Adaptive and Robust Control of Robot Manipulators: Theory and Comparative Experiments *

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Abstract

A new adaptive robust control scheme is developed for the trajectory tracking control of robot manipulators. The scheme has a desired compensation adaptation law structure with a dynamic sliding mode and achieves a guaranteed transient performance and final tracking accuracy in the presence of both parametric uncertainties and uncertain nonlinearities. In the absence of uncertain nonlinearities, the scheme also achieves asymptotic tracking. In addition, three conceptually different adaptive and robust control schemes — a simple nonlinear PID type robust control, a gain-based nonlinear PID type adaptive control, and a combined parameter and gain based adaptive robust control — are derived for comparison. All algorithms, as well as two benchmark adaptive schemes, are implemented and compared on a two-link direct-drive robot. Comparative experimental results show the importance of using both proper controller structure and parameter adaptation in designing high performance controllers. It is observed that in these experiments, the proposed scheme improves tracking performance without increasing control effort.

Keywords

Adaptive Control, Sliding Mode Control, Robust Control, Robot Manipulators, Motion Control

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1 Introduction

Industrial manipulators are commonly used in tasks such as painting, welding and material handing. In these tasks, their end-effectors are required to move from one place to another in a free workspace or to follow desired trajectories. In order to meet increased productivity requirement as well as tight tolerance requirements, it is essential for the manipulator to follow desired trajectories as close as possible at fast speeds. Thus, trajectory tracking control of robot manipulators is of practical significance. It is the simplest but most fundamental task in robot control [1, 2]. Practically, parameters of the system such as gravitational load vary from a task to another, and, may not be precisely known in advance. The system may also be subjected to uncertain nonlinearities such as external disturbances and joint friction. On the whole, a good control strategy should take into account both parametric uncertainties and uncertain nonlinearities. During the past decade, numerous robust control algorithms have been proposed, such as adaptive control [3, 4, 5, 6, 7], sliding mode control [8, 9, 10, 11], and, recently, adaptive robust control [12, 13, 14, 15].

The main advantage of the model-based (or parameter-based) adaptive algorithms [3, 4, 5, 6, 7, 16] lies in the fact that, through on-line *parameter adaptation*, the system can achieve *asymptotic tracking* without using infinite-gain feedback when the system is subjected to parametric uncertainties only. However, they all have some significant drawbacks. First, nothing is said about transient response of the system. In fact, experience in the adaptive control of linear systems [17] suggests that poor initial parameter estimates may result in unacceptably bad transient behavior, even while retaining perfect asymptotic performance. Second, uncertain nonlinearities are not considered, and the closed-loop system may lose stability even when small disturbances appear [18]. Although some modification techniques to the integral type adaptation law such as σ-modification [18, 19] can be employed to enhance the system robustness, tracking accuracy can no longer be guaranteed since the steady state tracking error can only be shown to stay within an unknown ball whose size depends on the disturbance.

On the other hand, deterministic robust control (DRC) such as sliding mode control (SMC) [20, 8, 10] employs certain *controller structures* to attenuate the effect of the model uncertainties coming from both parametric uncertainties and uncertain nonlinearities. In general, it can guarantee transient performance and certain final tracking accuracy. However, DRC does not discriminate between parametric uncertainties and uncertain nonlinearities and the control law uses fixed parameters. Model uncertainties coming from parametric uncertainties cannot be reduced. In order to reduce tracking errors, feedback gains must be increased, resulting in high-gain feedback and increased bandwidths of closed-loop systems. Theoretically, SMC can use discontinuous control laws, and some of the continuous DRC schemes [21, 22] can use infinite gain feedback control to achieve asymptotic tracking. However, those are impractical and unachievable solutions because of finite bandwidths of physical systems.

Recently, we proposed a new approach, adaptive robust control (ARC) [12, 14, 14, 15], which uses both means — *proper controller structure and parameter adaptation* — to reduce tracking errors. Departing from the model-based adaptive control, the approach also puts emphasis on the selection of controller structure as in DRC. Thus, the main problem of unknown transient performance and non-robustness to uncertain nonlinearities of adaptive control can be solved painlessly. Contrary to DRC, the approach discriminates the difference between parametric uncertainties and uncertain nonlinearities and uses parameter adaptation to reduce the parametric uncertainties. Thus an improved performance can be obtained–*asymptotic tracking* is achieved without using infinite-gain feedback as in adaptive control in the presence of parametric uncertainties. The approach differs fundamentally from the existing robust adaptive control approaches [18, 23, 24] in that it puts more emphasis on performance and the selection of controller structure and thus achieves a much stronger performance robustness (guaranteed transient and final tracking accuracy) in addition to stability robustness.

There are also some adaptive schemes [25, 26, 27] called performance-based (or direct) adaptive control [28], in which adaptation laws are used to adjust controller gains instead of physical parameters. These gain-based schemes are claimed to be simple, computationally efficient and require very little model information. Robustness to bounded disturbances is also guaranteed. However, they can only guarantee tracking errors within certain bounds even when the system is subject to parametric uncertainties only. Asymptotic stability is lost and the system may exhibit relatively larger final tracking errors as in DRC.

Some comparative experiments were carried out in [16] to test some of the model-based (or parameter-based) adaptive algorithms. However, the tested algorithms belonged to the same class. Facing so many algorithms and so many qualitatively different approaches, one has difficulty in choosing a suitable one for a particular application since each algorithm has its own claim. Thus, it is of practical significance to test qualitatively different approaches on the same machine to understand their fundamental advantages and drawbacks.

This paper serves for two purposes: one is to further improve performance of robot control systems and the other is to test qualitatively different algorithms experimentally to set up a standard with which various controllers can be compared. To achieve the first purpose, we propose a new adaptive robust control (ARC) scheme, in which the regressor is calculated by reference trajectory information only, and thus the resulting adaptation law is less sensitive to noisy velocity signals and has a better robustness in addition to largely reduced on-line computation. The idea of using the desired compensation adaptation law was proposed by Sadegh and Horowitz [5] and was experimentally demonstrated by Whitcomb, et al. [16] that it achieves a superior tracking performance among existing adaptive schemes. Theoretically, the main difference between the proposed approach and the desired compensation adaptive algorithm (DCAL) in [5] is that the proposed approach can guarantee a prescribed precision and transient performance even in the presence of uncertain nonlinearities. To serve for the second purpose, a very simple nonlinear PID scheme is first proposed, which can guarantee stability and requires little model information. By adjusting feedback gains on-line, a simple gain-based adaptive control is also suggested to remove the requirements in choosing feedback gains in the nonlinear PID scheme. By combining the design techniques of the gain-based adaptive control and the model-based ARC, a new adaptive robust scheme is also proposed to remove the conditions on the selection of the controller gains. Finally, all schemes, as well as two benchmark adaptive control schemes [4, 5], are implemented and compared. Experimental results are presented to show the advantages and the drawbacks of each method.

This paper is organized as follows. Section II establishes all the dynamic equations used and formulates the problem. Section III generalizes the adaptive sliding mode control (ASMC) scheme presented in [12]. Section IV-VII present the proposed desired compensation-type ARC (DCARC), nonlinear PID robust control (NPID), nonlinear PID-type adaptive control (PIDAC), and desired compensation adaptive robust control with adjustable gains (ARCAG) respectively. Section VIII shows the comparative experimental results and section IX draws conclusions.

2 Dynamic Model of Robot Manipulators

A dynamic equation of a general rigid link manipulator having n degrees of freedom in free space can be written as [2]

$$M(q,\beta)\ddot{q} \ + \ C(q,\dot{q},\beta)\dot{q} + G(q,\beta) + \tilde{f}(q,\dot{q},t) = u \tag{1}$$

where $q \in R^n$ is the joint displacement vector, $\beta \in R^p$ is the vector of a suitably selected set of the robot parameters, $u \in R^n$ is the applied joint torque, $M(q,\beta) \in R^{n\times n}$ is the inertia matrix, $C(q,\dot{q},\beta)\dot{q} \in R^n$ is the Coriolis and centrifugal force, $G(q,\beta) \in R^n$ is the gravitational force, and $\tilde{f}(q,\dot{q},t) \in R^n$ is the vector of unknown nonlinear functions such as external disturbances and unmodeled joint friction. Equation (1) has the following properties that will facilitate the controller design [2, 4, 7, 5].

Property 1 . *$M(q,\beta)$ is a symmetric positive definite (s.p.d.) matrix, and there exists $k_m > 0$ such that $k_m I_{n\times n} \leq M(q,\beta)$. Furthermore, for the robot with all joints revolute or prisma, there exists $k_M > 0$ so that $M(q,\beta) \leq k_M I_{n\times n}$. For a general robot, $M(q,\beta) \leq k_M I_{n\times n}$ is valid for any finite workspace $\Omega_q = \{q : \|q - q_0\| \leq q_{max}\}$ where q_0 and q_{max} are some constants.*

Property 2 . *The matrix $N(q,\dot{q},\beta) = \dot{M}(q,\beta) - 2C(q,\dot{q},\beta)$ is a skew-symmetric matrix.*

Property 3 . *$M(q,\beta), C(q,\dot{q},\beta)$, and $G(q,\beta)$ can be linearly parametrized in terms of β. Therefore, we can write*

$$M(q,\beta)\ddot{q}_r \ + \ C(q,\dot{q},\beta)\dot{q}_r + G(q,\beta) = f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta \tag{2}$$

where $Y \in R^{n\times l_\beta}$, $\dot{q}_r$ and $\ddot{q}_r$ are any reference vectors.

In general, the parameter vector β cannot be exactly known. For example, the payload of the robot depends on tasks. However, the extent of parametric uncertainties can be predicted. Therefore, we make the following assumptions about parametric uncertainties and uncertain nonlinearities

Assumption 1: *β lies in a known bounded set Ω_β and $\|\tilde{f}(q,\dot{q},t)\|$ can be bounded by a known function, i.e.,*

$$\begin{array}{l} \beta \in \Omega_\beta \triangleq \{\beta : \quad \beta_{min} < \beta < \beta_{max} \ \} \\ \|\tilde{f}(q,\dot{q},t)\| \leq h_f(q,\dot{q},t) \end{array} \tag{3}$$

where β_{min}, β_{max}, and $h_f(q, \dot{q}, t)$ are known (operation $<$ for vectors is defined elementwise and $\| \bullet \|$ denotes a norm of $\bullet$ which is a vector or a matrix).

We can now formulate the trajectory tracking control of robot manipulators as follows:

Suppose $q_d(t) \in R^n$ is given as the desired joint motion trajectory. Let $e = q(t) - q_d(t) \in R^n$ be the motion tracking error. For the robot manipulator described by (1), under the Assumption 1, design a control law u so that the system is stable and q tracks $q_d(t)$ as close as possible.

3 Adaptive Sliding Mode Control (ASMC)

In this section, the smooth adaptive sliding mode control scheme [12] is modified in the way that the resulting control law is more general and revealing. The results will also be utilized in the following sections' design. The scheme combines SMC with adaptive control to take advantages of the two methods while overcoming their drawbacks. A dynamic sliding mode is employed to eliminate the unpleasant reaching transient and to enhance the dynamic response of the system in sliding mode.

Let a dynamic compensator be

$$\begin{array}{ll} \dot{z} = A_z z + B_z e & z \in R^{n_c} \quad A_z \in R^{n_c \times n_c} \quad B_z \in R^{n_c \times n} \\ y_z = C_z z + D_z e & y_z \in R^n \quad C_z \in R^{n \times n_c} \quad D_z \in R^{n \times n} \end{array} \tag{4}$$

where (A_z, B_z, C_z, D_z) is controllable and observable and $e = q - q_d$ is as defined in the previous section. Define a switching-function-like term as

$$\begin{array}{rcll} \xi & = & \dot{e} + y_z & \xi \in R^n \\ & = & \dot{q} - \dot{q}_r & \dot{q}_r \triangleq \dot{q}_d(t) - y_z \end{array} \tag{5}$$

Transfer function from ξ to e is

$$e = G_\xi^{-1}(s)\xi \tag{6}$$

where

$$G_\xi(s) = sI_n + G_c(s) \qquad G_c(s) = C_z(sI_{n_c} - A_z)^{-1}B_z + D_z \tag{7}$$

It was shown in [12] that by suitably choosing dynamic compensator transfer function $G_c(s)$, the resulting dynamic sliding mode $\{\xi = 0\}$, i.e., free response of transfer function $G_\xi^{-1}(s)$, can be arbitrarily shaped to possess any exponentially fast converging rate. In addition, when C_z is of full column rank, the initial value $z(0)$ of the dynamic compensator (4) can be chosen to satisfy

$$C_z z(0) = -\dot{e}(0) - D_z e(0) \tag{8}$$

then $\xi(0) = 0$. It is shown in [12] that choosing the initial value $z(0)$ in such a way guarantees that the system is maintained in the sliding mode all the time and the reaching transient is eliminated when ideal sliding mode control is applied.

