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Smooth Robust Adaptive Sliding Mode Control of Manipulators With Guaranteed Transient Performance[1]

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A systematic way to combine adaptive control and sliding mode control (SMC) for trajectory tracking of robot manipulators in the presence of parametric uncertainties and uncertain nonlinearities is developed. Continuous sliding mode controllers without reaching transients and chattering problems are first developed by using a dynamic sliding mode. Transient performance is guaranteed and globally uniformly ultimately bounded (GUUB) stability is obtained. An adaptive scheme is also developed for comparison. With some modifications to the adaptation law, the control law is redesigned by combining the design methodologies of adaptive control and sliding mode control. The suggested controller preserves the advantages of both methods, namely, asymptotic stability of the adaptive system for parametric uncertainties and GUUB stability with guaranteed transient performance of sliding mode control for both parametric uncertainties and uncertain nonlinearities. The control law is continuous and the chattering problem of sliding mode control is avoided. A prior knowledge of bounds on parametric uncertainties and uncertain nonlinearities is assumed. Experimental results conducted on the UCB/NSK SCARA direct drive robot show that the combined method reduces the final tracking error to more than half of the smoothed SMC laws for a payload uncertainty of 6 kg, and validate the advantage of introducing parameter adaptation in the smoothed SMC laws.

I Introduction

Trajectory tracking control of robot manipulators has been extensively studied in the past decade. Earlier results, such as the computed torque method [1], require exact knowledge of the robot dynamics. In practice, parameters of the system, such as gravitational load, vary from one task to another and may not be precisely known in advance. The system may also be subjected to uncertain nonlinearities coming from joint friction, external disturbances, etc. To account for these effects, two nonlinear robust control methods have been popular: adaptive control [2, 3, 4, 5, 6, 7] and sliding mode control [8, 9, 10, 11].

A nonlinear adaptive method that guarantees asymptotic stability without any approximation of nonlinear dynamics was first developed by Craig et al. [2]. The requirement of acceleration measurements and invertibility of the estimate of the inertia matrix was later removed by Slotine and Li [3, 12], Wen and Bayard [6], Sadegh and Horowitz [4], and Middleton and Goodwin [13]. Sadegh and Horowitz presented an adaptive scheme [4] which uses feedforward reference trajectory information rather than actual state information, and a locally exponentially stable adaptive algorithm [5] under the assumption of (semi) persistent excitation. Adaptive constrained motion control and adaptive hybrid motion/force control in an unknown stiffness environment were studied by Yao and Tomizuka in [14] and [15] respectively. Recently, comparative experiments for different adaptive motion algorithms were presented by Whitcomb, et al. [16].

The advantages of the above adaptive algorithms lie in the system's guaranteeing asymptotic stability or exponential stability when the system is subjected to parametric uncertainties only. However, they all have significant drawbacks. First, nothing is said about the transient response of the system. Even local exponential stability needs the condition of (semi) persistent excitation, which depends on the specific desired trajectory used and is difficult to verify. In fact, experience in the adaptive control of linear systems [17] suggests that poor initial parameter estimates may result in unacceptably poor transient behavior, even while retaining perfect asymptotic performance. Second, uncertain nonlinearities coming from disturbances and unmodeled dynamics are not considered, and the system may lose stability even when a small disturbance appears [18]. Although some modification techniques to the integral type adaptation law such as σ-modification [18, 19] can be employed to enhance the system robustness, tracking accuracy can no longer be guaranteed since the steady state tracking error can only be shown to stay within an unknown ball whose size depends on the disturbances.

As an alternative robust approach, variable structure control (VSC) (or sliding mode control, SMC) [20, 8, 9] has received great attention because of its simplicity, strong adaptation to various perturbations from modeling errors and disturbances, and guaranteed transient performance. The underlying principle of SMC is to alter system dynamics along some surfaces in the state space so that the system state is attracted to these surfaces and maintained on them thereafter. Thus, during the sliding motion of the state on the surfaces, the system remains insensitive to parameter deviation and external disturbances when certain matching conditions are satisfied. Among VSC schemes for robot manipulators, there have been proposals to make each sliding surface attractive. This approach makes the problem complicated, resulting in a control law defined implicitly by a set of fairly complicated algebraic inequalities [8, 9]. By ex-

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ploiting the passivity of robot dynamics, other researchers obtained simple control laws, which make the state of the system attracted to the intersection of the surfaces without necessarily reaching each individual one [10, 21, 22, 23]. The sliding surfaces formulated in these schemes are fixed in the state space. Recently, a dynamic sliding mode controller, in which a dynamic compensator is introduced in forming the sliding surfaces, is employed by Yao et al. [11] to ensure that the system realizes control purposes, such as impedance control, hybrid motion/force control, and constrained motion control. Reaching transients are also eliminated so that the system is maintained in the sliding mode all the time.

The severe drawback of the VSC control law is its discontinuity across sliding surfaces. Such a control law leads in practice to control chattering, which involves high frequency control activity and may excite neglected high frequency dynamics. To remove control chattering, smoothing techniques such as a boundary layer [9, 24] have to be employed. However, such a modification can guarantee the tracking error only within a prescribed precision even when the system is subjected to parametric uncertainties alone.

Recently, some researchers have tried to overcome the above mentioned drawbacks of adaptive control and VSC control in one way or another. Qu, et al. [25] presents an exponentially stable trajectory following scheme to deal with the transient problem of the adaptive system; however, as pointed out and shown by simulation [25], the control law may approach a discontinuous control as time approaches infinity. Thus, it essentially shares the same property as VSC laws and the control chattering problem is not solved. The role of the adaptive part is not clear, and the result is locally valid under the continuous adaptation law being used. A combined direct, indirect, and variable structure adaptive method has been suggested by Narendra and Boskovic [26] for linear plants, and has been extended by Yu, et al. [27] for application to robot manipulators. However, transient performance is still not guaranteed and uncertain nonlinearities are not addressed. Datta and Ho [28] present an adaptive computed torque method with improved transient performance under parametric uncertainties, but the controller is quite complicated and requires the measurement of joint accelerations.

This paper will present a systematic way to combine adaptive control and sliding mode control for trajectory tracking control of robot manipulators in the presence of both parametric uncertainties and uncertain nonlinearities. Our goal is to preserve the advantages of the two methods—namely, asymptotic stability of adaptive systems in the presence of parametric uncertainties and guaranteed transient performance of VSC systems for both parametric uncertainties and disturbances—and, at the same time, overcome the drawbacks of the two methods, i.e., poor transient performance and poor robustness of adaptive systems and chattering problem of VSC systems.

This paper is organized as follows: Section II establishes all the dynamic equations used and formulates the problem. Section III presents the proposed sliding mode control. Section IV shows the adaptive control scheme in the absence of disturbances. Section V gives the suggested continuous adaptive robust control scheme. Section VI presents the experimental results and Section VII draws the conclusions.

II Dynamic Model of Robot Manipulators

The dynamic equation of a general rigid link manipulator having n degrees of freedom in free space can be written as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F_f(q, \dot{q}) + \tilde{f}(q, \dot{q}, t) = \tau \tag{1}$$

where $q \in R^n$ is the joint displacement vector, $\tau \in R^n$ is the applied joint torque, $M(q) \in R^{n\times n}$ is the inertia matrix, $C(q, \dot{q})\dot{q} \in R^n$ is the Coriolis and centrifugal force, $G(q) \in R^n$ is the gravitational force, $F_f(q, \dot{q}) = F_{fv}\dot{q} + F_{fq}$ sign $(\dot{q})$ is the joint friction force, in which F_{fv} and F_{fq} are diagonal damping matrix and diagonal Coulomb friction matrix respectively. sign $(\dot{q}) = [\text{sign}(\dot{q}_1), \ldots, \text{sign}(\dot{q}_n)]^T$, sign $(\cdot)$ represents the sign function, and $\tilde{f}(q, \dot{q}, t) \in R^n$ is the vector of unknown nonlinear functions such as external disturbances. Equation (1) has the following properties which will facilitate the controller design [29, 7, 4, 25].

Property 1. $M(q)$ is a symmetric positive definite (s.p.d.) matrix, and there exists $k' > 0$ such that $k' I_{n\times n} \le M(q)$. Furthermore, for the robot with all joints revolute or prismatic, there exists $k'' > 0$ so that $M(q) \le k'' I_{n\times n}$. For a general robot, $M(q) \le k'' I_{n\times n}$ is valid for any finite workspace $\Omega = \{q: \|q - q_0\| \le q_{\max}\}$ where q_0 and $q_{\max}$ are some constants.

Property 2. The matrix $N(q, \dot{q}) = \dot{M}(q) - 2C(q, \dot{q})$ is a skew-symmetric matrix.

Property 3. $M(q)$, $C(q, \dot{q})$, $G(q)$, $F_f(q, \dot{q})$ are linear in terms of a suitably selected set of the robot parameters $\beta \in R^k$. Therefore, we can write

$$M(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) + F_f(q, \dot{q}) = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\beta \quad Y \in R^{n\times k} \tag{2}$$

where $\dot{q}_r$ and $\ddot{q}_r$ are any reference vectors.

