Adaptive Control of Robot Manipulators in Constrained Motion—Controller Design[1]

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Adaptive motion and force control of manipulators in constrained motion in the presence of parametric uncertainties both in the robot and contact surfaces are considered in this paper. A new constrained dynamic model is obtained to account for the effect of contact surface friction. An adaptive law is suggested with unknown parameters updated by both the motion and force tracking errors to guarantee asymptotic motion and force tracking without any persistent excitation conditions to be satisfied. The suggested controller has the expected PI type force feedback control structure with a low proportional (P) force feedback gain. Detailed simulation results are given to show the effectiveness of the proposed controller.

1 Introduction

Many practical applications of robot manipulators involve tasks in which the robot end-effectors are required to make contact with environment. Typical examples of such tasks are contour following, grinding, scribing, as well as assembly related tasks. In these applications, contact forces between the manipulator end-effectors and the environment are generated. For successful execution of such tasks, simultaneous control of motion of the robot as well as contact force is required (Whitney, 1987; Raibert and Craig, 1981; Khatib, 1987; Hogan, 1985; Kazerooni et al., 1986; Yao et al., 1992b). Therein, many researchers have focused their efforts on the situation where the end-effector is in contact with rigid frictionless surfaces (McClamroch and Wang, 1988; Wang and McClamroch, 1989; Mills and Goldenberg, 1989; Kankaanranta and Koivo, 1988; Yoshikawa et al., 1988; Yun, 1988; Yao et al., 1992c; Cai and Goldeberg, 1989; Cole, 1989; Grabbe et al., 1993). In such cases, kinematic constraints corresponding to some algebraic constraints among the manipulator state variables are imposed on the manipulator motion, and motion of such a system is called constrained motion.

A general theoretical framework of constrained motion control has been rigorously developed by McClamroch and Wang (1988). Their controller is based on a modification of the computed torque method. Lyapunov's direct method is used in Wang and McClamroch (1989) to develop a class of decentralized position and force controllers. Mills and Goldenberg (1989) applied the descriptor theory to constrained motion control. The controller is derived based on a linearized dynamic model of the manipulator. State feedback control and dynamic state feedback control are utilized to linearize the robot dynamics with respect to motion and contact force by Yoshikawa et al. (1988) and Yun (1988), respectively.

The above methods are based on the exact model of constrained robot dynamics. From a practical point of view, parameters of the system such as gravitational loads vary from a task to another, and hence, may not be precisely known in advance. This motivated the use of adaptive controllers, and a number of schemes have been proposed to deal with the adaptive motion control of rigid robots (Craig et al., 1987; Slotine and Li, 1987b; Sadegh and Horowitz, 1990; Johansson, 1990; Ortega and Spong, 1989). However, only several adaptive constrained motion control methods have been proposed (Slotine and Li, 1987a; Carelli and Kelly, 1989; Su et al., 1990; Jean and Fu, 1991). The dynamic model proposed in (McClamroch and Wang, 1988) and a parameter adaption law similar to the one in (Slotine and Li, 1987b) were used by (Carelli and Kelly, 1989) to derive conditions to ensure asymptotic position tracking and bounded force error. The same conclusion was obtained by (Su et al., 1990) using an adaptive scheme without force feedback. (Jean and Fu, 1991) further discussed this problem with the consideration of computational efficiency and the conditions for velocity tracking and constrained force tracking. Basically, the above adaptive constrained motion control methods are all based on the reduced dynamic model proposed in McClamroch and Wang (1988), which enable motion and force controllers to be designed separately. It should be noted that this model is only valid for frictionless contact surfaces, while most real contact surfaces have friction. Furthermore, the previous parameter adaption laws proposed are only driven by motion tracking error. Thus, the force tracking error can be guaranteed to be only bounded unless some persistent excitation conditions are satisfied, which is difficult to verify and depends on specific desired motion trajectories. Although theoretically the force tracking error can be made small by using a large proportional force feedback gain (Carelli and Kelly, 1989; Jean and Fu, 1991), the gain for the proportional force feedback is severely limited in applications due to the acausality that arises since rigid body dynamics is assumed in the modeling of the robot (Paljug et al., 1992). In fact, recent one-dimensional force experimental results presented by Volpe and Khosla (1992) and Paljug et al. (1992) suggest that the best force tracking performance is achieved by integral (I) force feedback or PI force feedback control.

In this paper, we consider the adaptive motion and force control of robot manipulators in constrained motion in the presence of parametric uncertainties both in the robot and surface coefficient of dry friction. In the presence of surface friction, the reduced constrained dynamic model (McClamroch and Wang, 1988) cannot be obtained, and a new transformed constrained dynamic model that is suitable for controller design is proposed. An adaptation structure similar to those in motion control (Slotine and Li, 1987b; Sadegh and Horowitz, 1990) is adopted, but with unknown parameters updated by both motion and force tracking errors. The suggested control law can guarantee asymptotical motion and force tracking without persistent excitation condition, and has the expected PI type force feedback control structure with a low proportional force feedback gain. Simulation results will illustrate the proposed controller.

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This paper is organized as follows. Dynamic equations for constrained robots are given in Section 2. Section 3 presents the proposed adaptive motion and force controller. Simulation results are shown in Section 4 and conclusions are given in Section 5.

2 Dynamic Model of Constrained Robot Manipulators

Dynamic equation of a general rigid link manipulator having n degree of freedom can be written as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + J^T(q)F = \tau \qquad (1)$$

where $q \in R^n$ is the joint displacement vector, $\tau \in R^n$ is the applied joint torque, $M(q) \in R^{n\times n}$ is the inertia matrix, $C(q, \dot{q})\dot{q} \in R^n$ is the vector function characterizing Coriolis and Centrifugal forces, $g(q) \in R^n$ is the gravitational force, $J(q) = \partial x(q)/\partial q \in R^{n\times n}$ is the Jacobian matrix which is assumed to be nonsingular in finite work space Ω, and $x \in R^n$ is the position and angles of the end-effector in Cartesian space, $F \in R^n$ is the vector of forces/moments on the environment exerted by the robot at the end-effector. (Corresponding to x, forces are decomposed along the Cartesian axes, and moments are decomposed along the rotation axes defining the angles, which may not be orthogonal.)

Equation (1) has the following properties which will facilitate the controller design (Slotine and Li, 1987b; Sadegh and Horowitz, 1990; Ortega and Spong, 1989).

Property 1. $M(q)$ is a symmetric positive definite (s.p.d.) matrix, and there exist $k' > 0$ such that $k'I_n \le M(q)$. I_n represents an $n \times n$ identity matrix. Furthermore, for the robot with all joint revolute or prismatic, there exist $k'' > 0$ so that $M(q) \le k''I_n$. For a general robot, $M(q) \le k''I_n$ is valid for any finite workspace $\Omega = [q: \|q - q_0\| \le q_{\max}]$, where q_0, $q_{\max}$ are constant vector and scalar.

Property 2. The matrix $N(q, \dot{q}) = \dot{M}(q) - 2C(q, \dot{q})$ is a skew-symmetric matrix.

Property 3. $M(q)$, $C(q, \dot{q})$, $g(q)$ are linear in terms of a suitably selected set of the robot parameters $\beta \in R^k$.

It is assumed that the robot is nonredundant and the measurements of position, velocity, and constrained force are available. The robot end-effector in contact with rigid constraint surfaces will be considered. It is assumed that the end-effector is initially in contact with the constraint surface, and the control exercised over the constrained force is such that the force will always hold the end-effector on the constraint surface.

