Performance-Oriented Adaptive Robust Control of a Class of Nonlinear Systems Preceded by Unknown Dead Zone With Comparative Experimental Results

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Abstract—This paper presents an integrated direct/indirect adaptive robust control scheme for a class of nonlinear dynamic systems preceded by unknown nonsymmetric, nonequal slope dead-zone nonlinearity. Departing from existing approximate adaptive dead-zone compensations, this paper uses indirect parameter estimation algorithms along with on-line condition monitoring to obtain an accurate estimation of the unknown dead zone when certain relaxed persistent-excitation conditions are satisfied—a theoretical result that cannot be achieved with the existing methods. Such a result is obtained by making full use of the fact that though not being linearly parameterized globally, the unknown dead zone can still be linearly parameterized perfectly within certain known working ranges. With these accurate estimates of dead-zone parameters, perfect dead-zone compensation is then constructed and utilized in the development of a performance-oriented adaptive robust control algorithm for the overall system. Consequently, asymptotic output tracking is achieved even in the presence of unknown dead zone. In addition, the proposed algorithm achieves certain guaranteed robust transient performance and final tracking accuracy even when the entire system may be subjected to other uncertain nonlinearities and time-varying disturbances. The proposed algorithm is also experimentally tested on a linear motor drive system preceded by a simulated unknown nonsymmetric dead zone. Comparative experimental results obtained validate the effectiveness of dead-zone compensation and the high-performance nature of the proposed approach in practical implementation.

Index Terms—Adaptive control, nonlinear systems, parameter estimation, robust control, uncertainty.

I. INTRODUCTION

Nonsmooth nonlinearities such as dead zone, backlash, and hysteresis are ubiquitously present in hydraulic servo valves, dc servo motors, mechanical connections, and piezoelectric stages [1]–[4]. Among them, dead-zone nonlinearity is a relatively simple static input–output relationship which characterizes the insensitivity of the output-to-input values during certain working ranges. Should the parameters of the dead-zone nonlinearity be known, it is straightforward to construct a perfect inversion of the dead-zone nonlinearity to compensate for its effect completely [5]. However, in reality, the dead-zone output is often difficult to measure and the dead-zone parameters are seldom known completely or remain the same values over the entire lifespan of a machine. Thus, how to effectively compensate the effect of dead-zone nonlinearity in the presence of parametric uncertainties without direct measurement of dead-zone output has always been a practically important problem. The problem is also difficult to solve as it is unclear if exact inversion of the dead zone for perfect dead-zone compensation is possible or not in such a scenario. It has been shown that imperfect compensation of the dead zone in feedback control systems could severely deteriorate the achievable control performance, leading to undesirable control accuracy, limit cycles, and even instability [6].

During the past decades, extensive research has been devoted to solving the aforementioned problem in one way or another, with quite a large number of publications appeared [5], [7], [8], [9]. Specifically, an adaptive dead-zone inverse was first proposed in [5] under the unrealistic assumption of set certainty equivalence. Such an assumption was subsequently removed in [7] but only bounded output tracking errors are achieved. The work in [10] continued the previous research and did achieve perfect asymptotical adaptive cancelation of an unknown dead zone. However, a much stronger assumption of both the input and the output of the dead zone being measured was assumed, which cannot be met in most practical applications having dead zones. In [11]–[13], fuzzy-logic and neural network were used to provide alternative interpretations to the basis functions needed for adaptive dead-zone inversions [7]. However, no essential improvement on theoretically achievable results are obtained, i.e., only bounded output tracking errors are achieved. The work in [10] continued the previous research and did achieve perfect asymptotical adaptive cancelation of an unknown dead zone. However, a much stronger assumption of both the input and the output of the dead zone being measured was assumed, which cannot be met in most practical applications having dead zones. In [11]–[13], fuzzy-logic and neural network were used to provide alternative interpretations to the basis functions needed for adaptive dead-zone inversions [7]. However, no essential improvement on theoretically achievable results are obtained, i.e., only bounded output tracking errors are achieved. In [9], [14], and [15], the dead zone was modeled as a combination of a linear input with either an unknown constant gain for symmetric dead zones [14] or time-varying unknown gain for nonsymmetric ones [9], [15] and a bounded disturbance-like term. With this formulation, traditional robust adaptive control techniques can be conveniently applied to achieve bounded output tracking errors. In [8], smooth basis functions as opposed to the discontinuous ones in [7] and [11] were explored to provide some
approximate inversions of the dead-zone nonlinearity. Model-less approaches such as the disturbance observer [16] are also employed to compensate for nonsmooth nonlinearities [17].

It is noted that without assuming the dead-zone output being measured, even under the assumption that certain persistent-excitation (PE) conditions are satisfied, none of the aforementioned existing schemes can achieve perfect adaptive compensation of unknown dead zones for asymptotic output tracking, except resorting to the unrealistic chattering control inputs [18]. The reason for this might come from the fact that the hard dead-zone nonlinearity cannot be linearly parameterized globally. As the existing schemes are based on direct adaptive control designs, they all need uncertain nonlinearities to be linearly parameterized perfectly and globally for asymptotic output tracking, which is not possible for unknown dead zones. Departing from these approximate dead-zone compensations, in this paper, we will make full use of the fact that, though not being linearly parameterized globally, the unknown dead zone can still be linearly parameterized when restricted to some known ranges. As such, when on-line parameter adaptation is turned ON only during those working ranges, accurate estimations of all unknown dead-zone parameters could be obtained. Consequently, perfect adaptive compensation of unknown dead zones for asymptotic output tracking can be achieved, provided that certain relaxed PE conditions are satisfied. This is made possible in this paper through the use of indirect parameter estimation algorithms with on-line condition monitoring.

In [19], an integrated direct/indirect adaptive robust control (DIARC) scheme was presented, in which the construction of parameter adaptation law can be totally independent from the design of underline robust control law. This DIARC strategy allows various parameter adaptation algorithms having better convergence properties and practical modifications such as the explicit on-line monitoring of signal excitation levels to be used to improve the accuracy of parameter estimates in implementation. Such a DIARC strategy has been successfully applied to several applications such as the precision motion control of linear motors [20] and the coordinated contouring application [21].

This paper will make full use of the flexible parameter estimation process of the DIARC strategy and the dead-zone property of being linearly parameterized within certain known ranges to develop performance-oriented control algorithms for a class of uncertain nonlinear systems preceded by unknown dead zones. The DIARC schemes in [19] and [21] only consider systems with smooth unknown nonlinearities that can be linearly parameterized. As the dead-zone nonlinearity is nonsmooth and cannot be linearly parameterized globally, theoretically, the DIARC design in [19] cannot be straightforwardly applied to the type of systems studied in this paper. Appropriate nonsmooth dead-zone inverse has to be developed and novel ideas are needed to overcome the linear parametrization problem. It is shown that the proposed DIARC is able to achieve accurate estimates of all dead-zone parameters when certain relaxed PE conditions are satisfied. Consequently, perfect dead-zone compensation will be achieved and asymptotic output tracking without assuming the dead-zone output being measured is proven theoretically—a theoretical result that cannot be achieved by the existing methods. Furthermore, even when the relaxed PE conditions are not met and the overall system may be subjected to other uncertain nonlinearities and time-varying disturbances, the proposed DIARC algorithm still achieves a guaranteed robust transient performance and final tracking accuracy. Namely, the bounds of the output tracking errors are directly related to the controller parameters in a known form, not only in $L_2$ norm as in the traditional robust adaptive controls [8] but also in practically more meaningful $L_\infty$ norm.

