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FAULT DETECTION FOR A CLASS OF NONLINEAR SYSTEMS IN PRESENCE OF UNMODELED DYNAMICS AND PARAMETRIC UNCERTAINTIES USING ADAPTIVE ROBUST OBSERVERS

Shreekant Gayaka
Ray W. Herrick Labs
School of Mechanical Engineering
Purdue University
West Lafayette, Indiana 47907
Email: sgayaka@purdue.edu

Bin Yao
Ray W. Herrick Labs
School of Mechanical Engineering
Purdue University
West Lafayette, Indiana 47907
Email: byao@purdue.edu

ABSTRACT

The goal of this work is to present some theoretical results which can be used for increasing fault sensitivity of a detection scheme, without sacrificing robustness. Robustness against modeling uncertainties and fault sensitivity are two contradicting demands, and typically, one is achieved at the expense of the other. The main reason for this trade-off is the use of a worst case scenario bound for modeling uncertainties at the residual evaluation stage. Many robust fault detection algorithms have been proposed based on the assumption that an a priori *known functional bound exists for modeling uncertainties. In the present work, we look into the two main sources of modeling uncertainties, parametric uncertainties and unmodeled dynamics, and carefully examine their effect on residual evaluation. Finally, based on our analysis, and certain assumptions about the unmodeled dynamics and parametric uncertainties, we propose a threshold for residual generation and evaluation, and analytically prove its superior robustness and sensitivity properties.*

1 Introduction

The ability to take preventive measures in case of a fault is crucial for reliable functioning of any control system. A properly designed fault detection and diagnosis (FDD) module can quickly respond to any abnormal changes in the operating conditions owing to actuator or sensor failures. For this reason, it has become an integral part of many industrial processes and various other control systems.

Model-based FDD algorithms have found extensive use, as they are easy to implement and do not require any additional hardware. But, these algorithms rely on the assumption that an accurate mathematical model of the system is available. In reality, however, there are often significant differences between the actual system and the model. These modeling errors can deteriorate the performance of the FDD scheme. Robustness to modeling uncertainties guarantees the proper functioning of the FDD scheme in spite of modeling errors. Usually, a detection scheme is made robust by assuming a functional bound for modeling uncertainties and using it at the residual evaluation stage [1], [2]. But, the robustness comes at the cost of reduced fault sensitivity, which means small and incipient faults can go undetected. In order to improve sensitivity without the corresponding loss in robustness, we have to critically examine the assumptions on which the fault detection algorithms are designed.

In [3], [4] and [5], the authors present a detection architecture which uses parameter adaptation for reducing modeling uncertainties and improving sensitivity. The other main source of uncertainty is unmodeled dynamics, and in order to further improve sensitivity, it is essential to reduce the deviations caused by unmodeled dynamics. In the present work, we specifically focus on actuator fault detection in presence of input unmodeled dynamics (IUD) and parametric uncertainties. IUD has been considered by many researchers in the context of designing controllers [6], [7], [8]. But, its effect on fault detection has not been studied by many researchers. In [9], the authors present the significance of considering unmodeled dynamics in the scheme of fault detection and give a procedure to design a threshold function which can effectively differentiate between faults and deviations due to unmodeled dynamics. But, some of the implementation issues were left unaddressed in [9]. In this work, we analyze the case when there is parametric uncertainties in addition to unmodeled dynamics, and present a more practical way of tackling the problem by designing observers for the unmodeled states.

The paper is organized as follows. In the first section, we describe the class of system under consideration and the various assumptions associated with it. Then, we formulate the main problem that we solve in this work and the significance of the problem. In the next section, we describe the strategy used to tackle the two main problems in generating the residuals - unmodeled dynamics and parametric uncertainties. Design of adaptive robust observers for estimating the unmodeled states and adaptation law to reduce the extent of parametric uncertainties is explained in details. We also briefly describe the residual evaluation strategy. In the third section, we propose a threshold for residual evaluation and state the main performance results as theorems. In the last section, we state the main results and contributions of the present work, and conclude. For the sake of completeness and making this paper self-contained, some results which already exists in the literature have been mentioned along with proper references.

