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# NONLINEAR ADAPTIVE ROBUST OBSERVER FOR VELOCITY ESTIMATION OF HYDRAULIC CYLINDERS USING PRESSURE MEASUREMENT ONLY *

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## ABSTRACT

This paper considers the design of an observer to estimate the velocity of an electro-hydraulic system using pressure measurements only. The difficulties involved in the design of an observer for such a system include the highly nonlinear system dynamics, severe parametric uncertainties like large variation of inertial load and unmatched model uncertainties. In order to tackle these problems a nonlinear model based Adaptive Robust Observer (ARO) is designed which provides not only a robust estimate of the velocity of the system but also the robust identification of the system parameters. Online monitoring of certain persistence of excitation conditions allow us to improve the quality of the parameter and velocity estimates. Simulation and experimental results on the swing motion control of a three-degree-of freedom hydraulic robot arm demonstrate the effectiveness of the proposed observer.

## 1 Introduction

Hydraulic systems are widely used in industrial applications because of their size-to-power ratio and the ability to apply large forces and torques with fast response times. Some of the application areas of hydraulic systems include electro-hydraulic positioning systems (FitzSimons, 1996; Silva, 1998), active suspension control (Alleyne, 1996; Alleyne, 1995), material testing (Lee, 1990) and industrial hydraulic systems (Yao, 1998). For such systems, knowledge of the pressure in the forward and backward chambers of the cylinders, the displacement of the cylinder and the velocity of the cylinder is normally required in the design of position and velocity controllers.

For industrial mobile hydraulic systems such as excavators, (Yao, 1998), pressure measurements are common because of the relatively cheaper cost of installing pressure sensors. However, the lack of position and velocity feedback is common in such systems because of the prohibitive cost of the sensors as well as the high likelihood of failure of these sensors in harsh environments. Hence, design of observers which estimate the velocity from measurements of the cylinder pressure becomes very important if one still wants to control the cylinder velocity as in (DeBoer, 2001) or some sensor failure detection schemes are sought for the fragile position and velocity sensors, which is the focus of this paper.

Unfortunately, hydraulic systems have a number of characteristics that complicate the design of observers for such systems. One characteristic is that the dynamics of hydraulic systems are highly nonlinear (Merritt, 1967). These nonlinearities include: deadband and hysteresis existing in the control valves, nonlinear pressure/flow relations and variation in the fluid volumes due to the movement of the actuator. Apart from the nonlinearities, hydraulic systems have large extents of model uncertainties. The uncertainties can be classified into parametric uncertainties and uncertain nonlinearities. Examples of the parametric uncertain-

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ties include the large changes in the load seen by the system and the large variations in the hydraulic parameters due to change in temperature, pressure and component wear (Whatton, 1989). Other general nonlinearities such as external disturbances, leakage, and friction cannot be modelled exactly and the nonlinear functions that describe them are not known. These nonlinearities are called uncertain nonlinearities. These model uncertainties make the construction of observers for hydraulic systems rather difficult.

In the past, the design of observers was done by linearizing the system and applying linear observer designs to the linearized plant. Unfortunately, the hydraulic system is subjected to non-smooth and discontinuous nonlinearities due to control input saturation, directional change of valve opening, friction and valve overlap. Hence, the linearizing approach will not yield good results. Also, it was shown in (Hedrick, 1994), that the direct application of a Luenberger type observer would not work well for nonlinear systems with nonlinearities which are non-Lipschitz. Hence, a nonlinear model based observer which takes into account the system nonlinearities will likely perform better.

The other problem with hydraulic systems is the high degree of parametric uncertainty. In (Yanada, 1997) it was pointed out that when the difference between the system parameters of a plant and its observer model is significant, the observer with fixed parameters brings about large state estimation errors, particularly during the transients. This problem can be solved using adaptive observers. The researchers in (Rajamani, 1995) constructed an adaptive observer for a class of nonlinear systems with Lipschitz nonlinearities. Unfortunately, the adaptive observer designed cannot be directly used in closed loop control because in the presence of disturbances of large magnitude, the estimation error in (Rajamani, 1995) may become unbounded.

In this paper a novel nonlinear observer is proposed that estimates the velocity and parameters of an electro-hydraulic system using pressure measurements only. Similar to the design of adaptive robust controllers (ARC) in (Yao, 1997), the adaptive robust observer (ARO) is designed using robust filter structures to reduce the effect of unmodeled disturbances, while parameter adaptation is used to reduce the model uncertainty so that the state estimation error can be reduced. The observer that is designed has an extended filter structure so that on-line parameter adaptation can be utilized to reduce the effect of possible large nominal disturbances. Discontinuous projection mapping is used in the parameter adaptation process of the adaptive robust observer for a controlled adaptation process. Consequently, all the signals in the ARO, i.e., the state estimates and the parameter estimates can be guaranteed to be bounded even in the presence of bounded uncertain nonlinearities such as disturbances. As a result, the adaptive robust observer (ARO) can be safely used in real time control applications in addition to other targeted applications such as the reliable velocity estimates for the detection of position and velocity sensor failures.