The desired compensation idea in [22] has also been incorporated into the proposed DIARC dead-zone compensation strategy to synthesize high-performance motion controllers for electrical drive systems in [23]. In this paper, comparative experiments will be carried out on a different linear motor-driven stage preceded by a simulated unknown nonsymmetrical and nonequal slope dead-zone nonlinearity. In addition, the proposed algorithm will be compared with a typical example of the existing robust adaptive dead-zone compensations—the control algorithm in [18]—along with control algorithms with no dead-zone compensation, dead-zone compensation based on nominal values, and dead-zone compensation based on actual values. Comparative experimental results show that the proposed DIARC dead-zone compensation outperforms the one in [18] significantly and achieves almost the same steady-state tracking performance as that of the one with exact dead-zone compensation (i.e., assuming that the dead-zone parameters are known). Overall, the much better tracking performance of the proposed algorithm seen in the experiments validates the effectiveness of the proposed DIARC dead-zone compensation strategy in practical applications.

II. PROBLEM STATEMENT

A. System Model

This paper considers the same class of nonlinear dynamic systems preceded by unsymmetric dead zone as in [8], [14], and [15], which are described by

$$x^{(n)} = \sum_{i=1}^{p} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) + E w(t) + f_u$$

$$y = x(t), \quad w(t) = D(v(t))$$

(1)

where $Y_i, i = 1, 2, \ldots, p$, are some known continuous nonlinear functions, and parameters $a_i$ and $E$ represent unknown constants. $v(t)$ is the output from the controller, $w(t)$ is the actual input to the plant, and $y(t)$ is the output from the plant. $f_u$ represents the lumped uncertain nonlinearities including external disturbances. The actuator nonlinearity $D(v(t))$ is described as a dead-zone characteristic shown in Fig. 1(a).

With input $v(t)$ and output $w(t)$, the dead zone can be represented as in [7] as follows:

$$w(t) = D(v(t)) = \begin{cases} 
  m_r v(t) - m_r b_r, & \text{for } v(t) \geq b_r \\
  0, & \text{for } b_l < v(t) < b_r \\
  m_l v(t) - m_l b_l, & \text{for } v(t) \leq b_l 
\end{cases}$$

(2)
where the parameters \( m_r, m_l, b_r, \) and \( b_l \) are constants and stand for the right slope, left slope, right break-point, and left break-point of the dead-zone, respectively.

Let \( E = 1 \) for that its effect can be considered in the unknown slope \( m_r \) and \( m_l \). For notation simplicity, let \( \theta = [a_1, a_2, \ldots, a_p, m_r, m_l, b_r, m_l b_l]^T \). The control objective is to design a control law \( v(t) \) to ensure that all closed-loop signals are bounded and the plant state vector \( X = \{x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\} \) tracks the specified desired trajectory \( X_d = \{x_d(t), \dot{x}_d(t), \ldots, x^{(n-1)}_d(t)\} \) with certain guaranteed transient responses. The following practical assumptions are made.

**Assumption 1:** The dead-zone output \( w(t) \) is not measurable and the dead-zone parameters \( m_r, m_l, b_r, b_l \) are unknown, but their signs are known as \( m_r > 0, m_l > 0, b_r > 0, b_l < 0 \).

**Assumption 2:** The unknown parameter vector \( \theta \) is within a known bounded convex set \( \Omega_\theta \). Without loss of generality, it is assumed that \( \forall \theta \in \Omega_\theta, a_i \in [a_{i_{\min}}, a_{i_{\max}}], i = 1, 2, \ldots, p, \) and \( 0 < m_{i_{\min}} \leq m_i \leq m_{i_{\max}}, 0 < m_{l_{\min}} \leq m_l \leq m_{l_{\max}}, 0 < (m_{r_{i_{\min}}}, b_{i_{\min}}) \leq (m_{r_{i_{\max}}}, b_{i_{\max}}), (m_{l_{i_{\min}}}, b_{l_{i_{\min}}}) \leq (m_{l_{i_{\max}}}, b_{l_{i_{\max}}}) \) are all known constants.

**Assumption 3:** The uncertain nonlinearity \( f_u \) can be bounded by

\[
|f_u| \leq \delta(X)f_d(t)
\]

where \( \delta(X) \) is a known positive function and \( f_d(t) \) is an unknown but bounded positive time-varying function.

### B. Dead-Zone Compensation

The essence of compensating dead-zone effect is to employ a perfect dead-zone inverse function \( v(t) = D_f(w(t)) \) such that \( D(D_f(w(t))) = w(t), \forall w(t) \). However, as seen from Fig. 1(a), the dead-zone function is not a one-one mapping during the zero-output zone of \( v(t) \in [b_l, b_r] \). As such, there does not exist an unique inverse and various choices of a particular dead-zone inverse function in the controller design may have significantly different results in implementation. This problem has largely been ignored in all the previous publications. Typically, only a particular dead-zone inverse is provided without carefully examining its potential implementation problems. For example, in [7], the following underline perfect dead-zone inverse function was used:

\[
v(t) = D_f(w(t)) = \begin{cases} \frac{w + m_r b_r}{m_r}, & \text{if } w(t) \geq 0 \\ \frac{w + m_l b_l}{m_l}, & \text{if } w(t) < 0. \end{cases}
\]

Such an inverse function may suffer from the following implementation problem. In addition to chattering of the control input signal \( v(t) \) between two distinct values of \( b_l \) and \( b_r \) when \( w(t) = 0 \) due to the unavoidable any extremely small calculation errors of \( w(t) \) either by quantization or measurement noise, when the system comes to a stop in which \( w(t) \) continuously converges to zero from some negative values, unnecessary jumping of \( v(t) \) from \( b_l \) to \( b_r \) will occur as well. This tends to excite the neglected high-frequency dynamics and prolong the settling of the system response to the target value. To alleviate this implementation problem, in this paper, the following underline perfect dead-zone inverse will be used:

\[
v(t) = D_f(w) = \begin{cases} \frac{w(t) + m_r b_r}{m_r}, & \text{if } w(t) > 0 \\ \arg\min_{c \in [b_l, b_r]} \left| D_f(w(t_-)) - c \right|, & \text{if } w(t) = 0 \\ \frac{w(t) + m_l b_l}{m_l}, & \text{if } w(t) < 0. \end{cases}
\]

where \( w(t_-) \) represents the left limit of \( w(t) \). Such an inverse is graphically shown in Fig. 1(b) for \( w(t) \neq 0 \). Essentially, when \( w(t) = 0 \), the proposed dead-zone inverse is to keep the previous value of \( v(t_-) = D_f(w(t_-)) \) without jumping. Thus, in reality, for continuous \( w \), when \( w(t) = 0 \), depending on past history, \( v(t) \) will be either \( b_l \) or \( b_r \) only as \( v(t_-) = b_l \) when \( w \) reaches zero from \( w < 0 \) region and \( v(t_-) = b_r \) when \( w \) reaches zero from \( w > 0 \) region. In implementation, to avoid the erroneous switching due to any calculation error of \( w \), a small tolerance band would be used when checking conditions like \( w = 0 \). It is noted that the proposed dead-zone inverse has certain memory effect as opposed to the pure static nature of the dead-zone inverse (4).

