Desired Compensation Adaptive Robust Control

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A desired compensation adaptive robust control (DCARC) framework is presented for nonlinear systems having both parametric uncertainties and uncertain nonlinearities. The paper first considers a class of higher order nonlinear systems transformable to a normal form with matched model uncertainties. For this class of uncertain systems, the desired values of all states for tracking a known desired trajectory can be predetermined and the usual desired compensation concept can be used to synthesize DCARC laws. The paper then focuses on systems with unmatched model uncertainties, in which the desired values of the intermediate state variables for perfect output tracking of a known desired trajectory cannot be predetermined. A novel way of formulating desired compensation concept is proposed and a DCARC backstepping design is developed to overcome the design difficulties associated with unmatched model uncertainties. The proposed DCARC framework has the unique feature that the adaptive model compensation and the regressor depend on the reference output trajectory and on-line parameter estimates only. Such a structure has several implementation advantages. First, the adaptive model compensation is always bounded when projection type adaption law is used, and thus does not affect the closed-loop system stability. As a result, the interaction between the parameter adaptation and the robust control law is reduced, which may facilitate the controller gain tuning process considerably. Second, the effect of measurement noise on the adaptive model compensation and on the parameter adaptation law is minimized. Consequently, a faster adaptation rate can be chosen in implementation to speed up the transient response and to improve overall tracking performance. These claims have been verified in the comparative experimental studies of several applications. [DOI: 10.1115/1.3211087]

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1 Introduction

During the past 2 decades, a great deal of effort has been devoted to the control of uncertain nonlinear dynamics. The problem is motivated by the fact that almost every physical system is subjected to certain degrees of model uncertainties. The causes of model uncertainties can be classified into two distinct categories: (i) repeatable or constant unknown quantities such as the unknown physical parameters (e.g., the inertia load of any industrial drive systems), and (ii) nonrepeatable unknown quantities such as external disturbances and imprecise modeling of certain physical terms. Two nonlinear control methods have been popular and well documented: adaptive control (AC) or robust adaptive control (RAC) [1–4] for parametric uncertainties and deterministic robust control (DRC) such as sliding-mode control [5–8] for both parametric uncertainties and uncertain nonlinearities. Recently, as in the RAC of linear systems [4], much of the effort in nonlinear adaptive control area has been devoted to assuring robust stability of the adaptive backstepping designs [1] with respect to bounded disturbances and which significant progress has been made [9–11].

In Refs. [12–15], an adaptive robust control (ARC) approach has been proposed for the design of a new class of high-performance robust controllers. The approach effectively integrates DRC and AC. The resulting ARC controllers have the theoretical results of both DRC and AC, while naturally overcoming their practical performance limitations. Comparative experimental results for the motion control of robot manipulators [16], the high-speed/high-accuracy trajectory tracking control of machine tools [17], linear motor drive systems [18], and electrohydraulic servo systems [19] have demonstrated the substantially improved performance of the suggested ARC approach. Other applications include the motion and force control of robot manipulators in various contacting environment [20].

The proposed ARC approach was originally motivated by the conventional RAC [4,21,9]. However, it should be realized that there are some subtle but fundamental differences between the proposed ARC and the conventional RACs, even including the recently presented tuning function based RAC approach [10,11]. First, in terms of fundamental viewpoint, the proposed ARC [15] puts more emphasis on the robust control law design in achieving a guaranteed robust performance. In fact, the parameter adaptation law in ARC can be switched off at any time without affecting global stability and sacrificing the guaranteed transient performance result since the resulting controller becomes a deterministic robust controller. Second, in terms of the achievable performance, in the proposed ARC, the upper bound on the absolute value of the tracking error over entire time-history is given and is related to certain controller design parameters in a *known* form, which is more transparent than in RAC. Finally, in terms of specific approaches used for the controller design and the proof of achievable performance, the proposed ARC uses two Lyapunov functions; one the same as that in DRC and the other the same as that in adaptive control, while RAC uses the same Lyapunov function as in adaptive control only. Because of these subtle differences, the terminology of "*adaptive robust control*" is used for the proposed combined design method to differentiate the approach from the RAC approach and to reflect the strong emphasis on the robust control law design for a guaranteed robust performance as opposed to the goal of achieving robust stability only in RAC.

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For applications with relatively more transparent dynamics, one may have several options on the design of the robust control law and the parameter adaptation law under the proposed ARC framework. It is thus important to identify the desirable ARC controller structures so that one can select the most appropriate one for a particular application. One of them is the desired compensation ARC structure—the regressor in the model compensation and adaptation law depends on the reference trajectory only. The desired compensation adaptation law was initially proposed by Sadegh and Horowitz [22] for the trajectory tracking control of robot manipulators. The idea was then incorporated in the ARC design in Ref. [23], in which the resulting desired compensation ARC controller has the following desirable features: (a) The regressor can be calculated offline and thus on-line computation time can be reduced, and (b) the effect of measurement noise is minimized since the regressor does not depend on actual measurements. Consequently, a faster adaptation rate can be chosen in implementation to speed up the transient response and to improve overall tracking performance. These claims have been verified by the comparative experiments on the motion control of robot manipulators [16] and the linear motor drive systems [18].

