Output Feedback Adaptive Robust Control of Uncertain Linear Systems With Disturbances

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In this paper, a discontinuous projection based output feedback adaptive robust control (ARC) scheme is constructed for a class of linear systems subjected to both parametric uncertainties and disturbances that might be output dependent. An observer is first designed to provide exponentially convergent estimates of the unmeasured states. This observer has an extended filter structure so that on-line parameter adaptation can be utilized to reduce the effect of possible large disturbances that have known shapes but unknown amplitudes. Estimation errors that come from initial state estimates and uncompensated disturbances are dealt with via certain robust feedback at each step of the backstepping design. Compared to other existing output feedback robust adaptive control schemes, the proposed method explicitly takes into account the effect of disturbances and uses both parameter adaptation and robust feedback to attenuate their effects for an improved tracking performance. Experimental results on the control of an iron core linear motor are presented to illustrate the effectiveness and achievable performance of the proposed scheme. [DOI: 10.1115/1.2363413]

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1 Introduction

A great deal of effort has been devoted to the control of uncertain nonlinear systems [1–7] and some of the results have been extended to the output feedback control. Specifically, Kanellakopoulos et al. introduced backstepping procedure to a class of nonlinear systems in which nonlinearities depend only on the measured signals [8]. In [9], Krstić et al. presented an alternative approach to the adaptive control of linear systems with parametric uncertainties by using nonlinear methods such as tuning functions and nonlinear damping. The resulting controller possesses much better transient and steady state performance as compared to the traditional one. Parameter convergence properties of this controller were also analyzed by Zhang et al. [10]. Recently, Ikhouane and Krstić showed that by using a switching σ-modification [11] or smooth parameter projection [12] in the parameter adaptation law, the robustness of this scheme can be improved with respect to both unmodeled dynamics and bounded disturbances. A robust adaptive controller based on σ-modification was presented by Jiang and Praly in [13], in which asymptotic tracking is lost even in the presence of parametric uncertainties only.

In this paper, the adaptive backstepping approach developed in [9] is combined with the ARC design procedure [7] to construct a discontinuous projection based output feedback ARC controller for a class of linear systems subjected to both parametric uncertainties and bounded disturbances. As only output signal is measured, a Kreisselmeier observer [14] is first designed to provide exponentially convergent estimates of the unmeasured states. This observer has an extended filter structure so that parameter adaptation can be used to reduce the effect of possible large disturbances, which is very important for practical applications [15]. The destabilizing effect of estimation errors is dealt with using robust feedback at each step of the design procedure. Compared to the RAC approaches [11,12], the proposed scheme explicitly takes into account the effect of disturbances and uses both parameter adaptation and robust feedback to attenuate the effect of disturbances for an improved tracking performance. In addition, the design puts more emphasis on the robust control law design for a guaranteed robust performance in general. In fact, when the parameter adaptation law in ARC is switched off, the resulting controller becomes a deterministic robust controller. Furthermore, the proposed controller achieves a guaranteed transient performance and a prescribed final tracking accuracy in the sense that the upper bound on the tracking error over the entire time-history is given and related to certain controller design parameters in a *known* form, which is more transparent than those in RAC designs [16]. At the same time, proof of achievable performance is made simpler. The precision motion control of an iron core linear motor is used as an application case study and the experimental results are obtained to illustrate the effectiveness of the proposed method.

2 Problem Statement

The following single-input single-output (SISO) plant is considered:

$$y(t) = \frac{B(s)}{A(s)}u(t) + \frac{D(s)}{A(s)}\Delta(y,t) + d_y(t) \tag{1}$$

in which $A(s)=s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0$, $B(s)=b_m s^m+\cdots+b_1 s+b_0$, $D(s)=d_l s^l+\cdots+d_1 s+d_0$, and $m\le l<n$. The plant parameters a_i and b_i are unknown constants. For simplicity, d_i are assumed to be known but the results can be extended to the case where d_i are unknown constants without much difficulty. $d_y(t)$ represents the output disturbance, and $\Delta(y,t)$ represents any disturbance coming from the intermediate channels of the plant.

Unlike the RAC approaches [11,12], in the following, the disturbance $\Delta(y,t)$ will be explicitly taken into account and handled as follows: we first use the prior information about the nature of the disturbance to construct a nominal disturbance model $\Delta_n(y,t)=q(y,t)^T c$, in which $q(y,t)=[q_p(y,t),\ldots,q_1(y,t)]^T\in\mathbb{R}^p$ represents the vector of known basis shape functions, $c=[c_p,\ldots,c_1]^T$ represents the vector of unknown magnitudes. This

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nominal disturbance model will be used explicitly in the following controller design to improve the achievable output tracking performance, while the disturbance modeling error, $\tilde{\Delta}=\Delta-\Delta_n$, will be dealt with via certain robust feedback for a robust performance. A state space realization of the plant (1) is thus given by

$$\begin{aligned}
\dot{x}_1 &= x_2 - a_{n-1}x_1 \\
&\vdots \\
\dot{x}_{n-l-1} &= x_{n-l} - a_{l+1}x_1 \\
\dot{x}_{n-l} &= x_{n-l+1} - a_l x_1 + d_l q^T(y,t)c + d_l\tilde{\Delta} \\
&\vdots \\
\dot{x}_{\rho-1} &= x_\rho - a_{m+1}x_1 + d_{m+1}q^T(y,t)c + d_{m+1}\tilde{\Delta} \\
\dot{x}_\rho &= x_{\rho+1} - a_m x_1 + d_m q^T(y,t)c + d_m\tilde{\Delta} + b_m u(t) \\
&\vdots \\
\dot{x}_n &= -a_0 x_1 + d_0 q^T(y,t)c + d_0\tilde{\Delta} + b_0 u \\
y &= x_1 + d_y
\end{aligned} \tag{2}$$

where $\rho=n-m$ is the relative degree of the system.

For simplicity, the following notations are used: $\bullet_i$ for the ith component of the vector $\bullet$, $\bullet_{\min}$ for the minimum value of $\bullet$, and $\bullet_{\max}$ for the maximum value of $\bullet$. The operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors. Defining a vector of unknown parameters $\theta=[-a_{n-1},\ldots,-a_0,b_m,\ldots,b_0,c_p,\ldots,c_1]^T\in\mathbb{R}^{m+n+p+1}$, the following standard assumptions are made:

1. The nominal plant is of minimum phase, i.e., the polynomial $B(s)$ is Hurwitz. The plant order (n), relative degree $(\rho=n-m)$, and the sign of the high frequency gain $(\mathrm{sgn}(b_m))$ are known. ■
2. The extent of parametric uncertainties, disturbance modeling error $\tilde{\Delta}(t)$, output disturbance $d_y(t)$ as well as its derivative $\dot{d}_y(t)$ satisfy

$$\begin{aligned}
&\theta\in\Omega_\theta \triangleq \{\theta:\theta_{\min}<\theta<\theta_{\max}\} \\
&\tilde{\Delta}\in\Omega_\Delta \triangleq \{\tilde{\Delta}:|\tilde{\Delta}(y,t)|\le\delta(t)\} \\
&d_y\in\Omega_d \triangleq \{d_y:|d_y(t)|\le\delta_d(t)\} \\
&\dot{d}_y\in\Omega_f \triangleq \{\dot{d}_y:|\dot{d}_y(t)|\le\delta_f(t)\}
\end{aligned} \tag{3}$$

where $\theta_{\min}$ and $\theta_{\max}$ are known, and $\delta(t)$, $\delta_d(t)$ and $\delta_f(t)$ are unknown but bounded functions. ■

Given the reference trajectory $y_r(t)$, the objective is to synthesize a control input u such that the output y tracks $y_r(t)$ as closely as possible in spite of various model uncertainties. The reference signal y_r and its first ρ derivatives are assumed to be known and bounded. In addition, $y_r^{(\rho)}$ is piecewise continuous.

