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INTEGRATED DIRECT/INDIRECT ADAPTIVE ROBUST CONTROL (DIARC) OF HYDRAULIC ROBOTICS ARM WITH ACCURATE PARAMETER ESTIMATES

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ABSTRACT

In a general Adaptive Robust Control (ARC) framework, the emphasis is always on the guaranteed transient performance and accurate trajectory tracking in the presence of uncertain nonlinearity and parametric uncertainties. However, when secondary purposes such as system health monitoring and prognosis are of equal importance, intelligent integration of output tracking performance oriented direct adaptive robust control (DARC) and the recently proposed accurate parameter estimation-based indirect adaptive robust control (IARC) is required. In this paper, we will consider such a seamless integration for a hydraulic robotic arm.

The newly developed IARC design is first applied to the trajectory tracking for the robotic arm but with an improved estimation model, in which accurate parameter estimates are obtained through a parameter estimation algorithm that is based on physical dynamics rather than the tracking error dynamics. An integrated direct/indirect adaptive robust controller (DIARC) is then presented that preserves the excellent transient tracking performance of the direct ARC designs as well as the better parameter estimation process of the IARC design. The proposed Integrated Direct/ Indirect Adaptive Robust Controller (DIARC) achieves the controller-identifier separation, thus enabling certain modularity in the controller design.

INTRODUCTION

Robotic manipulators driven by hydraulic actuators have been widely used in the industry for tasks such as material handling and earth moving due to their high power density. These types of tasks typically require that the end-effectors of the manipulators follow certain prescribed desired trajectories in the working space. In order to meet the increasing requirement of productivity and performance of modern industry, the development of high speed and high accuracy trajectory tracking controllers for the coordinated motion of robot manipulator driven by hydraulic actuators is of practical importance.

In the past, much of the work in the control of hydraulic systems uses linear control theory [2], [3], [4] and feedback linearization techniques [5], [6]. In [7], Alleyne and Hedrick applied the nonlinear adaptive control to the force control of an active suspension driven by a double-rod cylinder. They demonstrated that nonlinear control schemes can achieve a much better performance than conventional linear controllers. They considered the parametric uncertainties of the cylinder only. In [8], [9], the direct ARC approach proposed by Yao and Tomizuka in [10], [11] was generalized to provide a rigorous theoretical framework for the high performance robust motion control of a double-rod hydraulic actuator by taking into account both nonlinearities and model uncertainties of the electro-hydraulic servo-systems. In [12], [13], the design is extended to the precision motion control of single-rod hydraulic actuators. In [14], an integrated direct/indirect adaptive robust controller (DIARC) is developed that combines the excellent tracking performance of the

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direct ARC designs with the good parameter estimation process of the indirect ARC designs. In the previously proposed direct ARC controller designs [12], [13], the parameter adaptation law is driven by the actual tracking errors. Although the output tracking performance is guaranteed in terms of both transient and final tracking accuracy, the parameter estimation may perform poorly even when the persistent exciting condition is satisfied, which is the nature of direct adaptive control designs where the design goal is to achieve perfect output tracking only [15]. For some applications, such a result might be satisfying. But for applications which demands simultaneous good tracking accuracy and parameter estimation, new design philosophy has to be sought, which is the focus of the paper. Specifically, an integrated direct/ indirect ARC (DIARC) design will be proposed for the precision control of electro-hydraulic systems. The parameter adaptation law of the proposed DIARC controller is based on the actual system dynamic model, rather than the tracking error dynamics used for the controller design. Such a design separation enables the designer to have the extra freedom of monitoring and manipulating the experimental conditions to have a better parameter estimation process. In addition, fast dynamic compensations which are similar to the direct ARC designs [12], [13] are employed to ensure the transient tracking performance in general. The end product is an ARC design that preserves both the excellent tracking performance of direct ARC designs [12], [13] and the good parameter estimation process of indirect adaptive designs [15], [16]. Theoretically, the proposed DIARC controller guarantees a prescribed output tracking transient performance and final tracking accuracy in the presence of both parametric uncertainties and disturbances while achieving asymptotic output tracking in the presence of parametric uncertainties only. Experimental results will be presented to show the good parameter estimation process and the high performance nature of the proposed DIARC design.

DYNAMIC MODEL AND PROBLEM FORMULATION

The problem under investigation is a 3-DOF electro-hydraulic robotic arm driven by three single rod hydraulic actuators, as depicted in Fig.1. The joint angles are represented by $q=[q_1,\ q_2,\ q_3]^T$ and $x=[x_{L1},\ x_{L2},\ x_{L3}]^T$ represents the displacement of the hydraulic cylinders, which is uniquely related to the joint angles due to static kinematic geometry of the robotic arm. Our control objective is to make joint angles q track any feasible desired motion trajectory as closely as possible. We also pursue the problem with additional objective of accurate parameter estimation for secondary purpose of health monitoring and prognosis. As a matter of fact, we would pursue our second goal more aggressively once we make sure that we have archived required accuracy in trajectory tracking.