Suppose that the environment is described by a set of m rigid hypersurfaces (McClamroch and Wang, 1988; Yoshikawa et al., 1988)

$$\Phi(x) = 0 \quad \Phi(x) = [\phi_1(x), \ldots, \phi_m(x)]^T \quad m \le n \qquad (2)$$

which are mutually independent, and $\phi_i(x)$ is assumed to be twice differentiable with respect to x. The interaction force F can be written as

$$\begin{aligned} F &= F_n + F_t = D^T(x)\lambda + A_t f_t(\mu, v_{\text{end}}, \lambda) \\ &= [D^T(x) + L^T(\mu, x, \dot{x})]\lambda \\ D(x) &= \frac{\partial \Phi(x)}{\partial x} \quad D, L \in R^{m\times n} \end{aligned} \qquad (3)$$

where $\lambda \in R^m$ is a vector of Lagrange multipliers associated with the constraints which usually represent normal contact force components, $F_n = D^T(x)\lambda$ represents constraint force, i.e., normal contact force in the Cartesian space, $F_t = A_t f_t(\mu, v_{\text{end}}, \lambda)$ is the vector of friction force, the direction of which is specified by A_t, the unit tangent direction of the surface, and its magnitude depends on F_n, i.e., λ, and the friction coefficient $\mu \in R^m$ with sign determined by the end-effector velocity v_{end}. L is linear with respect to the friction coefficient μ. In general, L is differentiable except at the point when v_{end} change direction on the surface. In the assumption of frictionless contact surface ($F_t = 0$), (3) reduces to the form given by McClamroch and Wang (1988) and Mills and Goldenberg (1989).

When motion of the robot is constrained to be on the surfaces (2), only $(n - m)$ coordinates of the position vector can be specified independently (Yao et al., 1992a). Control of all the position coordinates of the robot is unnecessary, and only $(n - m)$ position coordinates need to be controlled in the constrained motion of the robot. Therefore, motion control is in the $(n - m)$ mutually independent curvilinear coordinates, $\Psi(x) = [\psi_1(x), \ldots, \psi_{n-m}(x)]^T$. $\Psi(x)$ are assumed to be twice continuously differentiable and independent of $\Phi(x)$ in the finite workspace Ω. Thus, once $\Psi(x)$ is regulated to the desired value $\Psi_d(t)$, combining with the constraints (2), the configuration of robot is uniquely determined. Notice that $\Psi(x)$ can be selected as some joint angles q_i or some end-effector coordinates x_i which may be suitable for implementation. For example, since $D(x)$ is of full rank m, without the loss of generality, we can assume that the first m columns of $D(x)$ are independent. In this case, we can choose $\Psi(x) = [x_{m+1}, \ldots, x_n]^T$.

Define a set of curvilinear coordinates as (Yao et al., 1992b; Yoshikawa et al., 1988)

$$\begin{aligned} r = [r_f^T, r_p^T]^T \quad r_f &= [\phi_1(x), \ldots, \phi_m(x)]^T \\ r_p &= [\psi_1(x), \ldots, \psi_{n-m}(x)]^T \end{aligned} \qquad (4)$$

Differentiate (4)

$$\dot{r} = J_x\dot{x} = J_q\dot{q} \qquad (5)$$

where

$$J_x = \frac{\partial r(x)}{\partial x} \quad J_x = [D(x)^T \quad J_{xp}^T]^T \quad J_{xp} = \frac{\partial \Psi(x)}{\partial x} \in R^{(n-m)\times n}$$

$$J_q = \frac{\partial r(x(q))}{\partial q} \quad J_q = J_x(x(q))J(q) \quad J_q, J_x \in R^{n\times n} \qquad (6)$$

Using the transformations (4) and (5) in (1), then multiplying both sides by J_q^{-T}, dynamic equation (1) with the constraints (2) and the interaction force (3) can be expressed in terms of r as

$$\begin{aligned} M(r)\ddot{r} + C(r, \dot{r})\dot{r} + g(r) + B'(\mu, r, \dot{r})\lambda = T_r \\ r = \begin{bmatrix} 0 \\ r_p \end{bmatrix}, \quad B' = \begin{bmatrix} I_m \\ 0 \end{bmatrix} + B(\mu, r, \dot{r}) \end{aligned} \qquad (7)$$

or

$$\begin{aligned} M_{12}(r)\ddot{r}_p + C_{12}(r, \dot{r})\dot{r}_p + g_1(r) + (I_m + B_1)\lambda = T_{r1} \\ M_{22}(r)\ddot{r}_p + C_{22}(r, \dot{r})\dot{r}_p + g_2(r) + B_2\lambda = T_{r2} \end{aligned} \qquad (8)$$

where

$$M(r) = J_q^{-T}(q)M(q)J_q^{-1}(q) = \begin{bmatrix} M_{11}(r) & M_{12}(r) \\ M_{21}(r) & M_{22}(r) \end{bmatrix}$$

$$\begin{aligned} C(r, \dot{r}) &= J_q^{-T}(q)C(q, \dot{q})J_q^{-1}(q) \\ &\quad - J_q^{-T}(q)M(q)J_q^{-1}(q)\dot{J}_q(q)J_q^{-1}(q) \\ &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \end{aligned}$$

$$g(r) = J_q^{-T}g(q) = \begin{bmatrix} g_1(r) \\ g_2(r) \end{bmatrix}$$

$$B(\mu, r, \dot{r}) = J_x^{-T} L^T = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$T_r = J_q^{-T}(q)\tau = \begin{bmatrix} T_{r1} \\ T_{r2} \end{bmatrix} \quad (9)$$

in which the constraints are simply described by $r_f = 0$. Motion of the robot is thus uniquely determined by the coordinates r_p. Also, the constrained force F_n has a simple structure in the new coordinate system, i.e., $J_q^{-T} F_n = [I_m\ 0]^T \lambda$. In the absence of surface friction force, $B_1 = 0$, $B_2 = 0$, and thus constrained force does not appear in the second equation of (8). Therefore, motion control can be designed based on the this reduced order equation without considering force control. This is the basic strategy adopted by previous researchers in this area (McClamroch and Wang, 1988; Wang and McClamroch, 1989; Mills and Goldenberg, 1989; Kankaanranta and Koivo, 1988; Yoshikawa et al., 1988; Yun, 1988; Cai and Goldeberg, 1989; Cole, 1989; Grabbe et al., 1993). Clearly, in the presence of surface friction force, motion and force equations are coupled and a new strategy should be invented.

Let $K_f = \text{diag}\{k_{f1}, \ldots, k_{fm}\}$ and $G_f = \text{diag}\{g_{f1}, \ldots, g_{fm}\}$ be constant diagonal matrices with $k_{fi} > 0$, $g_{fi} \geq 0$, $i = 1, \ldots, m$. By adding $G_f\lambda$ to both sides of the first equation of (8), adding and subtracting $M_{21}(r)K_f\lambda$ to the left hand of the second equation of (8), and noticing $\lambda = K_f^{-1}K_f\lambda$, Equation (8) can be rewritten in a concise form as

$$H(r_p)v + C_h(r_p, \dot{r}_p)\dot{r} + g(r_p) + B_m(\mu, r_p, \dot{r}_p)\lambda = T_r + \bar{G}_f\lambda \quad (10)$$

where

$$v = \begin{bmatrix} K_f\lambda \\ \ddot{r}_p \end{bmatrix}$$

$$H(r_p) = \begin{bmatrix} (I_m + G_f)K_f^{-1} & M_{12}(r) \\ M_{21}(r) & M_{22}(r) \end{bmatrix}$$

$$C_h(r_p, \dot{r}_p) = \begin{bmatrix} 0 & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$B_m(\mu, r_p, \dot{r}_p) = B(\mu, r, \dot{r}) + B'_m(r_p)$$
$$B'_m(r_p) = -\begin{bmatrix} 0 \\ M_{21}(r)K_f \end{bmatrix}$$

$$\bar{G}_f = \begin{bmatrix} G_f \\ 0 \end{bmatrix} \quad (11)$$

Equation (10), which possesses some nice properties introduced in the following, is the basic equation for our controller design. The physical meaning of introducing K_f and G_f in (10) will become apparent later in controller design.

