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CHARACTERIZATION AND ATTENUATION OF SANDWICHED DEADBAND PROBLEM USING DESCRIBING FUNCTION ANALYSIS AND ITS APPLICATION TO ELECTRO-HYDRAULIC SYSTEMS CONTROLLED BY CLOSED-CENTER VALVES

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ABSTRACT

Sandwiched deadbands can be seen in a wide variety of systems, such as electro-hydraulic systems controlled by closed-center valves. In such a system, the deadband is between the plant and actuator dynamics and therefore can not be compensated directly like an input deadband. Though this sandwiched deadband problem may be attenuated to certain degree through sophisticated advanced control techniques, the increased cost and the necessity of actuator state feedback prohibit their widespread application in the industry. An economical and popular method is to add an inverse deadband function in the controller to cancel or compensate the highly nonlinear behavior of the deadband. However, such a solution requires that the dynamics before the deadband (eg. the valve dynamics) is fast enough to be neglected - a requirement that can not be met in reality unless the closed loop bandwidth of the overall system is limited very low. To raise the achievable closed loop bandwidth for a much improved control performance, it is essential to be able to precisely characterize the effect of this sandwiched deadband on the stability and performance of the overall closed-loop system, which is the main focus of the paper. Specifically, a describing function based nonlinear analysis will be conducted to predict when the instability will occur and how the resulting limit cycle depends on the actuator dynamics and the targeted closed-loop bandwidth. Based on the analysis, the optimal closed-loop bandwidth can be determined to maximize the achievable overall system performance. The technique is applied to an electro-hydraulic system controlled by closed-center valves to optimize the controller design.

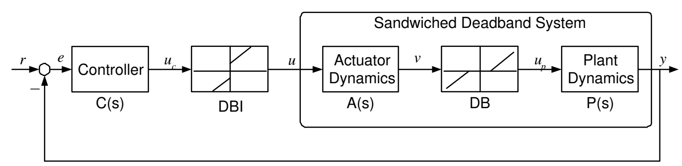

Figure 1. Sandwiched Deadband System with Direct Compensation.

INTRODUCTION

Sandwiched deadbands, as shown in Fig. 1, are common nuisances that exist in many systems, such as electro-hydraulic systems controlled by closed center valves, where the deadband of the valve is sandwiched by the valve and plant dynamics. Compared with the input deadband, which can be cancelled or compensated with an inverse deadband function, the sandwiched deadband is more difficult to deal with. The actuator dynamics in a sandwiched deadband system is usually not high enough to be neglected, otherwise, the sandwiched deadband problem can be simplified into the input deadband problem.

There are several solutions to this problem. One is to use a feed-forward controller to boost the actuator dynamics response, so that it would be sufficiently fast to be neglected, like that Bu

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and Yao have done in [1]. Once the actuator dynamics was neglected, the deadband became the input deadband, which could be compensated by an inverse deadband function. However, the success of such a strategy depends on the accuracy of the actuator dynamics. It can only achieve some limited improvements in practice due to the unavoidable uncertainties in any physical systems.

Another way to solve this problem is to use a local high gain feedback controller at the actuator level to attenuate the sandwiched deadband [1–4], like what Tao, etc. and Bu and Yao have done. However, to apply this technique, the feedback of the actuator states or output is required, which may significantly increase the system cost. For example, to apply the feedback compensation to electro-hydraulic systems controlled by closed-center valves, the feedback of the valve-spool position is required. Although the spool position feedback is available in some valves, it is not a general valve configuration and would definitely increase the system cost. In addition, the spool position measurement is normally too noisy to help increasing the actuator bandwidth significantly [1, 5].

A practical and industry-favorite solution is to simply neglect the actuator dynamics and use an inverse deadband function to compensate the deadband, shown in Fig. 1. This solution is simple, easy to implement, and is used in a lot of applications like [6]. Such a deadband compensation usually results in a very conservative controller design because the closed loop bandwidth has to be tuned very low to guarantee that the actuator dynamics can be safely neglected. When the closed loop bandwidth is increased to certain level, limit cycle is expected because the actuator dynamics makes the deadband compensation not perfect. Limit cycle is potentially very dangerous to systems with neglected vibration modes, because it may excite the vibration modes or even destabilize the system.

This paper focuses on the practical solution of the sandwiched deadband problem and uses describing function to systematically analyze and characterize the closed loop system. The analysis would give us a rough idea about when limit cycle would occur so that we can maximize the closed loop bandwidth without exciting limit cycle. Although this technique would not increase the theoretically achievable performance, it is helpful to make the system less conservative and to actually achieve the achievable performance in implementation.