The paper is organized as follows. Problem formulation and dynamic models are presented in section II. The proposed ARO is given in section III. Experimental setup and experimental results are presented in section IV. Conclusions are drawn in section V.

## 2 Problem Formulation and Dynamic Model

The schematic of a typical inertial load driven by a hydraulic cylinder is shown in Figure 1. The goal is to have the observer estimate the velocity of the cylinder as closely as possible with measurement of pressures only. The system can be thought of

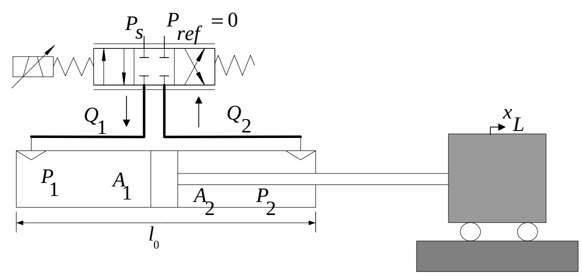


Figure 1. A ONE DOF ELECTRO-HYDRAULIC SYSTEM

as a single-rod hydraulic cylinder driving an inertial load at the end. The dynamics of the inertial load can be described as

$$m\ddot{x}_L = P_1A_1 - P_2A_2 - b\dot{x}_L - F_{fc}(\dot{x}_L) + \tilde{f}(t, x_L, \dot{x}_L) \qquad (1)$$

where $x_L$ and $m$ represent the displacement and the mass of the load respectively. $P_1$ and $P_2$ are the pressures of the two cylinder chambers respectively, $A_1$ and $A_2$ are the ram areas of the two cylinder chambers respectively, $b$ represents the combined coefficient of the modelled damping and viscous friction forces on the load and the cylinder rod, $F_{fc}$ represents the modelled Coulomb friction force and $\tilde{f}(t, x_L, \dot{x}_L)$ represents the lumped modelling error including the external disturbances and terms like the unmodeled friction forces.

The cylinder dynamics can be written as (Merritt, 1967):

$$\dot{P}_1 = \frac{\beta_e}{v_1(x_L)}(-A_1\dot{x}_L + Q_1 - \tilde{Q}_1) \qquad (2)$$

$$\dot{P}_2 = \frac{\beta_e}{v_2(x_L)}(A_2\dot{x}_L - Q_2 + \tilde{Q}_2) \qquad (3)$$

where $v_1(x_L) = v_{h1} + A_1x_L$ and $v_2(x_L) = v_{h2} - A_2x_L$ are the total volumes of the forward and return chamber respectively, $v_{h1}$ and $v_{h2}$ are the forward and return chamber volumes when $x_L = 0$, $\beta_e$ is the effective bulk modulus. $Q_1$ and $Q_2$ are defined as the modelled flows in and out of the head-end and rod-end of the cylinder and are related to the spool valve displacement of the servo-valve, $x_v$, by (Merritt, 1967)

$$Q_1 = k_{q1} x_v \sqrt{|\Delta P_1|}, \quad \Delta P_1 = \begin{cases} P_s - P_1 & \text{for } x_v \geq 0 \\ P_1 - P_r & \text{for } x_v < 0 \end{cases} \tag{4}$$

$$Q_2 = k_{q2} x_v \sqrt{|\Delta P_2|}, \quad \Delta P_2 = \begin{cases} P_2 - P_r & \text{for } x_v \geq 0 \\ P_s - P_2 & \text{for } x_v < 0 \end{cases} \tag{5}$$

where $k_{q1}$ and $k_{q2}$ are the flow gain coefficients for the forward and the return loop respectively, $P_s$ is the supply pressure of the pump and $P_r$ is the reference pressure in the return tank. $\tilde{Q}_1$ and $\tilde{Q}_2$ are the errors in the flow modelling including various leakage flows.

Define a set of state variables as $x = [x_1, x_2, x_3]^T = [\dot{x}_L, P_1, P_2]^T$. The entire system of equations (1)-(5), with control input $u = x_v$ can be expressed in state space form as:

$$\begin{aligned} \dot{x}_1 &= \frac{1}{m}(x_2 A_1 - x_3 A_2) + d, \quad d = \frac{1}{m}(\tilde{f}(t, x_L, x_1) - b x_1 - F_{fc}(x_1)) \\ \dot{x}_2 &= \frac{\beta_e}{v_1(x_L)}(-A_1 x_1 + g_2(x_2, sign(u))u) - \tilde{Q}_1 \\ \dot{x}_3 &= \frac{\beta_e}{v_2(x_L)}(A_2 x_1 - g_3(x_3, sign(u))u) + \tilde{Q}_2 \end{aligned} \tag{6}$$

where $g_2$ and $g_3$ are defined by

$$g_2 = k_{q1}\sqrt{|\Delta P_1|}, \quad g_3 = k_{q2}\sqrt{|\Delta P_2|} \tag{7}$$