In general, the parameter vector β cannot be exactly known. For example, the payload of the robot depends on the task. However, the extent of parametric uncertainties can be predicted. Therefore, we assume that β lies in a known bounded open convex set Ω_β. We also assume that $\tilde{f}$ can be bounded by

$$\|\tilde{f}(q, \dot{q}, t)\| \le h_f(q, \dot{q}, t) \tag{3}$$

where $h_f(q, \dot{q}, t)$ is a known scalar function. $\|\bullet\|$ denotes a norm of $\bullet$ which is a vector or a matrix. Without loss of generality, in this paper, $\|\bullet\|_2$ is used for a vector and its induced norm is used for a matrix, i.e., $\|A\| = \sigma_{\max}(A) = \lambda_{\max}^{1/2}(A^TA)$, where $\sigma(\bullet)$ denotes a singular value of $\bullet$, $\lambda(\bullet)$ an eigenvalue of $\bullet$, and $\bullet_{\max}$ (or $\bullet_{\min}$) is the maximum (or minimum) value of $\bullet$. Some techniques to determine these types of bounds can be found in [30]. Note that Assumption (3) is weaker than the bounded disturbance assumption generally used in the robust analysis of adaptive systems since disturbances in (3) can be unbounded. We can now formulate the robust motion control problem as follows:

Suppose $q_d(t) \in R^n$ is given as the desired joint motion trajectory with $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ being bounded. Let $e = q(t) - q_d(t) \in R^n$ be the motion tracking error. Consider the robot manipulator described by (1) with disturbances bounded by (3). With some or all of the robot parameters $\beta \in \Omega_\beta$ being unknown, design a control law and parameter adaptation laws so that the system is either GUUB stable or asymptotically stable and the tracking error e is as small as possible.

III Sliding Mode Control

In this section, a sliding mode control law will be designed to achieve the above robust control objective. A dynamic sliding mode will be employed to eliminate the reaching transient and to enhance the dynamic response of the system in sliding mode. Let a dynamic compensator with input e and output y_z be

$$\dot{z} = A_z z + B_z e \quad z \in R^{n_c}$$
$$y_z = C_z z + D_z e \quad y_z \in R^n \tag{4}$$

where constant matrices (A_z, B_z, C_z, D_z) will be chosen to ensure that the resulting dynamic sliding mode possesses the desired dynamic response and the system (4) is controllable and observable. The sliding mode controller will be designed to make the following quantity remain zero.

$$\xi = \dot{e} + y_z \quad \xi \in R^n$$
$$= \dot{q} - \dot{q}_r \quad \dot{q}_r \triangleq \dot{q}_d(t) - y_z \tag{5}$$

The transfer function from ξ to e is

$$e = G_\xi^{-1}(s)\xi \tag{6}$$

where

$$G_\xi(s) = sI_n + G_c(s) \quad G_c(s) = C_z(sI_{n_c} - A_z)^{-1}B_z + D_z \tag{7}$$

and I_n represents the $n \times n$ identity matrix. From (7), $G_\xi^{-1}(s)$ can be arbitrarily assigned by suitably choosing a dynamic compensator transfer function $G_c(s)$ as long as $G_\xi^{-1}(s)$ has relative degree one. Since during sliding mode, $\xi = 0$, the system response is governed by the free response of transfer function $G_\xi^{-1}(s)$. Therefore, as long as $G_\xi^{-1}(s)$ is stable, the resulting dynamic sliding mode will be stable and is invariant to various modelling errors. Furthermore, the sliding mode can be arbitrarily shaped to possess any exponentially fast convergence rate since poles of $G_\xi^{-1}(s)$ can be freely assigned. In addition, $G_\xi^{-1}(s)$ can be chosen to minimize the effect of ξ on e when the ideal sliding mode $\{\xi = 0\}$ cannot be exactly achieved in practice as in the case of smoothed VSC control law being used. Later, examples will be given to show how to achieve this.

The control torque can now be determined so that the system reaches the sliding mode in a finite time and has some prescribed reaching transient responses against the modelling errors. Noting (5) and Property 3, (1) can be rewritten as

$$M(q)\dot{\xi} + C(q,\dot{q})\xi + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta + \tilde{f}(q,\dot{q},t) = \tau \tag{8}$$

Let $\hat{\beta}$ be an estimate of β, and $\Omega_{\hat{\beta}}$ be a bounded set which contains β, i.e., $\beta \in \Omega_{\hat{\beta}}$. The vector $\tilde{\beta} = \hat{\beta} - \beta$ denotes the estimation error. For any $\hat{\beta} \in \Omega_{\hat{\beta}}$, we assume that the modelling error in (2) is bounded by a known scalar function $h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$, i.e.,

$$\|Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\beta}\|$$
$$= \|Y\hat{\beta} - Y\beta\| \leq h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \quad \forall \hat{\beta} \in \Omega_{\hat{\beta}} \tag{9}$$

Remark 1. In the above, $\Omega_{\hat{\beta}}$ can be simply chosen as Ω_β since we know that $\beta \in \Omega_\beta$. The reason that we did not make this restriction is that the result will be more general and can be used later in the design of adaptive sliding mode controllers. Since $\Omega_{\hat{\beta}}$ is a bounded set, there exists a $\beta_{\max}$ such that $\|\tilde{\beta}\| \leq \beta_{\max}$. h_β can thus be chosen as

$$h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r) = \|Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\|\beta_{\max} \tag{10}$$

This gives one way to choose h_β. However, there can be other ways since h_β is only required to be a bounding function. Making h_β a simple function reduces the computation time required in real time implementation. □

Theorem 1. For the robot manipulator described by Eq. (1) with the modelling errors (3) and (9), the system output follows the desired motion trajectory $q_d(t)$ if the following control torque is applied

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\beta} - K_\xi\xi + \tau'_v$$
$$\tau'_v = -(h + \epsilon)\frac{\xi}{\|\xi\|} \qquad h = h_\beta + h_f \tag{11}$$

where

$$\dot{q}_r = \dot{q}_d(t) - y_z$$
$$\ddot{q}_r = \ddot{q}_d(t) - \dot{y}_z = \ddot{q}_d(t) - (C_z\dot{z} + D_z\dot{e}) \tag{12}$$

K_ξ is any s.p.d. matrix, and ϵ is any positive scalar. Furthermore, the reaching time t_r when the system reaches the sliding mode satisfies

$$t_r \leq \frac{1}{c_1}\ln\left(1 + \frac{1}{c_2}\sqrt{\frac{2V_{s0}}{k'}}\right) \tag{13}$$

where

$$c_1 = \frac{\lambda_{\min}(K_\xi)}{k''}, \qquad c_2 = \frac{\epsilon}{\lambda_{\min}(K_\xi)}\sqrt{\frac{k''}{k'}}$$
$$V_{s0} = \frac{1}{2}\xi^T(0)M(q(0))\xi(0) \tag{14}$$

and the reaching transient response is bounded by

$$\|\xi(t)\| \leq \left(\sqrt{\frac{2V_{s0}}{k'}} + c_2\right)\exp(-c_1t) - c_2 \tag{15}$$

△

Proof: Noting Property 1, we choose a positive definite function as $V_s = (1/2)\xi^TM(q)\xi$ with $(1/2)k'\|\xi\|^2 \leq V_s \leq (1/2)k''\|\xi\|^2$. From Property 2, $(1/2)\xi^T\dot{M}(q)\xi = \xi^TC(q, \dot{q})\xi$. Noting (3) and (9), substituting (11) into (8) and differentiating V_s yields

$$\dot{V}_s = \xi^T[M(q)\dot{\xi} + C(q, \dot{q})\xi] = \xi^T[Y\tilde{\beta} - \tilde{f} - K_\xi\xi + \tau'_v]$$
$$\leq \|\xi\|(\|Y\tilde{\beta}\| + \|\tilde{f}\|) - \xi^TK_\xi\xi - [h_\beta + h_f + \epsilon]\|\xi\|$$
$$\leq -2c_1V_s - \sqrt{(2k')}c_1c_2\sqrt{V_s} \tag{16}$$

Theorem 1 can thus been proved by using the same techniques as in [11]. △

In the above theorem, ξ reaches zero exponentially with a reaching rate determined by the controller parameter K_ξ, and the reaching time t_r inversely depends on $\lambda_{\min}(K_\xi)$ and ϵ. Therefore, by suitably choosing K_ξ and ϵ, the reaching transient can be specified by prescribed quality. However, modelling errors do affect reaching transient and during reaching transients a large control torque may be required. The following corollary gives a way to eliminate the reaching transient.

Corollary 1. If C_z is of full row rank, then, by choosing the initial value $z(0)$ of the dynamic compensator (4) as

$$C_zz(0) = -\dot{e}(0) - D_ze(0), \tag{17}$$

Theorem 1 guarantees that the system is maintained in the sliding mode and the reaching transient is eliminated. △

Proof. If (17) is satisfied, $\xi(0) = 0$. From (13), $t_r = 0$. △

In general, τ'_v in the control law (11) is discontinuous across sliding surfaces. Because of possible neglected factors, such as sampling time, this type of control laws leads to control chattering in practice. To overcome this phenomenon, the above ideal VSC law should be smoothed. In the following, two methods based on the concept of boundary layer [9] are developed.

Smoothing Method 1. Control law (11) is smoothed to

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\beta} - K_\xi\xi + \tau'_s$$
$$\tau'_s = -(1 + \alpha_1h)h\frac{\xi}{\|\xi\| + \phi(t)} \tag{18}$$

where $\alpha_1 > 0$ is any positive scalar, and $\phi(t)$ is any bounded time-varying positive scalar, i.e., $0 \leq \phi(t) \leq \phi_{\max}$, which has the role of boundary layer thickness. The following theorem gives the performance of this smoothed control law.