CHARACTERIZATION OF SANDWICHED DEADBAND SYSTEM USING DESCRIBING FUNCTION ANALYSIS

A closed loop system with sandwiched deadband $DB(\cdot)$ between the linear actuator dynamics $A(s)$ and the plant dynamics $P(s)$, and a linear feedback controller $C(s)$ along with a direct deadband compensation $DBI(\cdot)$, shown in Fig. 1, is analyzed in this section. The analysis is not limited to linear systems and linear controllers as explained as follows. The limit cycle happens, if it does, due to the fact that the actuator can not travel through the deadband infinitely fast. As such, it happens only at the regulation period around the target point. Therefore, even though a nonlinear dynamic system with nonlinear controls will be investigated later in implementations, it is easy to linearize the system around the target equilibrium point and check the local behavior of the linearized system using the analysis of this section, as done later in the paper.

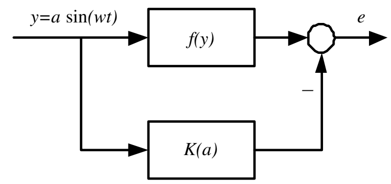

Figure 2. Describing Function of a Nonlinear Function.

Describing Functions of the Deadband and Its Inverse Function

Consider a nonlinear function $f(y)$ under a sinusoidal input $y = a\sin(\omega t)$, one can use a gain $K(a)$, which is a function of the magnitude of the input sinusoidal signal, to approximate the nonlinear function, Fig. 2. The Mean Square (MS) estimation error is:

$$\bar{e}^2 = \frac{1}{2\pi}\int_0^{2\pi} [f(a\sin(\omega t)) - Ka\sin(\omega t)]^2 \, d(\omega t) \qquad (1)$$

The solution of $K(a)$ to minimize the above MS error results in the describing function of the nonlinear function $f(\bullet)$ [7],

$$K(a) = \frac{1}{\pi a}\int_0^{2\pi} f(a\sin(\omega t))\sin(\omega t) d(\omega t) \qquad (2)$$

For simplicity, only symmetric deadband with slope equal to one is consider in this paper. Non-unity slope or asymmetric deadband can be worked out in the same way with minor modifications. Define the deadband and deadband inverse function as:

$$DB(\bullet) = \begin{cases} \bullet - d, & \bullet > d \\ 0, & -d \leq \bullet \leq d \\ \bullet + d, & \bullet < d \end{cases} \qquad (3)$$

$$DBI(\bullet) = \begin{cases} \bullet + d, & \bullet > 0 \\ 0, & \bullet = 0 \\ \bullet - d, & \bullet < 0 \end{cases} \qquad (4)$$

where d is the value of the deadband. Fig. 3 shows a sinusoidal signal with magnitude equal to a going through the deadband $DB(\bullet)$ and deadband inverse $DBI(\bullet)$.

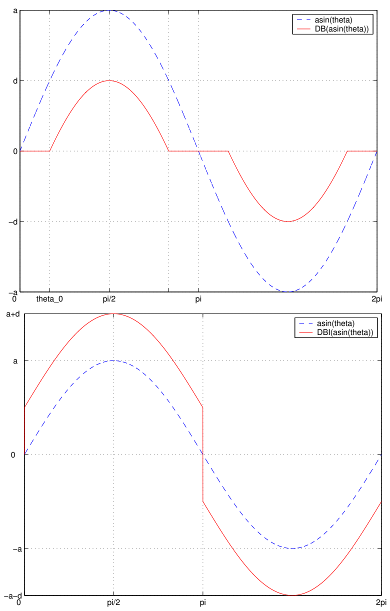

Figure 3. Deadband and Its Inverse Function.

Substituting the $DB(\bullet)$ and $DBI(\bullet)$ into (2), one can obtain the describing functions of the deadband function and its inverse function.

$$K_{DB}(a) = \begin{cases} 0, & a < d \\ 1 - \frac{2d}{\pi a}\sqrt{1-\frac{d^2}{a^2}} - \frac{2}{\pi}\sin^{-1}\frac{d}{a}, & a \geq d \end{cases} \quad (5)$$

$$K_{DBI}(a) = 1 + \frac{4d}{\pi a} \quad (6)$$

Characterization of the Closed Loop System

As already discussed, the limit cycle occurs, if it does, only around the equilibrium point. Therefore, without loss of generality, it is reasonable to assume $r = 0$ in Fig 1. Since the limit cycle means a periodic signal appear in the closed loop system, one can denote the signal right before the deadband inverse function $DBI(\bullet)$ as $u_c = a\sin(\omega t)$ with a being the magnitude and ω representing the fundamental frequency of the limit cycle when only its fundamental frequency component is considered.

