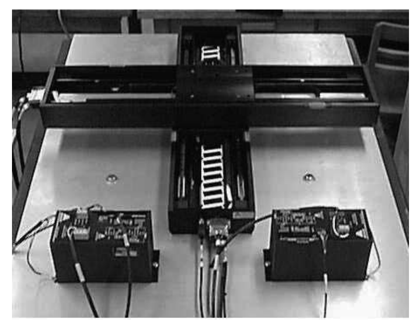

Fig. 7. Experimental setup.

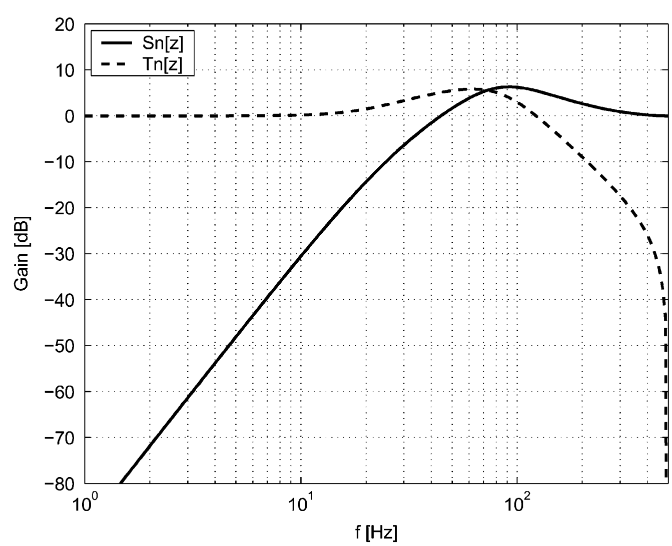

Fig. 8. Nominal sensitivity functions.

Theorem 2: If assumption 2 holds for the actual plant (25), the all signals are bounded. Furthermore, when $\hat{\boldsymbol{\theta}}$ converges to a constant value $\hat{\boldsymbol{\theta}}_\infty \neq \boldsymbol{\theta}_0$, the plant output follows:

$$y[i] = y_d[i] + \delta[i] \quad (31)$$

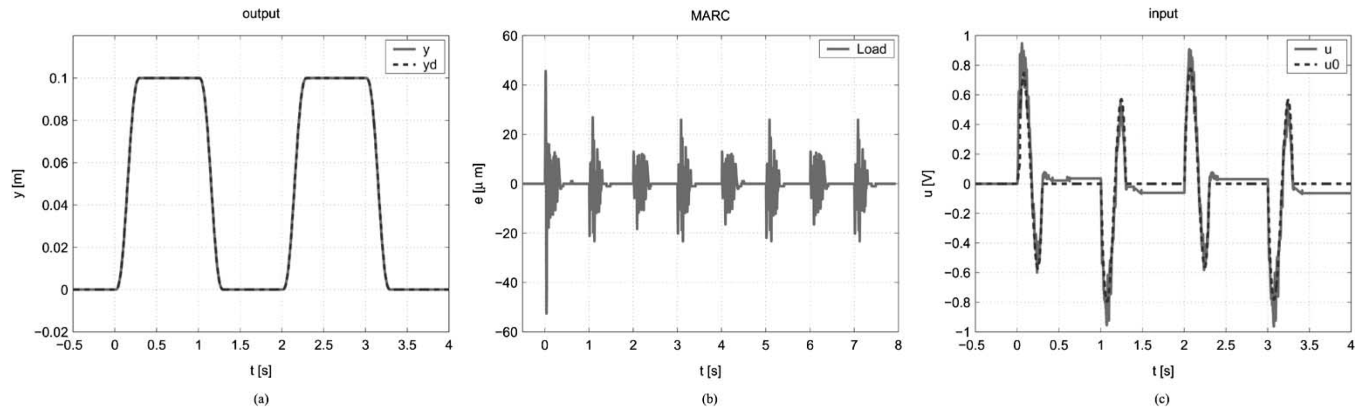

Fig. 9. Experimental results of proposed method ($T_u = 1$ [ms], with load-): (a) Output; (b) error; and (c) input.

where $y_d[i] := \boldsymbol{c}_c \boldsymbol{x}_d[i]$ and $\delta[i]$ is the sampled signal of (27) at every T_r. □

The proof is shown in the Appendix. As the PE condition is not assumed in this section, Theorem 2 can guarantee robust performance in terms of transient tracking error.