Let $\hat{\beta}_\pi$ be the smooth projection of $\hat{\beta}$, the estimate of β (The smooth projection is defined in [14] and is briefly reviewed in Appendix 1). Then $\hat{\beta}_\pi \in \Omega_{\hat{\beta}} = \{\hat{\beta}: \; \beta_{min} - \varepsilon_\beta \leq \hat{\beta} \leq \beta_{max} + \varepsilon_\beta\}$. Let $h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ be a bounding function satisfying

$$\|Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\beta}_\pi\| = \|Y\hat{\beta}_\pi - Y\beta\| \leq h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \qquad \forall \hat{\beta}_\pi \in \Omega_{\hat{\beta}} \tag{9}$$

For example, choose

$$h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r) = \|Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\| \; \beta_M \tag{10}$$

where $\beta_M = \|\beta_{max} - \beta_{min} + \varepsilon_\beta\|$. Define

$$h_s(q, \dot{q}, \dot{q}_r, \ddot{q}_r, t) = h_f(q, \dot{q}, t) + h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \tag{11}$$

Definition 1 *For any discontinuous vector like* $-h\frac{\bullet}{\|\bullet\|}$ *where* h *is a positive scalar function and* $\bullet$ *is a vector of functions, its continuous approximation,* $\hbar(-h\frac{\bullet}{\|\bullet\|})$*, with an approximation error* $\varepsilon(t)$ *is defined to be a vector of functions that satisfies the following two conditions:*

$$\begin{array}{ll} i. & \bullet^T \, \hbar(-h\frac{\bullet}{\|\bullet\|}) \leq 0 \\ ii. & h\| \bullet \| + \bullet^T \hbar(-h\frac{\bullet}{\|\bullet\|}) \leq \varepsilon(t) \end{array} \tag{12}$$

Examples of the approximation function $\hbar$ are given in the following remarks.

Remark 1 *A natural generalization of the concept of boundary layer [10] to multiple input/output cases is given by*

$$\hbar(-h\frac{\bullet}{\|\bullet\|}) = -(1 + \alpha_1 h)h\frac{\bullet}{\|\bullet\| + \phi(t)} \tag{13}$$

where $\alpha_1 > 0$ *is any positive scalar, and* $\phi(t)$ *is any bounded time-varying positive scalar, i.e.,* $0 \leq \phi(t) \leq \phi_{max}$*, which has the role of boundary layer thickness. It is easy to show [12] that (12) is satisfied for* $\varepsilon = \frac{\phi(t)}{4\alpha_1}$. ◊

Remark 2 *A smooth* $\hbar(-h\frac{\bullet}{\|\bullet\|}) = [\hbar_1, \ldots, \hbar_n]^T$ *is given by*

$$\hbar_i = -h \; tanh\left(\frac{h\bullet_i}{\phi_i(t)}\right) \tag{14}$$

By the properties of tanh function, condition i of (12) is satisfied and

$$\bullet^T \hbar = -\sum_{i=1}^n h \bullet_i tanh\left(\frac{h\bullet_i}{\phi_i}\right) \leq \sum_{i=1}^n (\kappa\phi_i - h| \bullet_i |) \leq \kappa \sum_{i=1}^n \phi_i - h\| \bullet \| \tag{15}$$

where $\kappa = 0.2785$. *Thus, condition ii of (12) is satisfied for* $\varepsilon = \kappa \sum_{i=1}^n \phi_i$. ◊

Remark 3 *In general, a time-varying $\phi(t)$ has to be employed in (13) or (14) to achieve a good tracking accuracy, which is quite complicated and is not easily implemented. To overcome this problem, the following modification is suggested:* [1]

$$\hbar(-h_s\frac{\xi}{\|\xi\|}) = \begin{cases} -K_s\xi & if \quad \|\xi\| \le \phi_h \quad \phi_h \triangleq \frac{\phi(t)}{h_s(q,\dot{q},\dot{q}_r,\ddot{q}_r,t)+\varepsilon_1} \\ -(1-c_1)K_s\xi - c_1 h_s\frac{\xi}{\|\xi\|} & \phi_h \le \|\xi\| \le (1+\varepsilon_2)\phi_h \\ -h_s\frac{\xi}{\|\xi\|} & \|\xi\| \ge (1+\varepsilon_2)\phi_h \end{cases} \tag{16}$$

where K_s is any s.p.d. matrix, $c_1 = \frac{\|\xi\|-\phi_h}{\varepsilon_2\phi_h}$, and ε_1 and ε_2 are any positive scalars. Using similar techniques as in [12], it can be shown that (12) is satisfied for $\varepsilon = (1+\varepsilon_2)\phi(t)$.

The above modification is quite simple and yet it provides the desired properties – namely, around sliding mode $\{\|\xi\| = 0\}$, a fixed feedback gain matrix is employed all the time and thus can be chosen near its allowable limit without inducing excessive control chattering. We can also tune the gain around each joint separately since it is a gain matrix instead of a nonlinear scalar gain. When the system is far away from sliding surfaces, the original nonlinear feedback control law is employed to guarantee the stability at large. It is shown in [12] by both simulation results and experimental results that the above modification can achieve a better tracking performance than (13). ◊

Theorem 1 *Choose a continuous control law as*

$$\begin{aligned} u &= u_a + u_s \\ u_a &= f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\beta}_\pi - K_\xi\xi \\ u_s &= \hbar(-h_s\frac{\xi}{\|\xi\|}) \end{aligned} \tag{17}$$

where K_ξ is any s.p.d. matrix, $\hbar(-h_s\frac{\xi}{\|\xi\|})$ is a continuous approximation of the ideal SMC control, $-h_s\frac{\xi}{\|\xi\|}$ with an approximation error $\varepsilon(t)$, and $\hat{\beta}$ is updated by

$$\dot{\hat{\beta}} = -\Gamma_\beta[l_\beta(\hat{\beta}) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)^T\xi] \tag{18}$$

where $l_\beta(\hat{\beta})$ is any vector of functions that satisfies the following conditions

$$\begin{array}{lll} i. \ \ l_\beta(\hat{\beta}) = 0 & if & \hat{\beta} \in \Omega_\beta \\ ii. \ \ \tilde{\beta}_\pi^T l_\beta(\hat{\beta}) \ge 0 & if & \hat{\beta} \notin \Omega_\beta \end{array} \tag{19}$$

Then, the following results hold:

A. *In general, all the signals in the system remain bounded and tracking errors, e and $\dot{e}$, exponentially converge to some balls with size proportional to ε. Furthermore, the tracking error ξ is bounded by*

$$\|\xi(t)\|^2 \le \frac{2}{k_m}[exp(-\lambda_V t)V(0) + \int_0^t exp(-\lambda_V(t-\nu))\varepsilon(\nu)d\nu] \tag{20}$$

[1] • is replaced by ξ here to make the presentation clear.

where $\lambda_V = \frac{2\lambda_{min}(K_\xi)}{k_M}$, *and* V *is a positive semi-definite (p.s.d.) function given by*

$$V = \frac{1}{2}\xi^T M(q,\beta)\xi \tag{21}$$

In addition, if (8) is satisfied, then $V(0) = 0$ *in (20).*

B. *If after a finite time,* $\tilde{f} = 0$, *then the following are true:*

a) $\xi \longrightarrow 0, \quad e \longrightarrow 0 \quad \dot{e} \longrightarrow 0$ *when* $t \longrightarrow \infty$ *i.e., the robot follows the desired motion trajectories asymptotically.*

b) *Additionally, if the desired motion trajectory satisfies the following persistent excitation condition*

$$\int_t^{t+T} Y^T(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d) Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d) d\nu \geq \varepsilon_d I_p \qquad \forall t \geq t_0 \tag{22}$$

where T, t_0 *and* ε_d *are some positive scalars, and* $q_d^{(3)}(t)$ *is bounded, then* $\tilde{\beta} \longrightarrow 0$ *when* $t \longrightarrow \infty$, *i.e., estimated parameters converge to their true values.* △

Proof of Theorem 1: The detailed proof can be found in [15] and is omitted here since it can be done in a similar way as in the proof of Theorem 2 in the following section. □.

Remark 4 *Note that, comparing to the adaptive control algorithms [3, 4, 5, 6, 7, 29, 30, 16], the above theorem guarantees transient performance and final tracking accuracy even in the presence of uncertain nonlinearities (Results in A). Comparing to deterministic robust control [10], the method achieves asymptotic tracking or zero final tracking error in the presence of parametric uncertainties (Results in B).* □

Remark 5 *Note that* $l_\beta = 0$ *is a trivial solution of (19). The reason of introducing* l_β *is to make the adaptation process more robust since* l_β *functions as a nonlinear damping. In this application, the adaptation law, the right hand side of (18), can be discontinuous since* $\hat{\beta}$ *is still continuous and the control law uses* $\hat{\beta}$ *only. It is shown in [12, 15] that the widely used projection method in adaptive systems [31, 32] can be used in defining the needed modification function* l_β. *With this discontinuous modification, the final form of the adaptation law is*

$$\dot{\hat{\beta}}_i = \begin{cases} 0 & if \quad \hat{\beta}_i = \beta_{imax} \ \ and \ \ (\Gamma_\beta Y^T \xi)_i < 0 \\ -(\Gamma_\beta Y^T \xi)_i & \begin{cases} \beta_{imin} < \hat{\beta}_i < \beta_{imax} \\ \hat{\beta}_i = \beta_{imax} \ \ and \ \ (\Gamma_\beta Y^T \xi)_i \geq 0 \\ \hat{\beta}_i = \beta_{imin} \ \ and \ \ (\Gamma_\beta Y^T \xi)_i \leq 0 \end{cases} \\ 0 & \hat{\beta}_i = \beta_{imin} \ \ and \ \ (\Gamma_\beta Y^T \xi)_i > 0 \end{cases} \tag{23}$$

and $\hat{\beta}_\pi = \hat{\beta}$ *since* $\hat{\beta} \in [\beta_{max}, \beta_{min}]$. *Example of continuous modification can also be found in [12, 15].* □

Remark 6 *The extra freedom in choosing the dynamic sliding mode $G_\xi^{-1}(s)$ can be utilized to minimize the effect of a non-zero ξ on the tracking error e. For example, if the system is mainly subject to some constant disturbances, a constant steady state ξ may appear. By including a differentiators in the numerator of $G_\xi^{-1}(s)$, e.g., $G_\xi^{-1}(s) = \frac{s}{s^2+k_p s+k_i} I_n$, which can be realized by choosing the dynamic compensator parameter as $C_z = I_n$, $A_z = 0$, $B_z = k_i I_n$, $D_z = k_p I_n$, a zero steady state tracking error $e(\infty)$ can be obtained.* ♢

Remark 7 *By setting $u_s = 0$ in (17), without using parameter projection and any modification to the adaptation law, and taking off the dynamic compensator (i.e., letting $C_z = 0, A_z = 0, B_z = 0, D_z > 0$ in (4)), the control law (17) reduces to Slotine and Li's well-known adaptive algorithm (SLAC), which is also tested later for comparison.* ♢

4 Desired Compensation Adaptive Robust Control (DCARC)

The regressor Y in the adaptation law (18) depends on the actual state. In [5], Sadegh and Horowitz proposed a desired compensation adaptation law (DCAL), in which the regressor is calculated by reference trajectory information only. By doing so, one obtains a resulting adaptation law that is less sensitive to noisy velocity signals and has a better robustness as well as a significantly reduced amount of on-line computation. Comparative experiments in [16] demonstrated the superior tracking performance of the DCAL. Inspired by these results, a desired compensation adaptive robust control (DCARC) is proposed in this section.

The state space realization of (6) is

$$\dot{x}_\xi = A_\xi x_\xi + B_\xi \xi \qquad y_\xi = C_\xi x_\xi \tag{24}$$

where $x_\xi = [z^T \ , \ e^T]^T \ \in R^{n_c+n}$ and

$$A_\xi = \begin{bmatrix} A_z & B_z \\ -C_z & -D_z \end{bmatrix} \qquad B_\xi = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \qquad C_\xi = [0 \ , \ I_n] \tag{25}$$

The equivalent results in state space about the dynamic sliding mode (6) can be stated as follows: there exists an s.p.d. solution P_ξ for any s.p.d. matrix Q_ξ for the following Lyapunov equation,

$$A_\xi^T P_\xi + P_\xi A_\xi = -Q_\xi \tag{26}$$

and the exponentially converging rate $\frac{\lambda_{min}(Q_\xi)}{\lambda_{max}(P_\xi)}$ can be any value by assigning the poles of A_ξ to the far left plane.