As the dead zone is assumed unknown, the aforementioned inverse can only be implemented with the estimates of the dead-zone parameters. Namely, let \( \hat{m}_r, (\hat{m}_r, b_r), \hat{m}_l, (\hat{m}_l b_l) \) be the estimates of \( m_r, m_l b_r, m_l b_l \), respectively, and \( w_d \) be the desired control signal that would achieve the stated control objective when there is no dead-zone effect. Then, viewing (5), a reasonable actual control input \( v(t) \) when the dead-zone (2) exists would be

\[
v(t) = \hat{D}_f(w_d) = \begin{cases} \frac{w_d(t) + (\hat{m}_r b_r)}{\hat{m}_r}, & \text{if } w_d(t) > 0 \\ \arg\min_{c \in [b_l, b_r]} \left| \hat{D}_f(w_d(t_-)) - c \right|, & \text{if } w_d(t) = 0 \\ \frac{w_d(t) + (\hat{m}_l b_l)}{\hat{m}_l}, & \text{if } w_d(t) < 0. \end{cases}
\]
where \( \hat{b}_r = \frac{\hat{m}_r b_r}{\hat{m}_r} \) and \( \hat{b}_l = \frac{\hat{m}_l b_l}{\hat{m}_l} \). With this dead-zone inverse, the resulting controlled system is shown in Fig. 2.

For notational simplicity, in the following, (6) is rewritten as

\[
v(t) = \frac{\kappa_+(w_d) w_d(t) + (\hat{m}_r b_r)}{\hat{m}_r} + \frac{\kappa_-(w_d) w_d(t) + (\hat{m}_l b_l)}{\hat{m}_l}
\]

where \( \kappa_+(w_d) \) and \( \kappa_-(w_d) \) are defined by

\[
\kappa_+(w_d)(t) = \begin{cases} 
1, & \text{if } (w_d(t) > 0 \text{ or } w_d(t) = 0 \text{ and } |v(t) - \hat{b}_r| \geq |v(t) - \hat{b}_l|) \\
0, & \text{else}
\end{cases}
\]

(8)

\[
\kappa_-(w_d)(t) = \begin{cases} 
1, & \text{if } (w_d(t) < 0 \text{ or } w_d(t) = 0 \text{ and } |v(t) - \hat{b}_r| < |v(t) - \hat{b}_l|) \\
0, & \text{else}
\end{cases}
\]

(9)

C. Projection-Type Adaptation Law Structure

One of the key elements of the ARC design is to use the practical available prior process information to construct the projection type adaptation law for a controlled adaptation process. For this purpose, the widely used projection mapping \( \text{Proj}_\theta \) will be used to keep the parameter estimates within the known bounded set \( \tilde{\Omega}_\theta \), the closure of the set \( \Omega_\theta \) as in [24]. The standard projection mapping [25] is as follows:

\[
\text{Proj}_\theta(\zeta) = \left\{ \begin{array}{ll}
\zeta, & \text{if } \hat{\theta} \in \tilde{\Omega}_\theta \text{ or } (\hat{\theta} \in \partial \Omega_\theta \text{ and } n^{\zeta} \hat{\theta} < 0) \\
\left(I - \Gamma \frac{n^{\hat{\theta}}}{n^{\hat{\theta}} \Gamma \frac{n^{\hat{\theta}}}{n^{\hat{\theta}}} \hat{\theta}} \right) \zeta, & \text{if } \hat{\theta} \in \partial \Omega_\theta \text{ and } n^{\zeta} \hat{\theta} > 0
\end{array} \right.
\]

(10)

where \( \zeta \in R^{p+4} \) is any function and \( \Gamma(t) \in R^{(p+4) \times (p+4)} \) can be any time-varying positive-definite symmetric matrix. In (10), \( \Omega_\theta \) and \( \partial \Omega_\theta \) denote the interior and the boundary of \( \Omega_\theta \), respectively, and \( n^{\hat{\theta}} \) represents the outward unit normal vector at \( \hat{\theta} \in \partial \Omega_\theta \). The parameter estimate vector \( \hat{\theta} \) is updated using the following projection-type adaptation law:

\[
\hat{\theta} = \text{Proj}_\theta(\Gamma \tau), \quad \hat{\theta}(0) \in \tilde{\Omega}_\theta
\]

(11)

where \( \tau \) is any estimation function and \( \Gamma(t) > 0 \) is any continuously differentiable positive symmetric adaptation rate matrix which would be detailedly presented in Section III-C. With this adaptation law structure, the following desirable properties hold.

\[ \text{(P1)}: \text{The parameter estimates are always within the known bounded set } \tilde{\Omega}_\theta, \text{i.e., } \hat{\theta}(t) \in \tilde{\Omega}_\theta, \forall t. \]

Thus, from Assumption 2, \( \forall t, \ a_{i, \min} \leq \hat{a}_i(t) \leq a_{i, \max}, \ i = 1, 2, \ldots, p, \)

\[ 0 < m_{r, \min} \leq \hat{m}_r(t) \leq m_{r, \max}, \quad 0 < m_{l, \min} \leq \hat{m}_l(t) \leq m_{l, \max}, \quad 0 < (m_r b_r)_{\min} \leq (\hat{m}_r b_r)(t) \leq (m_r b_r)_{\max}, \quad \text{and} \]

\[ (m_l b_l)_{\min} \leq (\hat{m}_l b_l)(t) \leq (m_l b_l)_{\max} < 0. \]

\[ \text{(P2):} \quad \hat{\theta}^T (\Gamma^{-1} \text{Proj}_\theta(\Gamma \tau)) - \tau \leq 0, \forall \tau. \]

D. Dead-Zone Compensation Error Characterization

Noting property P1 for the dead-zone parameter estimates, the following lemma can be obtained as in [23]:

Lemma 1: With the dead-zone inverse (7), the error between the actual dead-zone output \( w \) by (2) and the desired output \( w_d \) can be parameterized as

\[
w - w_d = (m_r b_r) \kappa_+(w_d) + (m_l b_l) \kappa_-(w_d)
\]

\[
- \frac{w_d + (m_r b_r)}{m_r} \kappa_+(w_d)
\]

\[
- \frac{w_d + (m_l b_l)}{m_l} \kappa_-(w_d) + d(t)
\]

(13)

where \( d(t) \) is a function defined as

\[
\begin{cases}
0, & \text{if } \kappa_+(w_d) = 1 \text{ and } v(t) \geq b_r \\
- \frac{w_d + (m_r b_r)}{m_r} m_r, & \text{if } \kappa_+(w_d) = 1 \text{ and } 0 < v < b_r \\
- \frac{w_d + (m_l b_l)}{m_l} m_l, & \text{if } \kappa_-(w_d) = 1 \text{ and } b_l < v < 0 \\
0, & \text{if } \kappa_-(w_d) = 1 \text{ and } v(t) \leq b_l
\end{cases}
\]

(14)

and bounded above by

\[
\begin{cases}
(m_r b_r)_{\max} - \frac{m_{r, \min}(m_r b_r)_{\min}}{m_{r, \max}}, & \text{if } \kappa_+(w_d) = 1 \\
(m_l b_l)_{\max} - \frac{m_{l, \min}(m_l b_l)_{\max}}{m_{l, \max}}, & \text{if } \kappa_-(w_d) = 1.
\end{cases}
\]

(15)

In addition, \( d(t) \in L^2[0, \infty) \) when \( \hat{\theta} \in L^2[0, \infty) \) and \( d(t) \rightarrow 0 \) when \( \hat{\theta}(t) \rightarrow 0 \) as \( t \rightarrow \infty \).