From Properties 1 and 2, the following properties can be obtained for Eq. (7) (Yao et al., 1990; Yao et al., 1992c).

Property 4. For the finite work space Ω in which J_q is nonsingular, $M(r)$ is a s.p.d. matrix with

$$k'_r I_n \leq M(r) \leq k''_r I_n \quad \forall q \in \Omega \quad (12)$$

where $k'_r = (k'/c_1^2)$, $k''_r = (k''/c_2^2)$, $c_l = \sup_{q\in\Omega} [\sigma_{\max}(J_q(q))]$ $c_2 = \inf_{q\in\Omega}[\sigma_{\min}(J_q(q))]$. $\sigma(\bullet)$ denotes singular values of $\bullet$, and $\bullet_{max}$ (or $\bullet_{\min}$) is the maximum (or minimum) value of $\bullet$.

Property 5. The matrix $N(r, \dot{r}) = \dot{M}(r) - 2C(r, \dot{r})$ *is a skew-symmetric matrix.*

From the above properties and property 3, Eq. (10) has the following properties.

Property 6. $H(r_p)$ is a s.p.d. matrix for sufficiently small $\lambda_{\max}(K_f) = \max_i k_{fi}$ where $\lambda(\bullet)$ denotes eigenvalues of $\bullet$.

Proof: Rewriting H as

$$H = \begin{bmatrix} (I + G_f)K_f^{-1} - M_{11}(r) & 0 \\ 0 & 0 \end{bmatrix} + M(r) \quad (13)$$

If $\lambda_{\max}(K_f) \leq 1/k''_r$, then, $\lambda_{\min}\{(I + G_f)K_f^{-1}\} \geq \lambda_{\min}\{K_f^{-1}\} = 1/\lambda_{\max}(K_f) \geq k''_r$. From property 4, $M_{11}(r) \leq k''_r I_m$. Thus, $(I + G_f)K_f^{-1} - M_{11}$ is a symmetric positive semidefinite (s.p.sd.) matrix. From (13), property 6 is established. □

Property 7. The matrix $N_h(r_p, \dot{r}_p) = \dot{H}(r_p) - 2C_h(r_p, \dot{r}_p)$ is a skew-symmetric matrix.

Proof: From (11) and (13)

$$N_h = \dot{M}(r) - 2C(r, \dot{r}) - \begin{bmatrix} \dot{M}_{11}(r) - 2C_{11}(r, \dot{r}) & 0 \\ 0 & 0 \end{bmatrix} \quad (14)$$

From property 5, $\dot{M}_{11} - 2C_{11}$ is a skew-symmetric matrix, which leads to property 7. □

Property 8. $H(r_p)$, $C_h(r_p, \dot{r}_p)$, $g(r_p)$, $B'_m(r_p)$ are linear in terms of the robot parameters β defined in property 3, i.e.,

$$H(r_p)z_v + C_h(r_p, \dot{r}_p)z_r + g(r_p) + B'_m(r_p)\lambda = Y_\beta(r_p, \dot{r}_p, z_r, z_v, \lambda)\beta \quad Y_\beta \in R^{n\times k} \quad (15)$$

where z_r, z_v are any reference values. $B(\mu, r, \dot{r})$ is linear in terms of friction coefficient μ, i.e.,

$$B(\mu, r, \dot{r})\lambda = Y_\mu(r_p, \dot{r}_p, \lambda)\mu \quad Y_\mu \in R^{n\times m} \quad (16)$$

Suppose $r_{pd}(t) = \Psi(x(q_d(t))) \in R^{n-m}$ is given as the desired robot motion trajectory, and $\lambda_d(t) \in R^m$ is the desired constrained force trajectory. Let

$$e_p(t) = r_p(t) - r_{pd}(t) \quad e_f(t) = \lambda(t) - \lambda_d(t) \quad (17)$$

be tracking errors of motion and constrained force. Consider the robot manipulator described by (10) with some or all of the robot and surface parameters β and μ unknown. The adaptive motion and force controller design problem is to design a control law for the actuator torque and an estimation law for the unknown parameters so that $e_p \to 0$, $e_f \to 0$ as $t \to \infty$.

3 Adaptive Motion and Force Control of Robot Manipulators

In this section, based on the particular structure of the constrained motion equation (10), an adaptive method is developed for solving the adaptive motion and force control problem. Define a vector $s \in R^n$ as

$$s = \begin{bmatrix} s_f \\ s_p \end{bmatrix} = \begin{bmatrix} K_f \int_0^t e_f(\nu)d\nu \\ \dot{e}_p + De_p \end{bmatrix} \quad (18)$$

which is a measure of motion and force tracking accuracy. The reference velocity and acceleration are defined as

$$z_r = \begin{bmatrix} z_{fr} \\ z_{pr} \end{bmatrix} = \begin{bmatrix} -s_f \\ \dot{r}_{pd} - De_p \end{bmatrix}$$

$$z_v = \begin{bmatrix} z_{vf} \\ z_{vp} \end{bmatrix} = \begin{bmatrix} K_f\lambda_d \\ \ddot{r}_{pd} - D\dot{e}_p \end{bmatrix} \quad (19)$$

Note $z_v \neq \dot{z}_r$. Let the constant parameter set be $\beta = [\beta_E^T, \beta_R^T]^T$, where β_E is the unknown parameter set needed to be estimated on-line and β_R is the known parameter set. Correspondingly, partition Y_β as $Y_\beta = [Y_{\beta E}, Y_{\beta R}]$. Let $\hat{\beta}_E$ be the estimate of β_E, and $\hat{H}(r_p, \hat{\beta})$, $\hat{C}_h(r_p, \dot{r}_p, \hat{\beta})$, $\hat{g}(r_p, \hat{\beta})$, $\hat{B}'_m(r_p,$

$\beta)$, $\hat{B}(r_p, \dot{r}_p, \hat{\mu})$ be the matrices obtained from the matrices $H(r_p)$, $C_h(r_p, \dot{r}_p)$, $g(r_p)$, $B'_m(r_p)$, $B(\mu, r_p, \dot{r}_p)$ in equation (10) by substituting the estimated $\hat{\beta} = [\hat{\beta}_E^T, \hat{\beta}_R^T]^T$ and $\hat{\mu}$ for the actual β and μ respectively. Then from property 8

$$\hat{H}(r_p, \hat{\beta})z_v + \hat{C}_h(r_p, \dot{r}_p, \hat{\beta})z_r + \hat{g}(r_p, \hat{\beta}) + \hat{B}'_m(r_p, \hat{\beta})\lambda = Y_\beta(r_p, \dot{r}_p, z_r, z_v, \lambda)\hat{\beta}$$

$$\hat{B}(r_p, \dot{r}_p, \hat{\mu})\lambda = Y_\mu(r_p, \dot{r}_p, \lambda)\hat{\mu} \quad (20)$$

and

$$\tilde{H}(r_p, \hat{\beta})z_v + \tilde{C}_h(r_p, \dot{r}_p, \hat{\beta})z_r + \tilde{g}(r_p, \hat{\beta}) + \tilde{B}'_m(r_p, \hat{\beta})\lambda = Y_\beta(r_p, \dot{r}_p, z_r, z_v, \lambda)\tilde{\beta}$$