It is shown in Appendix 2 that there are known non-negative bounded scalars $\gamma_1(t)$, $\gamma_2(t)$, $\gamma_3(t)$, and $\gamma_4(t)$, which depend on the reference trajectory and A_ξ only, such that the following inequality is satisfied

$$\begin{array}{l} \|f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta - f_0(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d) - Y(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d)\beta\| \\ \quad \le \gamma_1\|x_\xi\| + \gamma_2\|\xi\| + \gamma_3\|\xi\|\|x_\xi\| + \gamma_4\|x_\xi\|^2 \end{array} \tag{27}$$

$\gamma_1, \gamma_2, \gamma_3$, and γ_4 can be determined off-line. Similar to (9), there exists a known scalar function $h_\beta(q_d, \dot{q}_d, \ddot{q}_d)$ such that

$$\|Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\tilde{\beta}_\pi\| = \|Y\hat{\beta}_\pi - Y\beta\| \leq h_\beta(q_d, \dot{q}_d, \ddot{q}_d) \qquad \forall \hat{\beta}_\pi \in \Omega_{\hat{\beta}} \tag{28}$$

Since $h_\beta(q_d, \dot{q}_d, \ddot{q}_d)$ depends on reference trajectory only, it can be determined off-line, one of the advantages of this scheme. Similar to (11), define

$$h_s(q, \dot{q}, t) = h_f(q, \dot{q}, t) + h_\beta(q_d, \dot{q}_d, \ddot{q}_d) \tag{29}$$

Theorem 2 *Choose a continuous control law as*

$$\begin{array}{l} u = u_a + u_s \\ u_a = f_0(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d) + Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\hat{\beta}_\pi - K_\xi \xi - K_x x_\xi - \gamma_5 \|x_\xi\|^2 \xi \\ u_s = \hbar(-h_s \frac{\xi}{\|\xi\|}) \end{array} \tag{30}$$

where $K_\xi > 0$ *is an s.p.d. matrix,* γ_5 *is a positive scalar,* $K_x = B_\xi^T P_\xi$*, in which* P_ξ *is determined by (26),* $\hbar$ *is a continuous approximation of* $-h_s \frac{\xi}{\|\xi\|}$ *with an approximation error* ε*, and* $\hat{\beta}$ *is updated by*

$$\dot{\hat{\beta}} = -\Gamma[l_\beta(\hat{\beta}) + Y^T(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\xi] \tag{31}$$

If controller parameters K_ξ, Q_ξ*, and* γ_5 *are large enough such that*

$$\begin{array}{l} \lambda_{min}(K_\xi) \geq \varepsilon_3 + \gamma_2 + \gamma_6 + \gamma_8 \\ \lambda_{min}(Q_\xi) \geq 2(\varepsilon_3 + \gamma_7 + \gamma_{10}) \\ \gamma_5 \geq \gamma_9 + \gamma_{11} \end{array} \tag{32}$$

where ε_3 *is any positive scalar, and* $\gamma_6, \gamma_7, \gamma_8, \gamma_9, \gamma_{10}$*, and* γ_{11} *are any positive scalars satisfying*

$$\begin{array}{l} \gamma_6\gamma_7 = \frac{1}{4}\gamma_1^2 \\ \gamma_8\gamma_9 = \frac{1}{4}\gamma_3^2 \\ \gamma_{10}\gamma_{11} = \frac{1}{4}\gamma_4^2 \end{array} \tag{33}$$

then, the following results hold:

A. *In general, all the signals in the system remain bounded and tracking errors,* e *and* $\dot{e}$*, exponentially converge to some balls with size proportional to* ε*. Furthermore, the p.s.d function* V *defined by*

$$V = \frac{1}{2}\xi^T M(q, \beta)\xi + \frac{1}{2}x_\xi^T P_\xi x_\xi \tag{34}$$

is bounded by

$$V \leq exp(-\lambda_V t)V(0) + \int_0^t exp(-\lambda_V(t-\nu))\varepsilon(\nu)d\nu] \tag{35}$$

where λ_V *is a positive scalar satisfying*

$$\lambda_V \geq \frac{2\varepsilon_3}{max\{k_M \ , \ \lambda_{max}(P_\xi)\}} \tag{36}$$

B. *If after a finite time, $\tilde{f} = 0$, then the same results as in B of Theorem 1 can be obtained.* $\triangle$

Remark 8 *In (35), the exponentially converging rate λ_V can be any large value by choosing the controller parameters as follows. Noting λ_V is bounded below by (36) and k_M is a fixed constant, λ_V can be any large value as long as we can arbitrarily choose ε_3 and $\frac{\varepsilon_3}{\lambda_{max}(P_\xi)}$. Therefore, first set ε_3 to its desired value and let Q_ξ satisfy (32), in which γ_7 and γ_{10} can be any fixed values. Then, choose the dynamic compensator parameter A_ξ such that the solution P_ξ of (26) makes $\frac{\varepsilon_3}{\lambda_{max}(P_\xi)}$ big enough. $\gamma_1, \gamma_2, \gamma_3$, and γ_4 in (27) can then be determined, and $\gamma_6, \gamma_8, \gamma_9$ and γ_{11} can be calculated to satisfy (33). Finally, choose K_ξ and γ_5 such that (32) is satisfied. In this way, theoretically, any fast exponentially converging rate can be achieved.* $\diamondsuit$

Remark 9 *In the control law (30) and the adaptation law (31) the regressor $Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)$ is a function of the reference trajectory only and, thus, can be calculated off-line. In addition to the reduction of on-line computation time, this result also removes the problem of noise correlation between the estimation error and the adaptation signals, especially when the velocity measurement is noisy in implementation [5], and, thus, enhances the performance robustness of the resulting adaptive robust control law.* $\diamondsuit$

Remark 10 *By setting $u_s = 0$ in (30), without using parameter projection and any modification to the adaptation law, and taking off the dynamic compensator (i.e., letting $C_z = 0, A_z = 0, B_z = 0, D_z > 0$ in (4)), the control law (30) reduces to the well-known desired compensation adaptation law (DCAL) by Sadegh and Horowitz [5], which is also implemented for comparison.* $\diamondsuit$

Proof of Theorem 2: Noting (5) and Property 3, (1) can be rewritten as

$$M(q,\beta)\dot{\xi} + C(q,\dot{q},\beta)\xi + f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta + \tilde{f}(q,\dot{q},t) = u \tag{37}$$

From Property 1,

$$\frac{1}{2}k_m\|\xi\|^2 + \frac{1}{2}\lambda_{min}(P_\xi)\|x_\xi\|^2 \le V \le \frac{1}{2}k_M\|\xi\|^2 + \frac{1}{2}\lambda_{max}(P_\xi)\|x_\xi\|^2 \tag{38}$$

Noting (37), (24) and (26), differentiating V with respect to (w.r.t.) time yields

$$\begin{aligned} \dot{V} &= \xi^T[M(q,\beta)\dot{\xi} + C(q,\dot{q},\beta)\xi] + \tfrac{1}{2}x_\xi^T(A_\xi^T P_\xi + P_\xi A_\xi)x_\xi + x_\xi^T P_\xi B_\xi \xi \\ &= \xi^T[u - f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) - Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta - \tilde{f} + B_\xi^T P_\xi x_\xi] - \tfrac{1}{2}x_\xi^T Q_\xi x_\xi \end{aligned} \tag{39}$$

Substituting the control law (30) into (39) and noting (27), we can obtain

$$\begin{aligned} \dot{V} \;=\; & \xi^T[Y(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d)\tilde{\beta}_\pi + f_0(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d) - f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) + Y(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d)\beta \\ & -Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta - \tilde{f} - K_\xi\xi - \gamma_5\|x_\xi\|^2\xi + \hbar] - \tfrac{1}{2}x_\xi^T Q_\xi x_\xi \\ \le & -\xi^T K_\xi \xi - \gamma_5\|x_\xi\|^2\|\xi\|^2 - \tfrac{1}{2}x_\xi^T Q_\xi x_\xi + \xi^T Y(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d)\tilde{\beta}_\pi - \xi^T\tilde{f} + \xi^T\hbar \\ & +\gamma_1\|\xi\|\|x_\xi\| + \gamma_2\|\xi\|^2 + \gamma_3\|\xi\|^2\|x_\xi\| + \gamma_4\|x_\xi\|^2\|\xi\| \end{aligned} \tag{40}$$

Applying the inequality

$$w_1|y_1||y_2| \leq w_2 y_1^2 + w_3 y_2^2 \qquad \forall y_1, y_2 \in R \quad w_1, w_2, w_3 \geq 0 \tag{41}$$

where $4w_2w_3 = w_1^2$ to (40), we have,

$$\begin{aligned}\dot{V} &\leq -\xi^T K_\xi \xi - \gamma_5 \|x_\xi\|^2\|\xi\|^2 - \tfrac{1}{2}x_\xi^T Q_\xi x_\xi + \xi^T Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\tilde{\beta}_\pi - \xi^T \tilde{f} + \xi^T \hbar \\ &\quad +\gamma_6\|\xi\|^2 + \gamma_7\|x_\xi\|^2 + \gamma_2\|\xi\|^2 + \gamma_8\|\xi\|^2 + \gamma_9\|\xi\|^2\|x_\xi\|^2 + \gamma_{10}\|x_\xi\|^2 + \gamma_{11}\|x_\xi\|^2\|\xi\|^2 \\ &= -\xi^T[K_\xi - (\gamma_2 + \gamma_6 + \gamma_8)I_n]\xi - x_\xi^T[\tfrac{1}{2}Q_\xi - (\gamma_7 + \gamma_{10})I_{n+n_c}]x_\xi \\ &\quad -[\gamma_5 - (\gamma_9 + \gamma_{11})]\|x_\xi\|^2\|\xi\|^2 + \xi^T Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\tilde{\beta}_\pi - \xi^T \tilde{f} + \xi^T \hbar\end{aligned} \tag{42}$$

If (32) is satisfied, (42) becomes

$$\dot{V} \leq -\varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) + \xi^T Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\tilde{\beta}_\pi - \xi^T \tilde{f} + \xi^T \hbar \tag{43}$$

In general, when $\tilde{f} \neq 0$, from (3), (28), (29), (38), and condition ii of (12), (43) becomes

$$\dot{V} \leq -\varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) + h_s\|\xi\| + \xi^T \hbar \leq -\lambda_V V + \varepsilon \tag{44}$$

which leads to (35). In viewing (38), conclusion A of Theorem 2 is established.

Now consider the situation that $\tilde{f} = 0, \ \forall t \geq t_0$, for some finite t_0. Choose a p.s.d. function as

$$V_a = V + V_\beta(\tilde{\beta}, \beta) \tag{45}$$

where V_β is defined in (82). From (43), condition i of (12), (83), (31), and condition ii of (19), we have that $\forall t \geq t_0$,

$$\begin{aligned}\dot{V}_a &\leq -\varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) + \tilde{\beta}_\pi^T Y^T(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\xi + \tilde{\beta}_\pi^T \Gamma_\beta^{-1}\dot{\hat{\beta}} \\ &\leq -\varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) - \tilde{\beta}_\pi^T l_\beta \leq -W\end{aligned} \tag{46}$$

where $W = \varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2)$. Therefore, $W \in L_1$ and $V_a \in L_\infty$. Since all signals are bounded, it is easy to verify that $\dot{W}$ is bounded and thus uniformly continuous. By Barbalat's lemma [33], $W \longrightarrow 0$ as $t \longrightarrow \infty$, which implies the conclusion a) of B of Theorem 2. Furthermore, it is easy to verify that every term in (37) except $\dot{\xi}$ is uniformly continuous. Thus $\dot{\xi}$ is uniformly continuous, and thus $\dot{\xi} \longrightarrow 0$ by applying Barbalat's lemma. Substituting the control law into (37) and noting the above asymptotic convergence of error signals, it is easy to obtain that $Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\tilde{\beta}_\pi \longrightarrow 0$ as $t \longrightarrow \infty$. Thus, PE condition (22) will guarantee the convergence of parameter estimates, which leads to the conclusion b) of B of Theorem 2. □

5 Nonlinear PID Robust Control (NPID)

In this section, a simple robust control with nonlinear PID feedback structure is designed. The following simple control structure is suggested

$$u = f_c - (K_\xi(t) + \gamma_5\|x_\xi\|^2)\xi - K_x x_\xi \tag{47}$$

where f_c is any constant vector that is used to cancel the low frequency component, $K_\xi > 0$ is an s.p.d. matrix, γ_5 is a positive scalar, and $K_x = B_\xi^T P_\xi$, in which P_ξ is determined by (26).