Theorem 2. If the continuous control law (18) is applied to the robot manipulator described by Eq. (1) with the modelling errors (3) and (9), then, all the signals in the system remain bounded and tracking errors, $e(t)$ and $\dot{e}(t)$, exponentially converge to some balls, the sizes of which are proportional to $\phi_{\max}$. Furthermore, the tracking error ξ is bounded by

$$\|\xi(t)\|^2 \le \frac{2}{k'}\left[\exp(-\lambda_V t)V_{s0} + \frac{1}{4\alpha_1}\int_0^t \exp(-\lambda_V(t-\nu))\phi(\nu)d\nu\right] \quad (19)$$

where $\lambda_V = [2\lambda_{\min}(K_\xi)/k'']$. In addition, when $z(0)$ can be chosen to satisfy (17), $V_{s0} = 0$ and $\xi(t)$ remains within a ball with a size proportional to $\phi_{\max}$. △

Proof: Substituting (18) into (8) and following the same procedure as in proof of theorem 1, we can obtain

$$\dot{V}_s \le -\xi^T K_\xi \xi - \frac{1}{\|\xi\| + \phi(t)} \times \left[\left(\sqrt{\alpha_1}\|\xi\|h - \frac{1}{2\sqrt{\alpha_1}}\phi(t)\right)^2 - \frac{1}{4\alpha_1}\phi^2\right] \le -\lambda_V V_s + \frac{1}{4\alpha_1}\phi(t) \quad (20)$$

(20) implies (19), which means that ξ converges exponentially to a bounded ball, the size of which is proportional to $\phi_{\max}$. From Eq. (6), $e = G_\xi^{-1}(s)\xi$ and $\dot{e} = sG_\xi^{-1}(s)\xi$. Since $G_\xi^{-1}(s)$ is stable and has relative degree one, $sG_\xi^{-1}(s)$ is rational and stable. Therefore, from linear system theory, the exponential convergence of $\|\xi\|$ to a bounded ball O_ξ means the exponential convergence of e (or $\dot{e}$) to a ball O_e (or $O_{\dot{e}}$), whose size is proportional to O_ξ. △

Smoothing Method 2. Theorem 2 shows that after the ideal VSC law (11) is smoothed to (18), although the ideal sliding mode $\{\xi = 0\}$ no longer exists, an approximate sliding mode is obtained: i.e., ξ is guaranteed to converge exponentially to, or always stay inside, a neighborhood of ideal sliding mode with a thickness proportional to $\phi_{\max}$. A smaller ϕ will have a more accurate approximation but less robustness to the neglected factors and measurement noise since the resulting gain in τ_s' will be larger. This can be roughly explained as follows: τ_s' can be viewed as a proportional feedback control of ξ with a nonlinear scalar gain $(1 + \alpha_1 h)h/(\|\xi\| + \phi)$. Substituting (18) into (8), the error dynamics is

$$M(q)\dot{\xi} + C(q,\dot{q})\xi + \left[K_\xi + \frac{(1+\alpha_1 h)h}{\|\xi\| + \phi}\right]\xi = Y\tilde{\beta} - \tilde{f} \quad (21)$$

which can be considered as the first-order dynamics about ξ when ξ is small. Thus, a larger proportional feedback gain for ξ will have a faster response and a smaller steady state tracking error. However, because of the neglected factors in practice, the feedback gain has a finite upper limit around $\xi = 0$ for stability. Therefore, in choosing ϕ, a trade-off exists between the tracking accuracy that can be achieved and the robustness to neglected dynamics and measurement noise. The ideal control law (12) is the limiting case when $\phi \to 0$. In implementation, choosing a suitable $\phi(t)$ is not an easy job and normally requires trial and error since the extent of neglected dynamics and measurement noise may not be known in advance. To completely remove the control chattering problem around the sliding surface, a conservative large ϕ usually has to be chosen to make sure that the maximal possible time varying scalar gain $(1 + \alpha_1 h)h/(\|\xi\| + \phi) \approx (1 + \alpha_1 h)h/\phi$ is within its maximal allowable limit all the time. However, such a choice of ϕ normally results in a gain significantly below its allowable limit most of the time because of the time varying nature of h, and, thus, the results may be rather conservative. To fully utilize the capacity of the system, a time-varying $\phi(t)$ has to be chosen on-line based on $h(q, \dot{q}, \dot{q}_r, \ddot{q}_r, t)$ so that in the approximate sliding mode, the resulting gain $(1 + \alpha_1 h)h/\phi$ is around its limit all the time. Reference [9] has suggested such a choice of a dynamic ϕ, but the technique is quite complicated and is not easily implemented. Here, the following modification of the continuous control law is suggested:

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\beta} - K_\xi\xi + \tau_s''$$

$$\tau_s'' = \begin{cases} -K_{\tau_s}\xi & \text{if } \|\xi\| \le \phi_h, \qquad \phi_h \triangleq \dfrac{\phi(t)}{h + \epsilon_1} \\ -(1 - c_3)K_{\tau_s}\xi - c_3 h\dfrac{\xi}{\|\xi\|} & \text{if } \phi_h \le \|\xi\| \le (1 + \epsilon_2)\phi_h \\ -h\dfrac{\xi}{\|\xi\|} & \text{if } \|\xi\| \ge (1 + \epsilon_2)\phi_h \end{cases} \quad (22)$$

where K_{τ_s} is any s.p.d. matrix, $c_3 = (\|\xi\| - \phi_h)/\epsilon_2\phi_h$, ϵ_1 and ϵ_2 are any positive scalars, and the rest variables are the same as in (11).

The above modification is quite simple and yet it provides the desired properties—namely, around sliding mode $\{\|\xi\| = 0\}$, a fixed feedback gain matrix is employed all the time and thus can be chosen near its allowable limit without inducing control chattering. We can also tune the gain around each joint separately since it is a gain matrix instead of a nonlinear scalar gain. When the system is away from sliding surfaces, the original nonlinear feedback control law is employed to guarantee the stability at large. The following theorem gives the performance of the modified control law:

Theorem 3. If the continuous control law (22) is applied to the robot manipulator described by Eq. (1) with the modelling errors (3) and (9), all the signals in the system remain bounded and tracking errors, $e(t)$ and $\dot{e}(t)$, converge exponentially to some balls with sizes proportional to $\phi_{\max}$. Furthermore, the tracking error ξ is bounded by

$$\|\xi(t)\|^2 \le \frac{2}{k'}\left[\exp(-\lambda_V t)V_{s0} + (1 + \epsilon_2)\int_0^t \exp(-\lambda_V(t-\nu))\phi(\nu)d\nu\right] \quad (23)$$

In addition, when $z(0)$ can be chosen to satisfy (17), $V_{s0} = 0$ and $\xi(t)$ remains within a ball with a size proportional to $\phi_{\max}$.

Proof: From (22), it can be verified that

$$h\|\xi\| + \xi^T\tau_s'' \le (1 + \epsilon_2)\phi(t) \quad (24)$$

Theorem 3 can thus be proved by following the same procedure as in the proof of theorem 2. △

Remark 2. In Theorem 2 or 3, if $\phi(\infty) = 0$, from (19), and (23), $\xi \to 0$ as $t \to \infty$, which in turn guarantees $e \to 0$. In other words, by choosing $\phi(t)$ as a time-varying positive scalar converging to zero, asymptotic stability can be obtained. Furthermore, if $\phi(t)$ converges exponentially to zero, i.e., $\phi(t) \le \phi_0 \exp(-\lambda_\phi t)$ for some $\lambda_\phi > 0$ and $\phi_0 > 0$, from (19) (the result is also true for (23)),

$$\|\xi(t)\|^2 \le \frac{2}{k'}\left[\exp(-\lambda_V t)V_{s0} + \frac{\phi_0}{4\alpha_1(\lambda_V - \lambda_\phi)}(\exp(-\lambda_\phi t) - \exp(-\lambda_V t))\right] \quad (25)$$

where $\lambda_\phi \neq \lambda_V$ has been assumed for simplicity. Therefore, ξ, e and $\dot{e}$ exponentially converge to zero, and exponential stability is obtained. Notice that although the control laws in these cases are continuous for any finite time t, the control law (18) tends to the ideal control law (11) as $t \to \infty$. Therefore, control chattering will appear when $t \to \infty$ and it is not surprising that the ideal performance of VSC law is obtained. This result corresponds to some of the continuous robust control techniques (e.g., in [25]). However, it should be noted that although this type of control law is continuous in its form, the real problem of control chattering is still not solved. To truly remove control chattering, non-zero $\phi(t)$ must be used, and, it should not be too small. However, then only GUUB stability can be guaranteed and asymptotic stability is lost. We must try other ways to improve steady state tracking accuracy as done in the following section by adaptive control. □

Remark 3. Since ξ is only guaranteed to remain within a small region around ideal sliding mode when smoothed VSC law is applied, the significance of using dynamic sliding mode becomes apparent since the extra freedom in choosing $G_\xi^{-1}(s)$ can be utilized to minimize the effect of a non-zero ξ on the tracking error e. For example, if the system is mainly subject to some constant disturbances, a non-zero constant ξ may appear. By including a differentiator in the numerator of $G_\xi^{-1}(s)$, e.g., $G_\xi^{-1}(s) = s/(s^2 + k_p s + k_i)I_n$, which can be realized by choosing dynamic compensator parameter as $C_z = I_n$, $A_z = 0$, $B_z = k_i I_n$, $D_z = k_p I_n$, zero steady state tracking error $e(\infty)$ can be obtained. □

IV Adaptive Motion Control

As mentioned in Remark 2, poor steady state tracking accuracy of smoothed VSC laws has motivated researchers to seek other ways to improve the system tracking accuracy. In this section, adaptive control methods [3, 4, 5, 7] will be used to achieve this goal in the presence of parameter uncertainties only, i.e., $\tilde{f} = 0$ in (1).

Let the constant parameter set be partitioned as $\beta = [\beta_E^T, \beta_R^T]^T$, where β_E is the unknown parameter set which needs to be estimated on-line and β_R is the known parameter set. $\hat{\beta} = [\hat{\beta}_E^T, \beta_R^T]^T$ is its estimate. Correspondingly partition Y as $Y = [Y_E, Y_R]$. The control torque is suggested to be

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\beta} - K_\xi \xi \quad (26)$$

with the parameter adaptation law

$$\dot{\hat{\beta}}_E = -\Gamma_\beta Y_E^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\xi \quad (27)$$

where $\Gamma_\beta > 0$ and $K_\xi > 0$ are constant s.p.d. matrices, and ξ and $\dot{q}_r$, $\ddot{q}_r$ are defined by (5) and (12), respectively.