3 State Estimation

As only the output y is measured, the K-filters [2] are utilized to provide exponentially convergent estimates of the unmeasured states as follows. Rewrite (2) in the form

$$\begin{aligned}
\dot{x} &= A_0 x + (\bar{k}-a)x_1 + dq^T(y,t)c + bu + d\tilde{\Delta} \\
y &= x_1 + d_y
\end{aligned} \tag{4}$$

where

$$A_0=\begin{bmatrix} -k_1 & & \\ & I_{n-1} & \\ \vdots & & \\ -k_n & 0\cdots 0 & \end{bmatrix}\quad \bar{k}=\begin{bmatrix}k_1\\ \vdots\\ k_n\end{bmatrix}\quad a=\begin{bmatrix}a_{n-1}\\ \vdots\\ a_0\end{bmatrix}\quad b=\begin{bmatrix}0_{(\rho-1)\times 1}\\ \bar{b}\end{bmatrix}$$
$$\bar{b}=\begin{bmatrix}b_m\\ \vdots\\ b_0\end{bmatrix}\quad d=\begin{bmatrix}0_{(n-l-1)\times 1}\\ \bar{d}\end{bmatrix}\quad \bar{d}=\begin{bmatrix}d_l\\ \vdots\\ d_0\end{bmatrix} \tag{5}$$

By suitably choosing $\bar{k}$, the observer matrix A_0 will be stable. Thus, there exists a symmetric positive definite (s.p.d.) matrix P such that

$$PA_0 + A_0^T P = -I \quad P=P^T>0 \tag{6}$$

Following the design procedure of [2], the K-filters are given by

$$\begin{aligned}
&\dot{\xi}_n = A_0\xi_n + \bar{k}y \\
&\dot{\xi}_i = A_0\xi_i + e_{n-i}y \quad 0\le i\le n-1 \\
&\dot{\upsilon}_i = A_0\upsilon_i + e_{n-i}u \quad 0\le i\le m \\
&\dot{\psi}_i = A_0\psi_i + dq_i(y,t) \quad 1\le i\le p
\end{aligned} \tag{7}$$

where e_i denotes the ith standard basis vector in $\mathbb{R}^n$. Let η and λ be the n-dimensional state vectors of the filtered input and output by

$$\begin{aligned}
&\dot{\eta} = A_0\eta + e_n y \\
&\dot{\lambda} = A_0\lambda + e_n u
\end{aligned} \tag{8}$$

Then, the filter states ξ_i and υ_i in (7) can be obtained from the following algebraic expressions [2]

$$\begin{aligned}
&\xi_n = -A_0^n\eta \\
&\xi_i = A_0^i\eta \quad 0\le i\le n-1 \\
&\upsilon_i = A_0^i\lambda \quad 0\le i\le m
\end{aligned} \tag{9}$$

It is noted that the last equation of (7) is introduced so that parameter adaptation can be used to reduce the parametric uncertainty coming from c_i to achieve a better disturbance rejection capability. The state estimates can thus be represented by

$$\hat{x} = \xi_n - \sum_{i=0}^{n-1} a_i\xi_i + \sum_{i=0}^{m} b_i\upsilon_i + \sum_{i=1}^{p} c_i\psi_i \tag{10}$$

Let $\varepsilon_x=x-\hat{x}$ be the estimation error. From (4), (7), and (10), it can be verified that the observer error dynamics is given by

$$\dot{\varepsilon}_x = A_0\varepsilon_x + (a-\bar{k})d_y + d\tilde{\Delta} \tag{11}$$

The solution of this equation can be written as

$$\varepsilon_x = \varepsilon + \varepsilon_u \tag{12}$$

where ε is the zero input response satisfying $\dot{\varepsilon}=A_0\varepsilon$, and

$$\varepsilon_u = \int_0^t e^{A_0(t-\tau)}[(a-\bar{k})d_y(\tau) + d\tilde{\Delta}(y,\tau)]d\tau \quad t\ge 0 \tag{13}$$

is the zero state response. Noting assumption 2 and that matrix A_0 is stable, one has

$$\varepsilon_u\in\Omega_\varepsilon \triangleq \{\varepsilon_u:|\varepsilon_u(t)|\le\delta_\varepsilon(t)\} \tag{14}$$

where $\delta_\varepsilon(t)$ is a vector of unknown but bounded functions. In the following controller design, ε and ε_u will be treated as disturbances and dealt with using robust control functions to achieve a guaranteed robust performance.

4 Discontinuous Projection Based ARC Backstepping Design

4.1 Parameter Projection. Let $\hat{\theta}$ denote the estimate of θ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta}=\hat{\theta}-\theta$). From assumption 2, the discontinuous projection based ARC design [7] can be generalized to solve the robust tracking control problem for (1). Specifically, the parameter estimate $\hat{\theta}$ is updated through a parameter adaptation law having the form of

$$\dot{\hat{\theta}} = \mathrm{Proj}_{\hat{\theta}}(\Gamma\tau) \tag{15}$$

where $\Gamma>0$ is a diagonal matrix, and τ is an adaptation function to be synthesized later. The projection mapping $\mathrm{Proj}_{\hat{\theta}}(\bullet)=[\mathrm{Proj}_{\hat{\theta}_1}(\bullet_1),\ldots,\mathrm{Proj}_{\hat{\theta}_{m+n+p+1}}(\bullet_{m+n+p+1})]^T$ is defined in [17,5] as

$$\mathrm{Proj}_{\hat{\theta}_i}(\bullet_i)=\begin{cases}0 & \text{if } \hat{\theta}_i=\theta_{i\max} \text{ and } \bullet_i>0\\ 0 & \text{if } \hat{\theta}_i=\theta_{i\min} \text{ and } \bullet_i<0\\ \bullet_i & \text{otherwise}\end{cases} \tag{16}$$

It can be shown that for any adaptation function τ, the projection mapping (16) guarantees

$$\begin{aligned}&\mathbf{P1}\quad \hat{\theta}\in\bar{\Omega}_\theta=\{\hat{\theta}:\theta_{\min}\le\hat{\theta}\le\theta_{\max}\}\\ &\mathbf{P2}\quad \tilde{\theta}^T(\Gamma^{-1}\mathrm{Proj}_{\hat{\theta}}(\Gamma\tau)-\tau)\le 0,\quad \forall\ \tau\end{aligned} \tag{17}$$

4.2 Controller Design. The design combines the adaptive backstepping design [9] with the ARC design procedure [7,6]. In the following, the states of the system are replaced by their estimates and the estimation errors are dealt with at each step via robust feedback to achieve a guaranteed robust performance.