The robotic arm is considered to be a rigid body and its dy-

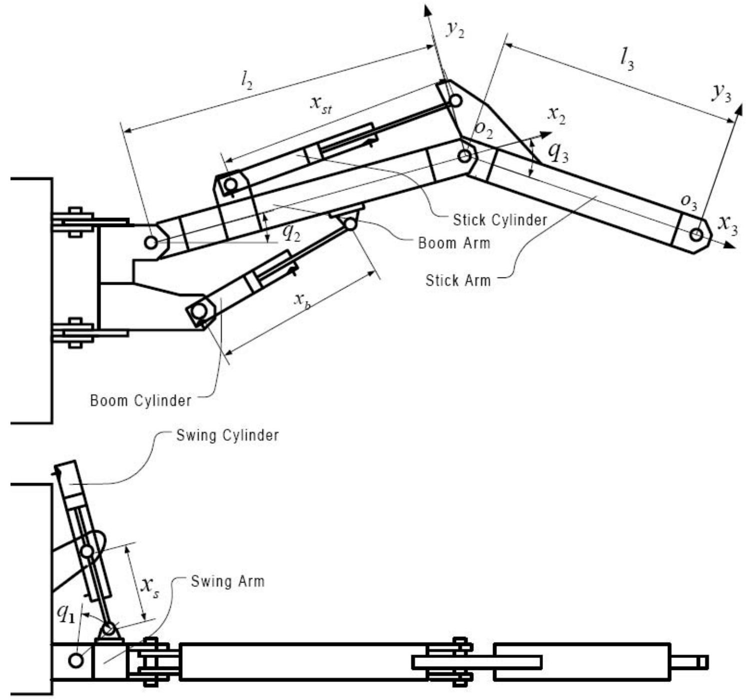

Figure 1. A Hydraulic Robot Arm

namics can be described by:

$$M(q)\ddot{x}+C(q,\dot{q})\dot{q}+G(q)=\frac{\partial x_l}{\partial q}(A_1P_1-A_2P_2)+T(t,q,\dot{q}) \quad (1)$$

where $P_1=[P_{11},\ P_{12},\ P_{13}]^T$ and $P_{1i}\quad i=(1,2,3)$ is the forward chamber pressures for i^{th} cylinder. Similarly, $P_1=[P_{21},\ P_{22},\ P_{23}]^T$ and $P_{2i}\quad i=(1,2,3)$ is the return chamber pressures for i^{th} cylinder. The ram areas of the two chambers driving each cylinder are given by following two diagonal matrices: $A_1=diag[A_{11},\ A_{12},\ A_{13}]$ and $A_1=diag[A_{21},\ A_{22},\ A_{23}]$ and $T(t,q,\dot{q})\in\mathbb{R}^3$ represents the lumped disturbance torque including external disturbances and terms like viscous and frictional torque.

Let m_L be the unknown payload mounted at the end of the nth arm, which is treated as a point mass for simplicity. Then, the inertial matrix $M(q)$, coriolis terms $C(q,\dot{q})$ and gravity terms $G(q)$ in (1) can be linearly parametrized with respect to the unknown mass m_L as

$$\begin{aligned}&M(q)=M_c(q)+M_L(q)m_L, G(q)=G_c(q)+G_L(q)m_L\\&C(q,\dot{q})=C_c(q,\dot{q})+C_L(q,\dot{q})m_L\end{aligned} \quad (2)$$

where $M_c(q),\ M_L(q),\ C_c(q,\dot{q}),\ C_L(q,\dot{q}),\ G_c(q),\ G_L(q)$ are known nonlinear functions of q and $\dot{q}$. One of the properties of the inertia matrix $M(q)$ is that its inverse can be written as:

$$M^{-1}(q) = \overline{M}(q)/|M(q)| \quad (3)$$

where $|M(q)|$ represents the determinant of $M(q)$, $\overline{M}(q)$ represents the adjoint matrix of $M(q)$. Furthermore, both $\overline{M}(q)$ and $|M(q)|$ can be written as

$$|M(q)| = I = I_c + \sum_{i=1}^{3} I_{si} m_L^i \quad \overline{M}(q) = \overline{M}_c + \sum_{i=1}^{2} \overline{M}_i m_L^i \quad (4)$$

where I_c, I_{si}, $\overline{M}_c$ and $\overline{M}_i$ are of the known functions of joint position q and I is a scalar.