Theorem 3 *If controller parameters K_ξ and γ_5 in (47), and Q_ξ in (26) are large enough such that*

$$\begin{aligned} &\lambda_{min}(K_\xi) \geq \varepsilon_3 + \gamma_2 + \gamma_6 + \gamma_8 + \tfrac{c_0^2}{4\varepsilon(t)} \\ &\lambda_{min}(Q_\xi) \geq 2(\varepsilon_3 + \gamma_7 + \gamma_{10}) \\ &\gamma_5 \geq \gamma_9 + \gamma_{11} \end{aligned} \tag{48}$$

where ε_3 is any positive scalar, $\gamma_6, \gamma_7, \gamma_8, \gamma_9, \gamma_{10}$, and γ_{11} are defined in Theorem 2, and

$$c_0 = \|f_c - f_0(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d) - Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\beta\| + h_f \tag{49}$$

then, the control law (47) guarantees that all signals in the system remain bounded and tracking errors, $e(t)$ and $\dot{e}(t)$, exponentially converge to some balls, the sizes of which are proportional to ε. Furthermore, V defined by (34) is bounded by (35). ♢

Remark 11 *By choosing the dynamic compensator as an integrator, x_ξ consists of e and $\int_0^t e$; thus, control law (47) may be considered as a nonlinear PID feedback control, which is quite easy to implement since it does not require any model information, except some bounds in choosing controller parameters.* ♢

Proof of Theorem 3: For the p.s.d. function V given by (34), its derivative is given by (39). Substituting control law (47) into (39) and following similar steps as in (40) and (42), we have

$$\begin{aligned} \dot{V} &= \xi^T[f_c - K_\xi\xi - \gamma_5\|x_\xi\|^2\xi - f_0(q, \dot{q}, \dot{q}_r, \ddot{q}_r) - Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\beta - \tilde{f}] - \tfrac{1}{2}x_\xi^T Q_\xi x_\xi \\ &\leq \xi^T[f_c - f_0(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d) - Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\beta - K_\xi\xi - \gamma_5\|x_\xi\|^2\xi - \tilde{f}] \\ &\quad -\tfrac{1}{2}x_\xi^T Q_\xi x_\xi + \gamma_1\|\xi\|\|x_\xi\| + \gamma_2\|\xi\|^2 + \gamma_3\|\xi\|^2\|x_\xi\| + \gamma_4\|x_\xi\|^2\|\xi\| \\ &\leq \|\xi\|c_0 - \xi^T K_\xi \xi - \gamma_5\|x_\xi\|^2\|\xi\|^2 - \tfrac{1}{2}x_\xi^T Q_\xi x_\xi + \gamma_6\|\xi\|^2 + \gamma_7\|x_\xi\|^2 + \gamma_2\|\xi\|^2 \\ &\quad +\gamma_8\|\xi\|^2 + \gamma_9\|\xi\|^2\|x_\xi\|^2 + \gamma_{10}\|x_\xi\|^2 + \gamma_{11}\|x_\xi\|^2\|\xi\|^2 \end{aligned} \tag{50}$$

If (48) is satisfied, (50) becomes

$$\begin{aligned} \dot{V} &\leq \|\xi\|c_0 - \tfrac{c_0^2}{4\varepsilon}\|\xi\|^2 - \varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) \\ &\leq \varepsilon - (\tfrac{c_0}{2\sqrt{\varepsilon}}\|\xi\| - \sqrt{\varepsilon} - \varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) \leq -\lambda_V V + \varepsilon \end{aligned} \tag{51}$$

which leads to Theorem 3. □

6 Nonlinear PID Adaptive Control (PIDAC)

Feedback gains in the nonlinear PID robust controller are required to satisfy the condition (48), in which the lower bounds are not quite straightforward to calculate. Although analytic formula exist to calculate them, as given in the above development, often the calculated lower bounds are so conservative and so large that they actually may not be used in implementation because of the limited bandwidth of physical systems. Also, the constant feedforward control term f_c may not quite match the low frequency component of the feedforward term because of parametric uncertainties. In this section, a gain-based nonlinear PID adaptive controller is proposed to solve these difficulties. We assume that only bounded disturbances appear — i.e., h_f in (3) is a constant instead of a function of states.

First, choose any $Q_\xi > 2\varepsilon_3 I$ and determine $K_x = B_\xi^T P_\xi$ by (26). There exist γ_7 and γ_{10} satisfying (48), and γ_6 and γ_{11} satisfying (33). Thus there exist constant $\bar{K}_\xi$ and $\bar{\gamma}_5$ such that (48) is satisfied. In the following, we do not need to calculate $\bar{K}_\xi$ and $\bar{\gamma}_5$, but only need to know their existence. The following control law is suggested:

$$u = \hat{f}_c - (\hat{K}_\xi + \hat{\gamma}_5 \|x_\xi\|^2)\xi - K_x x_\xi \tag{52}$$

Let β_K be the independent components of K_ξ. For example, if we want a diagonal K_ξ, β_K consists of the n diagonal elements only. $\hat{\beta}_K$ represents its estimate. Then we can write

$$\begin{array}{ll} \bar{K}_\xi \xi = Y_K(\xi)\bar{\beta}_K & \hat{K}_\xi \xi = Y_K(\xi)\hat{\beta}_K \\ \tilde{K}_\xi \xi = (\hat{K}_\xi - \bar{K}_\xi)\xi = Y_K(\xi)\tilde{\beta}_K & \tilde{\beta}_K = \hat{\beta}_K - \bar{\beta}_K \end{array} \tag{53}$$

where $Y_K(\xi)$ is a matrix of known functions. The gain adaptation law is chosen as

$$\begin{array}{l} \dot{\hat{f}}_c = \Gamma'_f[-\Gamma''_f(\hat{f}_c - f_{c0}) - \xi] \\ \dot{\hat{\beta}}_K = \Gamma'_{\beta K}[-\Gamma''_{\beta K}(\hat{\beta}_K - \beta_{K0}) + Y_K(\xi)^T \xi] \\ \dot{\hat{\gamma}}_5 = \Gamma'_\gamma[-\Gamma''_\gamma(\hat{\gamma}_5 - \gamma_{50}) + \|x_\xi\|^2 \|\xi\|^2] \end{array} \tag{54}$$

where $\Gamma'_f, \Gamma''_f, \Gamma'_{\beta K}, \Gamma''_{\beta K}, \Gamma'_\gamma$, and Γ''_γ are any constant s.p.d. matrices or scalars; f_{c0}, β_{K0}, and γ_{50} are the corresponding initial estimates. Choose a p.s.d. function as

$$V_a = V + \frac{1}{2}\tilde{f}_c^T {\Gamma'_f}^{-1} \tilde{f}_c + \frac{1}{2}\tilde{\beta}_K^T {\Gamma'_{\beta K}}^{-1} \tilde{\beta}_K + \frac{1}{2}\tilde{\gamma}_5^T {\Gamma'_\gamma}^{-1} \tilde{\gamma}_5 \tag{55}$$

where $\tilde{f}_c = \hat{f}_c - f_c$, $\tilde{\gamma}_5 = \hat{\gamma}_5 - \bar{\gamma}_5$, and V is defined by (34).

Theorem 4 *If the control law (52) with the gain adaptation law (54) is applied, then*

A . *In general, all signals in the system remain bounded and*

$$V_a \le \left[exp(-\lambda''_V t) V_a(0) + \int_0^t exp(-\lambda''_V (t-\nu)) \gamma_{17}(\nu) d\nu \right] \tag{56}$$

where γ_{17} and λ''_V are some positive scalars satisfying

$$\begin{array}{l} \gamma_{17} = \varepsilon + \frac{\|\Gamma''_f(f_c - f_{c0})\|^2}{4\gamma_{14}} + \frac{\|\Gamma''_{\beta K}(\bar{\beta}_K - \beta_{K0})\|^2}{4\gamma_{15}} + \frac{\|\Gamma''_\gamma(\bar{\gamma}_5 - \gamma_{50})\|^2}{4\gamma_{16}} \\ \lambda''_V \ge 2\frac{min\{\varepsilon_3,\ \lambda_{min}(\Gamma''_f) - \gamma_{14},\ \lambda_{min}(\Gamma''_{\beta K}) - \gamma_{15},\ \Gamma''_\gamma - \gamma_{16}\}}{max\{k_M,\ \lambda_{max}(P_\xi),\ \lambda_{max}({\Gamma'}_f^{-1}),\ \lambda_{max}({\Gamma'}_{\beta K}^{-1}),\ 1/\Gamma'_\gamma\}} \end{array} \tag{57}$$

in which γ_{14}, γ_{15}, and γ_{16} are any positive scalars such that $\gamma_{14} < \lambda_{min}(\Gamma''_f)$, $\gamma_{15} < \lambda_{min}(\Gamma''_{\beta K})$, $\gamma_{16} < \Gamma''_\gamma$.

B . *In addition, if the initial estimates β_{K0} and γ_{50} are large enough such that the condition (48) is satisfied for $f_c = f_{c0}$, then,*

$$V_a \le \left[exp(-\lambda'_V t)V_a(0) + \int_0^t exp(-\lambda'_V(t-\nu))\varepsilon(\nu)d\nu\right] \tag{58}$$

where the exponentially converging rate λ'_V satisfies

$$\lambda'_V \ge 2\frac{min\{\varepsilon_3,\ \lambda_{min}(\Gamma''_f),\ \lambda_{min}(\Gamma''_{\beta K}),\ \Gamma''_\gamma\}}{max\{k_M,\ \lambda_{max}(P_\xi),\ \lambda_{max}(\Gamma'^{-1}_f),\ \lambda_{max}(\Gamma'^{-1}_{\beta K}),\ 1/\Gamma'_\gamma\}} \tag{59}$$

i.e., tracking errors exponentially converge to some balls whose sizes are proportional to controller parameter ε. △

Remark 12 *The above adaptive controller does not require any model information and has a simple nonlinear PID feedback structure. Thus, it can be easily implemented and costs little computation time, however, bounded disturbances are assumed in the development, and asymptotic stability is not guaranteed even in the presence of parameter uncertainties only. Also, when the initial estimates do not satisfy the condition (48), the error bound γ_{17} in (56) is not guaranteed to be reduced by suitably choosing controller gains and theoretical performance may thus not be guaranteed.* ◇

Proof of Theorem 4: Rewrite (52) as

$$\begin{aligned} u &= \bar{u} + \tilde{f}_c - Y_K(\xi)\tilde{\beta}_K - \tilde{\gamma}_5\|x_\xi\|^2\xi \\ \bar{u} &= f_c - (\bar{K}_\xi + \bar{\gamma}_5\|x_\xi\|^2)\xi - K_x x_\xi \end{aligned} \tag{60}$$

Following similar derivations as in (50) and (51), and noting that $\dot{\tilde{\bullet}} = \dot{\hat{\bullet}}$ with adaptation law (54), we can obtain

$$\begin{aligned} \dot{V}_a \;=\;& \xi^T[\tilde{f}_c - Y_K(\xi)\tilde{\beta}_K - \tilde{\gamma}_5\|x_\xi\|^2\xi] + \xi^T[\bar{u} - Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta - \tilde{f} + B_\xi^T P_\xi x_\xi] \\ & -\tfrac{1}{2}x_\xi^T Q_\xi x_\xi + \tilde{f}_c^T[-\Gamma''_f(\hat{f}_c - f_{c0}) - \xi] + \tilde{\beta}_K^T[-\Gamma''_{\beta K}(\hat{\beta}_K - \beta_{K0}) + Y_K(\xi)^T\xi] \\ & +\tilde{\gamma}_5^T[-\Gamma''_\gamma(\hat{\gamma}_5 - \gamma_{50}) + \|x_\xi\|^2\|\xi\|^2] \\ \le\;& \varepsilon - \xi^T[\bar{K}_\xi - (\gamma_2 + \gamma_6 + \gamma_8 + \tfrac{c_0^2}{4\varepsilon})I_n]\xi - x_\xi^T[\tfrac{1}{2}Q_\xi - (\gamma_7 + \gamma_{10})I_{n+n_c}]x_\xi \\ & -[\bar{\gamma}_5 - (\gamma_9 + \gamma_{11})]\|x_\xi\|^2\|\xi\|^2 - \tilde{f}_c^T\Gamma''_f\tilde{f}_c - \tilde{f}_c^T\Gamma''_f(f_c - f_{c0}) - \tilde{\beta}_K^T\Gamma''_{\beta K}\tilde{\beta}_K \\ & -\tilde{\beta}_K^T\Gamma''_{\beta K}(\bar{\beta}_K - \beta_{K0}) - \tilde{\gamma}_5^T\Gamma''_\gamma\tilde{\gamma}_5 - \tilde{\gamma}_5^T\Gamma''_\gamma(\bar{\gamma}_5 - \gamma_{50}) \\ \le\;& \varepsilon - \varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) - \tilde{f}_c^T\Gamma''_f\tilde{f}_c - \tilde{f}_c^T\Gamma''_f(f_c - f_{c0}) - \tilde{\beta}_K^T\Gamma''_{\beta K}\tilde{\beta}_K \\ & -\tilde{\beta}_K^T\Gamma''_{\beta K}(\bar{\beta}_K - \beta_{K0}) - \tilde{\gamma}_5^T\Gamma''_\gamma\tilde{\gamma}_5 - \tilde{\gamma}_5^T\Gamma''_\gamma(\bar{\gamma}_5 - \gamma_{50}) \end{aligned} \tag{61}$$

In general, from (61):

$$\begin{aligned} \dot{V}_a \;\le\;& -\tilde{f}_c^T(\Gamma''_f - \gamma_{14}I_n)\tilde{f}_c - \tilde{\beta}_K^T(\Gamma''_{\beta K} - \gamma_{15}I)\tilde{\beta}_K - \tilde{\gamma}_5^T(\Gamma''_\gamma - \gamma_{16})\tilde{\gamma}_5 + \gamma_{17} \\ \le\;& -\lambda''_V V_a + \gamma_{17} \end{aligned} \tag{62}$$

which leads to Conclusion A of Theorem 4 by noting that γ_{17} is bounded.