2 Problem Formulation

The following nonlinear system will be considered in this paper,

$$\dot{x}_i = \theta^T F_{x_i}(x) + \Phi_i{}^T(x,\theta)v + \Delta_{x_i} + U(t-T_i) f_{a_i}(x,u,t) \quad (1)$$

$$\dot{\eta} = F_\eta(x,u)\theta + G_\eta(x)\eta + \Delta_\eta \quad (2)$$

$$v = u + \mu c \eta \quad (3)$$

where x_i is the i^{th} component of $x \in \mathbb{R}^n$. $u \in \mathbb{R}^m$ is the commanded or nominal control effort and v is the actual control effort. $\theta \in \mathbb{R}^p$ is the vector of unknown parameters. $\eta \in \mathbb{R}^\eta$ represents the unmodeled states. $F_{x_i} \in \mathbb{R}^{p\times 1}, \Phi_i \in \mathbb{R}^{m\times 1, F_\eta} \in \mathbb{R}^{\eta \times p}$ and $G_\eta \in \eta \times \eta$ are matrices or vectors of known smooth functions and $c \in \mathbb{R}^{m\times\eta}$ is constant matrix. μ and $\Delta_{(\bullet)}$ represents the extent of unmodeled dynamics and uncertain nonlinearities present in the system respectively. $U(t-T_i)$ represents a step function, and $f_{a_i}(x,u,t)$ represents the fault function.

As seen from (1), the dynamics of i^{th}-channel is not only affected by faults but, also by modeling errors. Ideally, it is expected that the detection scheme can completely isolate the effect of fault from that of unmodeled dynamics (η), parametric uncertainties (θ) and other bounded uncertainties (Δ_{x_i}). There are two main objectives of the present work - (a) to reconstruct the states in a way so that the effect of unmodeled dynamics and parametric uncertainties can be captured and generate residuals which are affected only by faults and bounded modeling errors and, (b) to design a threshold which can effectively differentiate between the effect of small/incipient faults and the bounded modeling errors.

Following are the assumptions associated with the class of systems under consideration,

Assumption 1: The unknown but constant parameters θ_i lie in a known bounded region Ω_{θ_i}, i.e.,

$$\theta_i \in \Omega_{\theta_i} = \{\theta_i : \theta_{i,min} < \theta_i < \theta_{i,max}\} \quad (4)$$

Assumption 2: The uncertain nonlinearities Δ_η and Δ_{x_i} are bounded by known constants i.e.,

$$\Delta_i \in \Omega_{\Delta_i} = \{\Delta_i : |\Delta(x,u,t)| \le \delta_i\} \quad (5)$$

where δ_i, $i = \eta, x_j$, are known constants.

Assumption 3: Control input $u(t)$ and system states $\eta(t), x_i(t)$ remain bounded before and after the occurrence of fault.

Remark 1: The assumption above implies that the states and control input remains bounded after the occurrence of fault, as no fault accommodation has been considered in this work.

Assumption 4: There exists a vector of design functions $\omega(x,\theta_\omega) \in \mathbb{R}^\eta, \theta_\omega \in \mathbb{R}^{p_\omega}$ such that,

$$\sum_{i=1}^{n} \frac{\partial \omega}{\partial x} \Phi_i^T(x,\theta) c = \sum_{i=1}^{n} \Psi_i(x) \quad (6)$$

where $\Psi_i(x)$ is a known function of x and is independent of θ. Moreover, $\Psi_i(x)$ and $\omega(x,\theta_\omega)$ are such that,

$$A_\xi(x) = G_\eta(x) - \mu \sum_{i=1}^{n} \Psi_i(x) \quad (7)$$

$$\omega(x,\theta_\omega) = \sigma(x)\theta_\omega \quad (8)$$

and A_ξ is exponentially stable i.e., there exists a positive constant ν_{A_ξ} such that $y^T(A_\xi^T + A_\xi)y \le -\nu_{A_\xi} y^T y, \quad \forall y \in \mathbb{R}^{n_y \times 1}$.

Assumption 5: The function $\Phi_i(x,\theta)$ can be linearly parameterized in terms of θ i.e.,

$$\Phi_i(x,\theta) = \theta^T \zeta_i(x) \quad (9)$$

The notation that has been used throughout the paper is as follows, unless otherwise mentioned. $\hat{(\bullet)}$ represents the estimated value of $(\bullet)$, $|(\bullet)|$ is the matrix norm of $(\bullet)$, $\overline{(\bullet)}$ represents a bounding functional for the regressor $(\bullet)$ and a capital Greek letter in subscript denotes a Kronecker product of two or more vectors of unknown parameters.

3 Fault Detection Architecture

In this section, we will explain the design of adaptive robust observers for estimating the unmodeled states and adaptation law to reduce parametric uncertainties. We will also describe the residual evaluation procedure.

Before we go into the details of the observer design, we would like to mention the following results related to Kronecker product. If $A \in \mathbb{R}^{n\times m}$ and $B \in \mathbb{R}^{r\times s}$, then the Kronecker product is given by [10],

$$A \otimes B = \begin{pmatrix} A_{11}B & A_{12}B & \dots & A_{1n}B \\ A_{21}B & A_{22}B & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ A_{m1}B & A_{m2}B & \dots & A_{mn}B \end{pmatrix} = matrix[A_{ij}B] \quad (10)$$

where $A \otimes B \in \mathbb{R}^{nr\times ms}$.