Assuming no cylinder leakages, the actuator (or the cylinder) dynamics can be written as [**?**],

$$\begin{array}{l} \frac{V_1(x)}{\beta_e}\dot{P}_1 = -A_1\dot{x} + Q_1 = -A_1\frac{\partial x}{\partial q}\dot{q} + Q_1 \\ \frac{V_2(x)}{\beta_e}\dot{P}_2 = A_2\dot{x} - Q_2 = A_2\frac{\partial x}{\partial q}\dot{q} - Q_2 \end{array} \quad (5)$$

where, $V_1(x) = V_{h1} + A_1 \cdot diag[x] \in R^{3\times3}$ and $V_2(x) = V_{h2} - A_2 \cdot diag[x]$ are the diagonal total control volume matrices of the two chambers of hydraulic cylinders respectively, which include the hose volume between the two chambers and the valves, $V_{h1} = diag[V_{h11}, V_{h12}, V_{h13}]$ and $V_{h2} = diag[V_{h21}, V_{h22}, V_{h23}]$ are the control volumes of the two chambers when $x = 0$, $diag[x] = diag[x_1, x_2, x_3]$, $\beta_e \in R$ is the effective bulk modulus, $Q_1 = [Q_{11}, Q_{12}, Q_{13}]^T$ is the vector of the supplied flow rates to the forward chambers of the driving cylinders, and $Q_2 = [Q_{21}, Q_{22}, Q_{23}]^T$ is the vector of the return flow rates from the return chambers of the cylinders.

Let $x_v = [x_{v1}, x_{v2}, x_{v3}]$ denotes the spool displacements of the valves in the hydraulic loops. Define the square roots of the pressure drops across the two ports of the first control valve as:

$$\begin{array}{l} g_{31}(P_{11}, sign(x_{v1})) = \begin{cases} \sqrt{P_s - P_{11}} & \text{for } x_{v1} \geqslant 0 \\ \sqrt{P_{11} - P_r} & x_{v1} < 0 \end{cases} \\ g_{41}(P_{21}, sign(x_{v1})) = \begin{cases} \sqrt{P_{21} - P_r} & \text{for } x_{v1} \geqslant 0 \\ \sqrt{P_s - P_{21}} & x_{v1} < 0 \end{cases} \end{array} \quad (6)$$

where P_s is the supply pressure of the pump, and P_r is the tank reference pressure. Similarly, let g_{3i} and g_{4i} be the square roots of the pressure drops for the ith hydraulic loop. For simplicity of notation, define the diagonal square root matrices of the pressure drops as:

$$\begin{array}{l} g_3(P_1, sign(x_v)) = diag[g_{31}(P_{11}, sign(x_{v1})), \ldots, g_{33}(P_{13}, sign(x_{v3}))] \\ g_4(P_2, sign(x_v)) = diag[g_{41}(P_{21}, sign(x_{v1})), \ldots, g_{43}(P_{23}, sign(x_{v3}))] \end{array} \quad (7)$$

Then, Q_1 and Q_2 in (5) are related to the spool displacements of the valves x_v by [?],

$$Q_1 = k_{q1} g_3(P_1, sign(x_v)) x_v \quad , Q_2 = k_{q2} g_4(P_2, sign(x_v)) x_v \quad (8)$$

where $k_{q1} = diag[k_{q11}, k_{q12}, k_{q13}]$ and $k_{q2} = diag[k_{q21}, k_{q22}, k_{q23}]$ are the *constant* flow gain coefficients matrices of the forward and return loops respectively.

Given the desired motion trajectory $q_d(t)$, the objective is to synthesize a control input $u = x_v$ such that the output $y = q$ tracks $q_d(t)$ as closely as possible in spite of various model uncertainties.

DIARC CONTROLLER DESIGN

In this paper, for simplicity, we consider the parametric uncertainties due to the unknown payload m_L, and the nominal value of the lumped disturbance T, T_n only. Other parametric ucnertainties can be dealt with in the same way if necessary. In order to use parameter adaptation to reduce parametric uncertainties to improve performance, it is necessary to linearly parametrize the system dynamics equation in terms of a set of unknown parameters. To achieve this, define the unknown parameter set as $\theta = [\theta_1, \theta_2^T]^T$ where $\theta_1 = m_L$ and $\theta_2 = T_n$. The system dynamic equations can thus be linearly parametrized in terms of θ as

$$\begin{array}{l} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \frac{\partial x}{\partial q}(A_1P_1 - A_2P_2) \\ \quad +\theta_2 + \tilde{T}(t,q,\dot{q}), \quad \tilde{T} = T(t,q,\dot{q}) - T_n \\ \dot{P}_1 = \beta_e V_1^{-1}(q)\left[-A_1\frac{\partial x}{\partial q}\dot{q} + Q_1(u, g_3(P_1, sign(u))\right] \\ \dot{P}_2 = \beta_e V_2^{-1}(q)\left[A_2\frac{\partial x}{\partial q}\dot{q} - Q_2(u, g_4(P_2, sign(u))\right] \end{array} \quad (9)$$

Parametric uncertainties and uncertain nonlinearities satisfy

$$\theta \in \Omega_\theta \triangleq \{\theta : \ \theta_{min} < \theta < \theta_{max}\} \quad (10)$$

$$|\tilde{T}(t,q,\dot{q})| \leqslant \delta_T(q,\dot{q},t) \quad (11)$$

where $\theta_{min} = [\theta_{1min}, ,\theta_{2min}]^T$, θ_{max}=$[\theta_{1max}, \theta_{2max}]^T$, $\delta_T(t,q,\dot{q})$ are known.