Now, let us consider the case that the initial estimates β_{K0} and γ_{50} satisfy the condition (48) for $f_c = f_{c0}$. Since the only condition in choosing $f_c, \bar{\beta}_K$, and $\bar{\gamma}_5$ is that they should satisfy the condition (48), we can choose $f_c = f_{c0}$, $\bar{\beta}_K = \beta_{K0}$ and $\bar{\gamma}_5 = \gamma_{50}$. In such a case, (61) becomes

$$\begin{aligned} \dot{V}_a &\leq \varepsilon - \varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) - \tilde{f}_c^T \Gamma_f'' \tilde{f}_c - \tilde{\beta}_K^T \Gamma_{\beta K}'' \tilde{\beta}_K - \tilde{\gamma}_5^T \Gamma_\gamma'' \tilde{\gamma}_5 \\ &\leq -\lambda_V' V_a + \varepsilon \end{aligned} \tag{63}$$

which leads to Conclusion B of Theorem 4. □

7 Desired Compensation Adaptive Robust Control with Adjustable Gains (ARCAG)

The DCARC scheme in section IV requires that feedback gains satisfy condition (32), which has the same drawback as the nonlinear PID robust control (NPID) scheme, as pointed out in the above section. In this section, by incorporating the gain-based adaptive control synthesis technique into the design of the DCARC scheme, a new adaptive robust controller is proposed to overcome this difficulty.

As in the above section, choosing any $Q_\xi > 2\varepsilon_3 I$ and obtaining $K_x = B_\xi^T P_\xi$ by (26), there exist constant $\bar{K}_\xi$ and $\bar{\gamma}_5$ such that (32) is satisfied. Since $\bar{K}_\xi$ and $\bar{\gamma}_5$ are unknown, instead of using constant feedback gains K_ξ and γ_5 in (30), we will adjust them as in the above gain-based adaptive control. The resulting control law is given by

$$\begin{aligned} u &= u_a + \hbar(-h_s \tfrac{\xi}{\|\xi\|}) \\ u_a &= f_0(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d) + Y(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\hat{\beta}_\pi - \hat{K}_\xi \xi - K_x x_\xi - \hat{\gamma}_5 \|x_\xi\|^2 \xi \end{aligned} \tag{64}$$

in which the parameter adaptation law for β is the same as in DCARC, and the gain adaptation laws are suggested as

$$\begin{aligned} \dot{\hat{\beta}}_K &= \Gamma_{\beta K}' [-\Gamma_{\beta K}''(\hat{\beta}_K - \beta_{K0}) + Y_K(\xi)^T \xi] \\ \dot{\hat{\gamma}}_5 &= \Gamma_\gamma' [-\Gamma_\gamma''(\hat{\gamma}_5 - \gamma_{50}) + \|x_\xi\|^2 \|\xi\|^2] \end{aligned} \tag{65}$$

Choose a positive definite (p.d.) function as

$$V_p = V + \frac{1}{2}\tilde{\beta}_K^T {\Gamma_{\beta K}'}^{-1} \tilde{\beta}_K + \frac{1}{2}\tilde{\gamma}_5^T {\Gamma_\gamma'}^{-1} \tilde{\gamma}_5 \tag{66}$$

where V is defined by (34).

Theorem 5 *If the control law (64) with adaptation laws (31) and (65) is applied,*

A. *In general, all signals in the system remain bounded and*

$$V_p \leq \left[exp(-\lambda_{V_p}'' t) V_p(0) + \int_0^t exp(-\lambda_{V_p}(t-\nu)) \gamma_{18}(\nu) d\nu \right] \tag{67}$$

where

$$\gamma_{18} = \varepsilon + \frac{\|\Gamma''_{\beta K}(\bar{\beta}_K-\beta_{K0})\|^2}{4\gamma_{15}} + \frac{\|\Gamma''_{\gamma}(\bar{\gamma}_5-\gamma_{50})\|^2}{4\gamma_{16}}$$
$$\lambda''_{V_p} \geq 2\frac{min\{\varepsilon_3,\ \lambda_{min}(\Gamma''_{\beta K})-\gamma_{15},\ \Gamma''_{\gamma}-\gamma_{16}\}}{max\{k'',\ \lambda_{max}(P_\xi),\ \ \lambda_{max}(\Gamma'^{-1}_{\beta K}),\ 1/\Gamma'_{\gamma}\}} \tag{68}$$

in which γ_{15} *and* γ_{16} *are any positive scalars such that* $\gamma_{15} < \lambda_{min}(\Gamma''_{\beta K})$ *and* $\gamma_{16} < \Gamma''_{\gamma}$

B. *In addition, if the initial estimates* β_{K0} *and* γ_{50} *are large enough such that the condition (32) is satisfied, then,*

$$V_p \leq \left[exp(-\lambda'_{V_p}t)V_p(0) + \int_0^t exp(-\lambda'_{V_p}(t-\nu))\varepsilon(\nu)d\nu\right] \tag{69}$$

where λ'_{V_p} *is a scalar satisfying*

$$\lambda'_{V_p} \geq 2\frac{min\{\varepsilon_3,\ \ \lambda_{min}(\Gamma''_{\beta K}),\ \Gamma''_{\gamma}\}}{max\{k_M,\ \lambda_{max}(P_\xi),\ \ \lambda_{max}(\Gamma'^{-1}_{\beta K}),\ 1/\Gamma'_{\gamma}\}} \tag{70}$$

and, thus, tracking errors exponentially converge to some balls whose sizes are proportional to the controller parameter ε. △

Proof of Theorem 5: Rewrite (64) as

$$\begin{aligned} u &= \bar{u} - Y_K(\xi)\tilde{\beta}_K - \tilde{\gamma}_5\|x_\xi\|^2\xi \\ \bar{u} &= f_0(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d) + Y(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d)\hat{\beta}_\pi - (\bar{K}_\xi + \bar{\gamma}_5\|x_\xi\|^2)\xi - K_x x_\xi + \hbar \end{aligned} \tag{71}$$

and define $\dot{V}\mid_{\bar{u}}$ as (actually the derivative of V under the control $\bar{u}$)

$$\dot{V}\mid_{\bar{u}} \quad = \xi^T[\bar{u} - f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) - Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta - \tilde{f} + B_\xi^T P_\xi x_\xi] - \tfrac{1}{2}x_\xi^T Q_\xi x_\xi \tag{72}$$

Noting (39) and (65), we have

$$\begin{aligned} \dot{V}_p &= \dot{V}\mid_{\bar{u}} + \xi^T[-Y_K(\xi)\tilde{\beta}_K - \tilde{\gamma}_5\|x_\xi\|^2\xi] + \tilde{\beta}_K^T{\Gamma'_{\beta K}}^{-1}\dot{\hat{\beta}}_K + \tilde{\gamma}_5^T{\Gamma'_{\gamma}}^{-1}\dot{\hat{\gamma}}_5 \\ &= \dot{V}\mid_{\bar{u}} - \tilde{\beta}_K^T\Gamma''_{\beta K}\tilde{\beta}_K - \tilde{\beta}_K^T\Gamma''_{\beta K}(\bar{\beta}_K - \beta_{K0}) - \tilde{\gamma}_5^T\Gamma''_{\gamma}\tilde{\gamma}_5 - \tilde{\gamma}_5^T\Gamma''_{\gamma}(\bar{\gamma}_5 - \gamma_{50}) \end{aligned} \tag{73}$$

Noting that $\dot{V}\mid_{\bar{u}}$ has the same form as $\dot{V}$ in (39) with u replaced by $\bar{u}$ and that $\bar{u}$ is the same as the control (30) used in DCARC with gains satisfying (32), all the derivations from (39) to (44) remain valid if we replace $\dot{V}$ by $\dot{V}\mid_{\bar{u}}$. Thus, in general, from (44),

$$\dot{V}\mid_{\bar{u}} \quad \leq -\varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) + \varepsilon \tag{74}$$

and when $\tilde{f} = 0$, from (43),

$$\dot{V}\mid_{\bar{u}} \quad \leq -\varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) + \tilde{\beta}_\pi^T Y^T(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d)\xi \tag{75}$$

From (74) and (73),

$$\begin{aligned} \dot{V}_p \quad \leq \quad & -\varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) + \varepsilon - \tilde{\beta}_K^T\Gamma''_{\beta K}\tilde{\beta}_K \\ & -\tilde{\beta}_K^T\Gamma''_{\beta K}(\bar{\beta}_K - \beta_{K0}) - \tilde{\gamma}_5^T\Gamma''_{\gamma}\tilde{\gamma}_5 - \tilde{\gamma}_5^T\Gamma''_{\gamma}(\bar{\gamma}_5 - \gamma_{50}) \end{aligned} \tag{76}$$

Following similar arguments as in the proof of Theorem 4, Theorem 5 can be obtained from (76). □

In the following, we will show that this controller can actually do more than what stated in the above theorem, a reasonable assertion in view of the great performance offered by its counterpart DCARC.

Theorem 6 *Consider the situation that after a finite time, $\tilde{f} = 0$ (no uncertain nonlinearities). If the initial gain estimates β_{K0} and γ_{50} satisfy condition (32), in addition to the results in B of Theorem 5, asymptotic tracking is also achieved.* △

Proof of Theorem 6: Since the initial gain estimates satisfy the condition (32), we can set $\bar{\beta}_K = \beta_{K0}$ and $\bar{\gamma}_5 = \gamma_{50}$. Thus, when $\tilde{f} = 0$, from (75) and (73),

$$\dot{V}_p \leq -\varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) + \tilde{\beta}_\pi^T Y^T(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)\xi - \tilde{\beta}_K^T \Gamma''_{\beta K} \tilde{\beta}_K - \tilde{\gamma}_5^T \Gamma''_\gamma \tilde{\gamma}_5 \tag{77}$$

Choose a p.s.d. function V_{pa} as $V_{pa} = V_p + V_\beta(\tilde{\beta}, \beta)$ where V_β is defined in (82). Following similar arguments as in (46), we can obtain that

$$\dot{V}_{pa} \leq -\varepsilon_3(\|\xi\|^2 + \|x_\xi\|^2) - \tilde{\beta}_K^T \Gamma''_{\beta K} \tilde{\beta}_K - \tilde{\gamma}_5^T \Gamma''_\gamma \tilde{\gamma}_5 \tag{78}$$

Thus, by applying Barbalat's lemma as in the proof of Theorem 2, Theorem 6 can be proved. □

8 Experimental Results

All schemes presented before are implemented and compared. In addition, Slotine and Li's adaptive algorithm [4] and Sadegh and Horowitz's DCAL [5], which achieves the best tracking performance in the experiments reported by Whitcomb, et al [16], are also implemented for comparison.

8.1 Experimental Setup

Experiments are conducted on the planar UCB/NSK two axis SCARA direct drive manipulator system. Fig. 1 shows the experimental set-up. A 486 PC equipped with IBM Data Acquisition and Control Adapters (DACA) board is used to control the entire setup. Each DACA board contains two 12 bit D/A and four 12 bit A/D converters. Motor position resolution is 153,600 pulses per revolution (or 4.09×10^{-5} rad). The velocity signal is then obtained by the difference of two consecutive position measurements with a first-order filter [2]. The real-time code is written in C language. Details of the experimental setup

[2] The robot is equipped with tachometers to measure the joint rotation velocities, which are fed to the 486 PC through the A/D channels of the IBM DACA board, but the signals are too noisy and not used.

and modeling can be found in [34]. The matrices in dynamic equation (1) are given by [34]

$$\begin{aligned} M(q,\beta) &= \begin{bmatrix} p_1 + 2p_3C_{q2} & p_2 + p_3C_{q2} \\ p_2 + p_3C_{q2} & p_2 \end{bmatrix} \\ C(q,\dot{q},\beta) &= \begin{bmatrix} -p_3\dot{q}_2S_{q2} & -p_3(\dot{q}_1 + \dot{q}_2)S_{q2} \\ p_3\dot{q}_1S_{q2} & 0 \end{bmatrix} \\ G(q,\beta) &= 0 \end{aligned} \tag{79}$$

where $C_{q2} = \cos(q_2)$, $S_{q2} = \sin(q_2)$, p_1, p_2, and p_3, the combined robot and payload parameters, are given by $p_1 = p_{a1} + 0.194m_p$, $p_2 = p_{a2} + 0.0644m_p$, and $p_3 = p_{a3} + 0.0864m_p$, respectively, m_p is the payload mass, and $p_{a1} = 3.1623$, $p_{a2} = 0.1062$, and $p_{a3} = 0.17285$ are the robot parameters. The friction term $F_f(q,\dot{q})$ is lumped into $\tilde{f}(q,\dot{q},t)$ and is bounded by (3), where $h_f = 9$. In the experiment, only payload mass m_p is unknown with the maximum payload, $m_{pmax} = 10kg$. Thus, letting $\beta = m_p$ and $\Omega_\beta = (\ -0.00001,\ \ m_{pmax} + 0.00001)$, (2) can be formed. Since all the controllers are supposed to deal with model uncertainties, the initial estimate of the payload is set to $9kg$, with an actual value in experiments being around $0.7kg$. All experiments are conducted with a sampling time $\Delta T = 1ms$.

8.2 Performance Indexes

Since we are interested in tracking performance, sinusoidal trajectories with a smoothed initial starting phase are adopted for each joint. In this experiment, the desired joint trajectories are $q_d = [1.5(1.181 - 0.3343exp(-5t) - cos(\pi t - 0.561))\ ,\ 1.3045 - 0.538exp(-5t) - cos(\frac{4}{3}\pi t - 0.697))]^T$ (rad), which are reasonably fast. Zero initial tracking errors are used and each experiment is run for ten seconds, i.e, $T_f = 10s$.