Theorem 4. In the absence of uncertain nonlinearities, i.e., $\tilde{f} = 0$, the following results hold if the control law (26) with update law (27) is applied:

a) $\tilde{\beta}_E = (\hat{\beta}_E - \beta_E) \in L_\infty^{k_E}$

b) $\xi \to 0$, $e \to 0$, and $\dot{e} \to 0$ when $t \to \infty$

i.e., the system is stable and the robot follows desired motion trajectories asymptotically.

Additionally, if the desired motion trajectory satisfies the following persistent excitation condition

$$\int_t^{t+T} Y_E^T(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)Y_E(q_d, \dot{q}_d, \dot{q}_d, \ddot{q}_d)d\nu \ge \epsilon_d I_{k_E} \quad \forall t \ge t_0 \quad (28)$$

where T, t_0 and ϵ_d are some positive scalars, then,

c) $\tilde{\beta}_E \to 0$ when $t \to \infty$

i.e., estimated parameters converge to their true values. △

Proof. Choose a positive definite function as

$$V_a = V_s + \tfrac{1}{2}\tilde{\beta}_E^T\Gamma_\beta^{-1}\tilde{\beta}_E \quad (29)$$

where V_s is as defined before. Substituting Eq. (26) into (8) and noting $\tilde{f} = 0$,

$$M(q)\dot{\xi} + C(q, \dot{q})\xi + K_\xi\xi = Y_E(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\beta}_E \quad (30)$$

Noting $\dot{\tilde{\beta}}_E = \dot{\hat{\beta}}_E$, differentiating V_a, we obtain

$$\dot{V}_a = \xi^T[M(q)\dot{\xi} + C(q, \dot{q})\xi] + \tilde{\beta}_E^T\Gamma_\beta^{-1}\dot{\tilde{\beta}}_E = -\xi^T K_\xi \xi \quad (31)$$

Conclusions (a) and (b) can thus be proved by applying Barbalat's lemma [24]. It can be checked out that all the terms in (30) except $\dot{\xi}$ are uniformly continuous. Thus, $\dot{\xi}$ is uniformly continuous. From Barbalat's lemma, $\dot{\xi} \to 0$. From (30), $Y_E\tilde{\beta}_E \to 0$. Then, PE condition (28) guarantees Conclusion (c). △

V Smooth Adaptive Sliding Mode Control With Guaranteed Transient Performance

The advantage of the adaptive system in section IV is that it updates $\hat{\beta}$ on line to reduce $Y\tilde{\beta}$ (in fact, →0) to obtain asymptotic stability or zero steady state tracking error. There are two main drawbacks associated with it. First, transient performance of the system is not clear. Second, uncertain nonlinearities are not considered, and it is well known that the integral type adaptation law (27) may suffer from parameter drifting and destabilize the system in the presence of even a small disturbance and measurement noise [18]. On the other hand, transient performance is guaranteed in the smooth VSC design in section III for both parametric uncertainties and uncertain nonlinearities. This result makes VSC design attractive, and its only drawback is a relatively larger steady state tracking error. As seen from (21), for any fixed control gains, the steady state tracking error is proportional to the model uncertainties, $(Y\tilde{\beta} - \tilde{f})(\infty)$. Thus, by introducing parameter adaptation in the VSC design to reduce $Y\tilde{\beta}$, it is possible to further improve the final tracking accuracy. In this section, we will combine the VSC design with adaptive control method to take advantages of the two methods and, at the same time, overcome their drawbacks mentioned above.

The suggested control law has the same form as the smoothed VSC law (18) but with the parameter estimate updated on-line and a new bounding function h_β in designing τ_s', i.e.,

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\beta}(t) - K_\xi\xi + \tau_s' \quad (32)$$

where

$$\tau_s' = -(1 + \alpha_1 h)h\frac{\xi}{\|\xi\| + \phi(t)}$$

$$h(q, \dot{q}, \dot{q}_r, \ddot{q}_r, t) = h_f(q, \dot{q}, t) + h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \quad (33)$$

$h_\beta \ge 0$ will be designed later to guarantee transient performance. Initial state of the dynamic compensator is chosen by (17) and thus in the following $\xi(0) = 0$ will be used. Define $\Omega_{\beta E} = \{\beta_E: [\beta_E^T, \beta_R^T]^T \in \Omega_\beta\}$, which is a bounded open convex set since Ω_β is a bounded open convex set. The adaptation law (27) is modified to

$$\dot{\hat{\beta}}_E = -\Gamma_\beta Y_E^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\xi - \Gamma_\beta l_\beta(\hat{\beta}_E), \quad \hat{\beta}(0) \in \Omega_\beta \quad (34)$$

where $l_\beta(\hat{\beta}_E)$ is any vector of functions satisfying the following conditions

i. $l_\beta(\hat{\beta}_E) = 0$ if $\hat{\beta}_E \in \Omega_{\beta E}$
ii. $\tilde{\beta}_E^T l_\beta(\hat{\beta}_E) \geq 0$ if $\hat{\beta}_E \notin \Omega_{\beta E}$
iii. The nonlinear damping $l_\beta(\hat{\beta}_E)$ should be chosen in such a way that estimated parameters belong to a known bounded set Ω_β all the time, i.e., $\hat{\beta} \in \Omega_\beta$.

There are several ways to choose $l_\beta(\hat{\beta}_E)$ to satisfy the above conditions. Two examples are given to illustrate this.

Example 1: Discontinuous Modification. Normally, the right hand side of adaptation law (34) can be discontinuous since it only causes $\dot{\hat{\beta}}$ to be discontinuous, and $\hat{\beta}$ is still continuous which is normally used in the control law. Therefore, the discontinuous modification law $l_\beta(\hat{\beta})$ may be allowed. In such a case, the widely used projection method in general adaptive systems [31, 32] can be employed. Define a set, $\Omega'_{\beta E} = \Gamma_\beta^{-1/2}(\Omega_{\beta E})$, which is a bounded open convex set. Let $\partial\Omega_\beta$ denote the boundary of Ω_β, $\bar{P}r(\bullet)$ the projection of the vector $\bullet$ onto the hyperplane tangent to $\partial\Omega'_{\beta E}$ at $\Gamma_\beta^{-1/2}\hat{\beta}_E$, and β_{perp} the unit vector perpendicular to the tangent hyperplane of $\Omega_{\beta E}$ at β_E, pointing outward. Then, $l_\beta(\hat{\beta}_E)$ is given by

$$l_\beta(\hat{\beta}_E) = \begin{cases} 0 & \text{if } \begin{cases} \hat{\beta}_E \in \Omega_{\beta E} \\ \hat{\beta}_E \in \partial\Omega_{\beta E} \text{ and } \beta_{\mathrm{perp}}^T\Gamma_\beta Y_E^T\xi \geq 0 \end{cases} \\ -Y_E^T\xi - \Gamma_\beta^{-1/2}\bar{P}r(-\Gamma_\beta^{1/2}Y_E^T\xi) & \text{if } \hat{\beta}_E \in \partial\Omega_{\beta E} \text{ and } \beta_{\mathrm{perp}}^T\Gamma_\beta Y_E^T\xi < 0 \end{cases} \quad (35)$$

The above choice of $l_\beta(\hat{\beta}_E)$ guarantees that $\hat{\beta}_E \in \bar{\Omega}_{\beta E}$ no matter what the control law is and what the error dynamics is as long as the initial estimates satisfy (34). $\bar{\Omega}_{\beta E}$ is the closure of $\Omega_{\beta E}$. This is because the resulting $\dot{\hat{\beta}}_E$ in (34) always points inside or along the tangent plane of $\bar{\Omega}_{\beta E}$ when $\hat{\beta}_E \in \partial\Omega_{\beta E}$. Therefore, by choosing $\Omega_\beta = \{[\hat{\beta}_E^T, \beta_R^T]^T: \hat{\beta}_E \in \bar{\Omega}_{\beta E}\}$, condition *iii* is satisfied. Clearly, condition *i* is satisfied. It is shown in Appendix 1 that condition *ii* is satisfied. Therefore, (35) satisfies all the required conditions.