This paper formalizes the desired compensation ARC (DCARC) designs in Refs. [23,16,18] and develops DCARC controllers for a much larger class of nonlinear systems including systems with unmatched model uncertainties. Specifically, the paper first considers a class of higher order nonlinear systems transformable to a normal form with matched model uncertainties. For this class of uncertain systems, the desired values of all states for tracking a known desired trajectory can be predetermined and the usual desired compensation concept [22] can be used to synthesize DCARC laws as in the existing DCARC designs [23,16,18]. The paper then focuses on systems with unmatched model uncertainties and develops a novel DCARC backstepping design to overcome the design difficulties associated with unmatched model uncertainties. As will be shown in the paper, in the presence of unmatched model uncertainties, the desired values of the intermediate state variables for perfect output tracking of a known desired trajectory *cannot* be predetermined. As such, the usual desired compensation concept cannot be used and a different set of viewpoints and design tools have to be developed. The paper will present an alternative way of formulating the desired compensation concept for systems with unmatched model uncertainties, namely, the best on-line estimates of the desired values of the intermediate state variables will be used for model compensation. By doing so, the resulting ARC law has the unique feature that the adaptive model compensation and regressor depends on the desired output trajectory and the parameter estimates only. Thus, the major benefits of conventional DCARCs are still preserved. The developed DCARC backstepping design significantly enlarges the applicable nonlinear systems (e.g., systems in the semistrict feedback forms studied in Refs. [13,14]) and enables one to synthesize DCARC law for practical applications where actuator dynamics have to be explicitly taken into account in the design stage (e.g., the control of electrohydraulic systems [19]).

2 Adaptive Robust Control (ARC)

In this section, as a motivation and building block for the proposed DCARC, tracking control of a simple first-order uncertain nonlinear system will be used to illustrate the conventional ARC designs [12,15]. The system is described by

$$\dot{x} = f(x,t) + u, \quad f = \varphi^T(x)\theta + \Delta(x,t) \tag{1}$$

where x, $u \in R$, and f is an unknown nonlinear function. In general, f can be approximated by a group of known basis functions $\varphi(x) \in R^p$ with unknown weights $\theta \in R^p$, and the approximation error is denoted by the unknown nonlinear function $\Delta(x,t)$. The objective is to let x track its desired trajectory $x_d(t)$ as closely as possible.

Throughout the paper, the following notations are used: $\bullet_i$ represents the ith component of the vector $\bullet$ and the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors. The following reasonable and practical assumptions are made [12,16,17]:

Assumption A1. The extent of parametric uncertainties and uncertain nonlinearities is known, i.e.,

$$\theta \in \Omega_\theta \triangleq \{\theta: \theta_{\min} < \theta < \theta_{\max}\}$$

$$\Delta \in \Omega_\Delta \triangleq \{\Delta: \|\Delta(x,t)\| \le \delta(x,t)\} \tag{2}$$

where $\theta_{\min}$, $\theta_{\max}$ and $\delta(x,t)$ are known. □

Under Assumption A1, the discontinuous projection based ARC design [12,15] can be applied to solve the robust tracking control problem for Eq. (1). Specifically, the parameter estimate $\hat{\theta}$ is updated through a projection type parameter adaptation law given by

$$\dot{\hat{\theta}} = \mathrm{Proj}_{\hat{\theta}}(\Gamma\tau) \tag{3}$$

where Γ is any diagonal symmetric positive definite (spd) adaptation rate matrix, τ is an adaptation function to be specified later, and the projection mapping $\mathrm{Proj}_{\hat{\theta}}(\bullet)$ is defined by

$$\mathrm{Proj}_{\hat{\theta}}(\bullet) = \begin{cases} 0 & \text{if} \begin{cases} \hat{\theta}_i = \hat{\theta}_{i\max} & \text{and} \quad \bullet > 0 \\ \hat{\theta}_i = \hat{\theta}_{i\min} & \text{and} \quad \bullet < 0 \end{cases} \\ \bullet & \qquad \text{otherwise} \end{cases} \tag{4}$$

It is shown [12] that the projection mapping has the following nice properties

$$\hat{\theta} \in \bar{\Omega}_\theta = \{\hat{\theta}: \theta_{\min} \le \hat{\theta} \le \theta_{\max}\} \quad \text{(P1)}$$

$$\tilde{\theta}^T(\Gamma^{-1}\mathrm{Proj}_{\hat{\theta}}(\Gamma\bullet) - \bullet) \le 0, \quad \forall\ \bullet \quad \text{(P2)} \tag{5}$$

The ARC control law consists of two parts given by

$$u = u_f + u_s, \quad u_f = \dot{x}_d(t) - \varphi^T\hat{\theta}$$

$$u_s = u_{s1} + u_{s2}, \quad u_{s1} = -kz \tag{6}$$

where $z = x - x_d$ is the tracking error. In Eq. (6), u_f is the adaptive model compensation needed for perfect output tracking, and u_s is the robust control law consisting of two parts: the nominal stabilizing feedback u_{s1}, which happens to be a simple proportional feedback in this case; and the robust feedback u_{s2} to attenuate the effect of model uncertainties for a guaranteed robust performance. u_{s2} is synthesized to satisfy the following two conditions:

$$z[-\varphi^T\tilde{\theta} + \Delta(x,t) + u_{s2}] \le \varepsilon \quad \text{(condition i)}$$

$$zu_{s2} \le 0 \quad \text{(condition ii)} \tag{7}$$

where $\tilde{\theta} = \hat{\theta} - \theta$ represents the parameter estimation error, and ε is a positive design parameter representing the attenuation level of the model uncertainties that one would like to achieve. In Eq. (7), condition i is used to represent the fact that u_{s2} is synthesized to dominate the model uncertainties coming from both the parametric uncertainties and uncertain nonlinearities to achieve a guaranteed level of attenuation ε, and the passivelike constraint ii is imposed to make sure that introducing u_{s2} does not interfere with the nominal identification process of parameter adaptation. The specific forms of u_{s2} satisfying conditions like Eq. (7) can be found in ARC designs in Refs. [13–15].