Lemma 1: If $A \in \mathbb{R}^{\eta \times p_\omega}, B \in \mathbb{R}^{p_\omega \times 1}, C \in \mathbb{R}^{1\times p}, D \in \mathbb{R}^{p\times 1}$, then the product $ABCD$ can be expressed as,

$$ABCD = (A \otimes D^T)\cdot(B \otimes C^T) \quad (11)$$

Proof: Follows by expanding the product using (10).▽

3.1 Adaptive Robust Observer Design

In this work, we will use the observer design technique proposed in [11]. For designing observers for unmodeled states, we will use the following coordinate transformation,

$$\xi = \eta - \omega(x, \theta_\omega) \quad (12)$$

and the dynamics of the transformed state is given by,

$$\dot{\xi} = \dot{\eta} - \sum_{i=1}^{n} \frac{\partial \omega}{\partial x_i}\dot{x}_i \quad (13)$$

$$\Rightarrow \dot{\xi} = F_\eta(x,u)\theta + G_\eta(x)\eta + \Delta_\eta - \sum_{i=1}^{n} \frac{\partial \omega}{\partial x_i}\{\theta^T F_{x_i}(x) + \Phi_i(x,\theta)v + \Delta_{x_i} + U(t-T_i)f_{a_i}(x,u,t)\} \quad (14)$$

$$\Rightarrow \dot{\xi} = \left\{G_\eta(x) - \mu\sum_{i=1}^{n}\frac{\partial \omega}{\partial x_i}\Phi_i(x,\theta)c\right\}\eta + F_\eta(x,u)\theta - \sum_{i=1}^{n}\frac{\partial \omega}{\partial x_i}\theta^T F_{x_i}(x) - \sum_{i=1}^{n}\frac{\partial \omega}{\partial x_i}\Phi_i(x,\theta)u + \left\{\Delta_\eta - \sum_{i=1}^{n}\frac{\partial \omega}{\partial x_i}\Delta_{x_i}\right\} - \sum_{i=1}^{n}\frac{\partial \omega}{\partial x_i}U(t-T_i)f_{a_i}(x,u,t) \quad (15)$$

$$\Rightarrow \dot{\xi} = A_\xi(x)\xi + \chi(x)\theta_\omega + F_\eta(x,u)\theta - \sum_{i=1}^{n}\varphi_i(x)\theta_{new} - \sum_{i=1}^{n}\rho_i(x)\theta_{new} - f_\xi(x,u,t) + \Delta_\xi \quad (16)$$

where we have used the following simplification in (16),

$$A_\xi(x)\sigma(x) = \chi(x) \quad (17)$$

$$\sum_{i=1}^{n}\frac{\partial \omega}{\partial x_i}U(t-T_i)f_{a_i}(x,u,t) = f_\xi(x,u,t) \quad (18)$$

$$\left\{\Delta_\eta - \sum_{i=1}^{n}\frac{\partial \omega}{\partial x_i}\Delta_{x_i}\right\} = \Delta_\xi \quad (19)$$

$$\begin{aligned}\sum_{i=1}^{n}\frac{\partial \omega}{\partial x_i}\theta^T F_{x_i}(x) &= \sum_{i=1}^{n}\frac{\partial \sigma}{\partial x_i}\theta_\omega\theta^T F_{x_i}(x) \\ &= \sum_{i=1}^{n}\left(\frac{\partial \sigma}{\partial x_i}\otimes F_{x_i}^T(x)\right)(\theta\otimes\theta_\omega) \\ &= \sum_{i=1}^{n}\varphi_i(x)\theta_{new}\end{aligned} \quad (20)$$

$$\begin{aligned}\sum_{i=1}^{n}\frac{\partial \omega}{\partial x_i}\Phi_i(x,\theta)u &= \sum_{i=1}^{n}\frac{\partial \sigma}{\partial x_i}\theta_\omega\theta^T(\zeta_i(x)u) \\ &= \sum_{i=1}^{n}\left\{\frac{\partial \sigma}{\partial x_i}\otimes(\zeta_i(x)u)^T\right\}(\theta\otimes\theta_\omega) \\ &= \sum_{i=1}^{n}\rho_i(x,u)\theta_{new}\end{aligned} \quad (21)$$