Because the nonlinear deadband inverse function or its describing function is a static function, it only changes the magnitude of its input signal without affecting the phase. Therefore, with the describing function approximation, the signal after the deadband inverse, denoted by u, would be:

$$u(t) = K_{DBI}(a) \cdot a\sin(\omega t) \quad (7)$$

The control signal $u(t)$ then enters the linear actuator dynamics $A(s)$ and leaves it with changes in both magnitude and phase angle. Denoting the signal after the actuator $A(s)$ as $v(t)$, $v(t)$ would be:

$$v(t) = K_{DBI}(a)|A(j\omega)| \cdot a\sin(\omega t + \phi_A(j\omega)) \quad (8)$$

where $\phi_A(j\omega)$ is the phase angle of $A(j\omega)$. As the signal continues along the deadband function $DB(\bullet)$ and plant dynamics $P(s)$, the signal u_p and y can be obtained:

$$\begin{aligned} u_p(t) = K_{DBI}(a) \cdot |A(j\omega)| \cdot K_{DB}\big(K_{DBI}(a)|A(j\omega)|a\big) \\ \cdot a\sin(\omega t + \phi_A(j\omega))) \end{aligned} \quad (9)$$

and

$$\begin{aligned} y(t) = K_{DBI}(a) \cdot |A(j\omega)| \cdot K_{DB}\big(K_{DBI}(a)|A(j\omega)|a\big) \cdot |P(j\omega)| \\ \cdot a\sin(\omega t + \phi_A(j\omega) + \phi_P(j\omega)) \end{aligned} \quad (10)$$

where $\phi_P(j\omega)$ is the phase angle of $P(j\omega)$.

Since $r = 0$, $U_c(s) = -C(s)Y(s)$ and the following equality is obtained at the steady-state:

$$\begin{aligned} u_c(t) = -|C(j\omega)| \cdot K_{DBI}(a) \cdot |A(j\omega)| \cdot K_{DB}\big(K_{DBI}(a)|A(j\omega)|a\big) \\ \cdot |P(j\omega)| \cdot a\sin(\omega t + \phi_A(j\omega) + \phi_P(j\omega) + \phi_C(j\omega)) \end{aligned} \quad (11)$$

where $\phi_C(j\omega)$ is the phase angle of the controller $C(j\omega)$.

Equation (11) results in the following relationship:

$$\begin{aligned} & K_{DBI}(a) \cdot K_{DB}\big(K_{DBI}(a)|A(j\omega)|a\big) \cdot |C(j\omega)| \cdot |A(j\omega)| \cdot |P(j\omega)| = 1 \\ & \phi_C(j\omega) + \phi_A(j\omega) + \phi_P(j\omega) = \pi + 2k\pi, \qquad k = 0, 1, 2, \ldots \end{aligned} \quad (12)$$

or simply in complex form:

$$K_{DBI}(a) \cdot K_{DB}\big(K_{DBI}(a)|A(j\omega)|a\big) \cdot C(j\omega) \cdot A(j\omega) \cdot P(j\omega) + 1 = 0 \quad (13)$$

It is obvious, if K_{DB} and K_{DBI} are constants, i.e., the DB and DBI blocks are linear systems, (13) is nothing but the conventional closed loop characteristic equation with $s = j\omega$, i.e., a pair of complex closed loop poles at $\pm j\omega$, a necessary condition for the closed loop system to exhibit a steady-state sinusoidal type limit cycle of frequency ω.

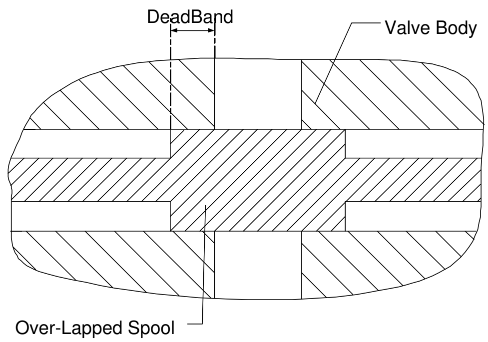

Figure 4. Over-Lapped Spool of Closed-Center Valves.