Commonly used performance measures, such as the rising time, damping and steady state error, are not adequate for nonlinear systems like robots. In [16], the scalar valued L^2 norm given by $L^2[e(t)] = (\frac{1}{T_f}\int_0^{T_f} \|e(t)\|^2 dt)^{1/2}$ is used as an objective numerical measure of tracking performance for an entire error curve $e(t)$. However, it is an average measure, and large errors during the initial transient stage cannot be predicted. Thus, the sum of the maximal absolute value of tracking error of each joint, $e_M = e_{1M} + e_{2M}$, is used as an index of measure of transient performance, in which $e_{iM} = \max_{t\in[0,T_f]}\{|e_i(t)|\}$. The maximal absolute value and the average tracking error of each joint during the last three seconds are defined by $e_{iF} = \max_{t\in[T_f-3,T_f]}\{|e_i(t)|\}$ and $L[e_{if}] = \frac{1}{3}\int_{T_f-3}^{T_f} |e_i|dt$ respectively. Then, $e_F = e_{1F} + e_{2F}$ and $L[e_f] = L[e_{1f}] + L[e_{2f}]$ are used as indexes to measure the steady state tracking error. The average control input of each joint, $L[u_i] = \frac{1}{T_f}\int_0^{T_f} |u_i|dt$, is used to evaluate the amount of control effort. The average of control input increments of each joint is defined by $L[\Delta u_i] = \frac{1}{10000}\sum_{k=1}^{10000} |u_i(k\Delta T) - u_i((k-1)\Delta T)|$. The sum of the normalized control variations of each joint, $c_u = \sum_{i=1}^{2} \frac{L[\Delta u_i]}{L[u_i]}$, is used to measure the degree of control chattering.

8.3 Controller Gains

The choice of feedback gains is crucial to achieve a good tracking performance for all controllers. A discussion of the gain tuning processes for each controller follows in detail. In general, the larger the feedback gains (especially, the gain K_ξ), the smaller the tracking errors. However, if the gains are too big, the robot will be subject to severe control chattering due to the measurement noise and the neglected high-frequency dynamics and a large noisy sound can be heard. After the gains exceed certain limits, the structural resonance is excited because of severe control chattering and the system goes unstable. Thus, in order to achieve a fair comparison, we will try to tune gains of each controller such that the tracking errors of each controller are minimized while maintaining the same degree of control chattering for all controllers.

ASMC: Adaptive Sliding Mode Control .

As explained in Remark 6 in Section III, a dynamic compensator ($n_c = 2$) is formed by (4), in which $A_z = 0I_2, B_z = 400I_2, C_z = I_2, D_z = 40I_2$ with initial values calculated on-line by (8). Such a choice of gains guarantees that the resulting sliding mode is critically damped with corner frequency $w_\xi = 20$ rad/sec.

The adaptation law is given by (18) with the modification (23) where $\Gamma_\beta = 10$. Thus, $\Omega_{\hat{\beta}} = \bar{\Omega}_\beta$ and $h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ can be determined by (10). The control law is then formed by (17), in which $K_\xi = diag\{40, 5\}$ and u_s is determined by (16) where $K_s = diag\{60, 4\}$, $\varepsilon_1 = 1$, $\varepsilon_2 = 0.5$, and $\phi = 300$.

SLAC: Slotine and Li's Adaptive Algorithm .

The control law is formed as explained in Remark 7 in Section III, in which $D_z = 20I_2$ is used to provide the same corner frequency w_ξ for the sliding mode as in ASMC. A large $K_\xi = diag\{180, 15\}$ is used to produce roughly the same degree of control chattering as ASMC. This gain is slightly larger than the combined feedback gain for ξ, $K_\xi + K_s$ in ASMC.

DCARC: Desired Compensation Adaptive Robust Control .

The same dynamic compensator as ASMC is used. Letting $Q_\xi = diag\{10^5, 10^4\}$, P_ξ is calculated from (26) and the resulting gain matrix K_x is $[120, 0, 1250, 0;\ 0, 12, 0, 125]$. The control law is given by (30), in which $K_\xi = diag\{100, 8\}$ and $\gamma_5 = 1000$. u_s is given by (16), in which $K_s = diag\{60, 4\}, \varepsilon_1 = 1, \varepsilon_2 = 0.5$, $\phi = 200$, and h_s is calculated by (29). The adaptation law is given by (31) with the modification (23) and $\Gamma_\beta = 10$.

DCAL: Sadegh and Horowitz's Desired Compensation Adaptation Law .

The control law is formed as explained in Remark 10 in Section IV, in which $D_z = 20I_2$ as in SLAC. By using the same Q_ξ as in DCARC, the resulting K_x is $[0, 0, 2500, 0; 0, 0, 0, 250]$. A large $K_\xi = diag\{170, 14\}$ is used to produce roughly the same degree of control chattering as DCARC and the rest of controller parameters are the same as in DCARC.

DCRC: Desired Compensation Robust Control .

The control law is the same as in DCARC except not to use the adaptation law. In such a case, the proposed DCARC reduces to a robust control (termed as DCRC(I) in the following).

To verify the effect of using a dynamic compensator, the same control law is applied, but without using the dynamic compensator, i.e., without the integrator, which is obtained by setting $C_z = 0, A_z = 0, B_z = 0, D_z = 20I$. Correspondingly, $K_x = [0, 0, 2500, 0; 0, 0, 0, 250]$ by using the same Q_ξ (termed DCRC(NI) in the following).

NPID: Nonlinear PID Robust Control .

The control law is given by (47) with the same γ_5 and K_x as in DCRC. $f_c = 0$. A large $K_\xi = diag\{160, 12\}$ is used. NPID(I) stands for integrator case and NPID(NI) for no integrator case as in DCRC.

PIDAC: Nonlinear PID Adaptive Control .

The control law is given by (52) with the same K_x as in DCRC and a diagonal $K_\xi = diag\{\beta_{K1},\ \beta_{K2}\}$. The gain adaptation law is given by (54), where $\beta_{K0} = [10, 1]^T$, $\gamma_{50} = 1000$, $f_{c0} = 0, \Gamma_f{}' = diag\{10, 2\}$, $\Gamma_f{}'' = diag\{0.1, 0.1\}$, $\Gamma'_{\beta K} = diag\{1000, 10\}$, $\Gamma''_{\beta K} = diag\{0.0002, 0.02\}$, $\Gamma'_\gamma = 10^4$, and $\Gamma''_\gamma = 2 \times 10^{-5}$.

ARCAG: Desired Compensation Adaptive Robust Control with Adjustable Gains

The control law is given by (64) with the same K_x, u_s and the parameter adaptation law as DCARC and a diagonal $K_\xi = diag\{\beta_{K1},\ \beta_{K2}\}$. The gain adaptation law is given by (65) where $\beta_{K0} = [20, 3]^T$, $\gamma_{50} = 500$. $\Gamma'_{\beta K} = diag\{1000, 10\}$, $\Gamma''_{\beta K} = diag\{0.00003, 0.003\}$, $\Gamma'_\gamma = 10^4$, and $\Gamma''_\gamma = 10^{-4}$.

8.4 Comparative Experimental Results

As in [16], we first test the reliability of the results by running the same controller several times. It is found that the standard deviation of the error from different runs is negligible.

The experimental results are shown in the following table (unit is rad for tracking errors and Nm for control input torques).

The above results are also displayed in Fig. 2 to Fig. 6. Based on the above experimental data, the following can be concluded:

a . **Parameter Adaptation Improves Tracking Accuracy**
If we compare the parameter-based adaptive controllers with their robust counterparts, i.e., DCARC versus DCRC(I), DCAL versus DCRC(NI), then we can see that, in terms of both final tracking accuracy (Fig. 3) and average tracking errors (Fig. 4), parameter adaptation reduces the tracking

Table 1: Experimental Results

Controller	e_M	e_F	$L[e_f]$	$L_2[e]$	$L[u_1]$	$L[u_2]$	c_u
ASMC	0.0301	0.0167	0.0058	0.0058	32.1	6.2	0.54
SLAC	0.0520	0.0325	0.0160	0.0133	32.8	6.2	0.55
DCARC	0.0201	0.0134	0.0039	0.0039	30.6	6.4	0.41
DCAL	0.0353	0.0199	0.0092	0.0081	30.3	6.3	0.43
DCRC(I)	0.0256	0.0227	0.0077	0.0081	30.3	6.3	0.42
DCRC(NI)	0.0690	0.0486	0.0175	0.0209	29.6	6.1	0.40
NPID(I)	0.0202	0.0195	0.0066	0.0061	30.5	6.4	0.41
NPID(NI)	0.0386	0.0346	0.0151	0.0145	29.8	6.3	0.40
PIDAC	0.0705	0.0158	0.0057	0.0070	30.4	6.3	0.44
ARCAG	0.0364	0.0119	0.0035	0.0045	30.2	6.3	0.42

errors around a factor of 2. The parameter-based adaptive controllers also have better transient performances (Fig. 2). The improvement comes from the fact that the estimated payloads approach their true values, which is shown in Fig. 7. This result verifies the advantage of introducing *parameter adaptation*. All controllers use almost the same amount of control effort and have the same degree of control chattering, as shown in Fig. 5 and Fig. 6, and thus the comparison is fair.

b . **Dynamic Compensator Improves Tracking Accuracy**

Comparing the controllers having dynamic compensators with their counterparts not employing dynamic compensators, i.e., DCRC(I) versus DCRC(NI) and NPID(I) versus NPID(NI), we can see that introducing dynamic compensators reduces the tracking errors by more than a factor of 2 in terms of all the performance indexes, as shown in Fig. 2 to Fig. 4. The comparison is fair, as shown by the control effort in Fig. 5, and the degree of control chattering in Fig. 6. This result supports the importance of employing *proper controller structure*.

c . **Desired Compensation Improves Tracking Accuracy**

Comparing the controllers having desired compensation with their counterparts using actual state in model compensation design, i.e., DCARC versus ASMC and DCAL versus SLAC, we can see that, in terms of all performance indexes (Fig. 2 to Fig. 4), the controllers with desired compensation have better tracking performances. They also have less degrees of control chattering, as shown in Fig. 6.

d . **Gain-based Adaptive Controllers via Fixed-gain Robust Controllers**

If we compare the gain-based adaptive controllers with their robust counterparts, i.e., PIDAC versus NPID(I) and ARCAG versus DCARC, we can see that gain-based adaptive controllers can have a large stability margin for the choice of feedback gains since they can use small initial gain estimates. Because of the small initial estimates, they have larger initial tracking errors or poorer transient response, as seen from Fig. 2. The estimated feedback gains (e.g., $\hat{K}_\xi$ shown in Fig. 8) increase quickly to some values that are slightly larger than the fixed feedback gains used in their robust counterparts (e.g., when $t = 10s$, $\hat{K}_\xi(t) = diag\{180,\ 12.6\}$ for PIDAC but $K_\xi = diag\{160, 12\}$ for NPID(I)). This is the reason that they achieve a slightly better final tracking accuracy, as shown in Fig. 3. We should keep in mind, however, that this advantage comes from the slightly increased degree of control chattering, as shown in Fig. 6. Therefore, in practice, gain-based adaptive controllers do not offer much advantage in improving tracking performance. They may be used in the initial gain-tuning process to obtain the lower bound of the stabilizing feedback gains instead of using a troublesome and conservative theoretical formula like (48). However, caution should be taken. Large dampings (e.g., $\Gamma''_{\beta K}$ and Γ''_{γ} in (54)) should be used; otherwise, the resulting final estimates may be too big that they may exceed the practical limits and destabilize the system because of their gain adaptation nature.

Since the proposed DCARC possesses all the desirable good qualities — parameter adaptation, dynamic compensator, and desired compensation — it is natural that it achieves the best tracking performance, as seen from Fig. 2 to Fig. 4, by using the same amount of control effort (Fig. 5) and control chattering (Fig. 6). These facts show again the importance of using the both means, parameter adaptation and proper controller structure, in designing high performance controllers, which is the main theme of the proposed ARC. Using either one of them alone is not enough — in fact, in these experiments, probably because the effect of link dynamics is not so severe and the disturbances and measurement noise are not so small, the proposed simple NPID robust controller out-performs DCAL, the adaptive controller that achieves the best tracking performance among existing adaptive controllers tested.

The tracking errors of DCARC are plotted in Fig. 9 and the control inputs are shown in Fig. 10. Those spikes of the tracking errors after the initial transient occur at the time when the joint velocities change their directions. Thus, they are mainly caused by the discontinuous Columb friction.