If all the parameters were known, the following observer could be used,

$$\dot{\hat{\xi}} = A_\xi(x)\hat{\xi} + \chi(x)\theta_\omega + F_\eta(x,u)\theta - \sum_{i=1}^{n}\varphi_i(x)\theta_{new} - \sum_{i=1}^{n}\rho_i(x)\theta_{new} \quad (22)$$

$$\Rightarrow \dot{\hat{\xi}} = A_\xi(x)\hat{\xi} + \Upsilon\theta_\Upsilon \quad (23)$$

where $\Upsilon = [F_\eta(x,u), -\sum_{i=1}^{n}\{\varphi_i(x)+\rho_i(x)\}, \chi(x)]$ and $\theta_\Upsilon = [\theta, \theta_{new}, \theta_\omega]^T$

Let $\tilde{\xi} = \hat{\xi} - \xi$. Then,

$$\dot{\tilde{\xi}} = \dot{\hat{\xi}} - \dot{\xi} \quad (24)$$

$$\Rightarrow \dot{\tilde{\xi}} = A_\xi(x)\tilde{\xi} - \Delta_\xi \quad (25)$$

in absence of any faults. But, the observer designed above is not implementable, as the parameters θ_Υ are not known. Hence, we will use the following nonlinear filter,

$$\dot{\tau}_{\theta_\Upsilon} = A_\xi(x)\tau_{\theta_\Upsilon} + \Upsilon \quad (26)$$

Using the above filter, the estimated transformed state ξ and unmodeled state η can be expressed as,

$$\hat{\xi} = \tau_{\theta_\Upsilon}\theta_\Upsilon \quad (27)$$

$$\eta = \tau_{\theta_\Upsilon}\theta_\Upsilon + \sigma(x)\theta_\omega - \tilde{\xi} = \tau_{\theta_\eta}\theta_\eta - \tilde{\xi} \quad (28)$$

where, $\tau_{\theta_\eta} = [\tau_{\theta_\Upsilon}, \sigma(x)]$ and $\theta_\eta = [\theta_\Upsilon, \theta_\omega]^T$.

3.2 Parameter Estimation Scheme

From (28), we still cannot estimate the unmodeled state η, as the parameters θ_η are unknown. In the present work, parameters are estimated using batched least square algorithm. The parameters are updated only when the regressor is rich enough i.e., if $R(kT) = \int_{(k-1)T}^{kT}\Omega(\tau)\Omega^T(\tau)d\tau$, then the adaptation law is given by,

$$\hat{\theta}_{(\bullet)}(kT) = \begin{cases} R(kT)^{-1}\int_{(k-1)T}^{kT}\Omega(\tau)z(\tau)d\tau & \text{if } R(kT) > \alpha(kT)I \\ \hat{\theta}_{(\bullet)}((k-1)T) & \text{otherwise} \end{cases} \quad (29)$$

where T is the window over which the regressor is monitored, $k = 1,2,3,..$ is an integer, $\alpha(kT)$ is a positive number, $(\bullet)$ represents any subscript and $z(t)$ is the static model used for parameter estimation, which we will be described later. Finally, the parameter estimate is given by,

$$\hat{\theta}_{(\bullet)}(t) = \hat{\theta}_{(\bullet)}((k-1)T) \quad (30)$$

We will now derive a model to estimate the unknown parameters. Let us rewrite the dynamics of the i^{th}-channel as follows,

$$\dot{x}_i = \theta^T F_{x_i}(x) + \theta^T\zeta_i(x)(u+\mu c\eta) + U(t-T_i)f_{a_i}(x,u,t) + \Delta_{x_i}$$
$$\Rightarrow = \Theta(x,u)\theta_\Theta - \theta^T(\mu\zeta_i(x)c\tilde{\xi}) + U(t-T_i)f_{a_i}(x,u,t) + \Delta_{x_i} \quad (31)$$

where we have used the following simplification,

$$\theta^T\zeta_i(x)\mu c\eta = \theta^T\zeta_i(x)\mu c(\tau_{\theta_\eta}\theta_\eta - \tilde{\xi}) = (\theta^T\otimes\theta_\eta^T)(\zeta_i(x)\mu\otimes(c\tau_{\theta_\eta})^T) - \theta^T(\mu\zeta_i(x)c\tilde{\xi}) = \theta_\Xi^T\Xi - \theta^T(\mu\zeta_i(x)c\tilde{\xi}) \quad (32)$$

and

$\Theta^T(x,u) = [F_{x_i}(x), \zeta_i(x)u, \Xi]$, $\theta_\Theta^T = [\theta^T, \theta^T, \theta_\Xi^T]$. But, as the parameters θ_Θ are not known, we shall use the following filters,

$$\dot{\Omega}^T = A\Omega^T + \Theta(x,u) \quad (33)$$

$$\dot{\Omega}_0 = A(\Omega_0 + x_i) \quad (34)$$

where A is any exponentially stable matrix i.e., there exists a ν_A such that $y^T(A^T+A)y \leq -\nu_A y^T y, \quad \forall y \in \mathbb{R}^{n_y\times 1}$. Define, $z = \Omega_0 + x_i$, which is calculable. Using equations (31) and (34), we obtain,

$$\dot{z} = Az + \Theta(x,u)\theta_\Theta + \Delta_z \quad (35)$$

where, $\Delta_z = \Delta_{x_i} - \theta^T(\mu\zeta_i(x)c\tilde{\xi})$ in absence of any faults.