Step 1: From (2), the derivative of the output tracking error $z_1=y-y_r$ is

$$\dot{z}_1 = x_2 - a_{n-1}y + a_{n-1}d_y + \dot{d}_y - \dot{y}_r \tag{18}$$

Since x_2 is not measured, it is replaced by its expression from (10)

$$x_2=\xi_{n,2}-\xi_{(2)}a+v_{(2)}\bar{b}+\psi_{(2)}c+\varepsilon_{x2} \tag{19}$$

where $\varepsilon_{x2}=\varepsilon_2+\varepsilon_{u2}$ is the estimation error of x_2, and

$$\begin{aligned}\xi_{(2)}&=[\xi_{n-1,2},\ldots,\xi_{0,2}]\quad v_{(2)}=[v_{m,2},\ldots,v_{0,2}]\\ \psi_{(2)}&=[\psi_{p,2},\ldots,\psi_{1,2}]\end{aligned} \tag{20}$$

in which $\bullet_{i,j}$ represents the jth element of $\bullet_i$. Substituting (19) into (18) gives

$$\dot{z}_1=b_m v_{m,2}+\xi_{n,2}+\theta^T\bar{\omega}-\dot{y}_r+\bar{\Delta}_1 \tag{21}$$

where $\omega^T=[\xi_{(2)},v_{(2)},\psi_{(2)}]+e_1^{*T}y$, $\bar{\omega}=\omega-e_{n+1}^{*}v_{m,2}$, $\bar{\Delta}_1=a_{n-1}d_y+\dot{d}_y+\varepsilon_2+\varepsilon_{u2}$, and e_i^* is the ith standard basis vector in $\mathbb{R}^{n+m+p+1}$. If $v_{m,2}$ were the input, one would synthesize a virtual control law α_1 for $v_{m,2}$ such that z_1 is as small as possible

$$\alpha_1(y,\eta,\bar{\lambda}_{m+1},\psi,\hat{\theta},t)=\alpha_{1a}+\alpha_{1s}\quad \alpha_{1a}=-\frac{1}{\hat{b}_m}\{\xi_{n,2}+\hat{\theta}^T\bar{\omega}-\dot{y}_r\} \tag{22}$$

where $\bar{\lambda}_i=[\lambda_1,\ldots,\lambda_i]^T$. In (22), α_{1a} functions as an adaptive model compensation law used to achieve an improved model compensation through on-line parameter adaptation given by (15), and α_{1s} is a robust control law to be synthesized later. Since the sign of b_m is known (see assumption 1), without loss of generality, one can assume $b_m>0$. It is thus reasonable to expect that the lower bound of b_m is positive, i.e., $(b_m)_{\min}=(\theta_{n+1})_{\min}>0$. Then, due to the use of the projection (15), from P1 of (17), $\hat{b}_m\ge(b_m)_{\min}>0$, which implies that the control function (22) is well defined. Let $z_2=v_{m,2}-\alpha_1$ denote the input discrepancy. Substituting (22) into (21) gives

$$\dot{z}_1=b_m(z_2+\alpha_{1s})-\tilde{\theta}^T\phi_1+\bar{\Delta}_1 \tag{23}$$

where $\phi_1\triangleq\bar{\omega}+e_{n+1}^*\alpha_{1a}$.

In the tuning function based backstepping adaptive control [2], one of the key points is to incorporate the adaptation function (or tuning function) in the construction of control functions to compensate for the destabilizing effect of the time-varying parameter estimates. Here, due to the use of discontinuous projection (16), the adaptation law (15) is discontinuous and thus *cannot* be used in the control law design at each step. Since the backstepping design needs the control function synthesized at each step to be sufficiently smooth in order to obtain its partial derivatives. In the following, it will be shown that this design difficulty can be overcome by strengthening the robust control law design. The robust control function α_{1s} consists of three terms given by

$$\alpha_{1s}=\alpha_{1s1}+\alpha_{1s2}+\alpha_{1s3}\quad \alpha_{1s1}=-\frac{1}{(b_m)_{\min}}k_{1s}z_1 \tag{24}$$

where α_{1s2} and α_{1s3} are robust control designed in the following and k_{1s} is any nonlinear feedback gain satisfying

$$k_{1s}\ge g_1+\|C_{\phi1}\Gamma\phi_1\|^2\quad g_1>0 \tag{25}$$

in which $C_{\phi1}$ is a positive definite constant diagonal matrix to be specified later. Substituting (24) into (23) gives

$$\dot{z}_1=b_m z_2-\frac{b_m}{(b_m)_{\min}}k_{1s}z_1+b_m(\alpha_{1s2}+\alpha_{1s3})-\tilde{\theta}^T\phi_1+\bar{\Delta}_1 \tag{26}$$

Define a positive semidefinite (p.s.d.) function V_1 as

$$V_1=\frac{1}{2}z_1^2 \tag{27}$$

In view of (26), its time derivative satisfies

$$\dot{V}_1\le b_m z_1 z_2-k_{1s}z_1^2+z_1(b_m\alpha_{1s2}-\tilde{\theta}^T\phi_1)+z_1(b_m\alpha_{1s3}+\bar{\Delta}_1) \tag{28}$$

From assumption 2, one has

$$\|\tilde{\theta}^T\phi_1\|\le\|\theta_M\|\|\phi_1\| \tag{29}$$

where $\theta_M=\theta_{\max}-\theta_{\min}$. Thus $\|\tilde{\theta}^T\phi_1\|$ is bounded by a known function, which ensures that there exists a robust control function satisfying the following conditions [18]:

$$\begin{aligned}&\text{(i)}\quad z_1\{b_m\alpha_{1s2}-\tilde{\theta}^T\phi_1\}\le\epsilon_{11}\\ &\text{(ii)}\quad z_1\alpha_{1s2}\le 0\end{aligned} \tag{30}$$

where ϵ_{11} is a positive design parameter. Essentially, condition (i) of (30) shows that α_{1s2} is synthesized to attenuate the effect of parametric uncertainties $\tilde{\theta}$ with the level of control accuracy being measured by ϵ_{11}, and condition (ii) is to make sure that α_{1s2} is dissipating in nature so that it does not interfere with the functionality of the adaptive control law α_{1a}.

Similarly, noting assumption 2 and (14), one may obtain

$$|\bar{\Delta}_1|\le\bar{\delta}_1(t)\triangleq|a_{n-1}|\delta_d(t)+\delta_f(t)+\varepsilon_2+\delta_{\varepsilon2}(t) \tag{31}$$

Note that $\bar{\delta}_1$ is an unknown but bounded function. In principle, the same strategy as in (30) can be used to design a robust control function α_{1s3} to attenuate $\bar{\Delta}_1$. However, since the bound of $\bar{\Delta}_1$ is unknown, it is impossible to prespecify the level of control accuracy. So a more relaxed requirement compared to the condition (i) of (30) is given

$$z_1(b_m\alpha_{1s3}+\bar{\Delta}_1)\le\epsilon_{12}\bar{\delta}_1^2 \tag{32}$$

where ϵ_{12} is a positive design parameter.