If the closed loop system does have a limit cycle of fundamental frequency ω, with the describing function approximation to the nonlinear elements, the fundamental frequency ω and the magnitude of the steady-state limit cycle a should satisfies (13). Thus, given the plant and actuator dynamics, $P(s)$ and $A(s)$, (13) can be used to provide a systematic way to analyze how the magnitude and frequency of the limit cycle, if it does exist, depends on the parameters of the controller $C(s)$, from which a well-informed decision could be made to select the specific controller parameters in implementation for an optimized control performance in practice.

Though the proposed describing function analysis is based on a linear system dynamics and linear controller, it is by no means limited to linear systems. Since our interest is the limit cycle, which happens around the equilibrium point, one can easily linearize a non-linear system around its equilibrium point and analyze its local behavior. In the next section an electro-hydraulic system controlled by closed-center valves is analyzed and a nonlinear controller with direct deadband compensation is optimized to achieve the best possible performance.

PERFORMANCE OPTIMIZATION OF ELECTRO-HYDRAULIC SYSTEMS CONTROLLED BY CLOSED-CENTER VALVES

Closed-center valves are widely used in industry for motion or velocity control due to their capability to hold position even when the hydraulic power is off. Such capacity is obtained through the over-lapped spool, which blocks the internal leakage when the spool is at neutral position, as shown is Fig. 4. Though the over-lapped spool is intentionally added and does provide desired properties, it adds a deadband to the system. It is obvious that the deadband is after the valve dynamics and therefore is sandwiched by two dynamic blocks — valve dynamics and plant dynamics.

From controller design point of view, the electro-hydraulic systems actuated by linear actuator (hydraulic cylinder) or rotary actuator (hydraulic motor) are basically same. Therefore, this paper would take the swing motion of the electro-hydraulic robot arm actuated by hydraulic cylinders, shown in Fig. 5, as an example to illustrate how to design a nonlinear controller and analyze the system with the describing function method.

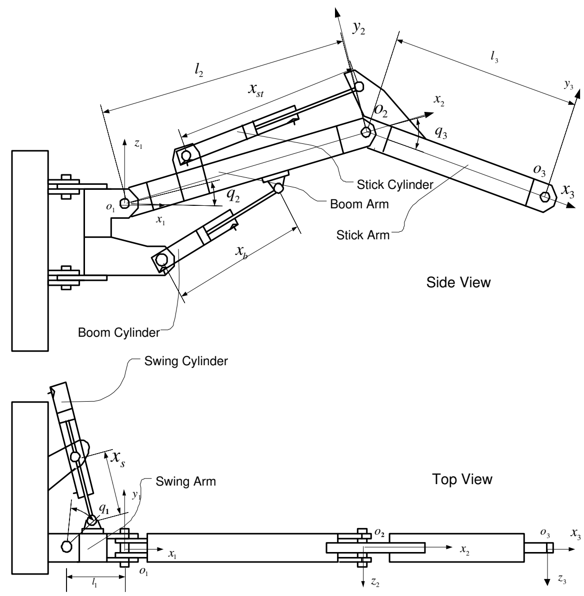

Figure 5. Three DOF Electro-Hydraulic Robot Arm.

Modern nonlinear controllers, such as adaptive robust control (ARC) technique developed by Yao and Tomozuka in [8,9], are able to deal with both uncertain nonlinearities and parametric uncertainties. The following analysis is done based on nominal plant dynamics only for simplicity, as the effect of external disturbances and parametric uncertainties can be effectively attenuated by the advanced nonlinear control techniques, such as ARC in [8,9].

Dynamic Model

A closed-center proportional directional control (PDC) valve, (*Vickers KBFDG4V-5-2C50N-Z-PE7-H7-10*), is used to control the swing motion. An off-line system identification reveals that the valve dynamics can be very well approximated below $10Hz$ by the following linear dynamics,

$$A(s) = \frac{X_{sp}(s)}{U(s)} = \frac{2387.44}{s^2 + 67.82s + 2652.90} \quad (14)$$

where x_{sp} is the spool displacement normalized to have same unit as the input signal, u represents the normalized control input to the valve ranging from -10 volts to +10 volts.

Due to the deadband x_{db}, the valve would not open until the spool displacement x_{sp} is larger than the deadband x_{db}. Therefore, a virtual spool displacement can be defined as:

$$x_{vr} = \begin{cases} x_{sp} - x_{db}, & x_{sp} > x_{db} \\ 0, & -x_{db} \leq x_{sp} \leq x_{db} \\ x_{sp} + x_{db}, & x_{sp} < x_{db} \end{cases} \tag{15}$$

All signals in the above equations x_{sp}, x_{vr} and x_{db} are normalized to have same unit as the input signal. The value of the deadband x_{db} is equivalent to 1.0 volt.