$$\tilde{B}(r_p, \dot{r}_p, \hat{\mu})\lambda = Y_\mu(r_p, \dot{r}_p, \lambda)\tilde{\mu} \quad (21)$$

where $\tilde{\bullet} = \hat{\bullet} - \bullet$ represents the estimation error of the matrix (vector) $\bullet$. The control torque is suggested to be

$$\begin{aligned} T_r &= \hat{H}(r_p, \hat{\beta})z_v + \hat{C}_h(r_p, \dot{r}_p, \hat{\beta})z_r + \hat{g}(r_p, \hat{\beta}) \\ &\quad + (\hat{B}_m(r_p, \dot{r}_p, \hat{\beta}, \hat{\mu}) - \bar{G}_f)\lambda - K_s(t)s - \bar{K}_p e_p \\ &= Y_\beta(r_p, \dot{r}_p, z_r, z_v, \lambda)\hat{\beta} + Y_\mu(r_p, \dot{r}_p, \lambda)\hat{\mu} \\ &\quad - \bar{G}_f\lambda - \bar{K}_p e_p - K_s(t)s \quad (22) \end{aligned}$$

and adaption law

$$\dot{\hat{\beta}}_E = -\Gamma_\beta Y_{\beta E}^T(r_p, \dot{r}_p, z_r, z_v, \lambda)s \quad (23)$$

$$\dot{\hat{\mu}} = -\Gamma_\mu Y_\mu^T(r_p, \dot{r}_p, \lambda)s \quad (24)$$

where $\bar{K}_p = [0, K_p]^T$, $K_p \in R^{(n-m)\times(n-m)}$, Γ_β, Γ_μ are constant s.p.d. matrices, $K_s(t)$ is a uniformly s.p.d. matrix, and s and z_r, z_v are defined by (18) and (19), respectively. D is chosen as $D = K_p^{-1}R$ and $R \in R^{(n-m)\times(n-m)}$ is any s.p.d. matrix.

Substituting the control law (22) into (10) and noticing (20) and (21), the error dynamics is obtained as

$$\begin{aligned} H\dot{s} + C_h s &= \tilde{H}(r_p, \hat{\beta})z_v + \tilde{C}_h(r_p, \dot{r}_p, \hat{\beta})z_r + \tilde{g}(r_p, \hat{\beta}) \\ &\quad + \tilde{B}_m(r_p, \dot{r}_p, \hat{\beta}, \hat{\mu})\lambda - \bar{K}_p e_p - K_s s \\ &= Y_{\beta E}(r_p, \dot{r}_p, z_r, z_v, \lambda)\tilde{\beta}_E + Y_\mu(r_p, \dot{r}_p, \lambda)\tilde{\mu} \\ &\quad - \bar{K}_p e_p - K_s s \quad (25) \end{aligned}$$

Since $\lambda = K_f^{-1}\dot{s}_f + \lambda_d$, Eq. (25) can be rewritten as

$$\begin{aligned} A(r_p, \dot{r}_p, \hat{\beta}, \hat{\mu})\dot{s} &= -C_h s + \tilde{H}(r_p, \hat{\beta})z_v + \tilde{C}_h(r_p, \dot{r}_p, \hat{\beta})z_r \\ &\quad + \tilde{g}(r_p, \hat{\beta}) + \tilde{B}_m(r_p, \dot{r}_p, \hat{\beta}, \hat{\mu})\lambda_d - \bar{K}_p e_p - K_s s \quad (26) \end{aligned}$$

where

$$A = \begin{bmatrix} (I + G_f - \tilde{B}_1)K_f^{-1} & M_{12}(r) \\ \hat{M}_{21}(r) - \tilde{B}_2 K_f^{-1} & M_{22}(r) \end{bmatrix}$$

$$= \begin{bmatrix} I & M_{12}M_{22}^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} Q(r_p, \dot{r}_p, \hat{\beta}, \hat{\mu})K_f^{-1} & 0 \\ \hat{M}_{21}(r) - \tilde{B}_2 K_f^{-1} & M_{22} \end{bmatrix}$$

$$Q = I + G_f - \tilde{B}_1 - M_{12}M_{22}^{-1}\tilde{B}_2 - M_{12}M_{22}^{-1}\hat{M}_{21}K_f \quad (27)$$

Theorem 1. For the constrained robot manipulator described by Eq. (10), in the finite workspace Ω studied, with a sufficiently small K_f, the following holds if the control law (22) with update law (23) and (24) is used:

(*a*) $\tilde{\beta} = (\hat{\beta} - \beta) \in L_\infty^k$, $\tilde{\mu} = (\hat{\mu} - \mu) \in L_\infty^m$
(*b*) $e_p \in L_\infty^{n-m} \cap L_2^{n-m}$ $s \in L_\infty^n \cap L_2^n$

Further, if $r_{pd}(t)$, $\dot{r}_{pd}(t)$, $\ddot{r}_{pd}(t)$, $\lambda_d(t)$ are bounded functions and Q in (27) is nonsingular, then,

(*c*) $e_p \to 0$, $\dot{e}_p \to 0$, $s \to 0$ when $t \to \infty$
(*d*) $e_f \in L_\infty^m$

i.e., the robot follows the desired motion trajectories and force tracking error is bounded with integral asymptotically converging to zero.

Additionally, if $r_{pd}^{(3)}(t)$, $\dot{\lambda}_d$ are also bounded functions, i.e., the desired motion and force trajectories are sufficient smooth, then,

(*e*) $e_f \to 0$, $\ddot{e}_p \to 0$ when $t \to \infty$

i.e., force tracking and motion acceleration tracking error asymptotically converge to zero.

Moreover, if the desired motion and force trajectory are chosen in such a way that the following persistent excitation conditions are satisfied

$$\int_t^{t+T} Y_d^T(\nu)Y_d(\nu)d\nu \geq \epsilon_d I_{k+m} \quad \forall t \geq t_0 \quad (28)$$

where $Y_d(t) \triangleq [Y_{\beta E}(r_{pd}(t), \dot{r}_{pd}(t), z_{rd}(t), z_{vd}(t), \lambda_d(t)), Y_\mu(r_{pd}(t), \dot{r}_{pd}(t), \lambda_d(t))]$, $z_{rd} \triangleq [0, \dot{r}_{pd}^T(t)]^T$, $z_{vd} \triangleq [(K_f\lambda_d(t))^T, \ddot{r}_{pd}^T]^T$, and T, t_0, ϵ_d are some positive scalars, then,

(*f*) $\tilde{\beta} \to 0$, $\tilde{\mu} \to 0$ when $t \to \infty$

i.e., estimated parameters converge to their true values. △

Proof: Noticing property 6, a positive definite function is chosen as

$$V = \tfrac{1}{2}s^T H(r_p)s + \tfrac{1}{2}e_p^T K_p e_p + \tfrac{1}{2}\tilde{\beta}_E^T\Gamma_\beta^{-1}\tilde{\beta}_E + \tfrac{1}{2}\tilde{\mu}^T\Gamma_\mu^{-1}\tilde{\mu} \quad (29)$$