III. INTEGRATED DIARC SYNTHESIS

In this section, using the dead-zone inverse (7) and certain conditional parameter adaptation, the integrated DIARC strategy in [19] will be generalized to solve the control problem posted in section II for the system (1).

A. Integrated DIARC Law

With the use of the projection type adaptation law structure (11), the parameter estimates are bounded with known bounds, regardless of the estimation function \( \tau \) to be used. In the following, this property will be used to synthesize an integrated DIARC control law with the dead-zone inverse (7) for the system (1) which achieves a guaranteed transient and steady-state
output tracking accuracy in general. Define
\[
s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x}(t)
\]
with \(\lambda > 0\) which can be rewritten as
\[
s(t) = \Lambda^T \tilde{X}(t)
\]
where \(\Lambda^T = \left[C_{n-1}^{-1} \lambda^{n-1}, C_{n-2}^{-1} \lambda^{n-2}, \ldots, C_{n-n}^{-1} \lambda^{-n}\right], \tilde{X} = X - X_d, \quad X = [x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)], \quad \text{and} \quad X_d = [x_d(t), \dot{x}_d(t), \ldots, x_d^{(n-1)}(t)]
\]
Noting (1) and (13) in Lemma 1, the derivative of \(s(t)\) can be derived as
\[
\dot{s}(t) = \Lambda^T \tilde{X}(t) + \tilde{X}^T(t)
\]
\[
= \Lambda^T \tilde{X}(t) - x_d^{(n)}(t) + \sum_{i=1}^{p} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t))
\]
\[
+ f_u + w(t)
\]
\[
= \Lambda^T \tilde{X}(t) - x_d^{(n)}(t) + \sum_{i=1}^{p} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t))
\]
\[
+ f_u + d(t) + w_d(t) + (\tilde{m}_r \tilde{b}_r) \kappa_+ (w_d) + (\tilde{m}_l \tilde{b}_l) \kappa_- (w_d)
\]
\[
- \tilde{w}_d + \left(\tilde{m}_r \tilde{b}_r\right) \tilde{m}_r \kappa_+ (w_d) - \tilde{w}_d + \left(\tilde{m}_l \tilde{b}_l\right) \tilde{m}_l \kappa_- (w_d)
\]
\[
= \Lambda^T \tilde{X}(t) - x_d^{(n)}(t) + \sum_{i=1}^{p} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t))
\]
\[
+ f_u + d(t) + \psi(t)
\]
\[
= f_u + d(t) + \dot{\psi}(t)
\]
\[
= f_u + d(t) + \dot{\psi}(t)
\]
From (18) and (19), the derivative of \(s(t)\) can be derived as
\[
\dot{s}(t) = f_u + d(t) - \sum_{i=1}^{p} \tilde{a}_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t))
\]
\[
+ \left[\tilde{m}_r (w_{da} + w_{da2}) - \tilde{m}_r \kappa_+ (w_d) \right] w_{da1}
\]
\[
+ \left[\tilde{m}_l (w_{da} + w_{da2}) - \tilde{m}_l \kappa_- (w_d) \right] w_{da2}
\]
\[
\psi(t) = -[Y_1, Y_2, \ldots, Y_p, w_{da1}, -1]^T.
\]
Conceptually, (21) lumps the original system uncertain non-linearity \(f_u\) with the model uncertainties due to physical parameter estimation errors and divides it into the static component \(d_c\) (or low-frequency component in reality) and the high-frequency components \(\Delta^* (t)\). In the following, the low-frequency component \(d_c\) will be compensated through fast adaptation similar to those in the direct ARC designs [24] as follows.

Let \(d_c \in \mathbb{R}^p\) be any preset bound and use this bound to construct the following projection-type adaptation law for \(\tilde{d}_c^\ast\):
\[
\tilde{d}_c^\ast = \text{Proj}_{\mathbb{R}^p} (\gamma s) = \begin{cases} 0, & \text{if } |\tilde{d}_c^\ast (t)| = d_c^\ast M \text{ and } \tilde{d}_c s > 0 \\ \gamma s, & \text{else} \end{cases}
\]
with \(\gamma > 0\) and \(\tilde{d}_c^\ast (0) = 0\). Such an adaptation law guarantees \(|\tilde{d}_c^\ast (t)| \leq d_c^\ast M, \forall t\). For notation simplicity, define
\[
k^* = \kappa_+ (w_d) \frac{m_r}{m_l} + \kappa_- (w_d) \frac{m_l}{m_l}
\]
which has a value of 1 in the absence of dead-zone slopes estimation error. Substituting (21) into (20) and noting (19),

In (19), \(w_{da1}\) represents the usual model compensation with the parameter adaptation algorithm with condition monitoring to be detailed in Section III-B. \(w_{da2}\) is a model compensation term similar to the fast dynamic compensation-type model compensation used in the DARC designs [19], in which \(\tilde{d}_c\) can be thought as the estimate of the low-frequency component of the lumped model uncertainties defined later. \(w_{d1}\) represents the robust control term in which \(w_{d1}\) is a simple proportional feedback to stabilize the nominal system with a constant gain \(k_{s1}\), and \(w_{d2}\) is a robust feedback term used to attenuate the effect of various model uncertainties for a guaranteed robust control performance in general.
\( s(t) \) can be written as
\[
s(t) = k^* (w_{ds} + w_{d2s}) + \bar{d}_c + \Delta^* (t)
\]
\[
= k^* w_{ds1} + k^* w_{ds2} - \bar{d}_c + (1 - k^*) \Delta_c + \Delta^* (t).
\]  
(24)

Noting Assumption 2, Assumption 3, and property P1, there exists a \( w_{ds2} \) such that the following two conditions are satisfied:

1) \( s w_{ds2} \leq 0 \)

2) \( s [k^* w_{ds2} - \bar{d}_c + (1 - k^*) \Delta_c + \Delta^* (t)] \leq \varepsilon + \varepsilon_d f_d^2 \)  
(25)

where \( \varepsilon \) and \( \varepsilon_d \) are parameters which can be arbitrarily small. Essentially, condition 2 of (25) shows that \( w_{ds2} \) is synthesized to dominate the model uncertainties coming from both parametric uncertainties and uncertain nonlinearities, and condition 1 of (25) is to make sure that \( w_{ds2} \) is dissipative in nature, so that it does not interfere with the functionality of the adaptive control part \( w_{ds} \).