Parameter Estimation Model

Let H_f be a stable low pass filter with relative degree no less than two. Applying the filter to (9), one obtains

$$H_f\left[\frac{\partial x}{\partial q}(A_1P_1 - A_2P_2)\right] = H_f\left[M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)\right] \quad (12)$$

Using above filter, various parameter estimation-based algorithm can be used by treating $H_f\left[\frac{\partial x}{\partial q}(A_1P_1 - A_2P_2)\right]$ as the virtual output and creating a static linear regressor model.Adaptation function and adaptation rate matrix are given by:

$$
\begin{aligned}
\tau &= \frac{1}{1+\nu\phi_f^T\Gamma\phi_f}\phi_f\varepsilon \\
\dot{\Gamma} &= \alpha\Gamma - \frac{1}{1+\nu\phi_f^T\Gamma\phi_f}\Gamma\phi_f\phi_f^T\Gamma
\end{aligned} \tag{13}
$$

Projection Type Adaptation Law with Rate Limits

As in [11], the widely used projection mapping $Proj_{\hat{\theta}}(\bullet)$ will be used to keep the parameter estimates with in the known bound Ω_θ. The standard projection mapping is given by:

$$
Proj_{\hat{\theta}} = \begin{cases} \zeta, & \hat{\theta} \in \overline{\Omega_\theta} \text{ or } n_{\hat{\theta}}^T\zeta \leqslant 0 \\ \left(I - \Gamma\frac{n_{\hat{\theta}}n_{\hat{\theta}}^T}{n_{\hat{\theta}}\Gamma n_{\hat{\theta}}}\right)\zeta, & \hat{\theta} \in \partial\Omega_\theta \text{ or } n_{\hat{\theta}}^T\zeta > 0 \end{cases} \tag{14}
$$

where $\zeta \in R_p$, $\Gamma(t) \in R_{p\times p}$, $\overline{\Omega}_\theta$ and $\partial\Omega_\theta$ denote the interior and the boundary of Ω_θ respectively, and $n_{\hat{\theta}}$ represents the outward unit normal vector at $\hat{\theta} \in \partial\Omega_\theta$.

The complete separation between the robust control term and estimator design, in addition to projection type parameter adaptation law, it is also necessary to use the preset adaptation rate limits. A saturation function is defined, for this purpose as :

$$
sat_{\dot{\theta}_M}(\zeta) = s_0\zeta, \qquad s_0 = \begin{cases} 1, & \|\zeta\| \leqslant \dot{\theta}_M \\ \frac{\dot{\theta}_M}{\|\zeta\|}, & \|\zeta\|\,\dot{\theta}_M \end{cases} \tag{15}
$$

Parameter update law is given by

$$
\dot{\hat{\theta}} = sat_{\dot{\theta}_M}\left(Proj_{\hat{\theta}}(\Gamma\tau)\right), \qquad \hat{\theta}(0) \in \Omega_\theta \tag{16}
$$

where, τ is an adaptation function and $\Gamma(t) > 0$ is any continuously differentiable positive symmetric adaptation rate matrix.

Controller Design using Overparametrizing

The design parallels the recursive backstepping design procedure via ARC Lyapunov functions [11] as follows.

Step 1

Define a switching-function-like quantity as $z_2 = \dot{z}_1 + k_1z_1 = \dot{q} - \dot{q}_r$, where $\dot{q}_r = \dot{q}_d - k_1z_1$, $z_1 = q - q_d(t)$, in which $q_d(t)$ is the reference trajectory and k_1 is a positive feedback gain. The design in this step is to make z_2 as small as possible with a guaranteed transient performance.

Define a positive semi definite (p.s.d) function as $V_2 = \frac{1}{2}z_2^TMz_2$, from(9) and the property that $\dot{M}(q) - 2C(q,\dot{q})$ is a skew symmetric matrix [?], its derivative is given by

$$
\begin{aligned}
\dot{V}_2 &= z_2^T(M\dot{z}_2 + \tfrac{1}{2}\dot{M}z_2) \\
&= z_2^T[-M\ddot{q}_r - C(\dot{q},q)\dot{q}_r - G(q) + \tfrac{\partial x}{\partial q}(A_1P_1 - A_2P_2) + \theta_2 + \tilde{T}]
\end{aligned} \tag{17}
$$

Define the load pressure as $P_L = A_1P_1 - A_2P_2$. If we treat P_L as the virtual control input to (17), a virtual control law P_{Ld} for P_L will be synthesized such that z_2 is as small as possible with a guaranteed transient performance. Since (17) has both parametric uncertainties θ_1 and θ_2 and uncertain nonlinearity $\tilde{T}$, the ARC approach proposed in [?] will be generalized to accomplish the objective.