Neglecting cylinder flow leakages, the hydraulic cylinder equations can be written as [10],

$$\begin{aligned} \frac{V_1(x_L)}{\beta_e}\dot{P}_1 &= -A_1\dot{x}_L + Q_1 = -A_1\frac{\partial x_L}{\partial q}\dot{q} + Q_1 \\ \frac{V_2(x_L)}{\beta_e}\dot{P}_2 &= +A_2\dot{x}_L - Q_2 = +A_2\frac{\partial x_L}{\partial q}\dot{q} - Q_2 \end{aligned} \tag{16}$$

where $V_1(x_L) = V_{h1} + A_1 x_L$ and $V_2(x_L) = V_{h2} - A_2 x_L$ are the total cylinder volumes of the head and rod ends including connecting hose volumes respectively, V_{h1} and V_{h2} are the initial control volumes when $x_L = 0$, β_e is the effective bulk modulus. Q_1 and Q_2 are the supply and return flows respectively, which satisfy the orifice equation.

$$\begin{aligned} Q_1 &= k_{q1} x_{vr}\sqrt{|\Delta P_1|} \\ Q_2 &= k_{q2} x_{vr}\sqrt{|\Delta P_2|} \end{aligned} \tag{17}$$

where k_{q1} and k_{q2} are orifice flow coefficients which can be obtained through off-line system identification, x_{vr} are the virtual valve spool displacement, and ΔP_1 and ΔP_2 are the pressure drops across the valve.

The nominal swing motion dynamics, neglecting external disturbances, can be described by [1, 11].

$$J \cdot \ddot{q} = \frac{\partial x_L}{\partial q}(P_1 A_1 - P_2 A_2) - D_f \cdot \dot{q} \tag{18}$$

where q represents the swing joint angle, J is the moment of inertia of the swing motion, x_L represents the swing hydraulic cylinder displacement, P_1 and P_2 are the head and rod end pressures of the cylinder respectively, A_1 and A_2 are the head and rod end ram areas of the cylinder respectively, D_f is the damping and viscous friction coefficient. The specific forms of J and G_c are given in [1].

Nonlinear Controller Design

A direct deadband inverse is used to compensate the sandwiched deadband as described in the previous section. Therefore a nonlinear controller will be design based on the plant (i.e., the cylinder and swing motion dynamics) only. The nonlinear controller design follows the standard backstepping design technique [12]. The objective is to let the system output q track a desired trajectory q_d as close as possible. Define a set of state variables $x_1 = q$, $x_2 = \dot{q}$ and $x_3 = P_1 A_1 - P_2 A_2$ as well as the control input $Q_L = \frac{A_1}{V_1} Q_1 + \frac{A_2}{V_2} Q_2$, the plant dynamics including swing motion dynamics and cylinder dynamics can be rewritten as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{J}\left[\frac{\partial x_L}{\partial q} x_3 - G_c(x_1) - D_f x_2\right] \\ \dot{x}_3 &= \beta_e\left[Q_L - \left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2}\right)\frac{\partial x_L}{\partial q} x_2\right] \end{aligned} \tag{19}$$

and the control signal Q_L can be written as a function of x_{vr}:

$$Q_L = \left(\frac{A_1}{V_1} k_{q1}\sqrt{|\Delta P_1|} + \frac{A_2}{V_2} k_{q2}\sqrt{|\Delta P_2|}\right) \cdot x_{vr} = f(x_{vr}) \tag{20}$$

Step 1

Define the tracking error as $z_1 \triangleq x_1 - x_{1d}$, where $x_{1d} = q_d$ is the desired trajectory. Differentiating z_1, one can get:

$$\dot{z_1} = \dot{x}_1 - \dot{x}_{1d} = x_2 - \dot{x}_{1d} \tag{21}$$

Since x_2 can be considered as the input to (21), a virtual control law α_1 for x_2 can be design to make z_1 exponentially converge to zero:

$$\alpha_1 = \dot{x}_{1d} - k_1 z_1 \tag{22}$$

Step 2

Define the discrepancy between the virtual control law α_1 and the system state x_2 as $z_2 \triangleq x_2 - \alpha_1$. Differentiating z_2 while noting (19),

$$\dot{z}_2 = \frac{1}{J}\left(\frac{\partial x_L}{\partial q} x_3 - D_f x_2\right) - \dot{\alpha}_1 \tag{23}$$