Remark 1. One smooth example of α_{1s2} satisfying (30) can be found in the following way. Let h_1 be any smooth function satisfying

$$h_1 \geq \|\theta_M\|^2 \|\phi_1\|^2 \tag{33}$$

Then, α_{1s2} can be chosen as [6,7]

$$\alpha_{1s2} = -\frac{h_1}{4(b_m)_{min}\epsilon_{11}} z_1 \tag{34}$$

An example of α_{1s3} satisfying (32) is given by [19]

$$\alpha_{1s3} = -\frac{1}{4(b_m)_{min}\epsilon_{12}} z_1 \tag{35}$$

Other smooth or continuous examples of the robust control functions satisfying conditions (30) can be worked out in the same way as in [6,7,19]. ■

Step 2: From (22), (9), and (8), and noting the rearrangements of (18)–(21), the derivative of α_1 is computed as

$$\dot{\alpha}_1 = \dot{\alpha}_{1c} + \dot{\alpha}_{1u}$$

$$\dot{\alpha}_{1c} = \frac{\partial \alpha_1}{\partial y}(\xi_{n,2} + \hat{\theta}^T \omega) + \frac{\partial \alpha_1}{\partial \eta}\dot{\eta} + \sum_{j=1}^{m+1} \frac{\partial \alpha_1}{\partial \lambda_j}\dot{\lambda}_j + \sum_{j=1}^{p} \frac{\partial \alpha_1}{\partial \psi_{j,2}}\dot{\psi}_{j,2} + \frac{\partial \alpha_1}{\partial t}$$

$$\dot{\alpha}_{1u} = \frac{\partial \alpha_1}{\partial y}(-\tilde{\theta}^T \omega + \bar{\Delta}_1) + \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} \tag{36}$$

Since $\dot{\eta}$, $\dot{\lambda}$, and $\dot{\psi}_j$ are given in (8) and (7), $\dot{\alpha}_{1c}$ is calculable and can be used in the design of control functions. However, $\dot{\alpha}_{1u}$ is not due to various uncertainties. Therefore, $\dot{\alpha}_{1u}$ has to be dealt with via robust feedback in this step. From (7) and (36), the derivative of $z_2 = \upsilon_{m,2} - \alpha_1$ is

$$\dot{z}_2 = \upsilon_{m,3} - k_2 \upsilon_{m,1} - \dot{\alpha}_{1c} - \dot{\alpha}_{1u} \tag{37}$$

Consider an augmented p.s.d. function given by

$$V_2 = V_1 + \frac{1}{2} z_2^2 \tag{38}$$

From (28) and (37), it follows that

$$\dot{V}_2 \leq \dot{V}_1\big|_{\alpha_1} + z_2\{b_m z_1 + \upsilon_{m,3} - k_2 \upsilon_{m,1} - \dot{\alpha}_{1c} - \dot{\alpha}_{1u}\} \tag{39}$$

where $\dot{V}_1|_{\alpha_1}$ is a shorthand notation for $-k_{1s}z_1^2 + z_1(b_m\alpha_{1s2} - \tilde{\theta}^T\phi_1) + z_1(b_m\alpha_{1s3} + \bar{\Delta}_1)$. Similar to (22), the ARC control function α_2 for the virtue control input $\upsilon_{m,3}$ in (37) consists of

$$\begin{aligned} \alpha_2(y, \eta, \bar{\lambda}_{m+2}, \psi, \hat{\theta}, t) &= \alpha_{2a} + \alpha_{2s} \\ \alpha_{2a} &= -\hat{b}_m z_1 + k_2 \upsilon_{m,1} + \dot{\alpha}_{1c} \\ \alpha_{2s} = \alpha_{2s1} + \alpha_{2s2} + \alpha_{2s3} &\quad \alpha_{2s1} = -k_{2s} z_2 \\ k_{2s} \geq g_2 + \left\| \frac{\partial \alpha_1}{\partial \hat{\theta}} C_{\theta 2} \right\|^2 &+ \|C_{\phi 2}\Gamma \phi_2\|^2 \end{aligned} \tag{40}$$

where $g_2 > 0$ is a constant, $C_{\theta 2}$ and $C_{\phi 2}$ are positive positive definite constant diagonal matrices, α_{2s2} and α_{2s3} are robust control functions to be chosen later. Substituting (40) and (36) into (39) and using similar techniques as in (23), one may have

$$\dot{V}_2 \leq \dot{V}_1\bigg|_{\alpha_1} + z_2 z_3 - k_{2s} z_2^2 + z_2(\alpha_{2s2} - \tilde{\theta}^T \phi_2) + z_2(\alpha_{2s3} + \bar{\Delta}_2) - z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} \tag{41}$$

in which $z_3 = \upsilon_{m,3} - \alpha_2$ represents the input discrepancy, and

$$\begin{aligned} \phi_2 &= e_{n+1}^* z_1 - \frac{\partial \alpha_1}{\partial y}\omega \\ \bar{\Delta}_2 &= -\frac{\partial \alpha_1}{\partial y}\bar{\Delta}_1 \end{aligned} \tag{42}$$

From (31), it follows that $|\bar{\Delta}_2| \leq |\partial \alpha_1 / \partial y| \bar{\delta}_1$. Similar to (30) and (32), the robust control functions α_{2s2} and α_{2s3} are chosen to satisfy

$$\begin{aligned} &\text{(i)} \quad z_2(\alpha_{2s2} - \tilde{\theta}^T \phi_2) \leq \epsilon_{21} \\ &\text{(ii)} \quad z_2(\alpha_{2s3} + \bar{\Delta}_2) \leq \epsilon_{22} \bar{\delta}_1^2 \\ &\text{(iii)} \quad z_2 \alpha_{2s2} \leq 0, \quad z_2 \alpha_{2s3} \leq 0 \end{aligned} \tag{43}$$

where ϵ_{21} and ϵ_{22} are positive design parameters. As in Remark 2, examples of α_{2s2} and α_{2s3} are given by

$$\begin{aligned} \alpha_{2s2} &= -\frac{h_2}{4\epsilon_{21}} z_2 \\ \alpha_{2s3} &= -\frac{1}{4\epsilon_{22}}\left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 \end{aligned} \tag{44}$$

in which h_2 is any smooth function satisfying

$$h_2 \geq \|\theta_M\|^2 \|\phi_2\|^2 \tag{45}$$

From (28) and (41), the derivative of V_2 satisfies the following inequality:

$$\dot{V}_2 \leq z_2 z_3 - \sum_{j=1}^{2} k_{js} z_j^2 + z_1\{b_m \alpha_{1s2} - \tilde{\theta}^T \phi_1\} + z_1\{b_m \alpha_{1s3} + + \bar{\Delta}_1\} + z_2\{\alpha_{2s2} - \tilde{\theta}^T \phi_2\} + z_2\{\alpha_{2s3} + \bar{\Delta}_2\} - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} z_2 \tag{46}$$

Step i $(3 \leq i < \rho)$: Mathematical induction will be used to prove the general results for all intermediate steps of the design. At step (i), the same ARC design as in the above two steps will be employed to construct an ARC control function α_i for the virtual input $\upsilon_{m,i+1}$. For step j, $\forall 3 \leq j \leq i-1$, let $z_j = \upsilon_{m,j} - \alpha_{j-1}$ and recursively define

$$\phi_j = -\frac{\partial \alpha_{j-1}}{\partial y}\omega \quad \bar{\Delta}_j = -\frac{\partial \alpha_{j-1}}{\partial y}\bar{\Delta}_1 \tag{47}$$

Lemma 1. *At step* i, *choose the desired ARC control function* α_i *as*

$$\begin{aligned} \alpha_i(y, \eta, \bar{\lambda}_{m+i}, \psi, \hat{\theta}, t) &= \alpha_{ia} + \alpha_{is} \\ \alpha_{ia} &= -z_{i-1} + k_i \upsilon_{m,1} + \dot{\alpha}_{(i-1)c} \\ \alpha_{is} = \alpha_{is1} + \alpha_{is2} + \alpha_{is3} \quad \alpha_{is1} &= -k_{is} z_i \quad k_{is} \geq g_i + \left\| \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} C_{\theta i} \Gamma \phi_i \right\|^2 \end{aligned} \tag{48}$$

where $g_i > 0$ *is a constant,* $C_{\theta i}$ *and* $C_{\phi i}$ *are positive definite constant diagonal matrices,* α_{is2} *and* α_{is3} *are robust control functions satisfying*

$$i \quad z_i(\alpha_{is2} - \tilde{\theta}^T \phi_i) \leq \epsilon_{i1}$$

$$ii \quad z_i(\alpha_{is3}+\bar{\Delta}_i) \le \epsilon_{i2}\bar{\delta}_1^2 \tag{49}$$

$$iii \quad z_i\alpha_{is2} \le 0, \quad z_i\alpha_{is3} \le 0$$

and

$$\dot{\alpha}_{(i-1)c} = \frac{\partial \alpha_{i-1}}{\partial y}(\xi_{n,2}+\hat{\theta}^T\omega) + \frac{\partial \alpha_{i-1}}{\partial \eta}\dot{\eta} + \sum_{j=1}^{m+i-1}\frac{\partial \alpha_{i-1}}{\partial \lambda_j}\dot{\lambda}_j + \sum_{j=1}^{p}\frac{\partial \alpha_{i-1}}{\partial \psi_j}\dot{\psi}_j + \frac{\partial \alpha_{i-1}}{\partial t} \tag{50}$$