The control function P_{Ld} consists of two parts given by

$$
\begin{aligned}
P_{Ld}(q,\dot{q},\hat{\theta}_1,\hat{\theta}_2,t) &= P_{Lda} + P_{Lds} \\
P_{Lda} &= (\tfrac{\partial x}{\partial q})^{-1}[\hat{M}\ddot{q}_r + \hat{C}(\dot{q},q)\dot{q}_r + \hat{G}(q) - \hat{\theta}_2 - K_2(t)z_2]
\end{aligned} \tag{18}
$$

where $K_2(t)$ is a positive feedback gain matrix and $\hat{M}(q) = M_c + M_L\hat{\theta}_1$, $\hat{C}(\dot{q},q) = C_c + C_L\hat{\theta}_1$, $\hat{G}(q) = G_c + G_L\hat{\theta}_1$. Substituting (18) into (17) and let $z_3 = P_L - P_{Ld}$ represent the input discrepancy, we will have

$$
\dot{V}_2 = z_2^T[\tfrac{\partial x}{\partial q}P_{Lds} - \phi_2\tilde{\theta} + \tilde{T}] - z_2^TK_2(t)z_2 + z_2^T\tfrac{\partial x}{\partial q}z_3 \tag{19}
$$

where $\phi_2 = [-M_L\ddot{q}_r - C_L(\dot{q},q)\dot{q}_r - G_L(q), I_{3\times3}]$. Then P_{Lds} can be chosen to satisfy:

$$
\begin{aligned}
&\text{condition i} && z_2^T[\tfrac{\partial x}{\partial q}P_{Lds} - \phi_2\tilde{\theta} + \tilde{T}] \leqslant \varepsilon_2 \\
&\text{condition ii} && z_2^T\tfrac{\partial x}{\partial q}P_{Lds} \leqslant 0
\end{aligned} \tag{20}
$$

where ε_2 is a design parameter which can be arbitrarily small. Essentially, condition i of (20) shows that P_{Lds} is synthesized to dominate the model uncertainties coming from both parametric uncertainties $\tilde{\theta}$ and uncertain nonlinearities $\tilde{T}$, and condition ii is to make sure that P_{Lds} is dissipating in nature so that it does not interfere with the functionality of the adaptive control part P_{Lda}.

Step 2

In this step, an actual control law will be synthesized so that z_3 converges to zero or a small value with a guaranteed transient performance and accuracy. If we were to use the backstepping design strategy via ARC Lyapunov function, then, the resulting ARC law would require the feedback of the joint acceleration $\ddot{q}$ since $\ddot{q}$ is needed in computing $\hat{\dot{P}}_{Ld}$, the calculable part of the derivative of the desired virtual control function P_{Ld}, for adaptive model compensation. In order to avoid the need for joint acceleration feedback, in the following, the property of the inertia matrix in (4) will be used as follows.

Multiply both side of first equation of (9) by $|M|M^{-1} = \bar{M}$ we will have

$$|M|\ddot{q} + \bar{M}C(\dot{q},q)\dot{q} + \bar{M}G = \bar{M}\frac{\partial x}{\partial q}P_L + \bar{M}\theta_2 + \bar{M}\tilde{T} \tag{21}$$

Define $C_t(\dot{q},q) = \bar{M}C(\dot{q},q)$, $G_t(q) = \bar{M}G$, $d_n = \bar{M}\theta_2$, $\tilde{d} = \bar{M}\tilde{T}$. Thus (21) could be expressed by

$$I\ddot{q} + C_t\dot{q} + G_t = \bar{M}\frac{\partial x}{\partial q}P_L + d_n + \tilde{d} \tag{22}$$

where I is a scalar. Similar to (2), C_t, G_t and d_n can be expressed by $C_t(\dot{q},q) = C_{tc} + \sum_{i=1}^{3} C_{ti}\theta_1^i$, $G_t(q) = G_{tc} + \sum_{i=1}^{3} G_{ti}\theta_1^i$, $d_n = \bar{M}_c\theta_2 + \sum_{i=1}^{2}\bar{M}_i\theta_1^i\theta_2$. Where C_{tc} and G_{tc} are of the known nonlinear functions of q and $\dot{q}$.

Redefine the unknown parameters as:$[\beta_1,\beta_2,\beta_3,\beta_4^T,\beta_5^T,\beta_6^T]$=$[\theta_1,\theta_1^2,\theta_1^3,\theta_2^T,\ \theta_1\theta_2^T,\theta_1^2\theta_2^T]$. From (9), the derivative of z_3 is given by

$$\begin{aligned}
&\dot{z}_3 = \dot{P}_L - \dot{P}_{Ld}\\
&\dot{P}_L = \beta_e[-(A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q} + (A_1V_1^{-1}Q_1 + A_2V_2^{-1}Q_2)]\\
&\dot{P}_{Ld} = \tfrac{\partial P_{Ld}}{\partial q}\dot{q} + \tfrac{\partial P_{Ld}}{\partial \dot{q}}\ddot{q} + \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \tfrac{\partial P_{Ld}}{\partial t}
\end{aligned} \tag{23}$$