Considering x_3 as the input of the (23), a virtual control law α_2 can be designed as:

$$\alpha_2 = \frac{1}{\frac{\partial x_L}{\partial q}}(D_f x_2 + J\dot{\alpha}_1) - J\frac{1}{\frac{\partial x_L}{\partial q}}(k_2 z_2 + z_1) \tag{24}$$

Step 3

Let $z_3 \stackrel{\triangle}{=} x_3 - \alpha_2$ denote the input discrepancy. Differentiating z_3 while noting (19),

$$\begin{aligned} \dot{z}_3 &= \dot{x}_3 - \dot{\alpha}_2 \\ &= \beta_e \left[Q_L - \left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2} \right) \frac{\partial x_L}{\partial q} x_2 \right] - \dot{\alpha}_2 \end{aligned} \tag{25}$$

In viewing (25), Q_L is the control input and a control law Q_{Ld} can be synthesized as:

$$Q_{Ld} = \left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2}\right) \frac{\partial x_L}{\partial q} x_2 + \frac{1}{\beta_e}\dot{\alpha}_2 - \frac{1}{\beta_e}\left(k_3 z_3 + \frac{1}{J}\frac{\partial x_L}{\partial q} z_2\right) \tag{26}$$

The k_i, $i = 1,\ 2,\ 3$ in (22), (24) and (26) are all positive gains chosen to make the closed loop system having desired bandwidth and ability to attenuate model uncertainties and reject disturbances.

Define a positive definite Lyapunov function $V(t) \stackrel{\triangle}{=} \frac{1}{2}(z_1^2 + z_2^2 + z_3^2)$. Differentiating $V(t)$, one can get:

$$\begin{aligned} \dot{V} &= z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 \\ &= z_1(z_2 + \alpha_1 - x_{1d}) + z_2 \left\{ \frac{1}{J} \left[\frac{\partial x_L}{\partial q}(z_3 + \alpha_2) - D_f x_2 \right] - \dot{\alpha}_1 \right\} \\ &\quad + z_3 \left\{ \beta_e \left[Q_{Ld} - \left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2} \right) \frac{\partial x_L}{\partial q} x_2 \right] - \dot{\alpha}_2 \right\} \\ &= z_1(-k_1 z_1 + z_2) + z_2\left(-z_1 - k_2 z_2 + \frac{1}{J}\frac{\partial x_L}{\partial q} z_3\right) \\ &\quad + z_3\left(-\frac{1}{J}\frac{\partial x_L}{\partial q} z_2 - k_3 z_3\right) \\ &= -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 \\ &\leq -2\min\{k_1, k_2, k_3\} V \end{aligned} \tag{27}$$

Therefore, the closed loop system without considering the deadband is asymptotically stable with a convergence rate no less than two times of the minimum of k_1, k_2 and k_3.

Step 4

Therefore, the desired spool displacement is going to be:

$$x_{vrd} = f^{-1}(Q_{Ld}) = \frac{Q_{Ld}}{\frac{A_1}{V_1} k_{q1}\sqrt{|\Delta P_1|} + \frac{A_2}{V_2} k_{q2}\sqrt{|\Delta P_2|}} \tag{28}$$

Closed Loop Description

Linearize the system and controller around the equilibrium point $x_{eq} = [0,\ 0,\ 0]^T$, one can obtain the following linearized system:

$$Q_L = f(x_{vr})\big|_{x_{eq}} = \left(\frac{A_1}{V_1} k_{q1}\sqrt{|\Delta P_1|} + \frac{A_2}{V_2} k_{q2}\sqrt{|\Delta P_2|} \right)_{x_{eq}} \cdot x_{vr} \tag{29}$$

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & \frac{D_f}{J} & \frac{1}{J}\frac{\partial x_L}{\partial q} \\ 0 & -\beta_e\left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2}\right)\frac{\partial x_L}{\partial q} & 0 \end{pmatrix}_{x_{eq}} x + \begin{pmatrix} 0 \\ 0 \\ \beta_e \end{pmatrix} Q_L \tag{30}$$

and the controller:

$$\begin{aligned} Q_{Ld} &= \frac{1}{\beta_e}\left[-\frac{\partial q}{\partial x_L} J(k_1k_2k_3 + k_3) - \frac{1}{J}\frac{\partial x_L}{\partial q} k_1 \right]_{x_{eq}} x_1 \\ &\quad + \Big\{ \left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2}\right)\frac{\partial x_L}{\partial q} + \frac{1}{\beta_e}\Big[\frac{\partial q}{\partial x_L}\left[D_f\left(k_1 + k_2 + k_3 - \frac{D_f}{J}\right)\right] \\ &\quad - J(k_1k_2 + k_2k_3 + k_3k_1 + 1) \Big] - \frac{1}{J}\frac{\partial x_L}{\partial q} \Big\}_{x_{eq}} x_2 \\ &\quad + \frac{1}{\beta_e}\left(\frac{D_f}{J} - k_1 - k_2 - k_3\right) x_3 \end{aligned} \tag{31}$$