Then the ith error subsystem is

$$\dot{z}_i = z_{i+1} - z_{i-1} - k_{is}z_i + (\alpha_{is2} - \tilde{\theta}^T\phi_i) + (\alpha_{is3}+\bar{\Delta}_i) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \tag{51}$$

and the derivative of the augmented p.s.d. function

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \tag{52}$$

satisfies

$$\dot{V}_i \le z_i z_{i+1} - \sum_{j=1}^{i} k_{js}z_j^2 + z_1(b_m\alpha_{1s2} - \tilde{\theta}^T\phi_1) + \sum_{j=2}^{i} z_j(\alpha_{js2} - \tilde{\theta}^T\phi_j) + z_1(b_m\alpha_{1s3}+\bar{\Delta}_1) + \sum_{j=2}^{i} z_j(\alpha_{js3}+\bar{\Delta}_j) - \sum_{j=2}^{i}\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}z_j \tag{53}$$

The proof of the Lemma is given in the Appendix.

Step ρ: This is the final design step, in which the actual control u will be synthesized such that $v_{m,\rho}$ tracks the desired ARC control function $\alpha_{\rho-1}$. The derivative of z_ρ can be obtained as

$$\dot{z}_\rho = v_{m,\rho+1} + u - k_\rho v_{m,1} - \dot{\alpha}_{(\rho-1)c} - \frac{\partial \alpha_{\rho-1}}{\partial y}(-\tilde{\theta}^T\omega + \bar{\Delta}_1) - \frac{\partial \alpha_{\rho-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \tag{54}$$

If $v_{m,\rho+1}+u$ were the virtual input, (54) would have the same form as the intermediate step i. Therefore, the general form (47)–(53) applies to Step ρ. Since u is the actual control input, it can be chosen as

$$u = \alpha_\rho - v_{m,\rho+1} \tag{55}$$

where α_ρ is given by (48). Then, $z_{\rho+1}=u+v_{m,\rho+1}-\alpha_\rho=0$.

THEOREM 1. *Let the parameter estimates be updated by the adaptation law* (15) *in which τ is chosen as*

$$\tau = \sum_{j=1}^{\rho}\phi_j z_j \tag{56}$$

If diagonal controller gain matrices $C_{\theta j}$ and $C_{\phi k}$ are chosen such that $c_{\phi kr}^2 \ge \frac{\rho}{4}\sum_{j=2}^{\rho}1/(c_{\theta jr}^2)$, where $c_{\theta jr}$ and $c_{\phi kr}$ are the rth diagonal element of $C_{\theta j}$ and $C_{\phi k}$, respectively. Then, the control law (55) *guarantees that*

(A) *In general, the control input and all internal signals are bounded. Furthermore, V_ρ is bounded above by*

$$V_\rho(t) \le \exp(-\lambda_\rho t)V_\rho(0) + \frac{\bar{\epsilon}_{\rho1}+\bar{\epsilon}_{\rho2}\|\bar{\delta}_1\|_\infty^2}{\lambda_\rho}[1-\exp(-\lambda_\rho t)] \tag{57}$$

where $\lambda_\rho=2\min\{g_1,\ldots,g_\rho\}$, $\bar{\epsilon}_{\rho1}=\sum_{j=1}^{\rho}\epsilon_{j1}$, $\bar{\epsilon}_{\rho2}=\sum_{j=1}^{\rho}\epsilon_{j2}$, and $\|\bar{\delta}_1\|_\infty$ stands for the infinity norm of $\bar{\delta}_1(t)$.

(B) *If after a finite time t_0, $\tilde{\Delta}=0$ and $d_y=0$ (i.e., in the presence of parametric uncertainties and modeled disturbance $\Delta_n = q(y,t)^Tc$ only), then, in addition to results in A, asymptotic output tracking (or zero final tracking error) is also achieved.* ■

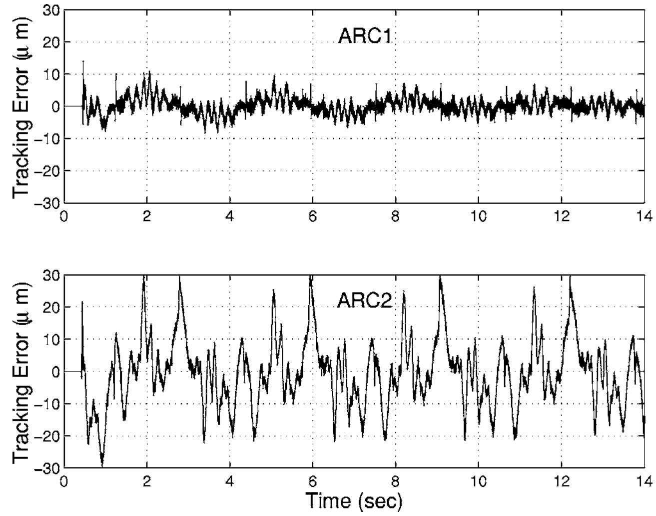

Fig. 1 Tracking errors for sinusoidal trajectory without load

Proof of the Theorem is given in the Appendix.

5 Experiment Results

To illustrate the above designs, experiment results are obtained for the tracking control of the x axis of a two-axis positioning stage detailed in [20] whose simplified model is

$$\dot{x}_1 = x_2 - a_1x_1$$
$$\dot{x}_2 = b_0u + \Delta(y,t) \tag{58}$$
$$y = x_1$$

where x_1 and x_2 are the states, y is the position of the inertia load of the linear motor, u is the control input, and $\Delta(y,t)$ represents the lumped disturbance consisting of the electromagnetic cogging force [21], friction force, and any external disturbances. The cogging force in $\Delta(y,t)$ is a periodic function of the position y. Its period depends on the motor magnet's pitch ($P=60$ mm). The cogging force can be approximated quite accurately by the first several harmonics [21,22]. For simplicity, in the proposed controller, only the fundamental and the third harmonics are used to approximate the periodic cogging force. Hence, the nominal disturbance model is chosen as $\Delta_n=q^Tc$, where $q(y) = [\cos(2\pi y/P), \sin(2\pi y/P), \cos(6\pi y/P), \sin(6\pi y/P)]^T$ is a vector of the known basis functions and $c=[c_4,c_3,c_2,c_1]^T$ is a vector of the unknown magnitudes. Thus the unknown parameter vector to be adapted is $\theta=[-a_1,b_0,c_4,c_3,c_2,c_1]^T$.