9 Conclusions

In this paper, the proposed adaptive robust control is applied to the trajectory tracking control of robot manipulators. Two schemes are developed: ASMC is based on the conventional adaptation structure and DCARC is based on the desired compensation adaptation structure. A dynamic sliding mode is used to enhance the system response. In addition, several conceptually different robust and adaptive controllers are also constructed for comparison — a simple nonlinear PID type robust control, and a simple gain-based adaptive control, which requires almost no model information, and a combined parameter and gain-based adaptive robust control. All algorithms, as well as two existing adaptive control algorithms, SLAC and DCAL, are implemented on a two-link SCARA type robot manipulator to study their advantages and disadvantages. Comparative experimental results show the importance of using both *proper controller structure* and *parameter adaptation* in designing high performance controllers, which is the main feature in the newly developed adaptive robust control [12, 14, 15]. It is observed that in these experiments, the proposed DCARC improves tracking performance without increasing control effort. Thus, the work in this paper serves for the two purposes: improving tracking performance of robot control systems and setting up a standard with which various control algorithms could be compared.

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Appendix 1: Smooth Projection

Let $\varepsilon_{\beta i}$ be an arbitrarily small positive real number. There exists a real-valued, sufficiently smooth nondecreasing function π_i such that

$$\begin{array}{ll} \pi_i(\nu_i) = \nu_i & \forall \nu_i \in (\beta_{imin}, \beta_{imax}) \\ \pi_i(\nu_i) \in [\beta_{imin} - \varepsilon_{\beta i}\ ,\ \beta_{imax} + \varepsilon_{\beta i}] & \forall \nu_i \in R \end{array} \tag{80}$$

with bounded derivatives up to a sufficiently high order. Define a smooth projection $\pi\ :\ R^p \rightarrow R^p$ by

$$\pi(\nu) = [\pi_1(\nu_1), \ldots, \pi_p(\nu_p)]^T \tag{81}$$

for each vector $\nu \in R^p$ with $\nu = [\nu_1, \ldots, \nu_p]^T$. Let $\hat{\beta}_\pi = \pi(\hat{\beta})$, $\tilde{\beta}_\pi = \hat{\beta}_\pi - \beta$, and

$$V_\beta(\tilde{\beta}, \beta) = \sum_{i=1}^{p} \frac{1}{\gamma_{\beta i}} \int_0^{\tilde{\beta}_i} (\pi_i(\nu_i + \beta_i) - \beta_i) d\nu_i \qquad \gamma_{\beta i} > 0 \tag{82}$$

Then V_β is positive definite w.r.t $\tilde{\beta}$ for each $\beta \in \Omega_\beta$. Furthermore,

$$\frac{\partial}{\partial \tilde{\beta}} V_\beta(\tilde{\beta}, \beta) = [\frac{1}{\gamma_{\beta 1}}(\pi_1(\hat{\beta}_1) - \beta_1),\ \ldots\ , \frac{1}{\gamma_{\beta p}}(\pi_p(\hat{\beta}_p) - \beta_p)] = \tilde{\beta}_\pi\ {}^T \Gamma_\beta^{-1} \tag{83}$$

where $\Gamma_\beta = diag\{\gamma_{\beta 1}, \ldots, \gamma_{\beta p}\}$

Appendix 2

For revolute joint robots, q appears in $M(q,\beta), C(q,\dot{q},\beta)$, and $G(q,\beta)$ in the form of $cos(q_i)$ or $sin(q_i)$ only. Therefore, $M(q,\beta), C(q,\dot{q},\beta)$, and $G(q,\beta)$ and their partial derivatives are uniformly bounded with respective to q. Applying the mean value theorem, there exist non-negative scalars, $\alpha_1(\ddot{q}_d), \alpha_3(\dot{q}_d), \alpha_4(\dot{q}_d)$, and α_7 such that

$$\begin{array}{l} \|M(q_d,\beta)\ddot{q}_d - M(q,\beta)\ddot{q}_d\| \leq \alpha_1(\ddot{q}_d)\|e\| \\ \|C(q,\dot{q}_d,\beta)\| \leq \alpha_4(\dot{q}_d) \\ \|C(q_d,\dot{q}_d,\beta)\dot{q}_d - C(q,\dot{q}_d,\beta)\dot{q}_d\| \leq \alpha_3(\dot{q}_d)\|e\| \\ \|G(q_d,\beta) - G(q,\beta)\| \leq \alpha_7\|e\| \end{array} \tag{84}$$

Since $C(q,\dot{q},\beta)$ is linear w.r.t. $\dot{q}$, there exist non-negative scalars, $\alpha_5(\dot{q}_d)$ and α_6 such that

$$\|C(q,\dot{q},\beta)\| = \|C(q,\dot{q}_d + \dot{e},\beta)\| \leq \alpha_5(\dot{q}_d) + \alpha_6\|\dot{e}\| \tag{85}$$

Let $\alpha_z = \|C_z\ , D_z\|$ and noticing that $\dot{e} = \xi - [C_z, D_z]x_\xi$, we have,

$$\|\dot{e}\| \leq \|\xi\| + \alpha_z\|x_\xi\| \tag{86}$$

Noting $C(q,\dot{q},\beta)\dot{q}_d = C(q,\dot{q}_d,\beta)\dot{q}$, from (84) to (86),

$$\begin{array}{l} \|M(q_d,\beta)\ddot{q}_d - M(q,\beta)\ddot{q}_r\| \leq \|M(q_d,\beta)\ddot{q}_d - M(q,\beta)\ddot{q}_d\| + \|M(q,\beta)(\ddot{q}_d - \ddot{q}_r)\| \\ \quad \leq \alpha_1(\ddot{q}_d)\|e\| + k''\|[C_z, D_z]A_\xi\|\|x_\xi\| + k''\|[C_z, D_z]B_\xi\|\|x_\xi\| \\ \|C(q_d,\dot{q}_d,\beta)\dot{q}_d - C(q,\dot{q},\beta)\dot{q}_r\| \leq \|C(q_d,\dot{q}_d,\beta)\dot{q}_d - C(q,\dot{q}_d,\beta)\dot{q}_d\| + \|C(q,\dot{q}_d,\beta)\dot{q}_d \\ \quad -C(q,\dot{q},\beta)\dot{q}_d\| + \|C(q,\dot{q},\beta)\dot{q}_d - C(q,\dot{q},\beta)\dot{q}_r\| \leq \alpha_3(\dot{q}_d)\|e\| \\ \quad +\alpha_4(\dot{q}_d)\|\dot{e}\| + \|C(q,\dot{q},\beta)\|\|[C_z, D_z]x_\xi\| \leq [\alpha_3(\dot{q}_d) + \alpha_4(\dot{q}_d)\alpha_z \\ \quad +\alpha_5(\dot{q}_d)\alpha_z]\|x_\xi\| + \alpha_4\|\xi\| + \alpha_6\alpha_z\|\xi\|\|x_\xi\| + \alpha_6\alpha_z^2\|x_\xi\|^2 \end{array} \tag{87}$$

From (84) and (87), it is clear that there are known constants, $\gamma_1, \gamma_2, \gamma_3$, and γ_4, such that (27) is satisfied.