Theorem 2.1. *If the adaptation function in Eq. (3) is chosen as*

$$\tau = \varphi(x)z \tag{8}$$

then, the ARC law (6) with the projection type parameter adaptation law (8) guarantees the following: (a) In general, all signals are bounded and the tracking error is bounded by

$$|z|^2 \le \exp(-2kt)|z(0)|^2 + \frac{\varepsilon}{k}[1-\exp(-2kt)] \tag{9}$$

i.e., the tracking error exponentially decays to a ball. The exponential converging rate $2k$ *and the size of the final tracking error* $(|z(\infty)| \le \sqrt{\varepsilon/k})$ *can be freely adjusted by the controller parameters* ε *and* k *in a known form. (b) If after a finite time, there exist parametric uncertainties only (i.e.,* $\Delta(x,t)=0$, $\forall t \ge t_0$*), then, in addition to the results in (a), asymptotic tracking or zero final tracking error is achieved, i.e,* $z \to 0$ *as* $t \to \infty$. □

3 Desired Compensation ARC

In the ARC design presented in Sec. 2, the regressor $\varphi(x)$ in the model compensation u_f in Eq. (6) and the parameter adaptation function (8) depends on the state x. Such an adaptation structure may have several potential implementation problems. First, the regressor $\varphi(x)$ has to be calculated online based on the actual measurement of the state x. Thus, the effect of measurement noise may be severe, and a slow adaptation rate may have to be used, which in turn reduces the effect of parameter adaptation. Second, despite that the intention of introducing u_f is for model compensation, because of $\varphi(x)$, u_f depends on the actual feedback of the state also. Although theoretically the effect of this added implicit feedback loop has been considered in the robust control law design as seen from condition i of Eq. (7), practically, there still exists certain interactions between the model compensation u_f and the robust control u_s. This may complicate the design of the robust control law and the controller gain tuning process in implementation. In the following, the idea of desired compensation adaptation law introduced in Ref. [22] will be combined with the proposed ARC design to obtain a DCARC controller structure to solve these practical problems.

For simplicity, denote the desired regressor as $\varphi_d(t)=\varphi(x_d(t))$. Let the regressor error be $\tilde{\varphi}=\varphi(x)-\varphi_d$. Noting that θ is unknown but with known bounded, there exists a known function $\delta_\phi(x,t)$ such that

$$|\tilde{\varphi}^T\theta| = |\varphi(x)^T\theta - \varphi(x_d)^T\theta| \le \delta_\phi(x,t)|z| \tag{10}$$

The proposed desired compensation ARC law and the adaptation function have the same forms as Eqs. (6) and (8), respectively, but with the desired regressor $\varphi_d(t)$ and a strengthened robust control u_s, which are given by

$$u = u_f + u_s, \quad u_f = \dot{x}_d(t) - \varphi_d^T(t)\hat{\theta}$$
$$u_s = u_{s1} + u_{s2}, \quad u_{s1} = -k_{s1}z$$
$$\tau = \varphi_d(t)z \tag{11}$$

where k_{s1} can be any nonlinear gain satisfying

$$k_{s1} \ge k + \delta_\phi(x,t) \tag{12}$$

and u_{s2} is required to satisfy conditions similar to Eq. (7) with a modified condition i as

$$z[-\varphi_d^T\tilde{\theta} + \Delta(x,t) + u_{s2}] \le \varepsilon \quad \text{(i)} \tag{13}$$

Theorem 3.1.[2] *If the DCARC law (11) is applied, the same results as stated in Theorem 2.1 are achieved.* □

Remark 3.1. The DCARC law (11) has the following advantages. (i) Since the regressor φ_d depends on the reference trajectory only, it is bounded and can be calculated offline to save on-line computation time if needed. (ii) Due to the use of projection mapping in Eq. (3), $\hat{\theta}$ is bounded as shown by P1 of Eq. (5). Thus the model compensation u_f in Eq. (11) is bounded no matter what type of adaptation law is going to be used. This implies that u_f does not affect the closed-loop system stability at all and the robust control function u_s can be synthesized totally independent from the design of parameter adaptation law for robust stability. (iii) Gain tuning process becomes simpler since some of the bounds like the bound of the first term inside the bracket of the left hand side of Eq. (13) can be estimated offline. (iv) The effect of measurement noise is reduced. □

[2]Proofs of all theorems are given in the Appendix.