For reference, standard least-square identification is performed to obtain the parameters of the system, and the nominal value of a_1 is 12.5 and b_0 is 50. To test the learning capability of the proposed ARC algorithms, a 9 kg load is mounted on the motor and the identified values of the parameters are $a_1=4.2$ and $b_0=16.7$. The bounds describing the uncertain ranges in (3) are chosen as $\theta_{\min}=[-30,10,-10,-10,-10,-10]^T$, and $\theta_{\max}=[-1.5,80,10,10,10,10]^T$. The control system is implemented using a dSPACE DS1103 controller board. The controller executes programs at a sampling rate of $f_s=2.5$ kHz. The following two controllers are compared:

ARC1: The output feedback ARC law synthesized in Sec. 4. All the roots of the observer polynomials are placed at $s=$

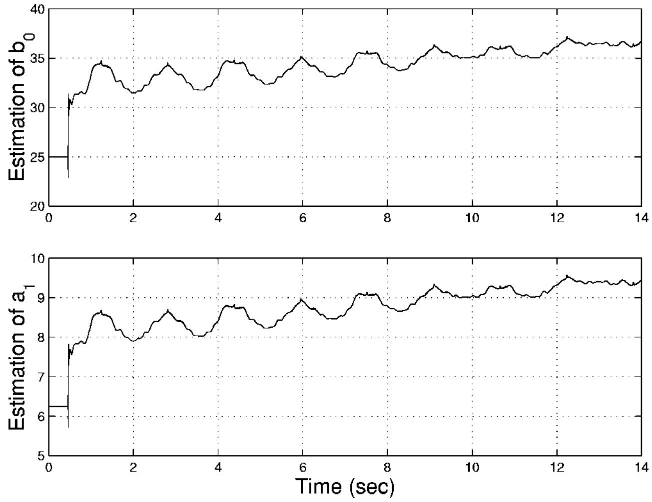

Fig. 2 Parameter estimation of ARC1

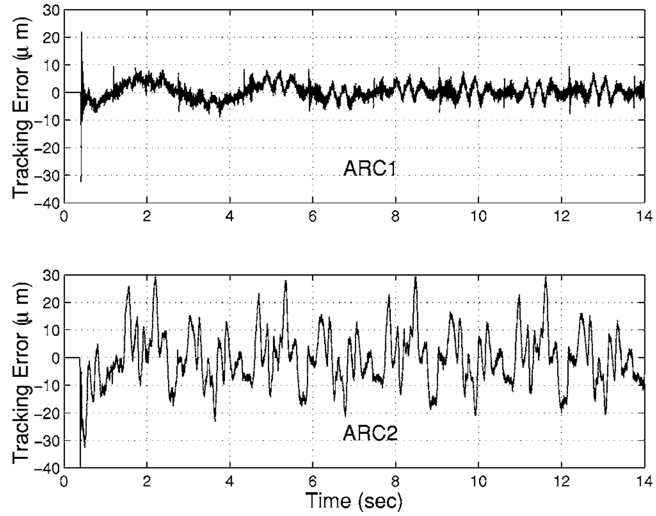

Fig. 4 Tracking errors for sinusoidal trajectory with load

−200, which leads to $k_1=400$ and $k_2=4\times10^4$. The controller parameters are: $g_1=200$, $\epsilon_{11}=\epsilon_{12}=5\times10^{-3}$, $C_{\phi1}=10^{-5}\cdot\text{diag}[5,0.5,10,10,10,10,10]$, $g_2=210$, $\epsilon_{21}=0.1$, $\epsilon_{22}=1$, $C_{\phi2}=C_{\phi1}$, $C_{\theta2}=10^5 I_3$. The adaptation rate is $\Gamma=10^6\cdot\text{diag}[2.5,1,100,100,100,100]$. The initial parameter estimates are: $\hat{\theta}(0)=[-6.25,25,0,0,0,0]^T$.

ARC2: The same control law as the ARC1 but without cogging force compensation, i.e., letting $\Gamma=10^6\cdot\text{diag}[2.5,1,0,0,0,0]$.

The motor is first commanded to track a sinusoidal trajectory $y_r=0.05\sin(2\pi t)$. The tracking errors are shown in Fig. 1. As seen, ARC1 achieves a better tracking performance than ARC2. This illustrates the effectiveness of cogging force compensation. Parameter estimation of ARC1 is given in Fig. 2. It shows that the adaptation algorithm is able to pick up the actual values of the two unknown parameters very quickly. The control inputs of the two controllers, which are well below the physical limits (±10 V), are given in Fig. 3. To test the performance robustness of the algorithms to parameter variations, the motor is then run with the 9 kg payload mounted on it. The tracking errors are given in Fig. 4. It shows that both controllers achieve good tracking performance in spite of the change of inertia load, and again, ARC1 performs better than ARC2.

The motor is then commanded to track a fast point-to-point motion trajectory shown in Fig. 5. Following this trajectory, the motor is expected to run back and forth between point A and point B, with a top velocity of ±1 m/s and a top acceleration of ±12 m/s^2. The tracking errors of both controllers are shown in Fig. 6. As seen, ARC1 outperforms ARC2. Furthermore, when the velocity is zero, the tracking accuracy of ARC1 is close to the optical encoders resolution.

Finally, the motor is commanded to track a slow point-to-point motion trajectory similar to the trajectory shown in Fig. 5, but with a top velocity of ±0.1 m/s and a top acceleration of ±2 m/s^2. The tracking errors are shown in Fig. 7. ARC1 again performs better than ARC2, which further illustrates the effectiveness of cogging force compensation in low-speed applications.

6 Conclusions

In this paper, an output feedback ARC scheme based on discontinuous projection is presented for a class of linear systems having both parametric uncertainties and disturbances that might be output dependent. In contrast to other existing robust adaptive control