Remark 4. If $\Omega_{\beta E} = \{\beta_E: \beta_{Ei} \in (\beta_{i\min}, \beta_{i\max}), i = 1, \ldots, k_E\}$, then, with the modification (35) in which Γ_β is a diagonal matrix, adaptation law (34) becomes

$$\dot{\hat{\beta}}_{Ei} = \begin{cases} 0 \text{ if } \hat{\beta}_{Ei} = \beta_{i\max} \text{ and } (\Gamma_\beta Y_E^T\xi)_i < 0 \\ -(\Gamma_\beta Y_E^T\xi)_i \quad \begin{cases} \beta_{i\min} < \hat{\beta}_{Ei} < \beta_{i\max} \\ \hat{\beta}_{Ei} = \beta_{i\max} \text{ and } (\Gamma_\beta Y_E^T\xi)_i \geq 0 \\ \hat{\beta}_{Ei} = \beta_{i\min} \text{ and } (\Gamma_\beta Y_E^T\xi)_i \leq 0 \end{cases} \\ 0 \text{ if } \hat{\beta}_{Ei} = \beta_{i\min} \text{ and } (\Gamma_\beta Y_E^T\xi)_i > 0 \end{cases} \quad \square \quad (36)$$

Example 2: Continuous Modification. If continuous adaptation laws are preferred as in some cases where $\dot{\hat{\beta}}$ will be used in the control law, any continuous modification which satisfies the conditions *i*, *ii*, and the following condition *iv* will meet the requirement.

iv. There exist known positive scalars k'_β and k''_β such that

$$\tilde{\beta}_E^T l_\beta(\hat{\beta}_E) \geq k'_\beta\|\tilde{\beta}_E\|^2 - k''_\beta \quad (37)$$

□

The reason is that condition *iv* implies condition *iii* as proved in the following. Noting (33), (34), and condition *iv*, substituting (32) into (8) and differentiating V_a defined by (29), we can obtain

$$\dot{V}_a = -\xi^T K_\xi \xi - \tilde{\beta}_E^T l_\beta(\hat{\beta}_E) + \frac{1}{4\alpha_1}\phi \leq -\xi^T K_\xi \xi - k'_\beta\|\tilde{\beta}_E\|^2 + \alpha_5 \leq -\lambda_{V_a}V_a + \alpha_5 \quad (38)$$

where $\alpha_5 = (1/4\alpha_1)\phi_{\max} + k''_\beta$, $\lambda_{V_a} = 2[\min\{\lambda_{\min}(K_\xi), k'_\beta\}/\max\{k'', \lambda_{\max}(\Gamma_\beta^{-1})\}]$, and similar techniques as in (20) have been used. Thus

$$V_a(t) \leq \exp(-\lambda_{V_a}t)V_a(0) + \frac{\alpha_5}{\lambda_{V_a}}[1 - \exp(-\lambda_{V_a}t)] \quad (39)$$

Since $\hat{\beta}_E(0) \in \Omega_{\beta E}$ and $\beta_E \in \Omega_{\beta E}$, then $\tilde{\beta}_E(0)$ is bounded by some known constants. Since $\xi(0) = 0$ by choosing $z(0)$, then, $V_a(0) = (1/2)\tilde{\beta}_E(0)^T\Gamma_\beta^{-1}\tilde{\beta}_E(0) \leq (1/2)\lambda_{\max}(\Gamma_\beta^{-1})\|\tilde{\beta}_E(0)\|^2$. Thus, there is a known constant V_{a0} such that $\forall\hat{\beta}_E(0) \in \Omega_{\beta E}$, and $\forall e(0) \in R^n$, $V_a(0) \leq V_{a0}$. From (39),

$$V_a(t) \leq V_{a\max}, \quad V_{a\max} = \max\left\{V_{a0}, \frac{\alpha_5}{\lambda_{V_a}}\right\} \quad (40)$$

So

$$\|\tilde{\beta}_E(t)\| \leq \sqrt{2V_{a\max}\lambda_{\max}(\Gamma_\beta)}, \quad \forall t \geq 0 \quad (41)$$

Defining $\Omega_\beta = \{\hat{\beta}: \|\tilde{\beta}_E\| \leq \sqrt{2V_{a\max}\lambda_{\max}(\Gamma_\beta)}\}$, condition iii is satisfied. This shows that condition *iv* implies condition iii.

Remark 5. As an example, suppose $\Omega_{\beta E} = \{\beta_E: \|\Gamma_w(\beta_E - \beta_a)\| < \delta_\beta\}$ where $\Gamma_w > 0$ is a known s.p.d. weighting matrix, β_a is the nominal value, and δ_β is a known positive scalar bound. Consider the following continuous modification

$$l_\beta(\hat{\beta}_E) = \begin{cases} 0 & \text{if } \|\Gamma_w(\hat{\beta}_E - \beta_a)\| \leq \delta_\beta \\ \gamma_\beta \dfrac{\|\Gamma_w(\hat{\beta}_E - \beta_a)\| - \delta_\beta}{\epsilon_3}\Gamma_w^2(\hat{\beta}_E - \beta_a) & \delta_\beta < \|\Gamma_w(\hat{\beta}_E - \beta_a)\| \leq \delta_\beta + \epsilon_3 \\ \gamma_\beta\Gamma_w^2(\hat{\beta}_E - \beta_a) & \|\Gamma_w(\hat{\beta}_E - \beta_a)\| > \delta_\beta + \epsilon_3 \end{cases} \quad (42)$$

where γ_β and ϵ_3 are any positive scalars. It can be shown as in [15] that (42) satisfies condition *i* and *ii*. It is shown in Appendix 2 that condition *iv* is satisfied and thus, (42) presents a needed continuous modification function. □

Since Ω_β is a known bounded set, $h_\beta(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ can be determined such that (9) is satisfied as in Remark 1. Thus, the control law (32) is completely designed. Its performance is revealed in the following theorem.

Theorem 5. If the control law (32) with (33) and (34) is applied to the manipulator described by (1), the following results hold

A In general, all the signals in the system remain bounded and tracking errors converge exponentially to some balls with size proportional to $\phi_{\max}$. Furthermore, the tracking error ξ is bounded by

$$\|\xi(t)\|^2 \leq \frac{1}{2k'\alpha_1}\int_0^t \exp(-\lambda_V(t-\nu))\phi(\nu)d\nu \quad (43)$$

B In addition to *A*, in the absence of uncertain nonlinearities, i.e., $\tilde{f} = 0$, the same results as in theorem 4 can be obtained—namely, the robot follows desired motion trajectories asymptotically, i.e., zero final tracking error in the presence of parametric uncertainties.

Proof: From condition iii, $\forall t \geq 0$, $\hat{\beta}(t) \in \Omega_{\hat{\beta}}$. Noting the similarity between the control (32) and the smoothed VSC control (18), and the choice of h_β, the same proof as in (20) can be used to show that the last inequality of (20) remains valid. Thus *A* of the above theorem is proved by noting $\xi(0) = 0$.

To prove B of Theorem 5, by virtue of conditions i and ii and $\tilde{f} = 0$, we can differentiate V_a defined by (29) to obtain

$$\begin{aligned}\dot{V}_a &= \xi^T[Y_E\tilde{\beta}_E - K_\xi\xi + \tau_s'] + \tilde{\beta}_E^T[-Y_E^T\xi - l_\beta(\hat{\beta}_E)] \\ &\leq -\xi^TK_\xi\xi - (1+\alpha_4 h)h\frac{\|\xi\|^2}{\|\xi\| + \phi(t)} - \tilde{\beta}_E^T l_\beta(\hat{\beta}_E) \\ &\leq -\xi^TK_\xi\xi \end{aligned} \quad (44)$$

Therefore, all the conclusions in theorem 4 remain valid by following the same subsequent proof. □

Remark 6. Theorem 5 shows that the same transient performance as in smoothed VSC design is obtained by the suggested control law. Asymptotic stability of conventional adaptive system is also preserved but with a guaranteed transient performance. Yet, the control law is continuous and thus avoids the unpleasant chattering problem. The analysis is qualitatively different from the robust adaptive control [18] for bounded disturbance in that not only robustness is obtained for a more general class of external disturbances but also performance robustness is guaranteed by the suggested controller, i.e., arbitrarily fast exponential convergence can be provided in the initial transient with the final tracking error within a precision which can be adjusted by the controller parameter ϕ and is independent of the magnitude of disturbances. Therefore, the control law effectively combines the VSC design with adaptive control design, and achieves the expected goal. In fact, when the adaptation law (34) is switched off, the control law (33) is the same as the continuous VSC law (18). □

Remark 7. In choosing $\phi(t)$ and dynamic compensator, the same comments as in section 3 and remark 3 apply.

Remark 8. The above design method can be easily extended to the case of bounded time-varying parameters. Similar results as in *A* of Theorem 5 can be obtained. Robustness to the neglected high frequency dynamics may also be obtained since exponential stability at large is guaranteed by the suggested method. Extension to task space trajectory tracking control is trivial.

Remark 9. In the above, τ is synthesized by the smoothing method 1 in VSC design. It can also be designed by the smoothing method 2 by following the same procedure as in the above. Namely, first choose τ the same form as the smoothed VSC law (22) but with $\hat{\beta}_E$ updated on-line by (34) and h_β determined later. Then, design the modified adaptation law (34) to ensure the conditions i–iii by using either discontinuous modification (35) or continuous modification (37) (In continuous case, α_5 in (40) is changed to $(1+\epsilon_2)\phi_{\max} + k''_\beta$ to estimate the set $\Omega_{\hat{\beta}}$). h_β can then be determined to satisfy (9) to complete the control law design. The same conclusions as in Theorem 5 can be obtained except the coefficient $1/2k'\alpha_1$ in (43) is changed to $(1+\epsilon_2)$.

VI Simulation and Experimental Results

6-1 Experimental Setup. Experiments are conducted on a planar UCB/NSK two axis SCARA direct drive manipulator. The robot consists of two NSK direct drive motors (Model 1410 for the first axis with maximum torque 245 Nm and Model 608 for the second axis with maximum torque 39.2 Nm), and two links with length 0.36 m and 0.24 m respectively. A 10-bit Resolver to Digital Converter (RDC) provides a motor position resolution of 153,600 pulses per revolution. The velocity signals are then obtained by the difference of the position signals with a first-order filter. An 486 PC equipped with a 12 bit IBM Data Acquisition and Control Adapters (DACA) board is used to control the entire setup. Detailed experimental setup and modeling can be found in [33].