4 DCARC of Systems in Normal Form

In this section, DCARC of high-order SISO nonlinear systems transformable to the following controllable canonical form with matched model uncertainties will be solved. The system under consideration is described by

$$\dot{x}_i = x_{i+1}, \quad i \le n-1$$
$$\dot{x}_n = \varphi^T(x)\theta + \Delta(x,t) + u$$
$$y = x_1 \tag{14}$$

where $x=[x_1,\ldots,x_n]^T \in R^n$ is the state, y is the output, and θ and $\Delta(x,t)$ are assumed to satisfy Eq. (2) as in Secs. 2 and 3. The objective is to design a bounded control law for the input u such that all signals are bounded and the output y tracks the desired output trajectory $y_d(t)$ as closely as possible. As such, if perfect output tracking were achieved (i.e., $y(t)=y_d(t)$, $\forall t$), from the first equation of Eq. (14), the values of the corresponding state variables would be $x_1=y=y_d(t)$, $x_2=\dot{x}_1=\dot{y}_d(t),\ldots,$ $x_n=x_1^{(n-1)}=y_d^{(n-1)}$. Therefore, define the desired state trajectory as $x_d(t) =[y_d(t),\dot{y}_d(t),\ldots,y_d^{(n-1)}]^T \in R^n$, which is known in advance. By doing so, we can define the desired regressor $\varphi_d=\varphi(x_d(t))$ and the regressor error $\tilde{\varphi}$ as in Sec. 3, and define the state tracking error as $e=x-x_d \in R^n$. Similar to Eq. (10), there exists a known vector function $\delta_\phi(x,t) \in R^n$ such that

$$|\tilde{\varphi}^T\theta| = |\varphi(x)^T\theta - \varphi(x_d)^T\theta| \le \delta_\phi(x,t)^T|e| \tag{15}$$

The system (14) has a relative degree of n and is in the semistrict feedback form studied in Ref. [13]. Thus, in principle, the backstepping designs may be applied to construct intermediate control functions for the first $n-1$ equations (i.e., state equations for $\bar{x}_{n-1}=[x_1,\ldots,x_{n-1}]^T$). However, since the system (14) has matched model uncertainties only. A simple sliding-modelike technique can be used to construct a control function for the first $n-1$ equations directly, which is adopted in the paper. Furthermore, a dynamic sliding mode can be employed to enhance the dynamic response of the system as in the control of robot manipulators [12,16,23]. The design proceeds as follows.

Let a dynamic compensator be

$$\dot{x}_c = A_c x_c + B_c e_1, \quad x_c \in R^{n_c}, \quad B_c \in R^{n_c\times 1}$$
$$y_c = C_c x_c, \quad y_c \in R \tag{16}$$

where (A_c,B_c,C_c) is controllable and observable and e_1 is the first element of e or the actual output tracking error. For simplicity, denote $\bar{e}_{n-1}$ as the first $n-1$ elements of e. Noting Eq. (14), $\dot{\bar{e}}_{n-1} =[e_2,\ldots,e_n]^T$, which is known. Define a switching-functionlike term as

$$\xi = L_\xi^T e + y_c = \bar{L}_{\xi n-1}^T\bar{e}_{n-1} + e_n + y_c = l_{\xi 1}e_1 + \cdots + l_{\xi n-1}e_1^{(n-2)} + e_1^{(n-1)} + y_c \tag{17}$$

where $L_\xi=[\bar{L}_{\xi n-1}^T,1]^T$, $\bar{L}_{\xi n-1}=[l_{\xi 1},\ldots,l_{\xi n-1}]^T$ is a constant vector to be chosen later. In frequency domain, from Eqs. (17) and (16), $e_1(s)$ is related to $\xi(s)$ by

$$e_1(s)=G_\xi(s)\xi(s), \quad G_\xi(s)=\frac{1}{s^{n-1}+l_{\xi n-1}s^{n-2}+\cdots+l_{\xi 1}+G_c(s)} \tag{18}$$

where $G_c(s)=C_c(sI_{n_c}-A_c)^{-1}B_c$. It is thus clear that poles of $G_\xi(s)$ can be arbitrarily assigned by suitably choosing dynamic compensator transfer function $G_c(s)$ and the constant vector L_ξ; $G_\xi(s)$ should be chosen such that the resulting dynamic sliding mode $\{\xi=0\}$ (i.e., free response of the transfer function $G_\xi(s)$) possesses fast enough exponentially converging rate and the effect of nonzero ξ on e_1 can be attenuated to a certain degree. In addition, the initial value $x_c(0)$ of the dynamic compensator (16) can be chosen to satisfy

$$C_c x_c(0)=-L_\xi^T e(0) \tag{19}$$

Then $\xi(0)=0$ and transient tracking error may be reduced.

Noting Eqs. (17) and (16), the state space representation of Eq. (18) is obtained as

$$\dot{x}_\xi=A_\xi x_\xi+B_\xi\xi, \quad y_\xi=C_\xi x_\xi \tag{20}$$

where $x_\xi=[x_c^T,\bar{e}_{n-1}^T]^T\in R^{n_c+n-1}$ and

$$A_\xi=\begin{bmatrix} A_c & B_c & 0_{n_c\times(n-2)} \\ 0_{(n-2)\times n_c} & 0_{(n-2)\times 1} & I_{n-2} \\ -C_c & & -\bar{L}_{\xi n-1}^T \end{bmatrix}$$

$$B_\xi=\begin{bmatrix} 0_{n_c\times 1} \\ 0_{(n-2)\times 1} \\ 1 \end{bmatrix}$$

$$C_\xi=[0_{1\times n_c},1,0_{1\times(n-2)}] \tag{21}$$

Since $G_\xi(s)$ is chosen to be stable, there exists a spd solution P_ξ for any spd matrix Q_ξ for the following Lyapunov equation:

$$A_\xi^T P_\xi+P_\xi A_\xi=-Q_\xi \tag{22}$$

Furthermore, the exponentially converging rate $\lambda_{\min}(Q_\xi)/\lambda_{\max}(P_\xi)$ can be any desired value by assigning the poles of A_ξ to the far left plane and suitably choosing Q_ξ.