From (27), $T[z_s]$ should be small in the noise frequency (normally high band) and $S[z_s]$ should be small in the disturbance frequency (normally low band) to make δ small. This can be done by the disturbance observer or robust control theory. The effect of modeling error and identification mismatch $\hat{P}[z_s] - P[z_s]$ can be eliminated by making $\hat{S}[z_s]$ small in the frequency of desired trajectory. However, if the parameter extent (4) is too big, it becomes difficult to make these functions small enough since the fixed robust controller $C_2[z_s]$ must be designed conservatively.

IV. Application to Precision Position Control of Linear Motor

In this section, the proposed MARC is applied to X-Y stage with liner motor shown in Fig. 7 [18]. In the experiments, only the Y-axis is utilized. The plant is a current controlled linear motor which can be modeled as

$$m\ddot{y} = u - b\dot{y} - d \tag{32}$$

where y is the position, m is the mass, b is the viscous friction coefficient, d is the external disturbance which is assumed to be a constant or slowly varying, and u is the control input voltage which corresponds to the force command. The unknown parameters (m, b, d) are estimated through the regression model defined as

$$\eta = \boldsymbol{\varphi}^T \boldsymbol{\theta} \tag{33}$$

where $\eta = F(s)u, \boldsymbol{\varphi}^T = F(s)[\ddot{y}, \dot{y}, 1], \boldsymbol{\theta} = [m, b, d]^T$, and $F(s) = \omega_f^2/(s+\omega_f)^2$ is the low-pass filter to realize the derivatives in $\boldsymbol{\varphi}$, i.e., $\varphi_1 = (\omega_f^2 s^2/(s+\omega_f)^2)y$. Since $y(t)$ and $u(t)$ are sampled at every T_y, the discrete-time RLS is utilized and the filter is implemented in discrete-time domain. The periods are set to $T_y = T_u = 1$ [ms] and $T_r = nT_u = 2$ [ms] because (32) is a second-order system.

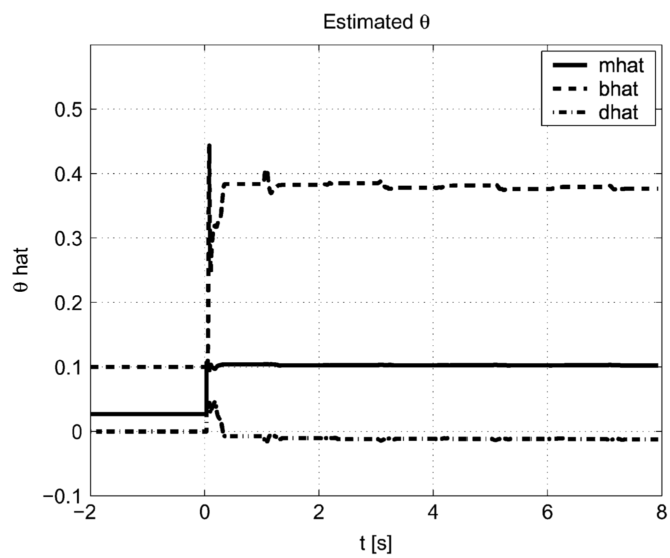

Fig. 10. Estimated parameters. ($T_u = 1$ [ms], with load, RLS forgetting factor $\lambda = 0.999$, and conditional updating.)

The extent of $\boldsymbol{\theta}$ is assumed to be as follows:

$$m \in (0.025, 0.2), \quad b \in (0, 1.0) \tag{34}$$

$$d \in (-10.0, 10.0). \tag{35}$$

The feedback controller is designed based on refined disturbance observer [19] given as

$$C_2(s) = \frac{Q(s)}{P_n(s)(1-Q(s))} \tag{36}$$

where the nominal plant is $P_n(s) = (1/(m_n s^2))$ and $m_n = 0.08$. The poles of nominal closed-loop system are set to $\omega_c = 2\pi 50$. Since the $P_n(s)$ has unstable poles, $Q(s)$ is selected appropriately to avoid the the unstable pole/zero cancellation between $P_n(s)$ and $C_2(s)$ [19]. From (36), the discrete-time controller $C_2[z_s]$ is discretized by Tustin transformation with $T_u = T_y = 1$ [ms]. Fig. 8 shows the sensitivity function $S[z] \simeq 1 - Q[z]$ and complementary sensitivity function $T[z] \simeq Q[z]$ for nominal plant. We numerically checked that this $C_2[z_s]$ stabilized the plant $P_\theta[z_s]$ for θ in (34).