**Remark 1:** One example of \( w_{ds2} \) satisfying (25) can be found in the following way. Let \( h \) be any function satisfying
\[
h \geq |k_{max}^*| ||w_{ds2}|| + ||\varrho_{max}|| ||\psi|| + |d(t)|_{max}
\]  
(26)

where \( k_{max}^* = \max \{ (m_{max} - m_{min}) / (m_{max} - m_{min}), (\varrho_{max} - \varrho_{min}) / (\varrho_{max} - \varrho_{min}) \}, \varrho_{max} = a_{max} - a_{min}, \varrho_{max} = a_{max} - a_{min}, \varrho_{max} = a_{max} - a_{min} \), \( (\varrho_1)_{max} = \max \{ (m_{max} - m_{min}) / (m_{max} - m_{min}), \max \{ (m_{max} - m_{min}) / (m_{max} - m_{min}) \} \}, \varrho_{max} = a_{max} - a_{min}, \varrho_{max} = a_{max} - a_{min} \)

\[
(\varrho_2)_{max} = \max \{ (r_{max} - r_{min}) / (r_{max} - r_{min}), (m_{max} - m_{min}) / (m_{max} - m_{min}) \}, |d(t)|_{max} = \max \{ ((m_{max} - m_{min}) / (m_{max} - m_{min})) \}, (m_{max} - m_{min}) / (m_{max} - m_{min}) \}. \]

Then, \( w_{ds2} \) can be chosen as
\[
w_{ds2} = - \left[ \frac{1}{4 \varepsilon k_{min}^*} h^2 + \frac{1}{4 \varepsilon_d k_{min}^*} \right] s  
\]  
(27)

where \( k_{min}^* = \min \{ (m_{min} / m_{max}), (\varrho_{min} / \varrho_{max}) \} \). Using the same techniques as in [19], it is easy to show that the aforementioned choice of \( w_{ds2} \) does satisfy (25).

The following theorem summarizes the theoretically achievable guaranteed performance.

**Theorem 1:** When the DIARC control law (19) with the dead-zone inverse (7) and the projection type adaptation law (11) is applied, regardless the estimation function \( \tau \) to be used, all the physical signals have been guaranteed to be bounded and guaranteed output-tracking transient and steady-state performance have been achieved. Thus, this section focuses on determining the specific parameter adaptation algorithms so that an improved steady-state tracking accuracy—asymptotic output tracking—can be obtained in the absence of uncertain nonlinearities [i.e., assuming \( f_d = 0 \) in (1)] with an emphasis on having a good parameter estimation process as well. For this purpose, rather than using any transformed tracking error dynamics to construct parameter estimation model as in the direct adaptive control designs, the actual plant model (1) will be directly used to construct specific estimation functions as detailed below.

As seen from (14), the model error \( d(t) \) in (13) is nonzero during some working ranges, revealing the fact that the dead-zone nonlinearity (2) cannot be globally linearly parameterized by its parameters \( m, m, b, \) and \( b \) with known basis functions. As such, if the system dynamics (1) is directly used to construct parameter estimation algorithm, one will suffer from the same problem as in all previous research that, theoretically, due to the existence of nonzero model error \( d(t) \), no asymptotic parameter estimation convergence can be achieved even when relevant PE conditions are satisfied. In the following, we will make full use of the fact that, though not being linearly parameterized globally, the unknown dead-zone nonlinearity can be perfectly linearly parameterized during the working regions of \( v \geq b, \) or \( v \leq b \) as \( d(t) = 0 \) in those cases. As such, if the parameter estimation is updated only when the control input \( v(t) \) inside the regions of \( v \geq b, \) or \( v \leq b \) (i.e., the parameter estimation is stopped when \( b \leq v \leq b \)), accurate estimations of all the unknown deadzone parameters can be achieved provided that some relaxed PE conditions are satisfied. For this purpose, explicit on-line condition monitoring will be used by noting the following result.

**Lemma 2:** [23]: Define a positive constant \( H = \max \{ (m_{max} - m_{min}), (m_{max} - m_{min}) \}, (m_{max} - m_{min}) \}. Then, when \( |w_{ds}(t)| \geq H \), the dead-zone input \( v(t) \) by the proposed dead-zone inverse (7) would satisfy \( v(t) \geq b, \) or \( v(t) \leq b \).

For notation simplicity, define
\[
\chi_+(\bullet) = \begin{cases} 1, & \text{if } \bullet \geq 0, \\ 0, & \text{else} \end{cases}, \quad \chi_-(\bullet) = \begin{cases} 1, & \text{if } \bullet \\ 0, & \text{else} \end{cases}  
\]  
(29)

and \( \chi_+(w_d(t)) = \chi_+(w_d(t) - H) + \chi_-(w_d(t) + H) \) which is 1 when \( |w_{ds}(t)| \geq H \) and is 0 when \( |w_{ds}(t)| < H \) by noting Lemma 2. Thus, multiplying both sides of the plant dynamics (1) by \( \chi(w_{ds}(t)) \) leads to
\[
\chi(w_{ds}(t)) x^{(n)} = \chi(w_{ds}(t)) \sum_{i=1}^{p} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) + (m_r v(t) - m_r b) \chi_+ (w_d(t) - H)  \\
+ (m_r v(t) - m_r b) \chi_- (w_d(t) + H)  
\]  
(30)
which is linearly parameterized by the system parameters. Accurate parameter estimation can, thus, be obtained by conducting on-line parameter estimation only using the measurement data when (30) is true. Namely, first group the measurement data into three different cases given next using the checkable condition in Lemma 2, and then perform on-line parameter estimation for relevant dead-zone parameters in each case separately as follows.

**Case I:** The data in this group consist of all the measurement data during the time intervals when \( w_d(t) \geq H \) only. For this dataset, \( \chi_+ (w_d(t) - H) = 1 \) and \( \chi_- (w_d(t) + H) = 0 \). Thus, the plant dynamics (30) are in the following linear regression model for parameter estimation

\[
y_r(t) = F_r^T(t) \theta_r
\]

where

\[
\theta_r = [a_1, a_2, \ldots, a_p, m_r, m_b, b_r]^T
\]

is the system parameter vector to be identified, and \( y_r(t) = x^{(n)}(t) \) and \( F_r^T(t) = [y_1(t), y_2(t), \ldots, y_p(t), v(t), -1] \) represent the corresponding model output and regressor vector, respectively. To avoid the potential noise problem associated with the calculation of \( x^{(n)} \) based on the state measurement data, appropriate filtering should be used. For example, let \( H_f(s) \) be the transfer function of any stable filter with a relative degree larger than or equal to 1 (e.g., \( H_f(s) = 1/(\omega fs + 1) \)). Applying the filter to both sides of (31), one obtains

\[
y_{rf}(t) = F_{rf}^T(t) \theta_r
\]

where \( y_{rf} = H_f y_r \) and \( F_{rf} = H_f F_r \). Noting that (33) is in the standard linear regression form for parameter estimation, various estimation algorithms exist. Among them, the least-squares estimation algorithm is known to have the best parameter estimation convergence property in general [26]. Thus, it is used to estimate the values of \( \theta_r \). With the covariance limiting modification, the resulting adaptation function \( \tau \) for the estimation of the parameter vector \( \theta_r \) using the projection type adaptation law structure (11) is