Define a p.s.d function as $V_3 = V_2 + \frac{1}{2}Iz_3^Tz_3$. The derivative of V_3 is given by

$$\begin{aligned}
\dot{V}_3 &= \dot{V}_2|_{z_3=0} + z_2^T\tfrac{\partial x}{\partial q}z_3 + Iz_3^T\dot{z}_3 + \tfrac{1}{2}\dot{I}z_3^Tz_3\\
&= \dot{V}_2|_{z_3=0} + z_3^T(\tfrac{\partial x}{\partial q}z_2 + I\dot{P}_L - I\dot{P}_{Ld} + \tfrac{1}{2}\dot{I}z_3)
\end{aligned} \tag{24}$$

where $\dot{V}_2|_{z_3=0}$ represents the derivative of V_2 when $z_3 = 0$ and $I\dot{P}_{Ld}$ can be expressed by

$$I\dot{P}_{Ld} = \widehat{I\dot{P}_{Ld}} + \widetilde{I\dot{P}_{Ld}} \tag{25}$$

where

$$\begin{aligned}
&\widehat{I\dot{P}_{Ld}} = \tfrac{\partial P_{Ld}}{\partial q}\hat{I}\dot{q} + \tfrac{\partial P_{Ld}}{\partial \dot{q}}(-\hat{C}_t\dot{q} - \hat{G}_t + \hat{\bar{M}}\tfrac{\partial x}{\partial q}P_L + \hat{d}_n) + \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\hat{I}\dot{\hat{\theta}} + \tfrac{\partial P_{Ld}}{\partial t}\hat{I}\\
&\widetilde{I\dot{P}_{Ld}} = (\tfrac{\partial P_{Ld}}{\partial q}\dot{q} + \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \tfrac{\partial P_{Ld}}{\partial t})(-\textstyle\sum_{i=1}^{3} I_{si}\tilde{\beta}_i) + \tfrac{\partial P_{Ld}}{\partial \dot{q}}[\textstyle\sum_{i=1}^{3}(C_{ti}\dot{q} + G_{ti})\tilde{\beta}_i\\
&\quad - \textstyle\sum_{i=1}^{2}\bar{M}_i\tfrac{\partial x}{\partial q}P_L\tilde{\beta}_i - \bar{M}_c\tilde{\beta}_4 - \textstyle\sum_{i=2}^{3}\bar{M}_{i-1}\tilde{\beta}_{3+i} + \tilde{d}]\\
&\hat{I} = I_c + \textstyle\sum_{i=1}^{3} I_{si}\hat{\beta}_i, \quad \hat{C}_t = C_{tc} + \textstyle\sum_{i=1}^{3} C_{ti}\hat{\beta}_i\\
&\hat{G}_t = G_{tc} + \textstyle\sum_{i=1}^{3} G_{ti}\hat{\beta}_i, \quad \hat{d}_n = \bar{M}_c\hat{\beta}_4 + \textstyle\sum_{i=2}^{3}\bar{M}_{i-1}\hat{\beta}_{3+i}\\
&\hat{\bar{M}} = \bar{M}_c + \textstyle\sum_{i=1}^{2}\bar{M}_i\hat{\beta}_i
\end{aligned} \tag{26}$$

$\widehat{I\dot{P}_{Ld}}$ represents the calculable part of $I\dot{P}_{Ld}$ and will be used in the model compensation part of the ARC control law in this step, $\widetilde{I\dot{P}_{Ld}}$ is the incalculable part of $I\dot{P}_{Ld}$ and will be attenuated by certain robust feedback.

Define $Q_L = A_1V_1^{-1}Q_1 + A_2V_2^{-1}Q_2$. From (24), Q_L can be treated as the virtual control input in this step and we will synthesize an ARC control function Q_{Ld} for Q_L such that P_L will track the desired control virtual control input P_{Ld} with a guaranteed transient and final tracking performance. Similar to the first step, Q_L is given by

$$\begin{aligned}
&Q_{Ld} = Q_{Lda} + Q_{Lds}\\
&Q_{Lda} = (A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q} + \tfrac{1}{\hat{I}\beta_e}(-\tfrac{\partial x}{\partial q}z_2 + \widehat{I\dot{P}_{Ld}} - \tfrac{1}{2}\hat{\dot{I}}z_3 - I_cK_3z_3)
\end{aligned} \tag{27}$$

where $\hat{\dot{I}} = \dot{I}_c + \sum_{i=1}^{3}\dot{I}_{si}\hat{\beta}_i$ and K_3 is a positive feedback gain matrix. Substituting (27) in (24), we have

$$\dot{V}_3 = \dot{V}_2|_{z_3=0} + z_3^T(I\beta_eQ_{Lds} - \phi_3\tilde{\beta} - \tfrac{\partial P_{Ld}}{\partial \dot{q}}\tilde{d}) - z_3^TI_cK_3z_3 \tag{28}$$

where $\phi_3 = [\phi_{3(1)},\cdots,\phi_{3(n-1)},\phi_{3n},\phi_{3(n+1)},\phi_{3(n+2)},\cdots,\phi_{3(2n)}]$.