If $\varepsilon = x_i + \Omega_0 - \Omega^T\theta_\Theta$, then its dynamics is given by,

$$\dot{\varepsilon} = A\varepsilon + \Delta_z \quad (36)$$

which is stable when $\Delta_z = 0$. Thus, the model used for estimating the parameters θ_Θ is,

$$z = \Omega^T\theta_\Theta + \varepsilon \quad (37)$$

where ε represents the cumulative effect of modeling errors and faults in the system. Now we shall state two lemmas which summarize the main results regarding the model developed for parameter estimation in this work. See [12], [4] and [5] for detailed proof and discussion about the following lemmas.

Lemma 2: The transformed state estimation error given by the solution of equation (25) is always bounded i.e., $\tilde{\xi} \in \mathcal{L}_\infty[0,\infty)$.

In absence of any modeling uncertainties i.e., $\Delta_\xi = 0$, the state estimation converges to zero with proper choice of the observer gain matrix $A_\xi(x)$, as seen from (25). In presence of bounded of uncertainties, however, it is easy to check that the estimation error remains bounded by using $\tilde{\xi}^2$ as a Lyapunov function.

Lemma 3: The model mismatch ε given by the solution of equation (36) remains bounded and satisfies the following inequality,

$$|\varepsilon(t)| \leq 2\frac{|\delta_z|}{\nu_A} \tag{38}$$

In absence of any modeling uncertainties i.e., when $\Delta_{x_i} = \Delta_\eta = 0$, ε is asymptotically stable, which makes the parameters converge to its exact value. But, in the presence of bounded uncertainties and bounded estimation error ξ, ε remains bounded with proper choice of the Hurwitz matrix A and observer gain matrix $A_\xi(x)$.

With these two lemmas in hand, we will now state the theorem which gives an explicit bound for the parameter estimation error.

Theorem 1: When the parameters are estimated using model (37) and adaptation law specified by (29), the estimation error remains bounded, and is given by,

$$|\tilde{\theta}_\Theta(t)| \leq \tilde{\theta}_{\Theta,max}(kT), \qquad \forall t \in [kT,(k+1)T) \tag{39}$$

where,

$$\tilde{\theta}_{\Theta,max}(0) = |\theta_{\Theta,max} - \theta_{\Theta,min}| \tag{40}$$

and

$$\tilde{\theta}_{\Theta,max}(kT) = \begin{cases} \frac{2|\delta_z|}{\alpha(kT)\upsilon_A}\int_{(k-1)T}^{kT}|\Omega(\tau)|d\tau & \text{if } R(kT) \geq \alpha(kT)I \\ \tilde{\theta}_{\Theta,max}((k-1)T) & \text{otherwise} \end{cases} \tag{41}$$

Proof: For the sake of completeness, we will outline the proof of the theorem, which has been proved previously in [12]. From (37), the parameter estimation error is given by,

$$\tilde{\theta}(t) = R(kT)^{-1}\int_{(k-1)T}^{kT}\Omega(\tau)\varepsilon(\tau)d\tau \tag{42}$$

where $\varepsilon(t)$ is bounded.

As parameter adaptation is carried out only when $\lambda_{min}(R(kT)) \geq \alpha(kT)$, we obtain,

$$\lambda_{max}(R(kT)) \geq \lambda_{min}(R(kT)) \geq \alpha(kT) \tag{43}$$

$$\Rightarrow \frac{1}{\lambda_{max}(R(kT))} \leq \frac{1}{\lambda_{min}(R(kT))} \leq \frac{1}{\alpha(kT)} \tag{44}$$

i.e., all eigenvalues of $R(kT)^{-1} \leq \frac{1}{\alpha(kT)}$. Therefore, using (42-44) for $\forall t \in [kT,(k+1)T)$ the estimation error satisfies,

$$\begin{aligned} |\tilde{\theta}(t)| &\leq \frac{1}{\alpha(kT)}\int_{(k-1)T}^{kT}|\Omega(\tau)\varepsilon(\tau)|d\tau \\ &\leq \frac{1}{\alpha(kT)}\frac{2|\delta_z|}{\nu_A}\int_{(k-1)T}^{kT}|\Omega(\tau)|d\tau \end{aligned} \tag{45}$$