\[
\tau = -\frac{1}{1 + \nu tr\{F_{rf}^T \Gamma F_{rf}\}} F_{rf} \zeta
\]

where \( \zeta \) and \( \Gamma \) are the prediction error and covariance matrix calculated by

\[
\zeta = F_{rf} \hat{\theta}_r - y_{rf}
\]

\[
\Gamma = \begin{cases} \alpha \Gamma - \frac{\Gamma F_{rf} F_{rf}^T \Gamma}{1 + \nu tr\{F_{rf}^T \Gamma F_{rf}\}}, & \text{if } \lambda_{max} (\Gamma(t)) \leq \rho_M \\ 0, & \text{otherwise} \end{cases}
\]

in which \( \alpha \geq 0 \) is the forgetting factor, \( \rho_M \) is the preset upper bound for the covariance matrix, and \( \nu \geq 0 \) with \( \nu = 0 \) leading to the unnormalized algorithm. It is known that such an estimator leads to the convergence of \( \hat{\theta}_r \) to its true value \( \theta_r \) so long as the following PE condition is satisfied [19], [26]:

\[
\Delta_{\lambda - T} F_{rf} F_{rf}^T d\tau \geq \beta I_p, \text{ for some } \beta > 0 \text{ and } T > 0.
\]

**Remark 2:** The parameter adaptation law in the existing robust adaptive dead-zone compensations is an integral part of the overall control design for closed-loop system stability. As such, the parameter adaptation needs to be turned on all the time to ensure the nominal stability of the closed-loop system. Subsequently, similar PE condition to (36) needs to hold true for all time for the convergence of parameter estimates, which is not realist at all in practical implementation. In contrast, the baseline control law presented in this paper is a robust control law not only guaranteeing robust stability but also robust performance regardless the convergence of the parameter adaptation as stated in Theorem 1. As such, parameter adaptation does not have to be turned on all the time and conditional parameter adaptation with explicit on-line monitoring of the signal excitation level can be used to ensure quality of the on-line parameter estimates. For example, in the previous development, the parameter estimation for \( \theta_r \) is conducted only using the data during the time intervals when \( w_d(t) \geq H \). Subsequently, the PE condition (36) only needs to hold true during those time intervals for the convergence of parameter estimates. In addition, one can even explicitly check if the PE condition (36) is satisfied or not and update the parameter estimates only when it is actually satisfied.

**Case II:** The data in this group consist of all the measurement data during the time intervals when \( w_d(t) \leq -H \) only. For this dataset, \( \chi_+ (w_d(t) - H) = 0 \) and \( \chi_- (w_d(t) + H) = 1 \), and the plant dynamics (30) are in the following linear regression model for parameter estimation:

\[
y_l(t) = F_l^T(t) \theta_l
\]

where \( y_l = x^{(n)}(t), F_l^T(t) = [y_1, y_2, \ldots, y_p, v(t), -1], \) and

\[
\theta_l = [a_1, a_2, \ldots, a_p, m_l, m_l b_l]^T.
\]

Thus, the same procedure as in Case I can be used to estimate the physical parameter vector \( \theta_l \), and the estimates \( \hat{\theta}_l \) converge to their true values when certain PE condition like (36) is satisfied. The details are omitted as they are identical to those in Case I.

**Case III:** This case consist of all the measurement data during the time intervals when \( |w_d(t)| \leq H \). Since the unknown dead-zone nonlinearity cannot be linearly parameterized in this case, no on-line parameter estimation would be used in this case.

Overall, parameter estimations in Cases I and II enable all unknown physical parameters, i.e., \( \theta = [a_1, a_2, \ldots, a_p, m_r, m_b, b_r]^T \), to be accurately estimated, provided that PE conditions like (36) are satisfied.

### C. Asymptotic Output Tracking

With the accurate parameter estimates obtained in Section III-B, in addition to the results stated in Theorem 1, an improved steady-state tracking performance is also obtained as summarized by the following theorem.

**Theorem 2:** In the presence of parametric uncertainties and unknown dead-zone nonlinearity only [i.e., assuming \( f_u = 0 \) in (1)], when accurate parameter estimation is obtained using the conditional parameter adaptation with explicit on-line monitoring of signal excitation levels, the proposed control law (19) with
the dead-zone inverse (7) achieves asymptotic output tracking as well, i.e., \( \bar{X}(t) \to 0 \) and \( s \to 0 \) as \( t \to \infty \).

**Proof:** See Appendix B.

### IV. COMPARATIVE EXPERIMENTS

The proposed DIARC control algorithm is implemented on a precision HIWIN stage driven by linear motors shown in Fig. 3 with additional 4.82-kg payload. Neglecting fast electrical dynamics and flexible modes of the mechanical systems, the dynamics of the linear motor preceded by simulated unknown dead zones can be described as follows:

\[
\begin{align*}
\ddot{x} &= Ew(t) - B \dot{x} - AS_f(\dot{x}) + f_u \\
y &= x(t), \quad w(t) = D(v(t))
\end{align*}
\]  
(39)

where \( x \) represents the position of the inertia load, \( v(t) \) is the input to the unknown dead zone, \( w(t) \) represents the control input voltage to the motor, \( y(t) \) is the position of the motor, \( E \) is the motor input gain, \( B \) is the equivalent viscous friction coefficient, \( AS_f(\dot{x}) \) represents the nonlinear Coulomb friction, in which the amplitude \( A \) may be unknown but the continuous shape function \( S_f(\dot{x}) \) is known, and \( f_u \) represents the lumped uncertain nonlinearities including various model approximation errors and external disturbances. Noting that the aforementioned linear motor dynamics is in the form of (1) with \( p = 2 \), \( a_1 = B \), \( a_2 = A \), and \( Y_1 = -\dot{x}, Y_2 = -S_f(\dot{x}) \).

The position sensor of the stage is a linear encoder with a resolution of 0.5 \( \mu \)m after quadrature. The velocity signal is obtained by the difference of two consecutive position measurements. Standard off-line least-squares identification is performed to obtain the parameters of the linear motor, and the identified values of the parameters are \( B = 2.2/s, A = 0.6/s, E = 2.4 m/s^2/V \), \( f_N = 0 m/s^2 \), where \( f_N \) is the nominal value of \( f_u \). All the control algorithms are implemented using a dSPACE DS1103 controller board. The controller executes programs at a sampling period of \( T_s = 0.2 \) ms, resulting in a velocity measurement resolution of 0.0025 m/s.

The control objective is to let the system state \([x, \dot{x}]^T\) follow their desired trajectory \([x_d, \dot{x}_d]^T\). Assuming

\[
x_d = 0.21 + 0.08\sin(\pi t - \pi/2) \\
+ 0.07\sin(0.8\pi t - \pi/2) + 0.06\sin(1.2\pi t - \pi/2).
\]  
(40)

Initial values of plant states are set as \( X(0) = [0, 0]^T \).