Define the transformed state error vector as $x_e=[x_\xi^T,\xi]^T=[x_c^T,\bar{e}_{n-1}^T,\xi]^T\in R^{n_c+n}$. The original state error vector e is related to x_e by

$$e=C_e x_e, \quad C_e=\begin{bmatrix} 0 & I_{n-1} & 0 \\ -C_c & -\bar{L}_{\xi n-1}^T & 1 \end{bmatrix} \tag{23}$$

Noting Eq. (15), there exists known nonlinear functions $\delta_{x_\xi}(x_e,t)$ and $\delta_\xi(x_e,t)$ such that

$$|\tilde{\varphi}^T\theta|\le\delta_{x_\xi}(x_e,t)\|x_\xi\|+\delta_\xi(x_e,t)|\xi| \tag{24}$$

The proposed DCARC law $\alpha(x_e,\hat{\theta},t)$ for Eq. (14) and the associated adaptation function $\tau_\alpha(x_e,t)$ have similar forms as Eq. (11) and are given by

$$\alpha=\alpha_f(\hat{\theta},t)+\alpha_s(x_e,t), \quad \alpha_s=\alpha_{s1}+\alpha_{s2}$$

$$\alpha_f=y_d^{(n)}(t)-\varphi_d^T(t)\hat{\theta}$$

$$\alpha_{s1}=-k_{x_e}(x_e,t)x_e=-k_{s1}\xi-C_c(A_c x_c+B_c e_1)-\bar{L}_{\xi n-1}^T\dot{\bar{e}}_{n-1}-B_\xi^T P_\xi x_\xi$$

$$\tau_\alpha=\varphi_d(t)\xi \tag{25}$$

In Eq. (25), $k_{s1}(x_e,t)$ is any nonlinear gain satisfying

$$k_{s1}\ge k+\delta_\xi+\frac{1}{2k_Q}\delta_{x_\xi}^2 \tag{26}$$

where k_Q is any gain less than $\lambda_{\min}(Q_\xi)$, and α_{s2} is required to satisfy constraints similar to Eq. (7)

$$\xi[-\varphi_d^T\tilde{\theta}+\Delta+\alpha_{s2}]\le\varepsilon \quad \text{(i)}$$

$$\xi\alpha_{s2}\le 0 \quad \text{(ii)} \tag{27}$$

Theorem 4.1. *If the DCARC law (25) is applied, i.e., $u=\alpha$ with $\hat{\theta}$ updated by Eq. (3) and $\tau=\tau_\alpha$, then, (a) in general, all signals are bounded. Furthermore, the non-negative function V_s defined by*

$$V_s=\tfrac{1}{2}x_\xi^T P_\xi x_\xi+\tfrac{1}{2}\xi^2 \tag{28}$$

is bounded above by

$$V_s\le\exp(-\lambda_V t)V_s(0)+\frac{\varepsilon}{\lambda_V}[1-\exp(-\lambda_V t)] \tag{29}$$

where $\lambda_V=\min\left\{\frac{\lambda_{\min}(Q_\xi)-k_Q}{\lambda_{\max}(P_\xi)}, \quad 2k\right\}$.

(b) If after a finite time, the system is subjected to parametric uncertainties only (i.e., $\Delta(x,t)=0$, $\forall t\ge t_0$), then, in addition to the results in (a), asymptotic tracking of all states and zero final output tracking error are achieved, i.e, $x_e\to 0$ and $e\to 0$ as $t\to\infty$. □

Remark 4.1. The DCARC law (25) has the structure that the model compensation α_f depend on the reference output trajectory and parameter estimate only, and the robust control term α_s does not depend on the parameter estimate. It thus has all the nice properties stated in Remark 3.1. In addition, from Eq. (25), $|\alpha_s|\le k_{\alpha s}(x_e,t)\|x_e\|$ for some function $k_{\alpha s}$. Thus, $\alpha_s\to 0$ as $\|x_e\|\to 0$, which indicates that α_f is indeed the desired control action that one needs for perfect output tracking in viewing the result (b) of the theorem. □

5 DCARC Backstepping Design

In this section, a DCARC backstepping design will be presented to overcome the design difficulties associated with higher "relative degrees" [13] and *unmatched* model uncertainties [1,7]. To keep the development concise, the system under consideration is obtained by augmenting the system (14) through a general first-order nonlinear input dynamics, which is described by

$$\dot{x}_i=x_{i+1}, \quad i\le n-1$$

$$\dot{x}_n=\varphi^T(x)\theta+\Delta(x,t)+u$$

$$\dot{u}=\varphi_u^T(x,u)\theta+\Delta_u(x,u,t)+v$$

$$y=x_1 \tag{30}$$

where v is the new input of the system and u becomes a measurable state variable. Similar to Eq. (2), the unknown nonlinear function Δ_u is assumed to be bounded by

$$|\Delta_u|\le\delta_u(x,u,t) \tag{31}$$

The goal is the same as in Sec. 4, i.e., we would like to have $y-y_d(t)$ as small as possible all the time.