Hence, the maximum parameter estimation error in presence of uncertain nonlinearities is given by (41).$\bigtriangledown$

Now that we have a mechanism for estimating the parameters θ_η, the estimated unmodeled state and estimation error is given by,

$$\hat{\eta} = \tau_{\theta_\eta}\hat{\theta}_\eta \tag{46}$$

$$\begin{aligned} \tilde{\eta} &= \hat{\eta} - \eta \\ &= \tau_{\theta_\eta}\tilde{\theta}_\eta + \tilde{\xi} \end{aligned} \tag{47}$$

3.3 Residual Evaluation Scheme

Using the adaptive robust state reconstruction scheme, the states can be estimated as follows,

$$\dot{\hat{x}}_i = -h_i(\hat{x}_i - x_i) + \hat{\theta}^T F_{x_i}(x) + \hat{\theta}^T\zeta_i(x)(u + \mu c\hat{\eta}) \tag{48}$$

$$\Rightarrow \dot{\hat{x}}_i = -h_i(\hat{x}_i - x_i) + \Theta(x,u)\hat{\theta}_\Theta \tag{49}$$

The state estimation error dynamics $\dot{\tilde{x}}_i = \dot{\hat{x}}_i - \dot{x}_i$ is given by,

$$\begin{aligned} \dot{\tilde{x}}_i(t) &= -h_i\tilde{x}_i + \Theta(x,u)\tilde{\theta}_\Theta + \theta^T(\zeta_i(x)\mu c\tilde{\xi}) \\ &\quad -U(t-T_i)f_{a_i}(x,u,t) - \Delta_{x_i} \end{aligned} \tag{50}$$

$$\begin{aligned} \tilde{x}_i(t) &= e^{-h_i(t-(k-1)T)}\tilde{x}_i((k-1)T) \\ &\quad + \int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\beta(x,u,\tau)d\tau, \quad \forall t \in [(k-1)T,kT) \end{aligned} \tag{51}$$

where $\beta(x,u,\tau) = \Theta(x,u)\tilde{\theta}_\Theta + \theta^T(\zeta_i(x)\mu c\tilde{\xi}) - U(t-T_i)f_{a_i}(x,u,t) - \Delta_{x_i}$. In the fault-free case, $U(t-T_i)f_{a_i}(x,u,t)$ term will be absent in the expression for β. It can be easily verified that the initial estimation error satisfies the following equation,

$$\tilde{x}_i(kT) = \sum_{j=1}^{k} e^{(j-k)h_iT} \int_{(j-1)T}^{jT} e^{-h_i(jT-\tau)}\beta(x,u,\tau)d\tau \quad (52)$$

4 Performance Results

In this section, we will design a threshold function for residual evaluation and prove the robustness and sensitivity properties of the fault detection scheme.

4.1 Robustness Analysis

Theorem 2: If the threshold for residual evaluation $\tilde{x}_i^0$ is chosen such that,

$$\begin{aligned}\tilde{x}_i^0(t) = {} & e^{-h_i(t-(k-1)T)}\Gamma_i((k-1)T) + \frac{\delta_i}{h_i}[1-e^{-h_i(t-(k-1)T)}] \\ & +\tilde{\theta}_{\Theta,max}((k-1)T)\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\overline{\Theta}(x,u)d\tau \\ & +\mu(\tilde{\theta}_{max}^T((k-1)T)+|\hat{\theta}^T((k-1)T)|)\cdot \\ & \int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\kappa_i(x,u,\tau)d\tau \end{aligned} \quad (53)$$

then the proposed actuator fault detection scheme is robust to input unmodeled dynamics and other modeling uncertainties and avoids false alarm.

Proof: We will first consider the following integrals.

$$\begin{aligned} I_1 &= |\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\theta^T\zeta_i(x)\mu c\tilde{\xi}d\tau| \\ &\le \mu\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}|\theta^T||\zeta_i(x)\mu c\tilde{\xi}|d\tau \\ &\le 2\mu\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}(|\hat{\theta}^T((k-1)T)|+ \\ &\quad \tilde{\theta}_{max}^T((k-1)T))\overline{\zeta_i(x)}|c|\xi_{max}d\tau \\ &= 2\mu(|\hat{\theta}^T((k-1)T)|+\tilde{\theta}_{max}^T((k-1)T))\cdot \\ &\quad \int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\kappa_i(x,u,\tau)d\tau \end{aligned} \quad (54)$$

where $|\xi(0)| \le \xi_0$ i.e., we assume that the initial estimation error for unmodeled states has a known bound and $\xi_{max} = max\left[\xi_0, \frac{|\delta_\xi|}{\nu_{A_\xi}}\right]$, $\kappa_i(x,u,t) = \overline{\zeta_i(x)}|c|\xi_{max}$.