#### A. Comparative Experiments I

The experiments are first conducted assuming that \( w(t) \) is the output of a simulated dead zone described by

\[
w = D(v) = \begin{cases} 
0.9(v - 1.2), & \text{for } v \geq 1.2 \\
0, & \text{for } -1 < v < 1.2 \\
1(v - (-1)), & \text{for } v \leq -1.
\end{cases}
\]  
(41)

Taking into account the motor gain of \( E = 2.4 m/s^2/V \), the parameter vector \( \theta \) in Section II is then given by \( \theta = [B, A, 0.9E, 0.9 \times 1.2E, E, -E]^T \), which has a value of \([2.2, 0.6, 2.16, 2.592, 2.4, -2.4]^T \) with the aforementioned off-line estimates of motor parameters. In the experiments, the actual dead-zone parameters are assumed to be unknown and an initial value of \( \theta_N = [1.5, 0.3, 1.5, 1.5, 1.5, 1.5]^T \) is used for \( \hat{\theta}(0) \) to account for the unknown dead-zone parameters. Furthermore, the bounds of the parametric variations are assumed to be

\[
\begin{align*}
\theta_{\text{min}} &= [1.5, 0.3, 1.5, 1.5, 1.5, 1.5]^T \\
\theta_{\text{max}} &= [3, 1.3, 3.5, 3.5, 3.5, 1.5]^T.
\end{align*}
\]  
(42)

To better illustrate the effectiveness of the proposed scheme, especially the dead-zone compensation, the following four control algorithms for the system (39) are implemented and compared.

- **C1:** The proposed DIARC adapting the estimates of physical parameters \( B, A, E, f_u \) and ignoring the existence of the unknown dead zone (41). This case represents the scenario where the existence of dead zone is totally ignored.
- **C2:** The proposed DIARC adapting the estimates of physical parameters \( B, A, E, f_u \) only and without updating the dead-zone parameter estimates. This case represents the scenario where the dead zone is compensated with their nominal values and effects of the unknown dead zone are ignored.
- **C3:** The proposed DIARC presented in Sections II and III.
- **C4:** The proposed DIARC in which only \( B, A, E, f_N \) are updated based on the proposed on-line parameter adaptation algorithm and the dead-zone (41) effect is compensated by (7) with actual dead-zone parameter values assuming the dead-zone parameters are known. Thus, this case represents the ideal scenario where the dead-zone effect can be completely compensated.

In C3, \( w_{ds2} \) in (19) is given in Section III-B. Theoretically, we should use the form of \( w_{ds2} = -k_{s2} s \) with \( k_{s2} \) being a nonlinear proportional feedback gain as given in (27) to satisfy the robust performance requirement (25) globally. In implementation, a large enough constant feedback gain \( k_{s2} \) is used instead to simplify the resulting control law. With such a simplification, though the robust performance condition (25) may not be guaranteed globally, the condition can still be satisfied in a large enough working range which might be acceptable to practical applications as done in [27]. With this simplification, noting (19), we choose \( w_{ds} = -k_s s \) in the experiments where \( k_s \) represents the
The experimental results in terms of all performance indexes after running the gantry for one period are given in Experiment I of Table I. Overall, C2, C3, and C4 achieve good steady-state tracking performance during movements that are better than C1. C3 and C4 achieve almost the same excellent tracking performances that are better than C2. The experimental results for all four controllers are shown in Figs. 4–8. Specifically, Fig. 4 shows the output tracking performance of all four cases, and Fig. 5 shows the corresponding tracking errors. From these plots, it is observed that all four cases have almost the same good initial output-tracking transient performance, illustrating the strong robust transient performance of the proposed DIARC strategy in general. It is also seen that steady-state output tracking errors of C3 and C4 are much less than that of C2. This result agrees with the prediction by theory in Section III that the proposed DIARC in C3 is able to achieve asymptotic convergence of dead-zone parameter estimates to their true values and the perfect dead-zone compensation of the proposed dead-zone inverse (7) when the estimates converge. This is also revealed by history of the system parameter estimates in Fig. 6. Furthermore, C2, C3, and C4 have much better output tracking performance than C1, which illustrates the necessity of dead-zone compensation. The estimate values of $\hat{B} = [2.25]$, $\hat{A} = 0.6$, $\hat{E} = 2.65$ shown in Fig. 7 are close to their off-line identified values, and the estimate values of $\hat{\theta} = [2.15, 0.7, 2.1, 2.2, -1.9]^T$ shown in Fig. 6 near the approximate accurate value of $\theta$, which verify the asymptotic convergence of system parameter estimates as well. The control inputs of all four cases shown in Fig. 8 reveal that the control input may exhibit some chattering phenomena during the periods when the motion is very small and the system is almost operating within the dead-zone region due to the use of the nonsmooth direct dead-zone inverse (7). In summary, the good output tracking performance of all four cases demonstrates the robust performance of the proposed DIARC algorithm. The much improved output tracking performance of C2, C3, and C4 over C1 illustrates the necessity of dead-zone compensation.

**TABLE I**

<table>
<thead>
<tr>
<th>Controller</th>
<th>Experiments I</th>
<th>Experiments II</th>
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<tbody>
<tr>
<td></td>
<td>C1</td>
<td>C2</td>
</tr>
<tr>
<td>(</td>
<td>z</td>
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<tr>
<td>(</td>
<td>z</td>
<td>_{\text{out}} \text{ (mm)})</td>
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</table>

Fig. 4. Output trajectory tracking of C1, C2, C3, and C4.

Fig. 5. Output tracking errors of C1, C2, C3, and C4.

Fig. 6. Parameter estimates of C3.
The much improved steady-state tracking performance of case C3 over C2 validates the need for accurate on-line dead-zone parameter estimates. And the almost same asymptotically converging steady-state output tracking performance of cases C3 and C4 verifies the asymptotic convergence of the dead-zone parameter estimates to their true values, and further validates the perfect compensation of unknown dead zones by the proposed DIARC algorithm.

It is noted that the proposed strategy has also been combined with the desired compensation-type ARC designs [22], [28], [29] for the precision motion control of linear motors in [23]. The comparative experimental results obtained on a linear motor-driven industrial gantry by Anorad (HERC-510-510-AA1-CC2) showed the same trends as in the previous experiments, revealing the performance consistencies of the proposed strategy in actual applications.

B. Comparative Experiments II

To further illustrate the effectiveness and merits of the proposed scheme, the robust adaptive control algorithm proposed in [18], a typical example of the previously published algorithms for systems with nonsmooth nonlinearities such as dead zones, is also implemented and compared with the proposed one. As the control algorithm in [18] is designed for systems with symmetric dead zones only, in the following, the simulated dead zone is set as equal slope and described by

\[
 w = D(v) = \begin{cases} 
 2.16v - 2.592, & \text{for } v \geq 1.2 \\
 0, & \text{for } -1 < v < 1.2 \\
 2.16v + 2.16, & \text{for } v \leq -1 
\end{cases}
\]

which leads to \( \theta = [2.2, 0.6, 2.16, 2.592, 2.16, -2.16]^T \). The bounds of the parametric variations are assumed to be the same as those in the previous section.