Differentiating V with respect to time yields

$$\begin{aligned} \dot{V} &= s^T H(r_p)\dot{s} + \tfrac{1}{2}s^T\dot{H}(r_p)s + e_p^T K_p\dot{e}_p \\ &\quad + \tilde{\beta}_E^T\Gamma_\beta^{-1}\dot{\tilde{\beta}}_E + \tilde{\mu}^T\Gamma_\mu^{-1}\dot{\tilde{\mu}} \\ &= s^T H(r_p)\dot{s} + s^T C_h(r_p, \dot{r}_p)s + e_p^T K_p(s_p - De_p) \\ &\quad + \tilde{\beta}_E^T\Gamma_\beta^{-1}\dot{\tilde{\beta}}_E + \tilde{\mu}^T\Gamma_\mu^{-1}\dot{\tilde{\mu}} \\ &= s^T[H(r_p)\dot{s} + C_h(r_p, \dot{r}_p)s + \bar{K}e_p] - e_p^T K_p De_p \\ &\quad + \tilde{\beta}_E^T\Gamma_\beta^{-1}\dot{\tilde{\beta}}_E + \tilde{\mu}^T\Gamma_\mu^{-1}\dot{\tilde{\mu}} \quad (30) \end{aligned}$$

where Property 7 has been used to eliminate the term $(1/2)s^T\dot{H}(r_p)s$. Noticing error dynamics (25), $\dot{\tilde{\beta}}_E = \dot{\hat{\beta}}_E$, $\dot{\tilde{\mu}} = \dot{\hat{\mu}}$ and updating law (23) and (24), we obtain

$$\begin{aligned} \dot{V} &= s^T Y_{\beta E}\tilde{\beta}_E + s^T Y_\mu\tilde{\mu} - s^T K_s(t)s - e_p^T Re_p \\ &\quad + \tilde{\beta}_E^T\Gamma_\beta^{-1}\dot{\hat{\beta}}_E + \tilde{\mu}^T\Gamma_\mu^{-1}\dot{\hat{\mu}} \\ &= -s^T K_s s - e_p^T Re_p + \tilde{\beta}_E^T[\Gamma_\beta^{-1}\dot{\hat{\beta}}_E + Y_{\beta E}s] \\ &\quad + \tilde{\mu}^T[\Gamma_\mu^{-1}\dot{\hat{\mu}} + Y_\mu s] \\ &= -s^T K_s s - e_p^T Re_p \quad (31) \end{aligned}$$

Equations (29) and (31) imply (*a*) and (*b*) of Theorem 1. The following is to prove (*c*) and (*d*).

From (*b*), $e_p, \dot{e}_p \in L_\infty^{(n-m)}$. Since $r_{pd}, \dot{r}_{pd}, \ddot{r}_{pd} \in L_\infty^{n-m}$ and $\lambda_d \in L_\infty^m$, *then*, $r_p, \dot{r}_p \in L_\infty^{n-m}$ and $z_r, z_v \in L_\infty^n$. Thus $\tilde{H}(r_p, \hat{\beta})$, $\tilde{C}_h(r_p, \dot{r}_p, \hat{\beta})$, $\tilde{g}(r_p, \hat{\beta})$, $\tilde{B}_m(r_p, \dot{r}_p, \hat{\beta}, \hat{\mu})$, $Q(r_p, \dot{r}_p, \hat{\beta}, \hat{\mu})$ are bounded matrices. Furthermore, from (27), nonsingularity and boundedness of Q imply the nonsingularity and boundedness of $A(r_p, \dot{r}_p, \hat{\beta}, \hat{\mu})$. From (26), we conclude $\dot{s} \in L_\infty^n$, which implies (*d*). Since $s \in L_\infty^n \cap L_2^n$, simply applying Barbalat's lemma (Slotine and Li, 1991), we obtain (*c*).

To prove (*e*), noticing that A and all the terms in the right hand of (26) are differentiable, differentiate (26)

$$\begin{aligned} A\ddot{s} &= -\dot{A}(r_p, \dot{r}_p, \ddot{r}_p, \hat{\beta}, \dot{\hat{\beta}}, \hat{\mu}, \dot{\hat{\mu}})\dot{s} - \dot{C}_h(r_p, \dot{r}_p, \ddot{r}_p, \hat{\beta}, \dot{\hat{\beta}})s \\ &\quad - C_h\dot{s} + \dot{\tilde{H}}(r_p, \dot{r}_p, \hat{\beta}, \dot{\hat{\beta}})z_v + \tilde{H}\dot{z}_v \end{aligned}$$

$$+ \dot{\tilde{C}}_h(r_p, \dot{r}_p, \ddot{r}_p, \hat{\beta}, \dot{\hat{\beta}})z_r + \tilde{C}_h\dot{z}_r + \dot{\tilde{g}}(r_p, \dot{r}_p, \hat{\beta}, \dot{\hat{\beta}})$$
$$+ \dot{\hat{B}}_m(r_p, \dot{r}_p, \ddot{r}_p, \hat{\beta}, \dot{\hat{\beta}}, \hat{\mu}, \dot{\hat{\mu}})\lambda_d + \tilde{B}_m\dot{\lambda}_d - \bar{K}_p\dot{e}_p$$
$$- K_s\dot{s} - \dot{K}_s s \quad (32)$$

Since $r_{pd}^{(3)} \in L_\infty^{n-m}$ and $\dot{\lambda}_d \in L_\infty^m$, we have, $\dot{z}_v \in L_\infty^n$. Noticing $s \to 0$, from (23) and (24), $\dot{\hat{\beta}} \to 0$ and $\dot{\hat{\mu}} \to 0$. Thus all the terms in the right hand side of (32) are bounded, which imply $\ddot{s} \in L_\infty^n$ and $\dot{s}$ is uniformly continuous. From Barbalat's lemma, we have $\dot{s} \to 0$, which imply (e). Rigorous proof of (f) is omitted since it is quite tedious and standard in adaptive control literature. Roughly, since $\dot{s} \to 0$ and (c) and (e), from (25), $Y_d(t)[\tilde{\beta}_E^T, \tilde{\mu}^T]^T \to 0$. Then, PE condition (28) will guarantee (f). Q.E.D. △

Remark 1. Theorem 1 requires that Q is nonsingular, which may be guaranteed in several ways. We classify them in the following three cases:

Case 1. In the case of known friction coefficient, instead of adaptation law (24), we set $\hat{\mu} = \mu$. Thus, $\tilde{B} = 0$. Since $\tilde{\beta} \in L_\infty^k$, $\hat{\beta} \in \Omega_\beta$ where Ω_β is a finite set. In the finite workspace $\Omega_r = r(\Omega)$ studied, by choosing a small weighing matrix K_f such that

$$\lambda_{\max}(K_f) < \frac{1}{\sup_{r\in\Omega_r, \hat{\beta}\in\Omega_\beta}\{\|M_{12}(r_p)M_{22}^{-1}(r_p)\hat{M}_{21}(r_p, \hat{\beta})\|\}} \quad (33)$$

where $\|\bullet\|$ denotes a norm of $\bullet$ which is a vector or a matrix (without loss of generality, in this paper, $\|\bullet\|_2$ is used, i.e., $\|A\| = \sigma_{\max}(A) = \lambda_{\max}^{1/2}(A^TA)$), $\forall y \in R^{n-m}$, $y \neq 0$, we have

$$\|Qy\| \geq \|(I + G_f)y\| - \|M_{12}M_{22}^{-1}\hat{M}_{21}K_f y\|$$
$$\geq (1 + \lambda_{\min}(G_f))\|y\| - \|M_{12}M_{22}^{-1}\hat{M}_{21}\|\lambda_{\max}(K_f)\|y\|$$
$$> \lambda_{\min}(G_f))\|y\| \quad (34)$$

Thus, $\sigma_{\min}(Q) > \lambda_{\min}(G_f) > 0$ and Q is nonsingular.

Case 2. In the above development, only the normal contact force F_n or λ is assumed to be measurable. If the total interaction force F between the end-effector and the contact surface can also be measured by the force sensor, F_t can be calculated from (3). Thus, $B\lambda = J_x^{-T}F_t$ can be directly obtained. The control law (22) can be simplified as

$$T_r = Y_\beta(r_p, \dot{r}_p, z_r, z_v, \lambda)\hat{\beta} + J_x^{-T}F_t - \bar{G}_f\lambda$$
$$- \bar{K}_p e_p - K_s s \quad (35)$$

with only $\hat{\beta}$ updated by (23). This case is then identical to the Case 1, in which nonsingularity of Q can be guaranteed by choosing a sufficient small weighing matrix K_f.