$$x_{vrd} = f^{-1}(Q_{Ld})|_{x_{eq}} = \frac{1}{\frac{A_1}{V_1} k_{q1}\sqrt{|\Delta P_1|} + \frac{A_2}{V_2} k_{q2}\sqrt{|\Delta P_2|}} \cdot Q_{Ld} \tag{32}$$

Summarize the above linearized system and controller as:

$$\begin{aligned} \dot{x} &= Ax + Bx_{vr} \\ -x_{vrd} &= Cx \end{aligned} \tag{33}$$

For simplicity, assume $k_1 = k_2 = k_3 = k$, the linearized relation from x_{vr} to x_{vrd} can be described by a transfer function $P(s)$ like follows:

$$\begin{aligned} P(s) &= \frac{-X_{vrd}(s)}{X_{vr}(s)} = C(SI - A)^{-1}B \\ &= \frac{(3k-0.4)s^2 + (3k^2-593)s + (k^3+k)}{s(s^2+0.4s+594)} \end{aligned} \tag{34}$$

The entire system with sandwiched deadband and direct deadband compensation then looks like Fig. 6.

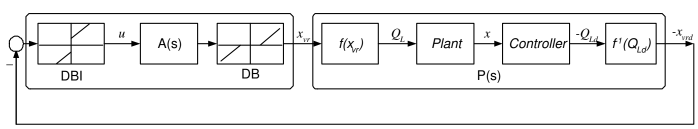

Figure 6. Equivalent Nonlinear Controlled Electro-Hydraulic System Block Diagram.

It is obvious that the entire electro-hydraulic system is now equivalent to the general sandwiched deadband system with a direct deadband inverse compensation. Apply the describing function analysis to this system, one can get:

$$K_{DBI}(a) \cdot K_{DB}(K_{DBI}(a)|A(j\omega)|a) \cdot A(j\omega) \cdot P(j\omega) + 1 = 0 \tag{35}$$

Note that the above equation only depends on the controller gain k. If k is already fixed, one can easily check wether limit cycle would happen by solving (35). One the other hand, one can optimize the controller design, i.e., choose the largest gain k (less conservative), by finding the range of k with which limit cycle would not happen.

SIMULATIONS AND EXPERIMENTS

While analytically solving (35) may be very difficult, it is very easy to numerically search a solution with the help modern numerical softwares, such as Matlab. A searching program is written in Matlab and it is found when k is greater than 17.5, limit cycle would happen.

Simulations are done to check wether the above theoretical predication is valid. The electro-hydraulic system with nonlinear controller and direct deadband compensation is to track a point-to-point trajectory shown in Fig. 7. The simulations are done with difference control gains, $k = 15$, $k = 18$ and $k = 20$. Simulation results, as shown in Fig. 8, indicate that strong oscillation happens when k is increased to 20, which is consistent with the theoretical prediction.

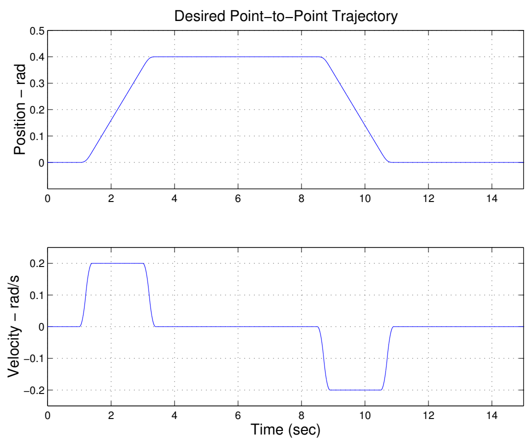

Figure 7. Desired Tracking Trajectory.

Experiments are also done to compare the closed loop performances of the electro-hydraulic system with/without sandwiched deadband. A critical-center servo valve (*Parker BD760AAAN10*) which has faster dynamic response and no deadband problem, is used to compare with the closed-center PDC valve. The proposed nonlinear controller with direct deadband compensation is applied to control the closed-center PDC valve while similar controller without the deadband compensation for the servo valve, the control gains are chosen to be $k = 20$ for PDC valve and $k = 30$ for servo valve. Experimental results, shown in Fig. 9, reveal that the PDC valve's performance is significantly improved and close to servo valve.