$$\begin{aligned} I_2 &= |\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\Theta(x,u)\tilde{\theta}_\Theta d\tau| \\ &\le \tilde{\theta}_{\Theta,max}\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\overline{\Theta}(x,u)d\tau \end{aligned} \quad (55)$$

$$\begin{aligned} I_3 &= |\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\Delta_i(x,u,\tau)d\tau| \\ &\le \int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\delta_i d\tau \\ &= \frac{\delta_i}{h_i}[1-e^{-h_i(t-(k-1)T)}] \end{aligned} \quad (56)$$

$$\begin{aligned} I_4 &= |\sum_{j=1}^{k} e^{(j-k)h_iT}\int_{(j-1)T}^{jT} e^{-h_i(jT-\tau)}\beta(x,u,\tau)d\tau| \\ &\le \sum_{j=1}^{k} e^{(j-k)h_iT}|\int_{(j-1)T}^{jT} e^{-h_i(jT-\tau)}\beta(x,u,\tau)d\tau| \end{aligned} \quad (57)$$

and

$$|\int_{(j-1)T}^{jT} e^{-h_i(jT-\tau)}\beta(x,u,\tau)d\tau| \le I_1+I_2+I_3 \quad (58)$$

when there are no faults and with the limits of the integrals being $(j-1)T$ to jT. Using equations (54-58), we obtain,

$$\begin{aligned} |\tilde{x}_i(kT)| &\le \Gamma_i(kT) \\ &= \sum_{j=1}^{k} e^{(j-k)h_iT}[\frac{\delta_i}{h_i}(1-e^{-h_iT}) \\ &\quad +\tilde{\theta}_{\Theta,max}((j-1)T)\int_{(j-1)T}^{jT} e^{-h_i(jT-\tau)}\overline{\Theta}(x,u)d\tau \\ &\quad \mu(|\hat{\theta}^T((j-1)T)|+\tilde{\theta}_{max}^T((j-1)T))\cdot \\ &\quad \int_{(j-1)T}^{jT} e^{-h_i(jT-\tau)}\kappa_i(x,u,\tau)d\tau] \end{aligned} \quad (59)$$

Now, from (51,52) and (54-58), we obtain the result of theorem 2 as given by equation (53). It follows immediately from the proof that the proposed scheme avoids false alarms in absence of any faults and is robust against modeling uncertainties.▽

4.2 Sensitivity Analysis

A fault detection scheme whichdoes not take into account the effect of unmodeled dynamics will be susceptible to false alarms, as the deviations in actual and nominal control effort can cause significant state-estimation errors. In order to make it robust to unmodeled dynamics, we have to use a worst case scenario bound for generating the threshold for residual evaluation. Similarly, if parameter adaptation is not considered in designing the detection scheme, we have to use the maximum value of parameter variation at residual evaluation stage to make it robust against parametric uncertainties. We will first derive the threshold which does not consider the unmodeled dynamics and then extend it to include the scenario where parameter adaptation is not considered either.

In absence of unmodeled dynamics, the nominal model of the system is given by,

$$\dot{x}_i = \theta^T F_{x_i}(x) + \phi_i(x,\theta)u \tag{60}$$

Based on the above equation, the state can be reconstructed as,

$$\dot{\hat{x}}_i = -h_i(\hat{x}_i - x_i) + \hat{\theta}^T F_{x_i}(x) + \hat{\theta}^T \zeta_i(x) u \tag{61}$$

$$\Rightarrow \dot{\hat{x}}_i = -h_i(\hat{x}_i - x_i) + \Theta(x,u)\hat{\theta}_\Theta - \hat{\theta}^T \zeta_i(x)\mu c\hat{\eta} \tag{62}$$

The state estimation error dynamics $\dot{\tilde{x}}_i = \dot{\hat{x}}_i - \dot{x}_i$ in this is given by,

$$\dot{\tilde{x}}_i = -h_i\tilde{x}_i + \Theta(x,u)\tilde{\theta}_\Theta - (\hat{\theta}^T \otimes \hat{\theta}_\eta^T)(\zeta_i(x)\mu \otimes (c\tau_\eta)^T) + \theta^T(\mu\zeta_i(x)c\tilde{\xi}) - U(t-T_i)f_{a_i}(x,u,t) - \Delta_{x_i} \tag{63}$$

$$\Rightarrow \dot{\tilde{x}}_i = -h_i\tilde{x}_i + \Theta(x,u)\tilde{\theta}_\Theta - \hat{\theta}_\Xi^T \Xi + \theta^T(\mu\zeta_i(x)c\tilde{\xi}) - U(t-T_i)f_{a_i}(x,u,t) - \Delta_{x_i} \tag{64}$$