Case 3. In the general case that only λ can be measured and μ have to be estimated by (24), since $\tilde{\mu} \in L_\infty^m$, $\hat{\mu} \in \Omega_\mu$ where Ω_μ is a finite set. By choosing

$$\lambda_{\min}(G_f) > -1 + \sup_{r\in\Omega_r, \hat{\mu}\in\Omega_\mu}\{\|\tilde{B}_1(r_p, \dot{r}_p, \hat{\mu})$$
$$+ M_{12}(r_p)M_{22}^{-1}(r_p)\tilde{B}_2(r_p, \dot{r}_p, \hat{\mu})\|\}$$
$$+ \lambda_{\max}(K_f)\sup_{r\in\Omega_r, \hat{\beta}\in\Omega_\beta}\{\|M_{12}(r_p)M_{22}^{-1}(r_p)\hat{M}_{21}(r_p, \hat{\beta})\|\},$$
$$(36)$$

we have, $\forall y \in R^m$, $y \neq 0$,

$$\|Qy\| \geq \|(I + G_f)y\| - \|(\tilde{B}_1 + M_{12}M_{22}^{-1}\tilde{B}_2)y\|$$
$$- \|M_{12}M_{22}^{-1}\hat{M}_{21}K_f y\|$$
$$\geq \|y\|[1 + \lambda_{\min}(G_f) - \|\tilde{B}_1 + M_{12}M_{22}^{-1}\tilde{B}_2\|$$
$$- \lambda_{\max}(K_f)\|M_{12}M_{22}^{-1}\hat{M}_{21}\|] > 0 \quad (37)$$

which implies the nonsingularity of Q. Thus, the nonsingularity of Q can be guaranteed by choosing a relatively large proportional (P) force feedback gain G_f. Normally, since μ and $\hat{\mu}$ are small, $\|\tilde{B}_1 + M_{12}M_{22}^{-1}\tilde{B}_2\| < 1$. By choosing a sufficiently small K_f, the right-hand side of (36) can be made less than zero so that no minimum limit is imposed on G_f and a small or zero force feedback P gain control is possible. □

Remark 2. Previous adaptive constrained motion control results can be considered as the special cases of the proposed controller. For example, in the absence of surface friction, letting $K_f = 0$, which can be considered as the limit case of $K_f \to 0$ in our controller, control law (22) is reduced to (term $\hat{H}(r_p, \hat{\beta})z_v$ is combined first to eliminate the appearance of K_f^{-1} in $\hat{H}$, and then letting $K_f \to 0$)

$$T_r = \begin{bmatrix} I_m + G_f \\ 0 \end{bmatrix}\lambda_d + \begin{bmatrix} \hat{M}_{12}(r_p, \hat{\beta}) \\ \hat{M}_{22}(r_p, \hat{\beta}) \end{bmatrix}(\ddot{r}_{pd} - D\dot{e}_p)$$
$$+ \begin{bmatrix} \hat{C}_{12}(r_p, \dot{r}_p, \hat{\beta}) \\ \hat{C}_{22}(r_p, \dot{r}_p, \hat{\beta}) \end{bmatrix}(\dot{r}_{pd} - De_p) + \hat{g}(r_p, \hat{\beta})$$
$$- \begin{bmatrix} G_f \\ 0 \end{bmatrix}\lambda - K_s s - \bar{K}_p e_p \quad (38)$$

which includes a proportional force feedback term only. Correspondingly, adaption law (23) is reduced to

$$\dot{\hat{\beta}}_E = -\Gamma_\beta Y_{\beta E}^T(r_p, \dot{r}_p, z_r, z_v, 0)s \quad (39)$$

in which z_r, z_v, s are functions of motion tracking error only. In this case, since $s = [0, s_p]^T$, by following the same steps in the proof for Theorem 1, we still have the conclusions (a), (b), (c), and (d) where $\hat{\mu} = \mu = 0$. However, we cannot have $e_f \to 0$ ((e) in Theorem 1) since s includes the motion tracking error only. Intuitively, since the adaptation law is driven by motion tracking error only, nothing can be said about force tracking unless certain persistent excitation conditions are satisfied such that the estimated parameters converge to their true values and exact feedforward compensation is achieved. Normally, this condition cannot be satisfied. Although a larger proportional force feedback gain G_f leads to a small force tracking error, its value will be severely limited in applications which will be demonstrated later in simulation. Further analysis on this point will be considered in a future paper. This result is equivalent to the result obtained in Carelli and Kelly (1989) and Jean and Fu (1991). Furthermore, by setting $G_f = 0$ in (38), the resulting control law consists of position control terms and force feedforward term only. No force feedback control is needed, which is equivalent to the result in Su et al. (1990). Note that all these results are based on the assumption that frictionless contact surfaces, which have the property that force does not affect the motion in position subspace, and, thus, motion control can be done separately and force can be regulated indirectly by pure motion control. □

Remark 3. From the discussions in Remark 2, we can see that by introducing the weighting matrix K_f in s, we can directly control the force tracking accuracy. This is especially important when friction surfaces appear, in which constrained force does affect the motion in position subspace. A larger K_f in the controller may provide better force tracking transient. However, it has a stability limit as required in Theorem 1. Also, a larger K_f may

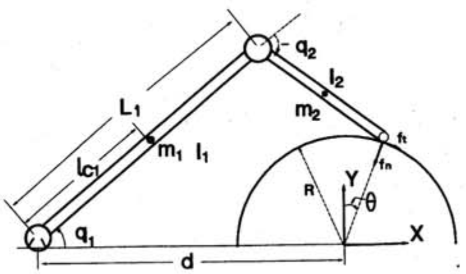

Fig. 1 Configuration of the robot moving on a semi-circle surface

result in a relatively larger proportional force feedback gain as $\hat{B}_m$ in (22) contains K_f. As analyzed in Yao and Tomizuka (1993), in digital implementation, the combined proportional force feedback gain $\hat{B}_m - \bar{G}_f$ in (22) will be severely limited. This also imposes a limit on the allowable size of K_f. □

4 Simulation

A two degree of freedom (DOF) direct drive planer SCARA robot in Fig. 1 is used in simulation. The matrices in dynamic equation (1) and forward kinematics are given by

$$M(q) = \begin{bmatrix} \beta_3 + 2\beta_1 C_{q2} & \beta_2 + \beta_1 C_{q2} \\ \beta_2 + \beta_1 C_{q2} & \beta_2 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} -\beta_1 \dot{q}_2 S_{q2} & -\beta_1(\dot{q}_1 + \dot{q}_2) S_{q2} \\ \beta_1 \dot{q}_1 S_{q2} & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 C_{q1} + l_2 C_{q12} - d \\ l_1 S_{q1} + l_2 S_{q12} \end{bmatrix}$$

$$J(q) = \begin{bmatrix} -l_1 S_{q1} - l_2 S_{q12} & -l_2 S_{q12} \\ l_1 C_{q1} + l_2 C_{q12} & l_2 C_{q12} \end{bmatrix} \quad (40)$$

where $C_{q1} = \cos(q_1)$, $C_{q2} = \cos(q_2)$, $C_{q12} = \cos(q_1 + q_2)$, $S_{q1} = \sin(q_1)$, $S_{q2} = \sin(q_2)$, and $S_{q12} = \sin(q_1 + q_2)$. $\beta = [\beta_1, \beta_2, \beta_3]^T$ is the robot physical parameters given by $\beta = [m_2 l_{c2} l_1, m_2 l_{c2}^2 + I_2, m_1 l_{c1}^2 + I_1 + m_2 l_1^2 + m_2 l_{c2}^2 + I_2]^T$. Actual parameter values of the robot are $l_1 = 0.36m$, $l_2 = 0.24m$, $\beta = [0.363, 0.353, 3.694]^T$, and $d = 0.5m$. The exact value of β is assumed to be unknown and its initial estimate is $\hat{\beta} = [0.18, 0.18, 1.8]^T$.