The matrices in dynamic equation (1) are given by [33]

$$M(q) = \begin{bmatrix} p_1 + 2p_3C_{q2} & p_2 + p_3C_{q2} \\ p_2 + p_3C_{q2} & p_2 \end{bmatrix},$$

$$C(q, \dot{q}) = \begin{bmatrix} -p_3\dot{q}_2S_{q2} & -p_3(\dot{q}_1 + \dot{q}_2)S_{q2} \\ p_3\dot{q}_1S_{q2} & 0 \end{bmatrix}$$

$$G(q) = 0 \quad (45)$$

where $C_{q2} = \cos(q_2)$, $S_{q2} = \sin(q_2)$, p_1, p_2, and p_3, the combined robot and payload parameters, are given by $p_1 = p_{a1} + 0.194m_p$, $p_2 = p_{a2} + 0.0644m_p$, and $p_3 = p_{a3} + 0.0864m_p$, respectively, m_p is the payload mass, and $p_{a1} = 3.1623$, $p_{a2} = 0.1062$, and $p_{a3} = 0.17285$ are the robot parameters. The friction term $F_f(q, \dot{q})$ is lumped into the uncertain nonlinearity term, $\tilde{f}$ and is bounded by (3) where $h_f = 7$. Letting $\beta = [p_{a1}, p_{a2}, p_{a2}, m_p]^T$, (2) can be formed. In the experiment, only payload mass m_p is unknown with the maximum payload, $m_{p\max} = 6$ kg. Thus, $\beta_R = [p_{a1}, p_{a2}, p_{a2}]^T$, $\beta_E = m_p$, and $\Omega_\beta = \{(p_{a1}, p_{a2}, p_{a2}, m_p)$: $p_{a1} = 3.1623$, $p_{a2} = 0.1062$, $p_{a3} = 0.17285$, $-0.00001 < m_p < m_{p\max} + 0.00001\}$. Since all the controllers are supposed to deal with model uncertainties, the initial estimate of payload is set to 6 kg, with an actual value in experiments being $m_p = 0$ kg (no payload). All experiments are conducted at a sampling time $\Delta T = 1$ ms.

6-2 Desired Trajectories and Performance Indexes. Since we are interested in tracking performance, sinusoidal trajectories with a smoothed initial starting phase are adopted for each joint. In this experiment, the desired joint trajectories are $q_d = [1.5(1.181 - 0.3343\exp(-5t) - \cos(\pi t - 0.561)), 1.2716 - 0.4851\exp(-5t) - \cos(1.25\pi t - 0.666))]^T$ (rad). Zero initial tracking errors are used and each experiment is run for ten seconds, i.e., $T_f = 10$ s.

Commonly used performance measures such as the rising time, damping and steady state error are not adequate for nonlinear systems such as robots. In [16], the scalar valued L^2 norm given by $L^2[e(t)] = ((1/T_f)\int_0^{T_f} \|e(t)\|^2 dt)^{1/2}$ is used as an objective numerical measure of tracking performance for an entire error curve $e(t)$. However, it is an average measure, and large errors during the initial transient stage cannot be reflected. Thus, we will use the maximal absolute value of tracking error of joint i, $e_i^M = \max_{t\in[0,T_f]}\{|e_i(t)|\}$, as an index of measure of transient performance. Also, we will use the maximal absolute value and the average tracking error of joint i during the last three seconds, $e_i^F = \max_{t\in[T_f-3,T_f]}\{|e_i(t)|\}$ and $L[e_{if}] = (1/3)\int_{T_f-3}^{T_f}|e_i|$, as indexes to measure the steady state tracking error. Finally, we will use the average control input $L[u_i] = (1/T_f)\int_0^{T_f}|u_i|\,dt$ to evaluate the amount of control effort, and the average of control input increments, $L[\Delta u_i] = (1/10000)\sum_{k=1}^{10000}|u_i(k\Delta T) - u_i((k-1)\Delta T)|$, as a measure of control chattering.

The choice of feedback gains is discussed in the following for each controller. In general, they are tuned near their allowable limits for each controller and they should not induce control chattering. Control gains are kept at same values if they appear in different controllers.

6-3 Control Laws. As explained in Remark 3, a dynamic compensator ($n_c = 2$) is formed by (4) in which $A_z = 0I_2$, $B_z = 400I_2$, $C_z = I_2$, $D_z = 40I_2$ with initial values calculated by (17). Five controllers are tested:

Smooth Adaptive Sliding Mode Controller (SASMC) I
The control law is given by (32) with the adaptation law given by (36). Thus, $\Omega_{\hat\beta} = \bar\Omega_\beta$ and h_β can be determined by (10) where $\beta_{\max} = m_{p\max}$.

Smooth Sliding Mode Controller (SSMC) I
The control law is the same as in SASMC I but without parameter adaptation.

Smooth Adaptive Sliding Mode Controller (SASMC) II
The control law is formed as in Remark 9 with the same adaptation law as in SASMC I.

Smooth Sliding Mode Controller (SSMC) II
The control law is the same as in SASMC II but without parameter adaptation.

Adaptive Controller (AC).
The control law is given by (26) with the adaptation law (27).

6-4 Simulation Results. We first run a simulation to test each controller in the ideal case in which the system is subject to parametric uncertainties only and without measurement noise. In the simulation, control inputs are updated at each sampling time only (zero order hold) and one sampling time delay is assumed. The following cases are tested:

SASMC I. Controller parameters are $K_\xi = \text{diag}\{300, 20\}$, $\alpha_1 = 0.01$, $\Gamma_\beta = 100$ and $\phi = 1.0$.

SSMC I. Two cases are run. One uses the same controller parameters as in SASMC I, which is referred to as SSMC I(i). Another uses a smaller boundary layer thickness, i.e., $\phi = 0.5$, which is referred to as SSMC I(ii).

SASMC II. K_ξ and Γ_β are the same as in SASMC I. $K_{\tau_s} = \text{diag}\{350, 40\}$, $\epsilon_1 = 1$, $\epsilon_2 = 0.5$, and $\phi = 20$.

SSMC II. The same controller parameters as in SASMC II.

AC. Two cases are run. AC(i) uses the same K_ξ and Γ_β as in SASMC I. AC(ii) uses a larger feedback gain, i.e., $K_\xi = \text{diag}\{650, 60\}$, which is the sum of K_ξ and K_{τ_s} used in SASMC II.

As shown in Fig. 3, all the controllers use almost the same amount of control effort except SSMC I(ii), which exhibits control chattering (seen by the performance index $L[\Delta u_2]$) because of a smaller ϕ. As seen from Figs. 1 and 2, all the controllers provide satisfactory tracking performance. However, the adaptive sliding mode controllers (SASMC I and II) have a much better final tracking accuracy (Fig. 2), and a better transient (Fig. 1) than their non-adaptive sliding mode counterparts (SSMC I and II). Also, SASMC II and SSMC II have a better transient (Fig. 1) and a better final tracking accuracy (Fig. 2) than SASMC I and SSMC I, respectively. This verifies that the controllers designed by the second smoothing technique are easier to tune. All the adaptive controllers (SASMC I and II, AC(i) and AC(ii)) have almost zero final tracking accuracy since the estimated payload converges to the true values quickly

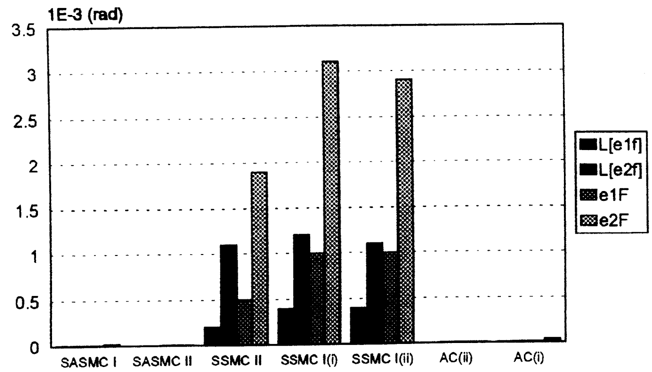

Fig. 2 Steady-state tracking performance (simulation. This figure shows that adaptive type controllers have far better final tracking accuracy than smoothed SMC type controllers.)

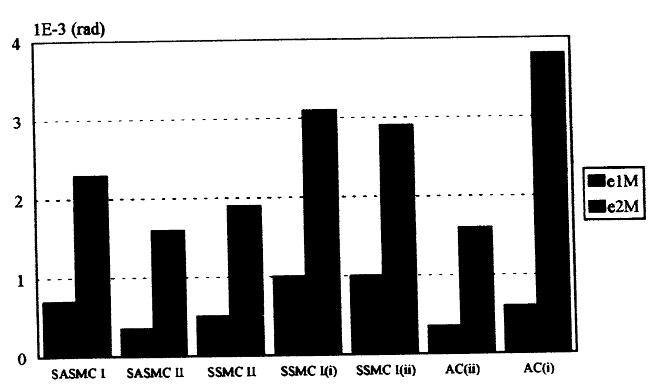

Fig. 1 Maximum tracking errors (simulation)

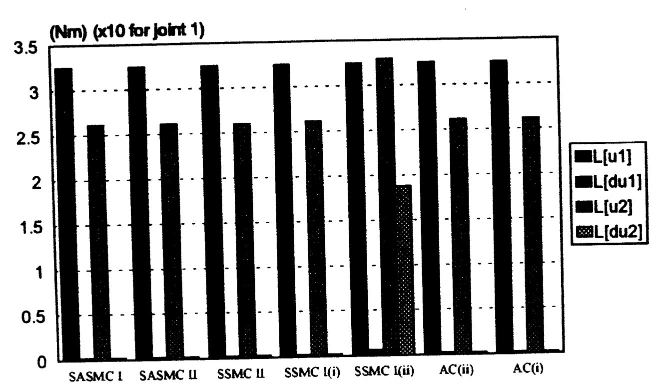

Fig. 3 Average control inputs and increments (simulation. This figure shows that all controllers use almost the same control effort except SSMC I(ii).)