$$\begin{aligned}
\phi_{3(1)} &= I_{s1}[\beta_eQ_{Lda} - \beta_e(A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q} - \tfrac{\partial P_{Ld}}{\partial q}\dot{q}\\
&\quad - \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \tfrac{\partial P_{Ld}}{\partial t}] + \tfrac{\partial P_{Ld}}{\partial \dot{q}}(C_{t1}\dot{q} + G_{t1} - \bar{M}_1\tfrac{\partial x}{\partial q}P_L) + \tfrac{1}{2}\dot{I}_{s1}z_3\\
\phi_{3(2)} &= I_{s2}[\beta_eQ_{Lda} - \beta_e(A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q}\\
&\quad - \tfrac{\partial P_{Ld}}{\partial q}\dot{q} - \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \tfrac{\partial P_{Ld}}{\partial t}] + \tfrac{\partial P_{Ld}}{\partial \dot{q}}(C_{t2}\dot{q} + G_{t2}\\
&\quad - \bar{M}_2\tfrac{\partial x}{\partial q}P_L) + \tfrac{1}{2}\dot{I}_{s2}z_3\\
\phi_{33} &= I_{s3}[\beta_eQ_{Lda} - \beta_e(A_1^2V_1^{-1} + A_2^2V_2^{-1})\tfrac{\partial x}{\partial q}\dot{q}\\
&\quad - \tfrac{\partial P_{Ld}}{\partial q}\dot{q} - \tfrac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \tfrac{\partial P_{Ld}}{\partial t}] + \tfrac{\partial P_{Ld}}{\partial \dot{q}}(C_{t3}\dot{q} + G_{t3}) + \tfrac{1}{2}\dot{I}_{s3}z_3\\
\phi_{3(4)} &= -\tfrac{\partial P_{Ld}}{\partial \dot{q}}\bar{M}_c, \quad \phi_{3(5)} = -\tfrac{\partial P_{Ld}}{\partial \dot{q}}\bar{M}_1\\
\phi_{3(6)} &= -\tfrac{\partial P_{Ld}}{\partial \dot{q}}\bar{M}_2
\end{aligned} \tag{29}$$

Thus Q_{Lds} could be chosen to satisfy:

$$\begin{aligned}
&\text{condition i} \quad z_3^T(I\beta_eQ_{Lds} - \phi_3\tilde{\beta} - \tfrac{\partial P_{Ld}}{\partial \dot{q}}\tilde{d}) \leqslant \varepsilon_3\\
&\text{condition ii} \quad z_3^TI\beta_eQ_{Lds} \leqslant 0
\end{aligned} \tag{30}$$

where ε_3 is a positive design parameter.

Once the control function Q_{Ld} for Q_L is synthesized as given by (27), the actual control input u can be backed out from the continuous one-to-one nonlinear load flow mapping as follows. Noting that the elements of the diagonal matrices g_3, g_4, V_1, and V_2 are all positive functions, u_i, the control input for the ith hydraulic loop, should have the same sign as Q_{Ldi}. Thus

$$\begin{aligned}
u_i = [&A_{1i}V_{1i}^{-1}k_{q1i}g_{3i}(P_{1i}, sign(Q_{Ldi})) +\\
&A_{2i}V_{2i}^{-1}k_{q2i}g_{4i}(P_{2i}, sign(Q_{Ldi}))]^{-1}Q_{Ldi}
\end{aligned} \tag{31}$$

where $i = 1,2,3$. The adaptation law in this step is given by

$$\dot{\hat{\beta}} = Proj(\Gamma_\beta \tau_\beta) \quad \tau_\beta = \phi_3 z_3 \tag{32}$$

Main Theoretical Results

Theorem 1. *Let the parameter estimates $\hat{\theta}$ and $\hat{\beta}$ be updated by the adaptation law (**??**) and (32) respectively. Then, the following results hold if the control law (31) is applied:*

A. *In general, the tracking errors, z_1, z_2 and z_3, are bounded. Furthermore, V_3, an index for the bound of the tracking errors, is bound above by*

$$V_3(t) \leqslant exp(-\lambda_V t)V_3(0) + \frac{\varepsilon_V}{\lambda_V}[1 - exp(-\lambda_V t)] \tag{33}$$

where $\lambda_V = \frac{2min\{k_2, k_3 I_c\}}{max\{k_M, I_M\}}$, $\varepsilon_V = \varepsilon_2 + \varepsilon_3$, k_M is the upper bound of the inertial matrix (i.e., $M(q) \leqslant k_M I_{3\times 3}$), , I_M is the upper bound of the determinant of inertial matrix, and $k_2 = \inf_{t>0} \lambda_{min}(K_2(t)), k_3 = \lambda_{min}(K_3)$.