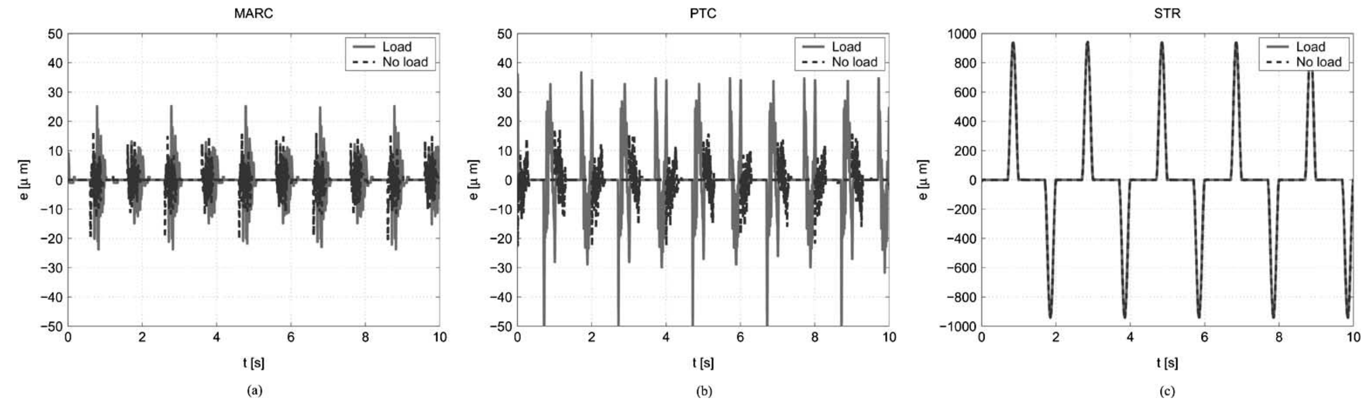

Fig. 11. Tracking error ($T_u = 1$ [ms]): (a) MARC; (b) PTC; and (c) STR.

TABLE I
MAXIMUM ERROR IN STEADY-STATE. ($T_u = 1$ [ms], $f_c = 50$ [Hz])

$\|e\|_\infty[\mu\text{m}]$	MARC	PTC	STR
w/ load	25.2	50.7	941
w/o load	20.2	21.4	943

Based on (18), the proposed MARC is adaptively tuned with $\hat{\boldsymbol{\theta}}[i]$ as

$$\hat{\boldsymbol{A}}_c := \begin{bmatrix} 0 & 1 \\ 0 & -\frac{\hat{b}}{\hat{m}} \end{bmatrix}, \quad \hat{\boldsymbol{b}}_c := \begin{bmatrix} 0 \\ \frac{1}{\hat{m}} \end{bmatrix} \tag{37}$$

$$\boldsymbol{c}_c := [1, 0], \quad \boldsymbol{x} := [y, \dot{y}]^T. \tag{38}$$

The $\hat{d}$ in $\hat{\boldsymbol{\theta}}$ is not used here since the $C_2[z_s]$ includes an integrator to reject step-type disturbance.

Fig. 9 shows the experimental results of proposed MARC. As shown in Fig. 9(a), the plant output follows the desired trajectory very well. Moreover, Fig. 9(c) shows that the control input is very smooth in spite of using multirate control. Thus, the proposed method has resolved the problem of the conventional zero-assignment method [6] which had oscillated multirate input. Fig. 10 shows the estimated parameters $\hat{\boldsymbol{\theta}}[k]$ when 20-lb load is mounted on the motor. The initial value is set to $\hat{\boldsymbol{\theta}}[0] = [0.027, 0.1, 0]^T$ which corresponds to no-load condition. The $\hat{\boldsymbol{\theta}}[k]$ converges to the parameters with load quickly by the RLS algorithm. Thus, the tracking error of Fig. 9(b) decreases immediately after the parameter convergence.