Figure 1: Experimental Setup

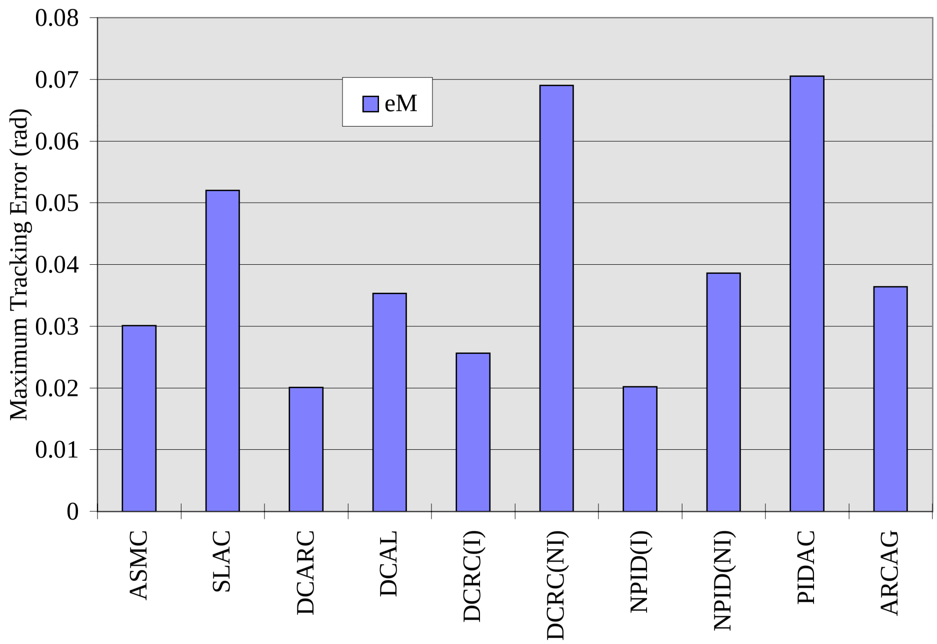

Figure 2: Transient Performance

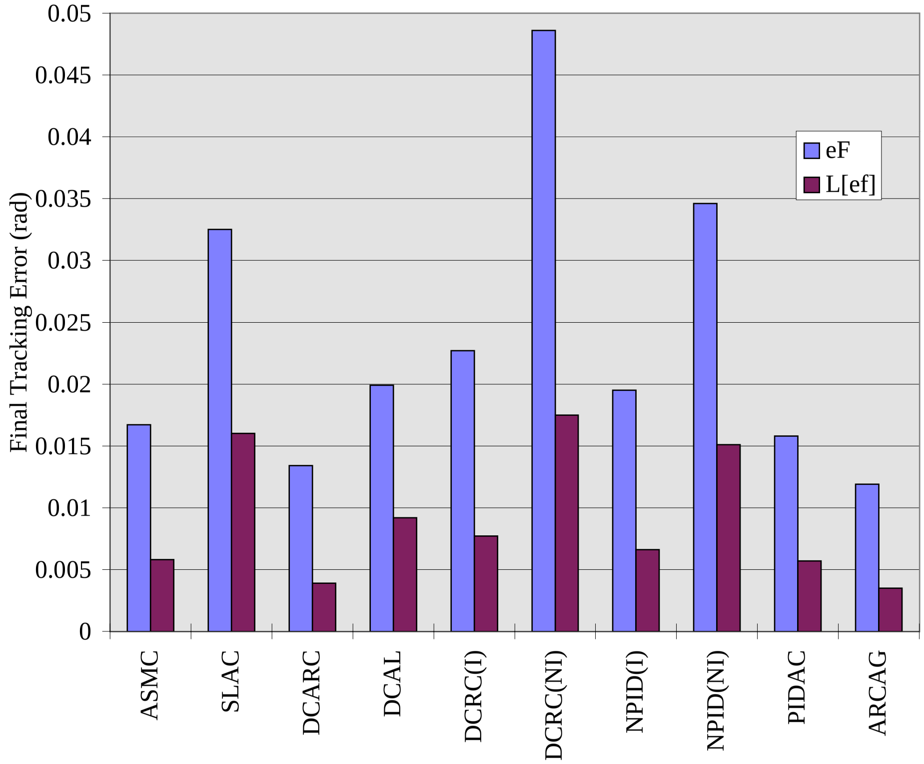

Figure 3: Final Tracking Accuracy

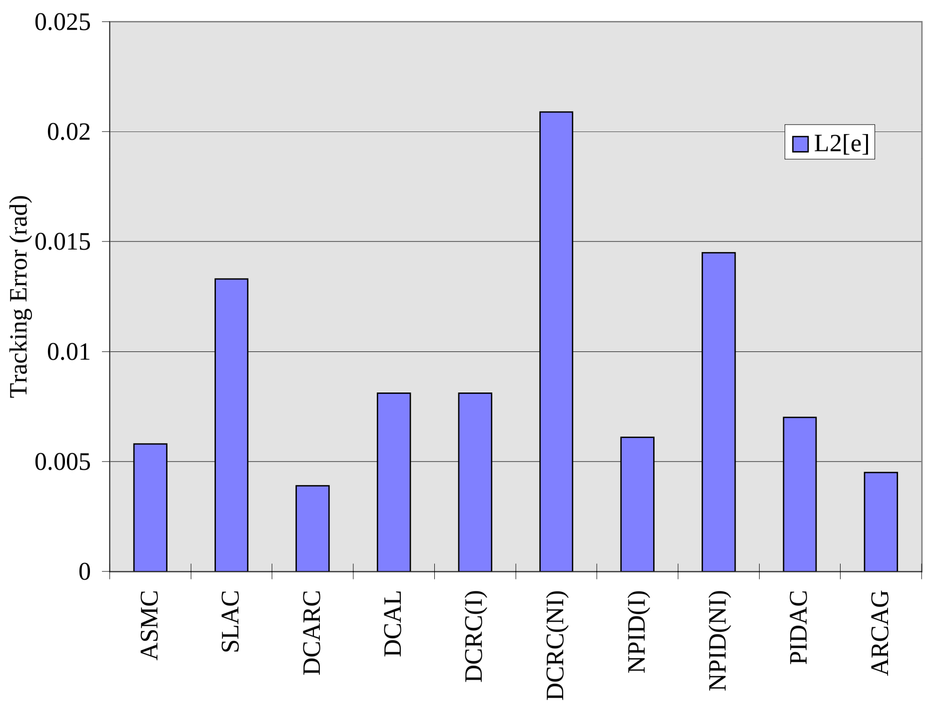

Figure 4: Average Tracking Errors

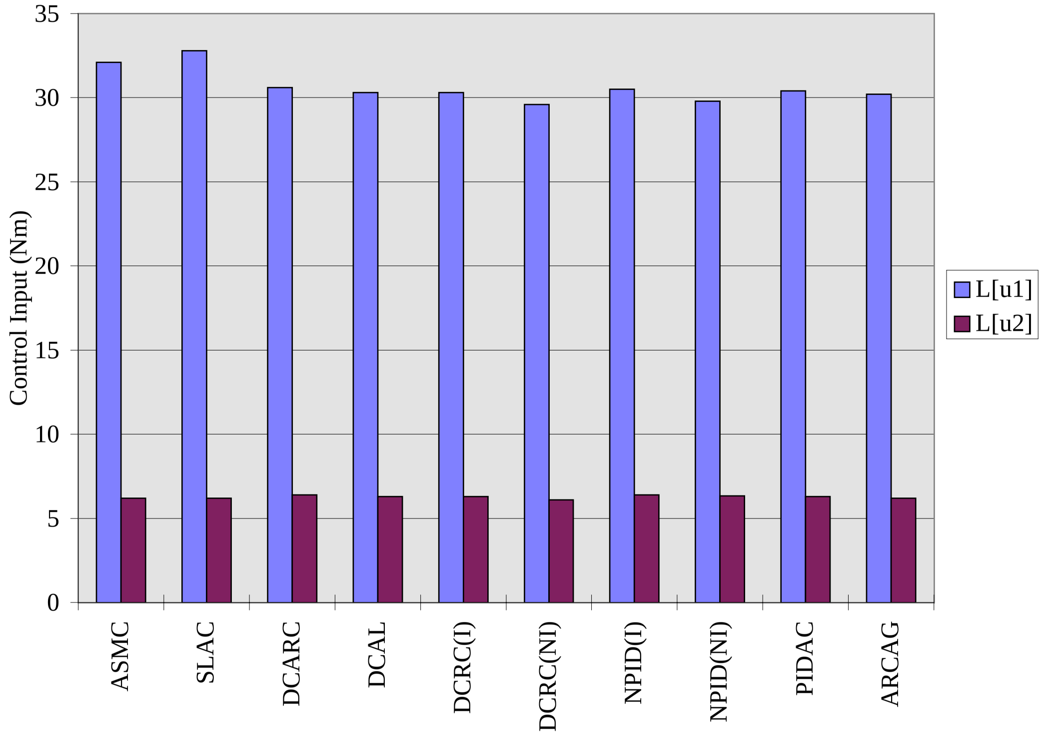

Figure 5: Control Effort

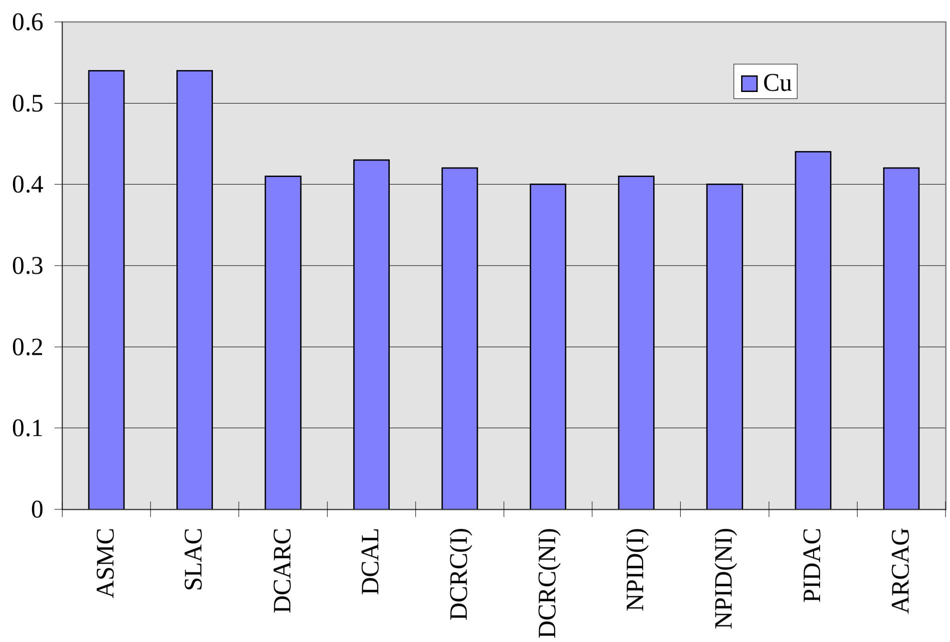

Figure 6: Control Chattering

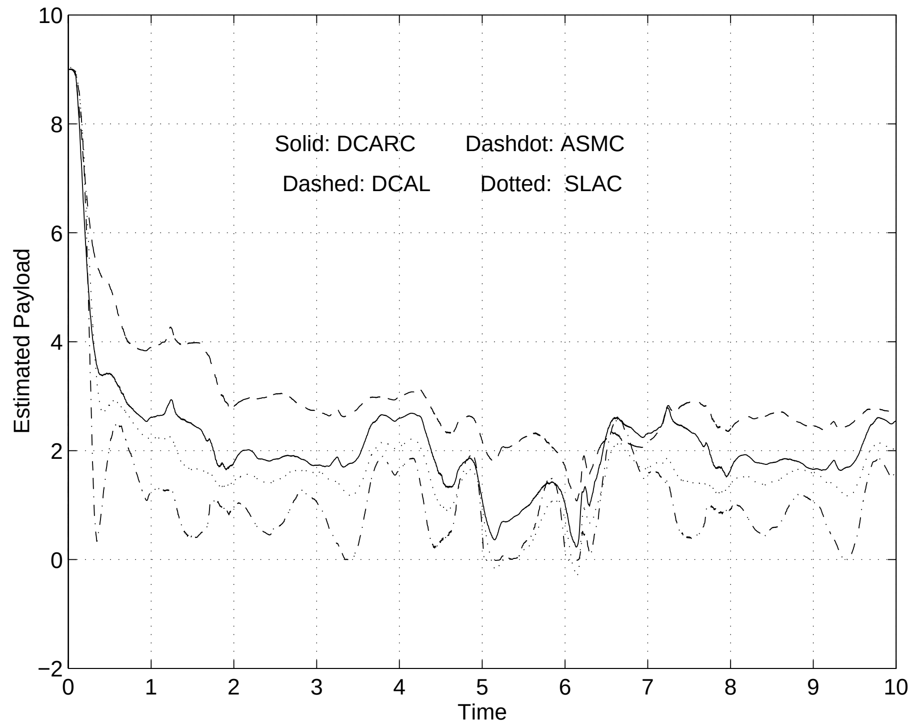

Figure 7: Estimated payloads approach their true values

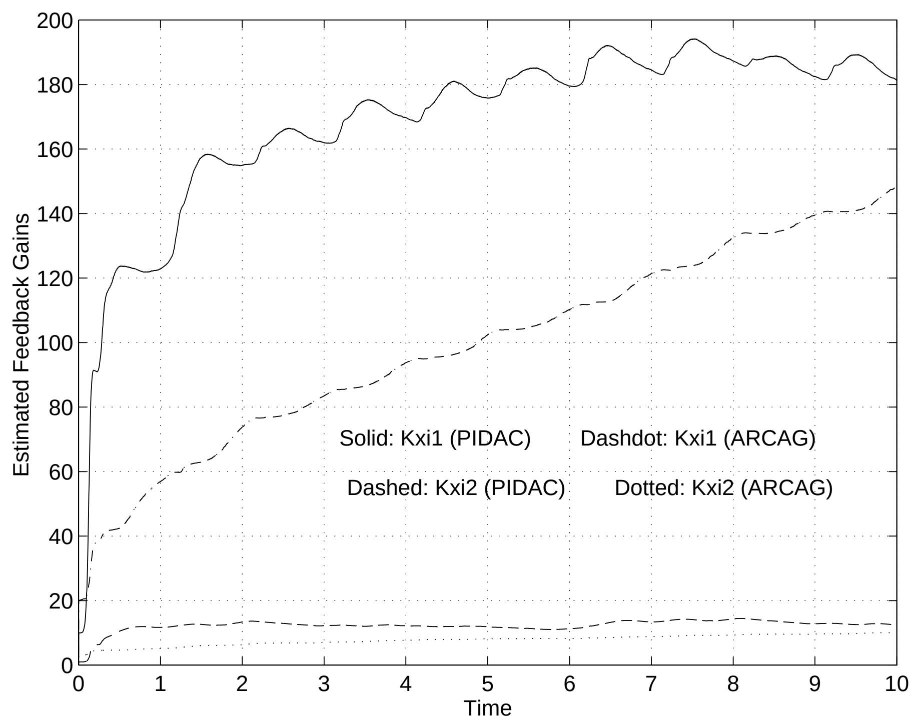

Figure 8: Estimated Feedback Gains $\hat{K}_{\xi}$

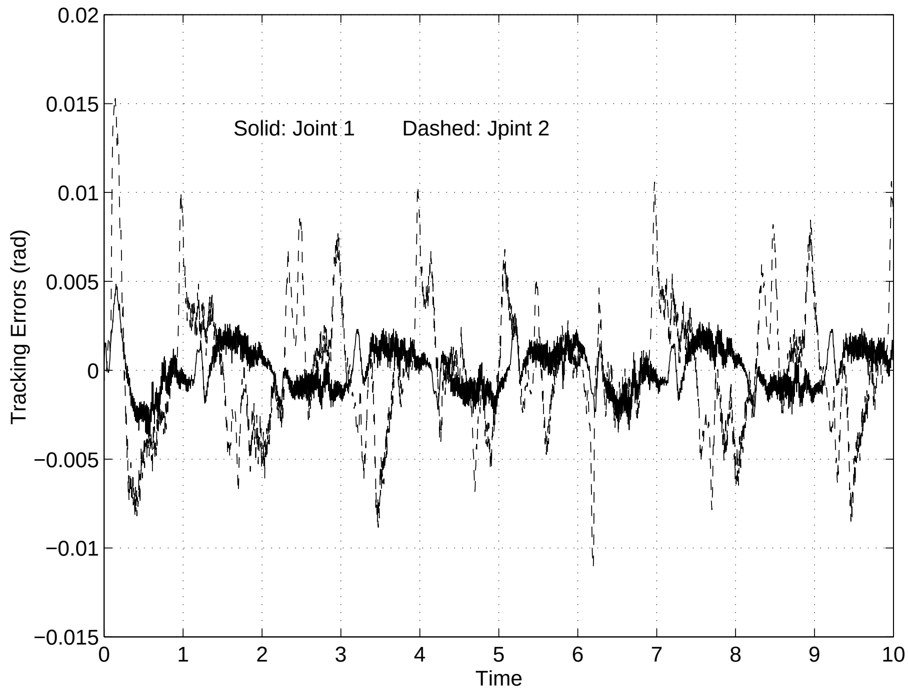

Figure 9: Joint Tracking Errors

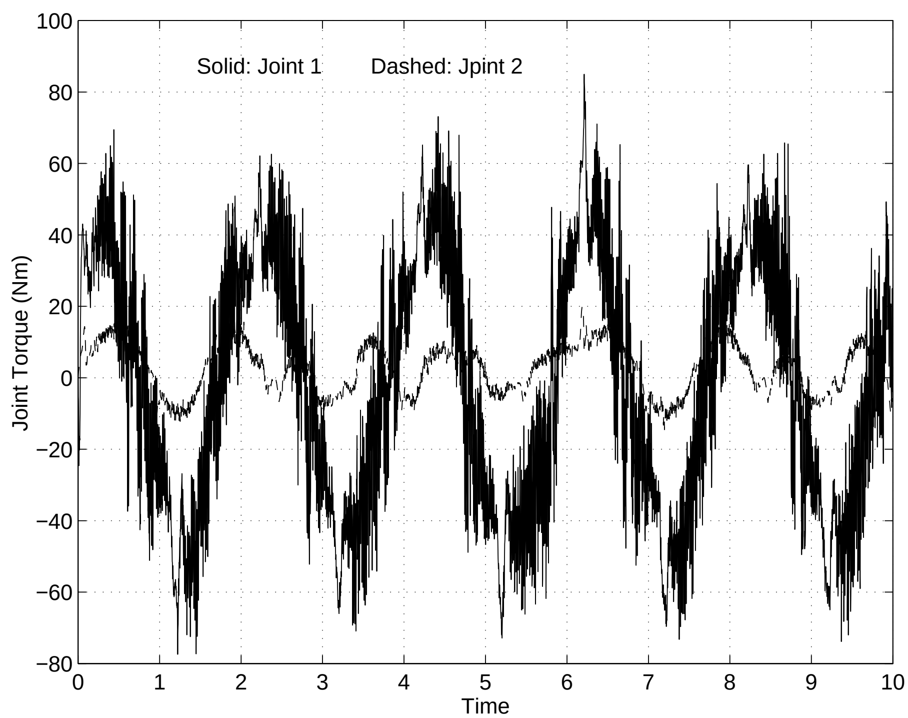

Figure 10: Joint Control Torque