The robot is in contact with a rigid semi-circle surface S as shown in Fig. 1. The surface S, with an unknown dry friction coefficient $\mu = 0.2$, is described by

$$\phi(x) = \sqrt{x^2 + y^2} - R = 0 \quad R = 0.2 \text{ m} \quad (41)$$

The task space is defined as

$$r = [r_f, r_p]^T \quad r_f = \sqrt{x^2 + y^2} - R \quad r_p = R\theta$$

$$\theta = \tan^{-1}(x/y) \quad (42)$$

Notice that r_p is orthogonal to the curvilinear coordinate of r_f. The interaction force on the surface is given by (3)

$$F = F_n + F_t \quad F_n = \begin{bmatrix} \sin(\theta) \\ \cos(\theta) \end{bmatrix} f_n$$

$$F_t = -\begin{bmatrix} \cos(\theta) \\ -\sin(\theta) \end{bmatrix} \mu \, \text{sgn}(\dot{r}_p) f_n \quad (43)$$

where $\lambda = f_n \in R$ represents the normal contact force component. Task space equations (7) can be obtained and transformed equations (10) are thus derived. The form of Y_β and Y_μ are obtained from (15) and (16).

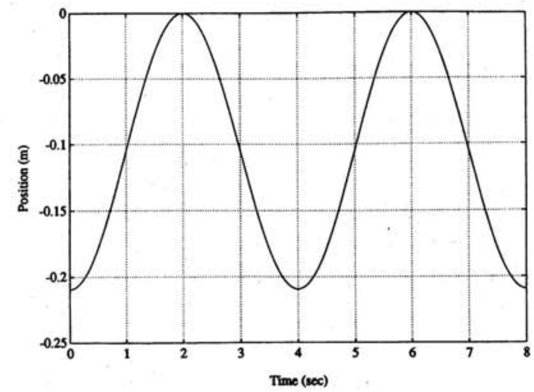

Fig. 2 Time response of position coordinate r_p

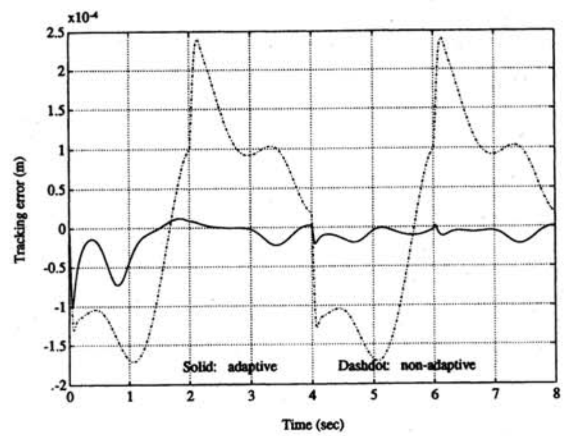

Fig. 3 Time response of position tracking error e_p

The tracking error s is given by (18) where $K_f = 0.02$, and $D = 50$. The control torque can then be calculated from (22) with adaptation law (23) and (24). Parameter values of the controller are $G_f = 0$, $\bar{K}_p = [0, 5000]^T$, $K_s = \text{diag}\{500, 500\}$, $\Gamma_\beta = \text{diag}\{150, 150, 150\}$, and $\Gamma_\mu = 5$. The desired motion and force trajectories are $r_{pd} = -(R\pi/6)(1 + \cos(0.5\pi t))$ and $f_{nd} = -15 + 5\cos(\pi t)$. Sampling time is 0.005 s.

Simulation results are shown in Figs. 2–8. The time response of position r_p and its tracking error e_p are shown in Fig. 2 and Fig. 3, respectively. The suggested adaptive controller has a good position tracking ability. Fig. 4 and Fig. 5 present the time response of contact force and normal force tracking error e_f, respectively. The system also exhibits good force tracking ability. Estimated robot parameter vector β and surface friction

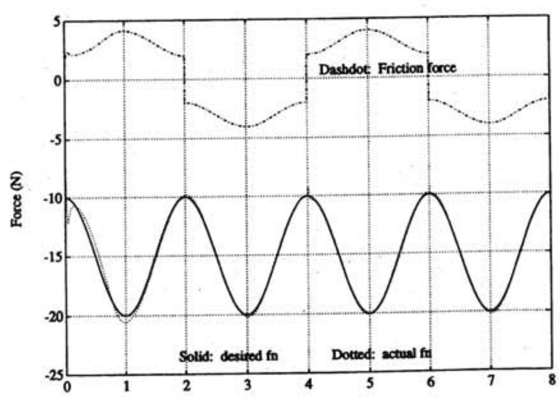

Fig. 4 Time response of interaction forces

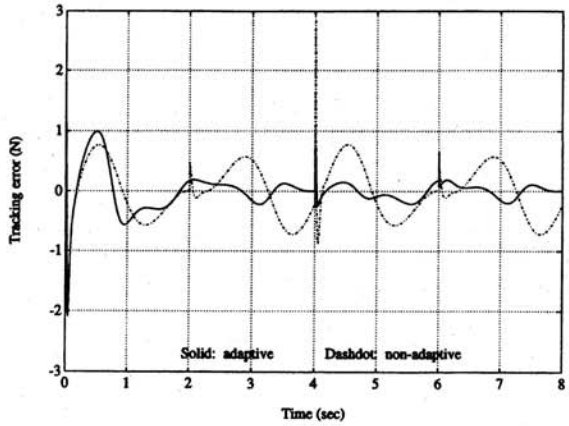

Fig. 5 Time response of force tracking error e_f

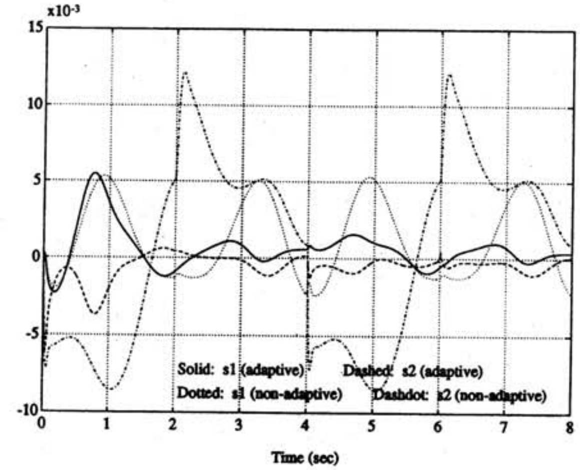

Fig. 8 Time response of tracking error s

force μ are shown in Fig. 6 and Fig. 7, respectively. The estimated parameters do not converge to their true values with $\hat{\mu}$ approaching its true value 0.2 since normally persistent excitation condition (28) is difficult to be satisfied. The time response of tracking error s is shown in Fig. 8, and the applied joint torque in Fig. 9. The sudden changes occurring at about $t = 2s$ in these figures are caused by a sudden change of surface friction force because of the direction change of the robot end-effector movement on the surface.

For comparison, the simulation is also run with the same controller parameters but without using the adaptation law (23) and (24). The tracing errors e_p, e_f, and s are shown in Figs. 3, 5, and 8, respectively. The system exhibits larger tracking errors comparing to the adaptive case.