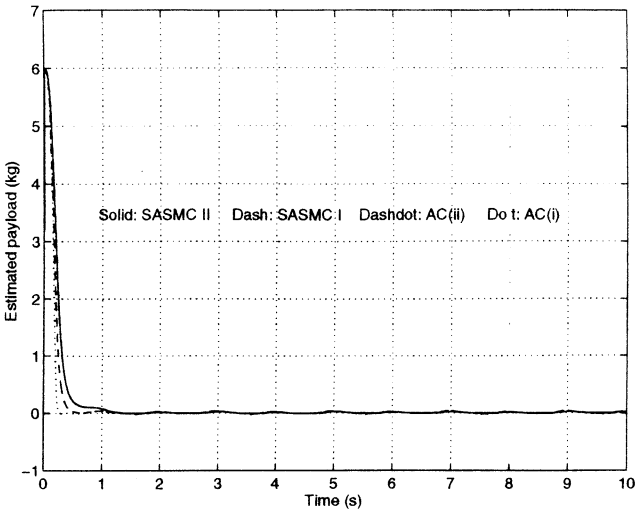

Fig. 4 Estimated parameters converge to their true values quickly (simulation)

(Fig. 4). Notice that the persistent excitation condition (28) is satisfied in this simulation.

6-5 Experimental Results. All the control laws proposed were implemented. Mainly because of the measurement noise and the neglected flexible modes, allowable gains in experiments are much smaller than those used in the simulation as follows,

SASMC I and SSMC I. $K_\xi = \text{diag}\{30, 4\}$, $\Gamma_\beta = 5$ and $\phi = 5.0$.

SASMC II and SSMC II. K_ξ and Γ_β are the same as in SASMC I. $K_{\tau_s} = \text{diag}\{40, 6\}$ and $\phi = 150$.

AC. AC(i) uses the same K_ξ and Γ_β as in SASMC I. AC(ii) uses a larger feedback gain, i.e., $K_\xi = \text{diag}\{70, 10\}$, which is the sum of K_ξ and K_{τ_s} used in SASMC II.

As shown in Figs. 5–8, we have basically the same qualitative results as in the simulation. All the controllers use almost the same amount of control effort but with different degrees of control chattering resulting from measurement noise. For example, the control inputs of SASMC II are shown in Fig. 9. As seen from the performance index $L[\Delta u_i]$, the adaptive sliding mode controllers (SASMC I and II) have a reduced degree of control chattering than their non-adaptive sliding mode counterparts (SSMC I and II). The adaptive controllers no longer have almost zero final tracking errors because of the presence of measurement noise and disturbances although the estimated payload still approaches the true values quickly as shown in Fig. 10. However, in terms of $L[e_f]$, final tracking errors of the adaptive sliding mode controllers (SASMC I and II) are less than half of those of their non-adaptive sliding mode counterparts (SSMC I and II). All these results support the advantage of introducing parameter adaptation in the smoothed sliding mode controller.

In this experiment, the best performance that the adaptive controller achieves (AC(ii)) is comparable to SASMC II with a slightly larger $L_2[e]$ (Fig. 5). This is because in this experiment disturbances are not severe and the persistent excitation condition (28) is satisfied so that the parameter drifting problem in adaptive control does not appear. However, even under these

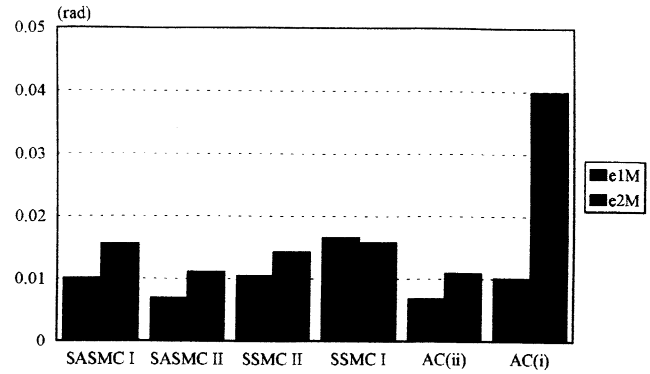

Fig. 6 Maximum tracking errors. (This figure shows that SASMCs have better transient performances than SSMCs.)

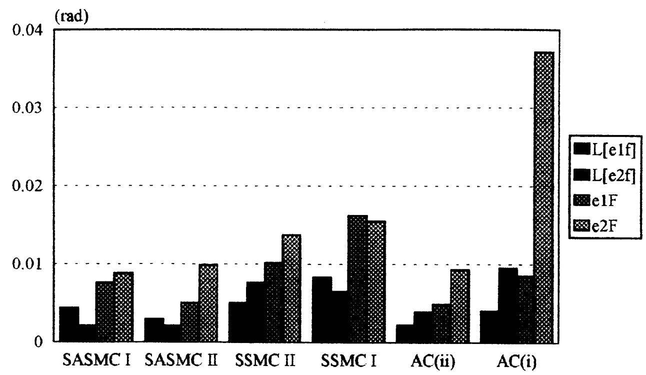

Fig. 7 Steady-state tracking performance. (This figure shows that SASMCs have better final tracking accuracy than SSMCs.)

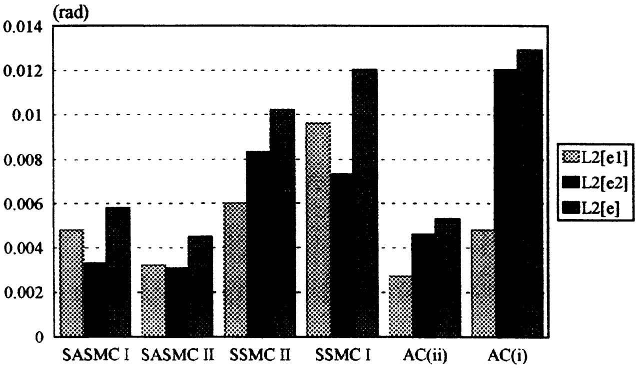

Fig. 5 Average tracking performance. (This figure shows that SASMCs have better tracking performances than SSMCs.)

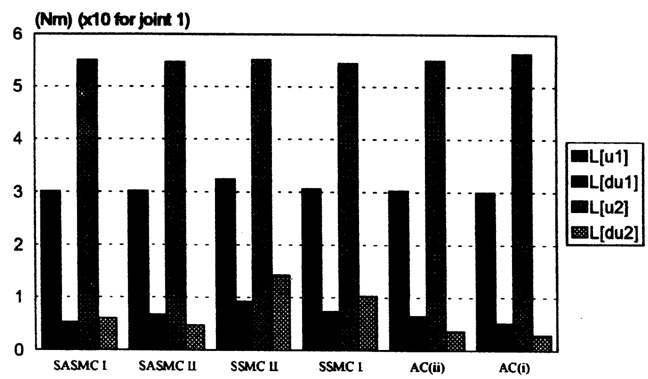

Fig. 8 Average control inputs and increments

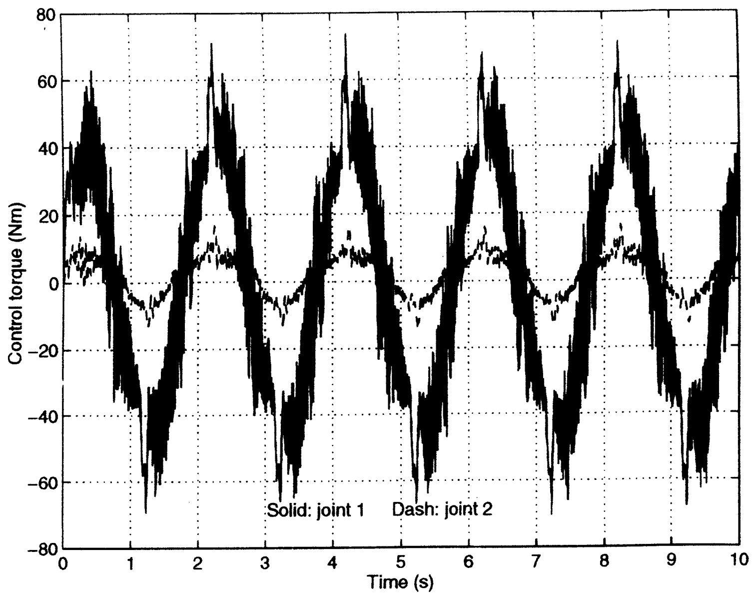

Fig. 9 Control input of SASMC II

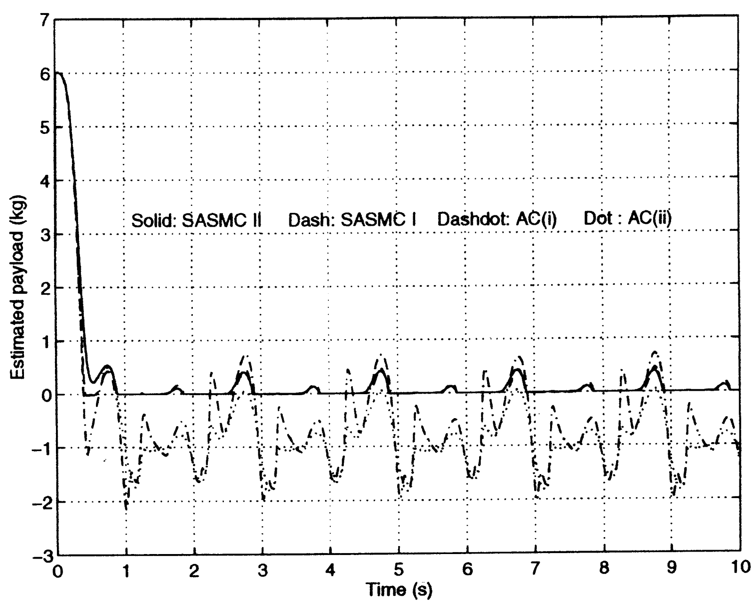

Fig. 10 Estimated parameters approach but do not converge to their true values quickly

conditions, the parameter adaptation shown in Fig. 10 reveals that the estimated payload in adaptive control approaches a wrong value. In general, when multiple parameters are adapted, this problem may be more serious. It can also be seen from Figs. 5–7 that a larger gain K_ξ provides better tracking performance than a smaller gain, as long as the high-frequency dynamics are not excited. This partially supports the important role of sliding mode type feedback terms, which act as nonlinear proportional feedback controllers.