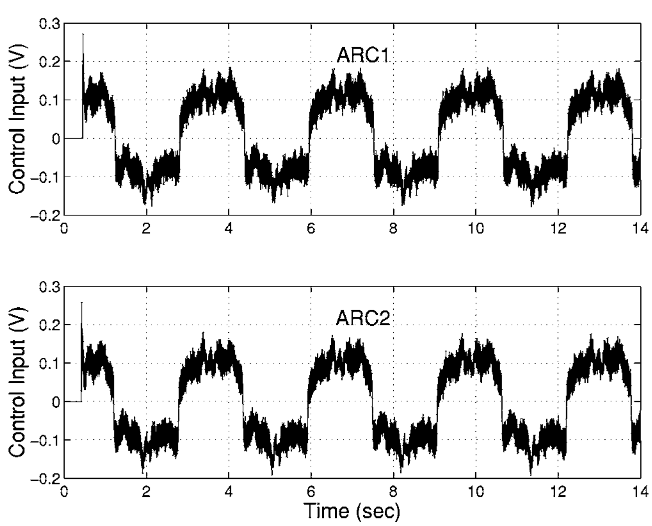

Fig. 3 Control inputs for sinusoidal trajectory without load

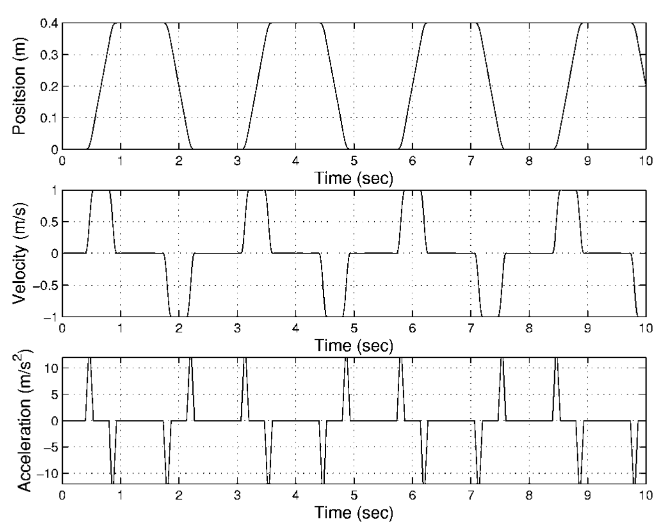

Fig. 5 Fast point-to-point motion trajectory

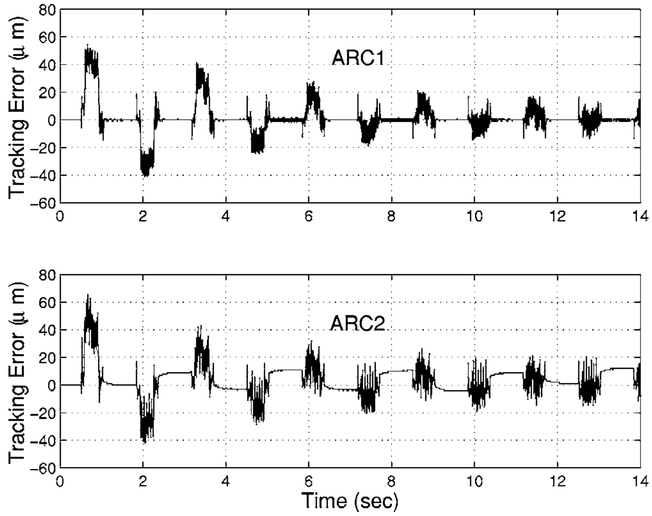

Fig. 6 Tracking errors for the fast point-to-point motion trajectory

schemes, the proposed controller uses on-line parameter adaptation to compensate for the nonlinear disturbances that can be modeled. The uncompensated disturbances and the estimation errors of unmeasured states are effectively handled via certain robust feedback to achieve a robust performance. The resulting controller achieves a guaranteed transient performance and a prescribed final tracking accuracy in the presence of both parametric uncertainties and bounded disturbances. In the presence of parametric uncertainties only, asymptotic output tracking is achieved without using switching or infinite-gain feedback. Experimental results verified the effectiveness of the proposed scheme.

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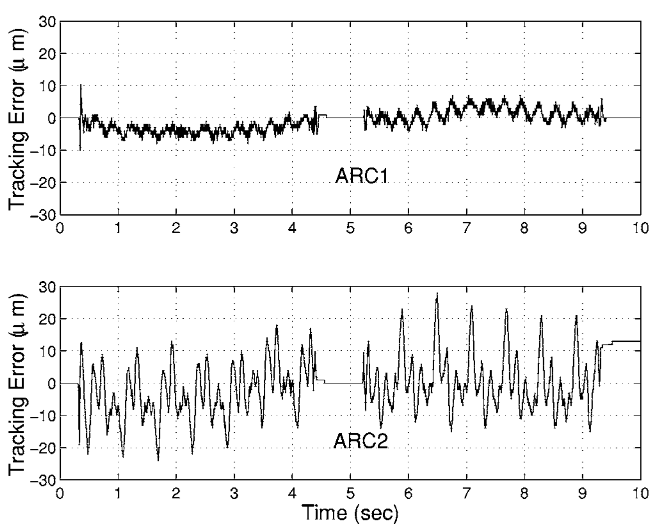

Fig. 7 Tracking errors for the slow point-to-point motion trajectory

Appendix

Proof of Lemma 1

It is easy to check that the first two steps satisfy the Lemma. So let us assume that the lemma is valid for step j, $\forall j \le i-1$, and show that it is also true for step i to complete the induction process. From (47)

$$|\bar{\Delta}_i| \le \left|\frac{\partial \alpha_{i-1}}{\partial y}\right| \bar{\delta}_1 \tag{A1}$$

Thus, there exist α_{is2} and α_{is3} satisfying (49) [7,19]. The control law (48) can then be formed. The derivative of z_i is given by

$$\dot{z}_i = v_{m,i+1} - k_i v_{m,1} - \dot{\alpha}_{(i-1)c} - \frac{\partial \alpha_{i-1}}{\partial y}(-\tilde{\theta}^T \omega + \bar{\Delta}_1) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \tag{A2}$$

After substituting $v_{m,i+1} = z_{i+1} + \alpha_i$ and (48) into (A2), it is straightforward to verify that (51) and (53) are satisfied for step i. This completes the induction process. ■

Proof of Theorem 1

Noting $z_{\rho+1}=0$, from (53), (30), (32), (43), and (49), the derivative of the Lyapunov function satisfies

$$\dot{V}_\rho \le \sum_{j=1}^{\rho} \left\{ -\left(g_j + \left\| \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} C_{\theta j} \right\|^2 + \|C_{\phi j}\Gamma \phi_j\|^2 \right) z_j^2 + \epsilon_{j1} + \epsilon_{j2}\bar{\delta}_1^2 - z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \right\} \tag{A3}$$

in which the fact $\partial \alpha_0 / \partial \hat{\theta} = 0$ is used. By completion of square, it follows that

$$\begin{aligned} -\sum_{j=2}^{\rho} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} &\le \sum_{j=2}^{\rho} |z_j| \left\| \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} C_{\theta j} C_{\theta j}^{-1}\dot{\hat{\theta}} \right| \\ &\le \sum_{j=2}^{\rho} \left(\left\| \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} z_j^2 + \frac{1}{4} \|C_{\theta j}^{-1}\dot{\hat{\theta}} \right\|^2 \right) \end{aligned} \tag{A4}$$

Noting that $C_{\theta j}^{-1}$ and Γ are diagonal matrices, from (15) and (56), one further has

$$\begin{aligned} \sum_{j=2}^{\rho} \left\| C_{\theta j}^{-1}\dot{\hat{\theta}} \right\|^2 &= \sum_{j=2}^{\rho} \left\| C_{\theta j}^{-1} \operatorname{Proj}_{\hat{\theta}}(\Gamma\tau) \right\|^2 \le \sum_{j=2}^{\rho} \|C_{\theta j}^{-1}\Gamma\tau\|^2 \\ &\le \sum_{j=2}^{\rho} \left(\sum_{k=1}^{\rho} \|C_{\theta j}^{-1}\Gamma\phi_k z_k\| \right)^2 \le \rho \sum_{j=2}^{\rho} \left(\sum_{k=1}^{\rho} \|C_{\theta j}^{-1}\Gamma\phi_k\|^2 z_k^2 \right) \end{aligned} \tag{A5}$$

If $C_{\theta j}$ and $C_{\phi k}$ satisfy the conditions in the theorem, substituting (A5) into (A4) gives

$$\begin{aligned} -\sum_{j=2}^{\rho} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} &\le \sum_{j=2}^{\rho} \left\| \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} C_{\theta j} \right\|^2 z_j^2 + \frac{\rho}{4} \sum_{k=1}^{\rho} \sum_{j=2}^{\rho} \|C_{\theta j}^{-1}\Gamma\phi_k\|^2 z_k^2 \\ &\le \sum_{j=2}^{\rho} \left\| \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} C_{\theta j} \right\|^2 z_j^2 + \sum_{k=1}^{\rho} \|C_{\phi k}\Gamma\phi_k\|^2 z_k^2 \end{aligned} \tag{A6}$$

In view of (A6), (A3) becomes

$$\dot{V}_\rho \le -\sum_{j=1}^{\rho} g_j z_j^2 + \sum_{j=1}^{\rho} (\epsilon_{j1} + \epsilon_{j2}\overline{\delta}_1^2) \le -\lambda_\rho V_\rho + \overline{\epsilon}_{\rho 1} + \overline{\epsilon}_{\rho 2}\overline{\delta}_1^2 \tag{A7}$$

which leads to (57). Since $B(s)$ is Hurwitz, the zero dynamics is stable. Following the standard adaptive control arguments as in [2], it can be proved that all internal signals are bounded. A of the theorem is thus proved.