The same as in the system (14) in Sec. 4, the desired values for the first n state variables for perfect output tracking is known in advance and given by $x_d(t)=[y_d,\ldots,y_d^{(n-1)}]^T$. However, if we use the same idea to obtain the desired trajectory for the added state variable u, in the absence of uncertain nonlinearities (i.e., $\Delta(x,t)=0$), the resulting desired value would be $u=\dot{x}_n-\varphi^T(x)\theta=y_d^{(n)}-\varphi^T(x_d)\theta$, which is unknown due to the appearance of the unknown parameters θ. It is thus clear that, in the presence of unmatched model uncertainties, the desired actions cannot be obtained in the same way as in the conventional desired compensation ARC presented before. A new way of defining the desired control actions has to be sought and new design tools have to be developed to deal with this added difficulty. The details are given below.

Noting that the best estimate of the desired action for u is given by $\alpha_f(\hat{\theta},t)$ defined in Eq. (25), we define the desired trajectory $u_d(t)$ for u to be $u_d=\alpha_f(\hat{\theta},t)$. Though this definition of u_d precludes it being calculated offline based on the desired trajectory only, u_d still has the desirable feature that it depends on the reference trajectory and on-line parameter estimates only to minimize the effect of measurement noises. The desired value of the function $\varphi_u(x,u)$ can thus be calculated as $\varphi_{ud} = \varphi_u(x_d(t),u_d(\hat{\theta},t))$. Similar to Eqs. (15) and (24), there exist known functions $\delta_{\varphi_u 1}$ and $\delta_{\varphi_u 2}$ such that

$$|\tilde{\varphi}_u^T\theta| = |\varphi_u(x,u)^T\theta - \varphi_{ud}(x_d,u_d)^T\theta| \leq \delta_{\varphi_u 1}\|x_e\| + \delta_{\varphi_u 2}|u-u_d| \tag{32}$$

Denote the input discrepancy for the first two equations of Eq. (30) as $z_u=u-\alpha$, in which α is defined by Eq. (25). As shown in Theorem 4.1, if $z_u=0$, output tracking would be achieved. Thus, the backstepping design in this section is essentially to synthesize a DCARC law for the actual input v such that z_u converges to a small value with a guaranteed transient performance as follows.

From Eq. (25), $|\alpha_s| \leq k_{\alpha s}(x_e,t)\|x_e\|$ for some known function $k_{\alpha s}$. Noting that $|u-u_d|=|z_u+\alpha_s| \leq |z_u|+|\alpha_s|$, from Eq. (32), there exist known functions $\delta_{\varphi_u 3}$ and $\delta_{\varphi_u 4}$ such that

$$|\tilde{\varphi}_u^T\theta| \leq \delta_{\varphi_u 3}\|x_e\| + \delta_{\varphi_u 4}|z_u| \tag{33}$$

The proposed DCARC backstepping law has the following structural form

$$v = v_f(\hat{\theta},t) + v_s(x_e,z_u,\hat{\theta},t), \quad v_s = v_{s1} + v_{s2} + v_{s3}$$

$$v_f = \frac{\partial \alpha_f(\hat{\theta},t)}{\partial t} - \varphi_{ud}^T(\hat{\theta},t)\hat{\theta}$$

$$v_{s1} = -k_{su}z_u - \xi + \frac{\partial \alpha_s}{\partial t} + \frac{\partial \alpha_s}{\partial x_\xi}(A_\xi x_\xi + B_\xi \xi) + \frac{\partial \alpha_s}{\partial \xi}(z_u - k_{s1}\xi - B_\xi^T P_\xi x_\xi + \alpha_{s2}) \tag{34}$$

where v_f is the model compensation depending on the reference trajectory and the parameter estimate only, v_{s1} is a robust feedback term having the same functionality as that in Eq. (25), v_{s2} is synthesized in the following to attenuate the effect of model uncertainties, and v_{s3} is an additional robust control action term synthesized in the following to handle the effect of time-varying parameter estimate.

Noting the particular form of Eq. (25), from Eqs. (30) and (34), it can be checked out that

$$\begin{aligned}\dot{z}_u &= \dot{u} - \dot{\alpha} = \varphi_u^T\theta + \Delta_u + v - \left[\frac{\partial \alpha_f}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{\partial \alpha_f}{\partial t} + \frac{\partial \alpha_s}{\partial x_\xi}\dot{x}_\xi + \frac{\partial \alpha_s}{\partial \xi}\dot{\xi} + \frac{\partial \alpha_s}{\partial t}\right] \\ &= -k_{su}z_u - \xi + \tilde{\varphi}_u^T\theta - \frac{\partial \alpha_s}{\partial \xi}\tilde{\varphi}^T\theta + v_{s2} - \phi_u^T\tilde{\theta} + \Delta_u - \frac{\partial \alpha_s}{\partial \xi}\Delta + v_{s3} \\ &\quad - \frac{\partial \alpha_f}{\partial \hat{\theta}}\dot{\hat{\theta}}\end{aligned} \tag{35}$$

where

$$\phi_u = \varphi_{ud}(\hat{\theta},t) - \frac{\partial \alpha_s}{\partial \xi}\varphi_d(t) \tag{36}$$