The proposed DIARC presented in Sections II and III and the robust adaptive control algorithm presented in [18] are then implemented and compared. They are referred to as C3 and C5 in the following, respectively. For the proposed DIARC algorithm (or C3), the control parameters are the same as those in the previous section with the initial values of all parameter estimates set as \( \theta_N = [1.5, 0.3, 1.5, 1.5, 1.5, -1.5]^T \). For case C5, the robust adaptive control law is designed following exactly the same procedure as that in [18]

\[
 v = \hat{e}w_d \\
 w_d = \ddot{x}_d(t) - c_1 \dot{x}_1 - \varphi \hat{\theta}_p - c_2 z_2 - z_1 - \text{sgn}(z_2) \hat{D} \\
 \hat{e} = -\beta w_d z_2, \quad \hat{\theta}_p = \Gamma \varphi^T z_2, \quad \hat{D} = \mu |z_2| 
\]

where \( \dot{z}_1 = \ddot{x}(t) \) and \( \ddot{z}_2 = \dot{z}_1 + c_1 z_1; c_1 = 100, c_2 = 50, \mu = 1000, \Gamma = 10 I_2, \) and \( \beta = 10 \). The initial values are \( \hat{e}(0) = 1/1.5, \hat{D}(0) = 0, \hat{\theta}_p(0) = [1.5, 0.3]^T, \) and \( v(0) = 0 \).

The experimental results in terms of all performance indexes after running the gantry for one period are given in Experiment II of Table I. It can be seen from these results that the proposed DIARC algorithm outperforms the robust adaptive control algorithm in [18] significantly, both in terms of the steady-state tracking performance and the transient tracking errors. The experimental results for the two controllers are also shown in Fig. 9 in terms of the output tracking performance and in Fig. 10 for the control inputs. It is seen from Fig. 9 that the steady-state output tracking error of the proposed DIARC is mostly less
than 20 μm and around 0, much less than that of C5. This result verifies the asymptotical output tracking performance of the proposed DIARC in the presence of parametric uncertainties and unknown dead zones. The control inputs shown in Fig. 10 reveal that the robust adaptive control algorithm in [18] exhibits the control input chattering phenomena during the entire motion due to the use of the discontinuous control action [the last term in \( w_d \) in (44)], while the proposed DIARC only has the chattering input during the periods when the motion is very small and the system is essentially operating at the dead-zone region only. All these results further validate the effectiveness and the high-performance nature of the proposed DIARC algorithm in practical applications.

V. CONCLUSION

In this paper, an integrated DIARC scheme has been developed for a class of single-input-single-output uncertain nonlinear systems preceded by nonsymmetric and nonequal slope dead-zone nonlinearity. By making full use of the dead-zone characteristics that it can be perfectly linearly parameterized within certain known working ranges, the proposed controller uses indirect parameter estimation algorithms with on-line condition monitoring for an accurate estimation of the unknown dead-zone nonlinearity. With these accurate estimates of dead-zone parameters, perfect asymptotic dead-zone compensation is then constructed and employed in the development of an integrated DIARC algorithm for the overall system. Consequently, asymptotic output tracking has been achieved even in the presence of unknown dead-zone nonlinearity. Furthermore, regardless of the convergence of the on-line parameter estimates to their true values, the proposed algorithm also achieves certain guaranteed robust transient performance and steady-state tracking accuracy even when the overall system may be subjected to other uncertain nonlinearities and time-varying disturbances as well. Comparative experimental results have been obtained on a linear motor drive system preceded by a simulated unknown dead-zone nonlinearity. The results validate the effectiveness of the proposed dead-zone compensation scheme. The excellent output tracking performances obtained in the experiments also verify the high-performance nature of the proposed DIARC strategy in actual applications.

APPENDIX A

**Proof of Theorem 1**: Noting (24) and (25), the derivative of

\[
\dot{V} = k^* w_d s + s[\kappa^* w_{d1} - \hat{d}_c + (1 - \kappa^*)\hat{d}_c + \Delta^*(t)]
\]

\[
\leq -k^* \kappa s^2 + \varepsilon + \varepsilon \dot{d}_f^2
\]

\[
\leq -\frac{\lambda_c}{2} s^2 + \varepsilon + \varepsilon \dot{d}_f^2.
\]

Thus

\[
\dot{V} \leq -\lambda_c V + \varepsilon + \varepsilon \dot{d}_f^2.
\]

Using the Comparison Lemma, (28) is true. The theorem can, then, be proved by noting that all on-line parameter estimates are guaranteed to be bounded regardless of the estimation function to be used shown in P1.

APPENDIX B

**Proof of Theorem 2**: When the PE conditions are satisfied, \( \hat{\theta} \in L_2 \) and \( \theta(t) \to 0 \) as \( t \to \infty \). Thus, noting Lemma 1, we have \( d \in L_2 \) as well. Noting (19), (20), and (23), the derivative of \( V = \frac{1}{2} s^2 \) when \( f_u = 0 \) is

\[
\dot{V} = k^* w_d s - \hat{d}_c s + d(t) s + \xi s
\]

where

\[
\xi = \left[ \left( \frac{\bar{m}_r}{m_r} \hat{d}_c - \frac{\bar{m}_r}{m_r} w_{d1} \right) \kappa_+ (w_d) + \left( \frac{\bar{m}_l}{m_l} \hat{d}_c - \frac{\bar{m}_l}{m_l} w_{d1} \right) \kappa_- (w_d) - \sum_{i=1}^{p} \bar{a}_i Y_i (x(t), \hat{x}(t), \ldots, x^{(n-i)}(t)) \right]
\]

\[
+ \left( \left( \frac{\bar{m}_r}{m_r} b_1 \right) - \frac{\bar{m}_r}{m_r} \left( \frac{\bar{m}_r}{m_r} b_1 \right) \right) \kappa_+ (w_d)
\]

\[
+ \left( \left( \frac{\bar{m}_l}{m_l} b_1 \right) - \frac{\bar{m}_l}{m_l} \left( \frac{\bar{m}_l}{m_l} b_1 \right) \right) \kappa_- (w_d).
\]

Choose a positive-definite function as

\[
V_a = V + \frac{1}{2\gamma} \hat{d}_c^2.
\]

Then, the derivative of \( V_a \) is

\[
\dot{V}_a = \dot{V} + \gamma^{-1} \hat{d}_c \ddot{d}_c
\]

\[
= k^* w_d s + d(t) s + \xi s + (\gamma^{-1} \text{Proj}_{\hat{d}_c} (\gamma s) - s) \hat{d}_c
\]

\[
\leq -k^* \kappa s^2 + k^* w_{d1} s + d(t) s + \xi s
\]

where P2 of the projection mapping [i.e., (12)] is used in deriving the inequality of the last step. As \( \xi \) defined by (48) is linear w.r.t. to the parameter estimation error \( \hat{\theta} \) with all coefficients being uniformly bounded by Theorem 1, the fact that \( \hat{\theta} \in L_2 \)
implies that $\xi \in L_2$. From condition 1 of (25), $k^\star w(t) s(t) \leq 0$. Thus, (50) implies that $s \in L_2$ as $\xi \in L_2$ and $d(t) \in L_2$. It is easy to verify that $s$ is uniformly continuous. Thus, by Barbalat’s lemma, $s \to 0$ as $t \to \infty$. Noting (47), $\hat{X}(t) \to 0$ as $t \to \infty$, which completes the proof of the theorem.

References:


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