SASMC II is also run with a non-zero initial tracking error $e(0)$ to test its transient performance. Fig. 11 and Fig. 12 show the joint tracking errors and switching functions respectively. It can be seen that the tracking error e decreases quickly, and the switching functions always start from zero. Note that the reaching transient is eliminated by the dynamic compensator with a properly adjusted initial condition.

VII Conclusions

In this paper, we have shown how to combine adaptive control and sliding mode control for trajectory tracking control of robot manipulators in the presence of both parameter uncertainties and external disturbances. The approach preserves the advantages of the two control methods while eliminating the drawbacks of each. Several smoothed SMC laws and smooth robust adaptive SMC laws are constructed. Extensive simulation results and experimental results are given to compare different controllers. Experimental results conducted on the UCB/NSK SCARA direct drive robot show that the suggested combined methods can reduce the final tracking errors by more than a factor of two compared to the smoothed SMC laws and have less degrees of control chattering for a payload uncertainty of 6 kg. These results validate the advantage of introducing parameter adaptation in the smoothed SMC laws.

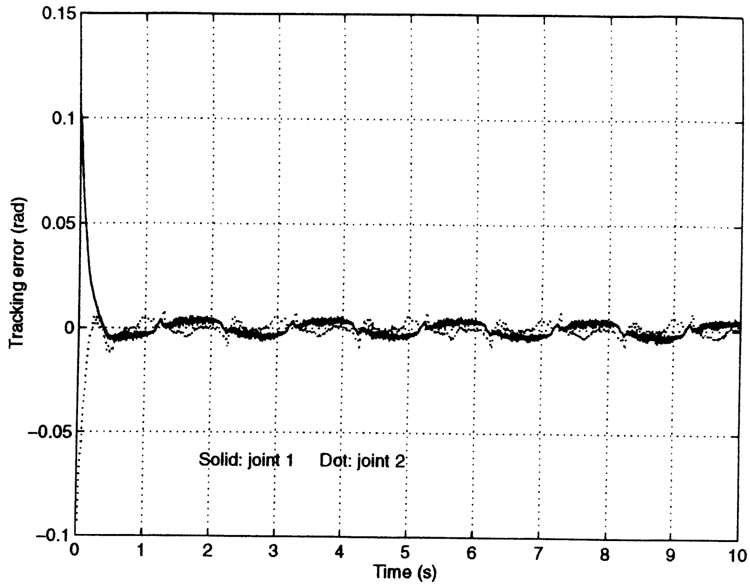

Fig. 11 Tracking error of SASMC II for nonzero initial tracking error

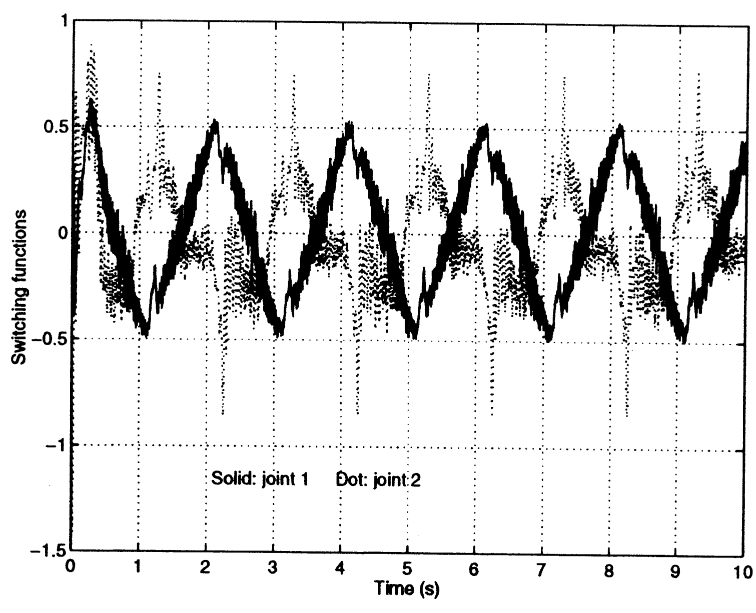

Fig. 12 This figure shows that reaching transient is eliminated for nonzero initial tracking error

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APPENDIX 1

When $\hat{\beta}_E \in \partial\Omega_{\beta E}$, by the definition of β_{perp},

$$(\Gamma_\beta^{1/2}\beta_{\text{perp}})^T\Gamma_\beta^{-(1/2)}\tilde{\beta}_E = \beta_{\text{perp}}^T\tilde{\beta}_E \geq 0 \quad \forall\beta_E \in \Omega_{\beta_E} \quad (46)$$

Thus, $\Gamma_\beta^{1/2}\beta_{\text{perp}}$ is along the outward normal direction of $\partial\Omega'_{\beta E}$ at $\Gamma_\beta^{-(1/2)}\hat{\beta}_E$, and $\bar{P}r(-\Gamma_\beta^{1/2}Y_E^T\xi)$ is given by

$$\bar{P}r(-\Gamma_\beta^{1/2}Y_E^T\xi) = -\Gamma_\beta^{1/2}Y_E^T\xi - c_n\Gamma_\beta^{1/2}\beta_{\text{perp}} \quad (47)$$

where scalar c_n is

$$c_n = \frac{(\Gamma_\beta^{1/2}\beta_{\text{perp}})^T(-\Gamma_\beta^{1/2}Y_E^T\xi)}{\|\Gamma_\beta^{1/2}\beta_{\text{perp}}\|^2} = \frac{-\beta_{\text{perp}}^T\Gamma_\beta Y_E^T\xi}{\|\Gamma_\beta^{1/2}\beta_{\text{perp}}\|^2} \quad (48)$$

Thus, when $\hat{\beta}_E \in \partial\Omega_{\beta E}$ and $\beta_{\text{perp}}^T\Gamma_\beta Y^T\xi < 0$, from (48), $c_n > 0$, and, from (35) and (47),

$$\begin{aligned}\tilde{\beta}_E l_\beta(\hat{\beta}_E) &= \tilde{\beta}_E[-Y_E^T\xi - \Gamma_\beta^{-(1/2)}\bar{P}r(-\Gamma_\beta^{1/2}Y_E^T\xi)] \\ &= c_n\tilde{\beta}_E^T\beta_{\text{perp}} \geq 0 \qquad (49)\end{aligned}$$

Since $\hat{\beta}_E \in \bar{\Omega}_{\beta E}\ \forall t \geq 0$ and $\Omega_{\beta E}$ is open, $\hat{\beta}_E \notin \Omega_{\beta E}$ is equivalent to $\hat{\beta}_E \in \partial\Omega_{\beta E}$. Thus condition *ii* is satisfied by noting (35) and (49). □

APPENDIX 2

When $\|\Gamma_w(\hat{\beta}_E - \beta_a)\| > \delta_\beta + \epsilon_3$,

$$\begin{aligned}\tilde{\beta}_E^T l_\beta(\hat{\beta}_E) &= \gamma_\beta\tilde{\beta}_E^T\Gamma_w^2[\tilde{\beta}_E + \beta_E - \beta_a] \\ &\geq \gamma_\beta\|\Gamma_w\tilde{\beta}_E\|^2 - \gamma_\beta\|\Gamma_w\tilde{\beta}_E\|\delta_\beta \\ &\geq (\gamma_\beta - \alpha_6)\|\Gamma_w\tilde{\beta}_E\|^2 - \frac{\gamma_\beta^2\delta_\beta^2}{4\alpha_6} \\ &\geq (\gamma_\beta - \alpha_6)\lambda_{\min}^2(\Gamma_w)\|\tilde{\beta}_E\|^2 - \frac{\gamma_\beta^2\delta_\beta^2}{4\alpha_6} \qquad (50)\end{aligned}$$

where α_6 is any constant such that $0 < \alpha_6 < \gamma_\beta$. The inequality

$$w_1|y_1||y_2| \leq w_2y_1^2 + w_3y_2^2, \qquad \forall y_1, y_2 \in R, \quad w_1 \geq 0,$$
$$w_2 \geq 0, \quad w_3 \geq 0, \quad \text{and} \quad w_2w_3 = \tfrac{1}{4}w_1^2 \qquad (51)$$

has been used in the above proof. When $\|\Gamma_w(\hat{\beta}_E - \beta_a)\| \leq \delta_\beta + \epsilon_3$,

$$\begin{aligned}\tilde{\beta}_E^T l_\beta(\hat{\beta}_E) &\geq 0 \geq (\gamma_\beta - \alpha_6)[\|\Gamma_w\tilde{\beta}_E\|^2 \\ &\quad - (\|\Gamma_w(\hat{\beta}_E - \beta_a)\| + \|\Gamma_w(\beta_E - \beta_a)\|)^2] \\ &\geq (\gamma_\beta - \alpha_6)\lambda_{\min}^2(\Gamma_w)\|\tilde{\beta}_E\|^2 - (\gamma_\beta - \alpha_6)(2\delta_\beta + \epsilon_3)^2 \qquad (52)\end{aligned}$$

Let $k'_\beta = (\gamma_\beta - \alpha_6)\lambda_{\min}^2(\Gamma_w)$ and $k''_\beta = \max\{\gamma_\beta^2\delta_\beta^2/4\alpha_6, (\gamma_\beta - \alpha_6)(2\delta_\beta + \epsilon_3)^2\}$, conditions iv is satisfied.