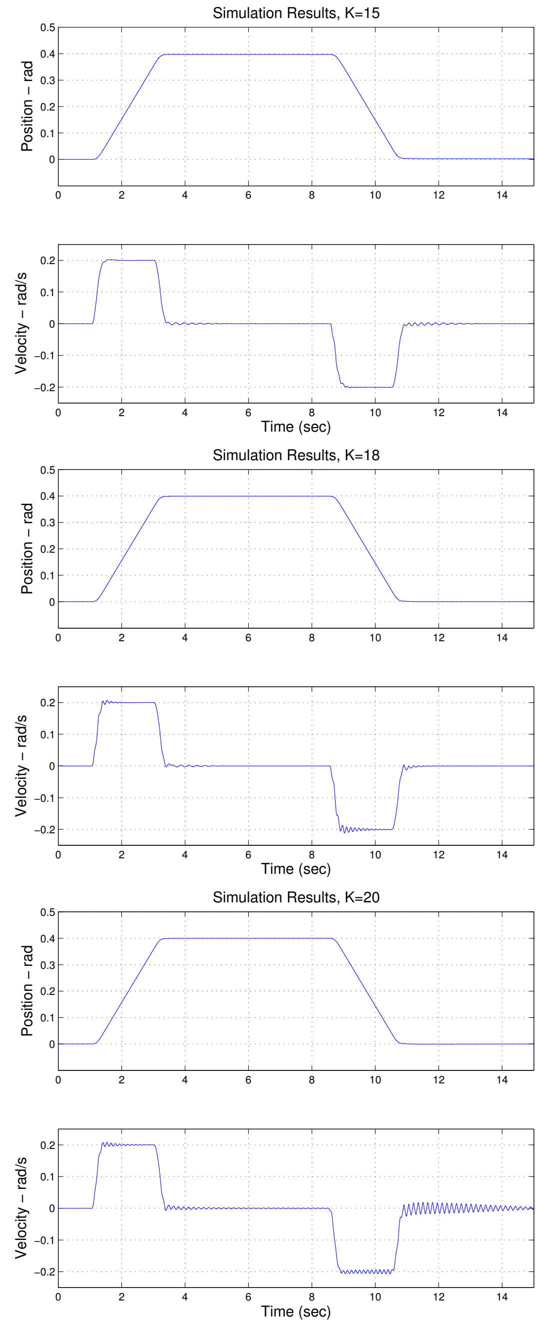

Figure 8. Simulation Results for Different K value.

CONCLUSIONS

Though the sandwiched deadband problem can be solved via sophisticated advanced control techniques, direct compensation is still a widespread solution for such problems, due to its low cost, easiness to design controller and implementation. However, the directly compensated systems are usually conservative, i.e, having very low bandwidth so that the actuator dynamics would not interfere with the system performance. This paper proposed a systematic way to analyze and characterize the directly compensated sandwiched deadband system to reveal when the limit cycle would happen. The method can be used to optimize the controller design for a less conservative closed loop system. An applica-

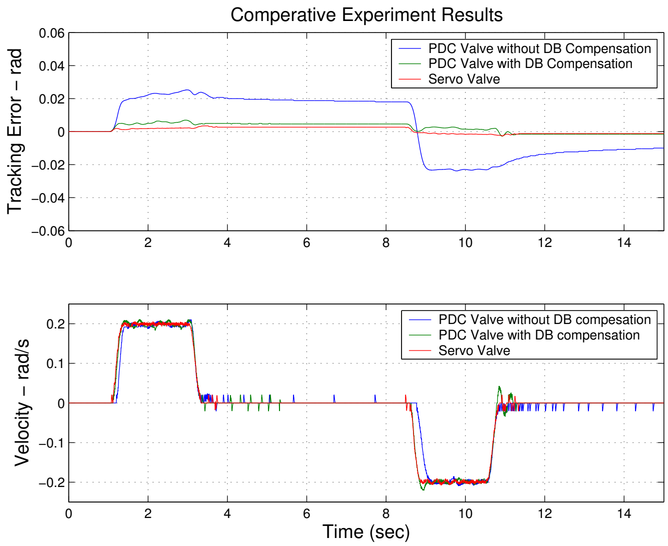

Figure 9. Comparative Experiment Results.

tion of this describing function analysis to the highly nonlinear electro-hydraulic system shows the effectiveness of the proposed analysis.

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