The following is to prove B of the theorem. In the presence of parametric uncertainties only (i.e., $\tilde{\Delta}=0$ and $d_y=0$), $\bar{\Delta}_1=\varepsilon_2$. From (43) and (49), it is easy to check that $|z_1(b_m\alpha_{1s3}+\bar{\Delta}_1)|\le\epsilon_{12}\varepsilon_2^2$ and $|z_j(\alpha_{js3}+\bar{\Delta}_j)|\le\epsilon_{j2}\varepsilon_2^2$, $j=2,\ldots,\rho$. Thus, noting (56) and condition **ii** of (30), (43), and (49) from (53) and (A6), one has

$$\dot{V}_\rho \le \sum_{j=1}^{\rho} (-\tilde{\theta}^T\phi_j z_j - g_j z_j^2 + \epsilon_{j2}\varepsilon_2^2) = -\sum_{j=1}^{\rho} g_j z_j^2 - \tilde{\theta}^T\tau + \overline{\epsilon}_{\rho 2}\varepsilon_2^2 \tag{A8}$$

Define a new p.d. function V_θ as

$$V_\theta = V_\rho + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \gamma\varepsilon^T P\varepsilon \tag{A9}$$

where $\gamma\ge\overline{\epsilon}_{\rho 2}$. Noting P2 of (17) and the fact that $\dot{\varepsilon}=A_0\varepsilon$, from (A8) and (6), the derivative of V_θ is

$$\dot{V}_\theta \le -\sum_{j=1}^{\rho} g_j z_j^2 - \tilde{\theta}^T\tau + \overline{\epsilon}_{\rho 2}\varepsilon_2^2 + \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} - \gamma\|\varepsilon\|^2 \le -\sum_{j=1}^{\rho} g_j z_j^2 \tag{A10}$$

in which P2 of (17) is used. Therefore, $z\in\mathcal{L}_2^2$. It is also easy to check that $\dot{z}$ is bounded. So, $z\to 0$ as $t\to\infty$ by the Barbalat's lemma, which leads to B of Theorem 1. ■

References

[1] Polycarpou, M. M., and Ioannou, P. A., 1993, "A Robust Adaptive Nonlinear Control Design," in *Proceedings of the American Control Conference*, pp. 1365–1369.

[2] Krstić, M., Kanellakopoulos, I., and Kokotović, P. V., 1995, *Nonlinear and Adaptive Control Design*, Wiley, New York.

[3] Freeman, R. A., Krstić, M., and Kokotović, P. V., 1996, "Robustness of Adaptive Nonlinear Control to Bounded Uncertainites," in IFAC World Congress, Vol. F, pp. 329–334.

[4] Pan, Z., and Basar, T., 1996, "Adaptive Controller Design for Tracking and Disturbance Attenuation in Parametric-Strict-Feedback Nonlinear Systems," in IFAC World Congress, Vol. F, pp. 323–328.

[5] Yao, B., and Tomizuka, M., 1996, "Smooth Robust Adaptive Sliding Mode Control of Manipulators With Guaranteed Transient Performance," ASME J. Dyn. Syst., Meas., Control, **118**(4), pp. 764–775.

[6] Yao, B., and Tomizuka, M., 1997, "Adaptive Robust Control of SISO Nonlinear Systems in a Semi-Strict Feedback Form," Automatica, **33**(5), pp. 893–900.

[7] Yao, B., 1997, "High Performance Adaptive Robust Control of Nonlinear Systems: A General Framework and New Schemes," in *Proceedings of the IEEE Conference on Decision and Control*, pp. 2489–2494.

[8] Kanellakopoulos, I., Kokotović, P. V., and Morse, A. S., 1991, "Adaptive Output-Feedback Control of Systems With Output Nonlinearities," in *Foundations of Adaptive Control*, P. V. Kokotović, ed., Springer-Verlag, Berlin, pp. 495–525.

[9] Krstić, M., Kanellakopoulos, I., and Kokotović, P. V., 1994, "Nonlinear Design of Adaptive Controllers for Linear Systems," IEEE Trans. Autom. Control, **39**, pp. 738–752.

[10] Zhang, Y., Ioannou, P. A., and Chien, C. C., 1996, "Parameter Convergence of a New Class of Adaptive Controllers," IEEE Trans. Autom. Control, **41**(10), pp. 1489–1493.

[11] Ikhouane, F., and Krstić, M., 1995, "Robustness of the Tuning Functions Adaptive Backstepping Design for Linear Systems," in *Proceedings of IEEE Conference on Decision and Control*, pp. 159–164.

[12] Ikhouane, F., and Krstić, M., 1998, "Adaptive Backstepping with Parameter Projection: Robustness and Asymptotic Performance," Automatica, **34**(4), pp. 429–435.

[13] Jiang, Z. P., and Praly, L., 1998, "Design of Robust Adaptive Controllers for Nonlinear Systems with Dynamic Uncertainties," Automatica, **34**(7), pp. 825–840.

[14] Kreisselmeier, G., 1977, "Adaptive Observers with Exponential Rate of Convergence," IEEE Trans. Autom. Control, **22**(4), pp. 2–8.

[15] Yao, B., Al-Majed, M.and Tomizuka, M., 1997, "High Performance Robust Motion Control of Machine Tools: An Adaptive Robust Control Approach and Comparative Experiments," IEEE/ASME Trans. Mechatron., **2**(2), pp. 63–76.

[16] Ioannou, P. A., and Sun, J., 1996, *Robust Adaptive Control*, Prentice-Hall, New Jersey.

[17] Sastry, S., and Bodson, M., 1989, *Adaptive Control: Stability, Convergence and Robustness*, Prentice-Hall, Englewood Cliffs, NJ.

[18] Yao, B., and Tomizuka, M., 2001, "Adaptive Robust Control of MIMO Nonlinear Systems in Semi-Strict Feedback Forms," Automatica, **37**(9), pp. 1305–1321. Parts of the paper were presented in the IEEE Conference on Decision and Control, pp. 2346–2351, 1995, and the IFAC World Congress, Vol. F, pp. 335–340, 1996.

[19] Yao, B., and Tomizuka, M., 1997, "Adaptive Robust Control of Nonlinear Systems: Effective Use of Information," in *Proceedings of the 11th IFAC Symposium on System Identification*, pp. 913–918 (invited).

[20] Xu, L., and Yao, B., 2001, "Adaptive Robust Precision Motion Control of Linear Motors with Negligible Electrical Dynamics: Theory and Experiments," in IEEE/ASME Trans. Mechatron., **6**(4), pp. 444–452.

[21] Braembussche, P. V., Swevers, J., Brussel, H. V., and Vanherck, P., 1996, "Accurate Tracking Control of Linear Synchronous Motor Machine Tool Axes," Mechatronics, **6**(5), pp. 507–521.

[22] Yao, B., and Xu, L., 1999, "Adaptive Robust Control of Linear Motors for Precision Manufacturing," in *Proceedings of the IFAC'99 World Congress*, Vol. A, pp. 25–30.