B *If after a finite time t_0, $\tilde{T} = 0$, i.e., in the presence of parametric uncertainties only, in addition to results in A, asymptotic output tracking is also obtained.* $\triangle$

EXPERIMENTAL RESULT

The experimental set-up is a three-link robot arm (a scaled down version of industrial backhoe loader arm) driven by three single-rod hydraulic cylinders as described in [13]. For the purpose of this paper, experiments will be conducted on the swing motion control of the arm (or the first joint) with the other two joints fixed. During the experiment, a 22.5kg external load is added at the end of the arm, which corresponds to value of $\theta_1 = 22.5$. The least square method and gradient method were used to estimate parameters.

The desired trajectory is a typical back and forth point-to-point motion trajectory as shown in Fig.2. As shown in Fig.3, the proposed DIARC controllers have good output tracking performance in terms of both transient and steady-state; the tracking error of DIARC with least-square estimation is within 0.8mm during the entire operation and is around 0.07mm during the periods when the system is at the two-end points. As seen from Fig.4, the load inertia estimate converges to its true value rather quickly, which results in the decreasing tracking errors shown in Fig.3. It is also evident that the parameter estimates using the least square method converge faster than that of gradient method, which in turn leads to a better tracking performance shown in Fig.3. The control input shown in Fig.4 is regular.

CONCLUSION

In this paper, an integrated direct/indirect adaptive robust control (DIARC) algorithm is proposed for the precision control of electro-hydraulic system with accurate parameter estimates. Motion control of a 3 DOF hydraulic robot arm is used as an experimental case study. Compared with existing direct adaptive robust control designs, the parameter adaptation law of DIARC is based on the model of the system to be identified, rather than the tracking error dynamics. Such a separation between parameter estimation and controller design enables the designer to have a better control of experimental conditions for parameter estimation. As a result, accurate parameter estimation is made possible. In addition, fast dynamic compensations similar to the direct adaptive robust control (ARC) design are employed to preserve the excellent tracking performance of direct ARC designs. Experimental results are presented to illustrate the effectiveness of the proposed DIARC algorithm for electro-hydraulic systems.

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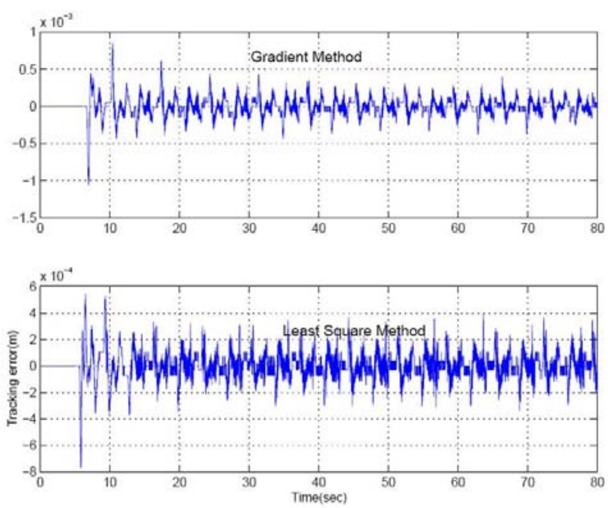

Figure 2. Tracking Errors

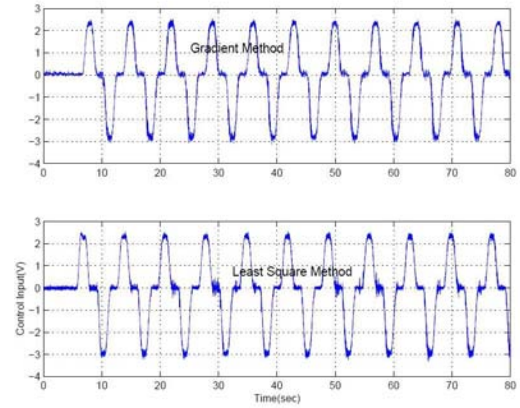

Figure 3. Control Input

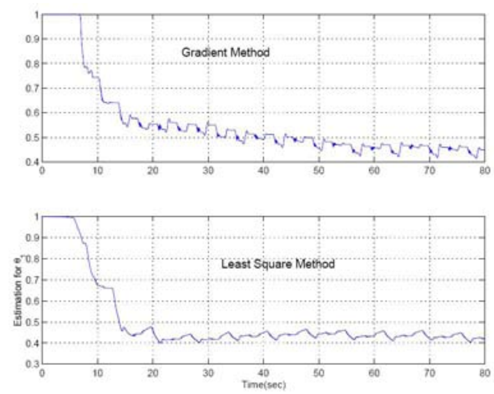

Figure 4. Load Inertia Estimates

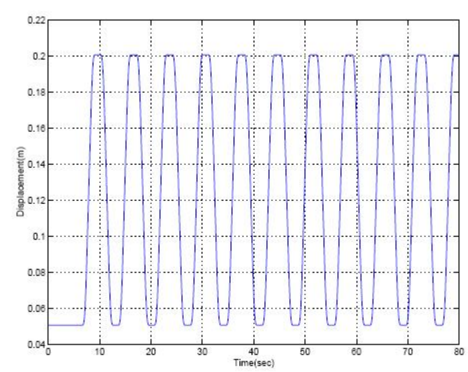

Figure 5. Desired Motion Trajectory