Theorem 3: If the threshold $\tilde{x}_i{}^0(t)$ is chosen such that,

1. an adaptive robust observer $\hat{\eta}$ is used for the unmodeled state η and

2. the parameter adaptation rate $\alpha(kT)$ is such that,

$$\alpha(kT) \geq \frac{2|\delta_z|}{|\theta_{max} - \theta_{min}|\nu_A} \int_{(k-1)T}^{kT} |\Omega(\tau)| d\tau \tag{65}$$

then the actuator fault detection scheme is more sensitive to incipient and small faults i.e., $\tilde{x}_i{}^0(t) \leq \mu_i^0(t)$.

Proof: Let us first consider the following integral,

$$I_5 = \left| \int_{(k-1)T}^{t} e^{-h_i(t-\tau)} \hat{\theta}_\Xi^T(\tau)\Xi d\tau \right| \leq |\hat{\theta}_\Xi^T(k-1)T| \int_{(k-1)T}^{t} e^{-h_i(t-\tau)} \overline{\Xi} d\tau \tag{66}$$

In absence of any faults, the state estimation error can be bounded as,

$$|\tilde{x}_i(t)| \leq e^{-h_i(t-(k-1)T)}|\tilde{x}_i((k-1)T)| + \left|\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\beta(x,u,\tau)d\tau\right| + \left|\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\hat{\theta}_\Xi^T(\tau)\Xi d\tau\right| \tag{67}$$

Hence, a threshold which is robust against unmodeled dynamics must be such that,

$$|\tilde{x}_i(t)| \leq \mu_i^0 = e^{-h_i(t-(k-1)T)}\Gamma_i'((k-1)T) + \frac{\delta_i}{h_i}[1 - e^{-h_i(t-(k-1)T)}] + \tilde{\theta}_{\Theta,max}((k-1)T)\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\overline{\Theta}(x,u)d\tau + |\hat{\theta}_\Xi^T(k-1)T|\int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\overline{\Xi}d\tau + \mu(\tilde{\theta}_{max}^T((k-1)T) + |\hat{\theta}^T((k-1)T)|)\cdot \int_{(k-1)T}^{t} e^{-h_i(t-\tau)}\kappa_i(x,u,\tau)d\tau \tag{68}$$

where,

$$\Gamma_i'(kT) = \Gamma_i(kT) + \sum_{j=1}^{k} e^{(j-k)h_iT}|\hat{\theta}_\Xi^T((j-1)T)|\int_{(j-1)T}^{jT} e^{-h_i(jT-\tau)}\overline{\Xi}d\tau \tag{69}$$

It is obvious from the derivations of $\tilde{x}_i^0$ and μ_i^0 that $\mu_i^0 \geq \tilde{x}_i^0$.

Also, if parameter adaptation law is such that $\alpha(kT)$ is chosen according to equation (65), then using the result of theorem 1, we obtain,

$$|\theta_{max} - \theta_{min}| \geq \frac{2|\delta_z|}{\alpha(kT)}\int_{(k-1)T}^{kT}|\Omega(\tau)|d\tau = \tilde{\theta}_{max}((k-1)T) \tag{70}$$

Hence, if parameter adaptation is not used, we will replace $\tilde{\theta}_{(\bullet,max)}(kT)$ by $\tilde{\theta}_{(\bullet,max)}(0)$ and $\hat{\theta}_{(\bullet)}$ by $\theta_{(\bullet,max)}$, which further increases the threshold value μ_i^0. This completes the proof. ▽

For details regarding sensitivity analysis in the presence of parametric uncertainties only, and detailed proof of the second part of the theorem, refer to [5] and [12].

5 Conclusions

In this paper, we presented a fault detection scheme which improves fault sensitivity without losing robustness against modeling uncertainties. The underlying assumption which allows us to meet the conflicting demands is that the uncertainties have known structure.

As a first step towards designing a sensitive yet robust algorithm, we use adaptive robust state reconstruction for reducing the effect of various uncertainties in estimating the states. Next, we use the structural information about the uncertainties to design adaptive robust observers for estimating unmodeled states and adaptation laws to reduce the extent of parametric uncertainties. These estimates are then incorporated in the design of a threshold for residual evaluation. The performance results for the proposed scheme are analytically proved and stated as theorems.

The main contributions of this work are:

1. Design of adaptive robust observers for estimating unmodeled states.
2. Design of a threshold for residual evaluation which reduces uncertainties due to unmodeled dynamics and unknown parameters.
3. Design of a fault detection scheme for nonlinear systems, which is sensitive to incipient and small faults but, robust to modeling uncertainties.

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