Define an augmented non-negative function as

$$V_{st} = V_s + \tfrac{1}{2}z_u^2 \tag{37}$$

where V_s is defined by Eq. (28). Noting Eqs. (24) and (33), from Eqs. (35), it is straightforward to show that

$$\begin{aligned}\dot{V}_{st} &\leq -\frac{1}{2}(\lambda_{\min}(Q_\xi) - k_Q)\|x_\xi\|^2 - k\xi^2 + \xi[-\varphi_d^T\tilde{\theta} + \Delta + \alpha_{s2}] - (k_{su} \\ &\quad - \delta_{\varphi_u 4})z_u^2 + \left[\delta_{\varphi_u 3}\|x_e\| + \left|\frac{\partial \alpha_s}{\partial \xi}\right|(\delta_{x_\xi}\|x_\xi\| + \delta_\xi|\xi|)\right]|z_u| + z_u\left[v_{s2}\right. \\ &\quad \left. - \phi_u^T\tilde{\theta} + \Delta_u - \frac{\partial \alpha_s}{\partial \xi}\Delta\right] + z_u\left[v_{s3} - \frac{\partial \alpha_f}{\partial \hat{\theta}}\dot{\hat{\theta}}\right]\end{aligned} \tag{38}$$

v_{s2} is now chosen to satisfy conditions similar to Eq. (27)

$$z_u\left[v_{s2} - \phi_u^T\tilde{\theta} + \Delta_u - \frac{\partial \alpha_s}{\partial \xi}\Delta\right] \leq \varepsilon_u \quad \text{(i)}$$

$$z_u v_{s2} \leq 0 \qquad \text{(ii)} \tag{39}$$

Specific form of v_{s2} can be obtained using the techniques in Refs. [13,15]. For example,

$$v_{s2} = -\frac{1}{4\varepsilon_u}h_u^2 z_u \tag{40}$$

where h_u is any function satisfying $h_u \geq \|\phi_u\|\|\theta_M\| + \delta_u + |\partial\alpha_s/\partial\xi|\delta$. Let the adaptation function be

$$\tau = \tau_a + \phi_u z_u = \varphi_d \xi + \phi_u z_u \tag{41}$$

v_{s3} can now be chosen as

$$v_{s3} = \frac{\partial \alpha_f}{\partial \hat{\theta}}\dot{\hat{\theta}} = \frac{\partial \alpha_f}{\partial \hat{\theta}}\text{Proj}_{\hat{\theta}}(\Gamma\tau) \tag{42}$$

to cancel the effect of the time-varying parameter estimate as seen from Eq. (38). Note that v_{s3}, given by Eq. (42), may experience possible finite jumps since the projection mapping is discontinuous at certain boundary points. If this poses a problem (e.g., if further backstepping design is needed as in Refs. [13–15]), then, instead of the perfect cancellation by Eq. (42), the technique in Ref. [15] can be used to construct a smooth v_{s3} to dominate the effect of the time-varying parameter estimate. The details are quite tedious and omitted here.

Theorem 5.1. *Consider the DCARC law (34) and the adaptation function (41) for the system (30). When the controller gain k_{su} in Eq. (34) is chosen large enough such that*

$$k_{su} \geq d_1 + \delta_{\varphi_u 4} + \frac{1}{2d_2}\left(\delta_{\varphi_u 3} + \left|\frac{\partial \alpha_s}{\partial \xi}\right|\delta_{x_\xi}\right)^2 + \frac{1}{4d_3}\left(\delta_{\varphi_u 3} + \left|\frac{\partial \alpha_s}{\partial \xi}\right|\delta_\xi\right)^2 \tag{43}$$

where d_1 is any positive scalar, d_2 and d_3 are any positive nonlinear gains satisfying $d_2 < \lambda_{\min}(Q_\xi) - k_Q$ and $d_3 < k$, respectively, then,

(a) In general, all signals are bounded. Furthermore, the nonnegative function V_{st} defined by (37) is bounded above by

$$V_{st} \leq \exp(-\lambda_V t)V_{st}(0) + \frac{\varepsilon_V}{\lambda_V}[1 - \exp(-\lambda_V t)] \tag{44}$$

where $\lambda_V = \min\{\lambda_{\min}(Q_\xi) - k_Q - d_2/\lambda_{\max}(P_\xi),\ 2(k-d_3),\ 2d_1\}$, and $\varepsilon_V = \varepsilon + \varepsilon_u$.

(b) If after a finite time, the system is subjected to parametric uncertainties only (i.e., $\Delta=0$ and $\Delta_u=0$). Then, in addition to the results in (a), asymptotic tracking of states and zero final output tracking error are achieved, i.e, $x_e \to 0$, $z_u \to 0$, and $e \to 0$ as $t \to \infty$. □

Remark 5.1. It is easy to verify that the DCARC law (34) has the unique structure that the model compensation v_f depends on the reference output trajectory and parameter estimate only. In addition, the robust control term v_s vanishes whenever the state tracking errors x_e and z_u converge to zero, i.e., $v_s \to 0$ when $x_e \to 0$ and $z_u \to 0$, which implies that the proposed DCARC control law is rather smooth. In the presence of parametric uncertainties only, in viewing the result (b) of the theorem, $v_s \to 0$ as $t \to 0$, which indicates that v_f is indeed the desired control action needed for perfect output tracking. □

6 Conclusions

A general framework on the DCARC has been presented for a class of nonlinear systems having both parametric uncertainties and uncertain nonlinearities. For systems with matched model uncertainties, the resulting DCARC controllers have the desirable feature that the regressor used in the adaptive model compensation and the adaptation law depends on the desired output reference trajectory only and can be precomputed to save on-line computation time. For systems with unmatched model uncertainties, the resulting DCARC controllers has the unique feature that the adaptive model compensation and essential part of the regressor in framework depend on the desired output trajectory and the on-line parameter estimates only. These features make the resulting DCARC controller significantly less sensitive to measurement noises. Consequently, a faster adaptation rate can be used in implementation to significantly improve the tracking performance. Though not presented in this paper, the proposed DCARC framework has also been applied to the precision motion control of linear motor drive systems having matched model uncertainties [18] and electrohydraulic systems having unmatched model uncertainties [19]. Extensive comparative experimental results obtained in both applications [18,24,25] have verified the above claims on the significantly improved control performance in implementation.