The simulation is then run with the same controller parameters as the first simulation except for K_f. Tracking error e_p and e_f for the three cases, (i). $K_f = 0$, which is equivalent to the case of no force feedback control with the adaptation law driven by only the position tracking error, (ii). $K_f = 0.02$, and (iii). $K_f = 0.04$, are shown in Fig. 10 and Fig. 11, respectively. The results agree with Remark 3 that larger K_f will have a better force tracking ability.

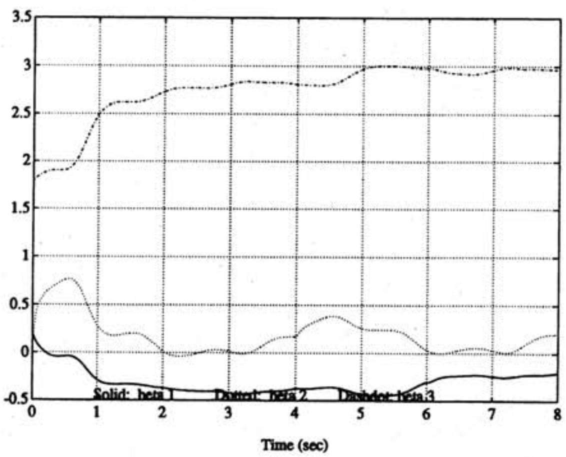

Fig. 6 Estimated robot parameter β

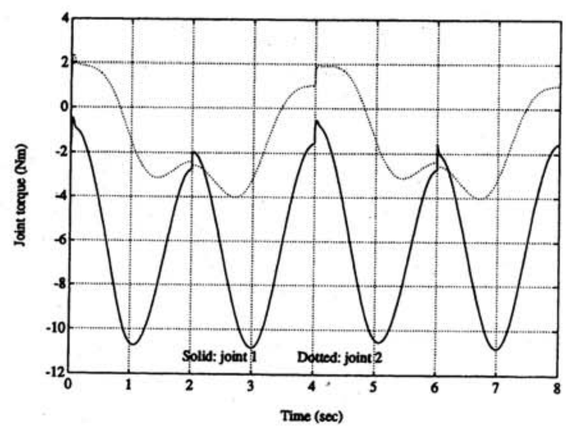

Fig. 9 Joint torque of the robot

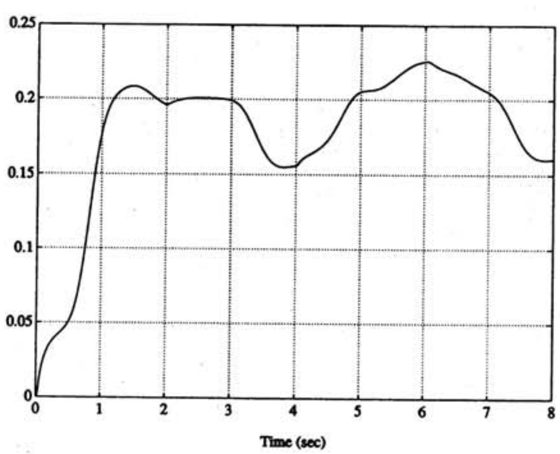

Fig. 7 Estimated surface friction coefficient μ

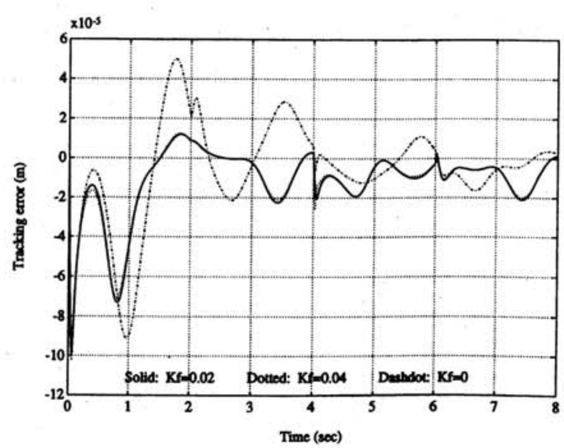

Fig. 10 Time response of position tracking error e_p

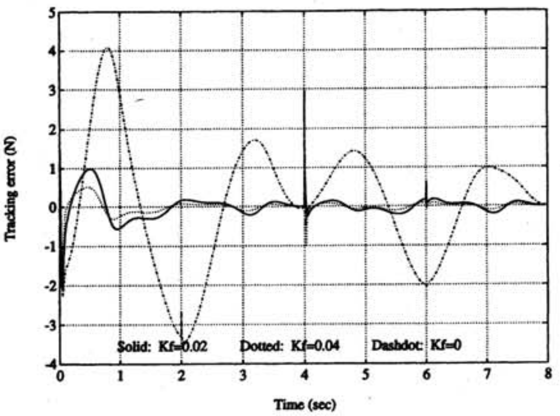

Fig. 11 Time response of force tracking error e_f

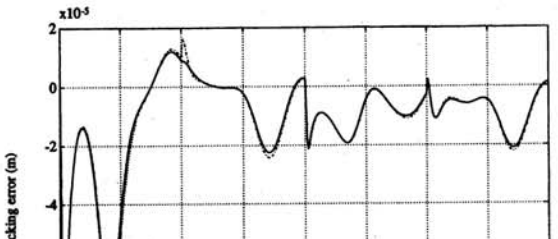

Fig. 12 Time response of position tracking error e_p

In the last simulation, the system is run with the same controller parameters as the first simulation except for G_f. Three cases, (i). $G_f = 0$, (ii). $G_f = 0.5$, and (iii) $G_f = 1$, are examined. Tracking error e_p and e_f for the three cases are presented in Fig. 12 and Fig. 13, respectively. A large G_f has little effect on position tracking error but provides a better force tracking transient. This is desirable especially in the initial stage where the force tracking error is large. However, with $G_f = 1$, the system suddenly goes to unstable around $t = 0.8s$ as seen from Fig. 13. This illustrates that the allowable gain G_f will be severely limited in applications. The control system should not rely on G_f to guarantee the force tracking

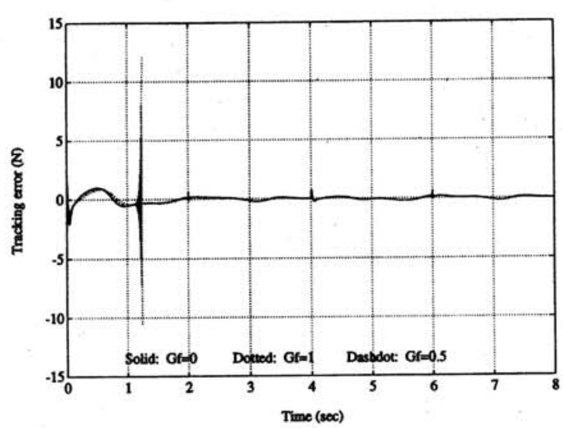

Figure 13: Time response of force tracking error e_f

Fig. 13 Time response of force tracking error e_f

accuracy. This particular phenomenon will be further studied in a future paper.

5 Conclusions

Adaptive constrained motion control of robot manipulators in the presence of parametric uncertainties both in the robot and contact surface was considered. Instead of the reduced constrained dynamic model obtained for frictionless contact surfaces, a new transformed constrained dynamic model, which is suitable for the controller design, has been proposed for the contact surfaces with or without friction. An adaptive control law with unknown parameters updated by both the motion and force tracking errors has been suggested to guarantee asymptotic motion and force tracking without any persistent excitation condition to be satisfied. The suggested control law has the expected PI type force feedback control structure with a low P-gain to avoid the acausality problem (Yao and Tomizuka, 1993). Extensive simulation results were given to show the effectiveness of the proposed method.

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