Fig. 11 and Table I show the comparison results between the proposed method (MARC), feedforward self-tuning regulator (STR) without pole/zero cancellation [1], and non adaptive perfect tracking controller (PTC) which is represented by (18) with fixed parameter $\hat{\boldsymbol{\theta}}[0]$. These three methods utilize the same feedback controller designed by (36). The STR has larger tracking error than the proposed MARC because the effect of unstable zero remains in the feedforward characteristics of STR. While PTC without adaptive scheme has good performance $\|e\|_\infty = 21.4$ [μm] in the no-load case, the error becomes big (50.7 μm) when the model in feedforward controller includes the parameter mismatch. This drawback is overcome by the proposed adaptive scheme as $\|e\|_\infty = 25.2$ [μm].

V. CONCLUSION

A novel multirate adaptive robust controller using multirate feedforward control was proposed. The advantage of this method is that the feedforward controller can be designed without considering the unstable zero problem. Thus, it is applicable to discrete-time non-minimum phase systems. Moreover, by combining the proposed feedforward controller with robust feedback controller and adaptive scheme, high robust tracking performance is obtained. Finally, the proposed scheme is applied to motion control of linear motor, and the advantages of this approach were demonstrated through experiments.

APPENDIX

A. Proof of Theorem 1

Proof: Since the $\hat{\boldsymbol{\theta}}$ is bounded by (22) [11] $(\hat{\boldsymbol{A}}_c, \boldsymbol{b}_c)$ is a controllable pair where $\hat{\boldsymbol{A}}_c$ is defined in (6) with $\hat{\boldsymbol{\theta}}$. Thus, $(\hat{\boldsymbol{A}}_s, \hat{\boldsymbol{b}}_s)$ is a controllable pair for almost every period T_y [5], [20]. Because $\hat{\boldsymbol{B}}$ is a controllability matrix of $(\hat{\boldsymbol{A}}_s, \hat{\boldsymbol{b}}_s)$ from (15), $\hat{\boldsymbol{B}}$ is nonsingular in (18).

Here, the state–space representation of (18) is given by

$$\boldsymbol{u}_0[i] = \left[\begin{array}{c|c} \boldsymbol{O} & -\hat{\boldsymbol{A}} \\ \hline \hat{\boldsymbol{B}}^{-1} & \hat{\boldsymbol{B}}^{-1} \end{array}\right] \boldsymbol{x}_d[i+1] \tag{39}$$

which is a stable system because all poles are located at origin. Thus, if $\boldsymbol{x}_d[i]$ is bounded, the feedforward input $\boldsymbol{u}_0$ is bounded, which means the overall system is internal stable.

When $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_0, \hat{\boldsymbol{A}} = \boldsymbol{A}$ and $\hat{\boldsymbol{B}} = \boldsymbol{B}$. From (11), (18), and (24), we find $\boldsymbol{x}[i_0+1] = \boldsymbol{x}_d[i_0+1]$ if $\boldsymbol{x}[i_0] = \boldsymbol{x}_d[i_0]$ and $\boldsymbol{u}_2[i_0] = \boldsymbol{0}$. Subsequently, it can be shown that $\boldsymbol{x}[i] = \boldsymbol{x}_d[i]$ for $\forall i \geq i_0$. □

B. Proof of Theorem 2

Proof: Even in the case of (25), the discontinuous projection can guarantee that $\hat{\boldsymbol{\theta}}$ is bounded [11]. Thus, in the same way as the previous proof, we can prove that $u_0[k]$ and $y_0[k]$ are bounded from the nonsingularity of $\hat{\boldsymbol{B}}$ and stability of (18). Therefore, all signals are bounded from the assumption 2 for $P(s)$.

The first term of (26) means that the fixed plant $\hat{P}[z_s]$ with $\hat{\boldsymbol{\theta}}_\infty$ is controlled by $u_0[k]$ as (18) with $\hat{\boldsymbol{\theta}}_\infty$. Thus, the output of $\hat{P}[z_s]u_0[k]$ perfectly tracks to the desired trajectory at every T_r from the Theorem 1. Therefore, (31) has been proven. □

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