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Appendix

1 Proof of Theorem 3.1. Substituting Eq. (11) into Eq. (1), the error equation is

$$\dot{z} + k_{s1} z = \tilde{\varphi}^T \theta - \varphi_d^T \tilde{\theta} + \Delta(x,t) + u_{s2} \tag{A1}$$

Noting Eqs. (10) and (12), the derivative of a non-negative function $V_s = \frac{1}{2} z^2$ is given by

$$\dot{V}_s \le -k_{s1} z^2 + |z||\tilde{\varphi}^T \theta| + z[-\varphi_d^T \tilde{\theta} + \Delta + u_{s2}] \le -kz^2 + z[-\varphi_d^T \tilde{\theta} + \Delta + u_{s2}] \tag{A2}$$

Thus, from Eq. (13),

$$\dot{V}_s \le -kz^2 + \varepsilon \le -2kV_s + \varepsilon \tag{A3}$$

which leads to Eq. (9) and proves results in (a) of the theorem.

Now consider the situation in (b) of the theorem, i.e., $\Delta = 0$, $t \ge t_0$. Choose a non-negative function V_a as

$$V_a = V_s + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{A4}$$

Noticing Eqs. (A2) and (11), condition ii of Eq. (7), and P2 of Eq. (5),

$$\dot{V}_a = \dot{V}_s + \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} \le -kz^2 + \tilde{\theta}^T \Gamma^{-1} (\dot{\hat{\theta}} - \Gamma \tau) \le -kz^2 + \tilde{\theta}^T (\Gamma^{-1} \mathrm{Proj}_{\hat{\theta}}(\Gamma \tau) - \tau) \le -kz^2 \tag{A5}$$

Therefore, $z \in L_2$. It is easy to check that $\dot{z}$ is bounded. So, z is uniformly continuous. By Barbalat's lemma, $z \to 0$ as $t \to \infty$, which proves (b) of the theorem. □

2 Proof of Theorem 4.1. In order for the results in this section to be conveniently used in the DCARC backstepping design in the Sec. 5, formulas are derived for the general case that u might be different from the control function α first. Denote the input discrepancy as $z_u = u - \alpha$, from Eqs. (14), (17), and (25), it can be checked out that

$$\dot{\xi} = z_u - k_{s1}\xi - B_\xi^T P_\xi x_\xi + \tilde{\varphi}^T \theta - \varphi_d^T \tilde{\theta} + \Delta + \alpha_{s2} \tag{A6}$$

Noting Eqs. (20), (22), (A6), and (24), the derivative of V_s given by Eq. (28) is

$$\dot{V}_s = -\frac{1}{2} x_\xi^T Q_\xi x_\xi - k_{s1}\xi^2 + \xi \tilde{\varphi}^T \theta + \xi[-\varphi_d^T \tilde{\theta} + \Delta + \alpha_{s2}] + \xi z_u \le -\frac{1}{2}(\lambda_{\min}(Q_\xi) - k_Q)\|x_\xi\|^2 - k\xi^2 + \xi[-\varphi_d^T \tilde{\theta} + \Delta + \alpha_{s2}] + \xi z_u \tag{A7}$$

in which $\xi \tilde{\varphi}^T \theta \le \delta_{x_\xi} \|x_\xi\| |\xi| + \delta_\xi \xi^2 \le 1/2k_Q \|x_\xi\|^2 + (1/2k_Q \delta_{x_\xi}^2 + \delta_\xi)\xi^2$ has been used. Thus, if $u = \alpha$ or $z_u = 0$, noting (i) of Eq. (27),

$$\dot{V}_s \le -\lambda_V V_s + \varepsilon \tag{A8}$$

which leads to Eq. (29) and proves the results in (a) of the theorem. Noting (ii) of Eq. (27) and (b) of the theorem can be proved by using a positive definite function V_a of the form Eq. (A4) and the same techniques as in Eq. (A5). □

3 Proof of Theorem 5.1. If Eq. (43) is satisfied, by using the completion of square and noting Eq. (42), from Eq. (38), it is easy to show that

$$\dot{V}_{st} \le -\frac{1}{2}(\lambda_{\min}(Q_\xi) - k_Q - d_2)\|x_\xi\|^2 - (k - d_3)\xi^2 - d_1 z_u^2 + \xi[-\varphi_d^T \tilde{\theta} + \Delta + \alpha_{s2}] + z_u \left[v_{s2} - \phi_u^T \tilde{\theta} + \Delta_u - \frac{\partial \alpha_s}{\partial \xi} \Delta \right] \tag{A9}$$

Noting (i)of Eqs. (27) and (39),

$$\dot{V}_{st} \le -\lambda_V V_{st} + \varepsilon_V \tag{A10}$$

which leads to (a) of the theorem.

When $\Delta = 0$ and $\Delta_u = 0$, noting Eq. (41) and (ii) of Eqs. (27) and (39),

$$\dot{V}_{st} \le -\lambda_V V_{st} - \tau^T \tilde{\theta} \tag{A11}$$

Thus, (b) of the theorem can be proved by using a positive definite function $V_{at} = V_{st} + 1/2 \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$ and the same techniques as in Eq. (A5). □

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