*Given the desired velocity trajectory $x_{1d}(t)$ and the measurement of the pressures $x_2$ and $x_3$, the objective is to design an observer that tracks output velocity $x_1(t)$ as closely as possible in spite of various parametric uncertainties and uncertain nonlinearities.*

## 3 Adaptive Robust Observer Design

### 3.1 Design Model and Issues to be Addressed

The system is subjected to parametric uncertainties due to the variations of $m$, $\beta_e$, $b$ and $F_{fc}$. In this paper, for simplicity, only the major parametric uncertainties due to $m$, the bulk modulus $\beta_e$, and $d_n$, the nominal value of the lumped modelling error $d$ in equation (6), are considered. In addition, since one does not have information about the cylinder displacement and the volumes of the fluid in the hoses dominates over the variations in the volume of the fluid in the cylinder chambers caused by the cylinder movement, the hose volumes, $v_{h1}$ and $v_{h2}$ are used in the design of the observer with the variations of the chamber volumes being lumped into some modelling uncertainty terms as detailed in the following.

Let $l_1 = \frac{A_1}{v_{h1}}$, $l_2 = \frac{A_2}{v_{h2}}$, $r_1(x_2, sign(u)) = \frac{g_2(x_2, sign(u))}{v_{h1}}$, $r_2(x_3, sign(u)) = \frac{g_3(x_3, sign(u))}{v_{h2}}$, $\bar{A} = \frac{A_2}{A_1}$ and define the unknown parameter set $\theta = [\theta_1, \theta_2, \theta_3]^T$ as $\theta_1 = A_1/m$, $\theta_2 = d_n$, and $\theta_3 = \beta_e$, the system dynamics (6) can be simplified to the following form:

$$\begin{aligned} \dot{x}_1 &= \theta_1(x_2 - \bar{A}x_3) + \theta_2 + \Delta_1(t, x_L, x_1) \\ \dot{x}_2 &= \theta_3(-l_1 x_1 + r_1(x_2, sign(u))u) + \Delta_2(t, x_2, x_3) \\ \dot{x}_3 &= \theta_3(l_2 x_1 - r_2(x_3, sign(u))u) + \Delta_3(t, x_2, x_3) \end{aligned} \tag{8}$$

where,

$$\begin{aligned} \Delta_1(t, x_L, x_1) &= \tilde{d} = d(t, x_L, x_1) - d_n \\ \Delta_2(t, x_2, x_3) &= -\tilde{Q}_1 - \frac{\beta_e(v_1(x_L) - v_{1h})}{v_1 v_{1h}}(-A_1 x_1 + g_2 u) \\ \Delta_3(t, x_2, x_3) &= \tilde{Q}_2 - \frac{\beta_e(v_2(x_L) - v_{2h})}{v_2 v_{2h}}(-A_2 x_1 + g_3 u) \end{aligned} \tag{9}$$

in which the last term in the lumped uncertainties $\Delta_2$ and $\Delta_3$ represents the effect of the variation of fluid volumes due to cylinder movement.

At this stage, it is easy to see that the major difficulties in the design of observers for the system described by equation (8) are:

(i) The system dynamics are highly nonlinear, due to the inherent nonlinearities of hydraulic dynamics such as the nonlinear flow gains (represented by $r_1(x_2, sign(u))$ and $r_2(x_3, sign(u))$.

(ii) The system has severe parametric uncertainties like large variations of the inertial load, the change of bulk modulus due to entrapped air or temperature, etc., as represented by the unknown $\theta$.

(iii) The model uncertainties are mismatched, i.e., both parametric uncertainties and uncertain nonlinearities appear in the dynamic equations which are not directly related to the control input $u = x_v$.

To address the above challenges, the following general strategies are adopted:

a. Nonlinear model based analysis and synthesis will be used in the observer design to deal with the nonlinearity of the system directly. Specifically, the subsequent design will be based on the actual system dynamic model (8).
b. Adaptive and robust approaches will be integrated and applied to design observers which are less sensitive to model uncertainties.
c. An adaptation law which is based on the state model of the system will be designed to get better parameter estimates.

Discontinuous projection mapping is used to guarantee that the observer estimates remain bounded even in the presence of disturbances.

d. The parameter estimates are updated only when certain persistent excitation conditions are satisfied. This leads to better parameter estimates which can be used in fault diagnostics and on line machine performance monitoring.

Using the above ideas, the nonlinear adaptive robust observer design is detailed in the following sections. The general structure of the proposed adaptive robust observer is shown in Figure 2 which shows the flow of information in the ARO structure.

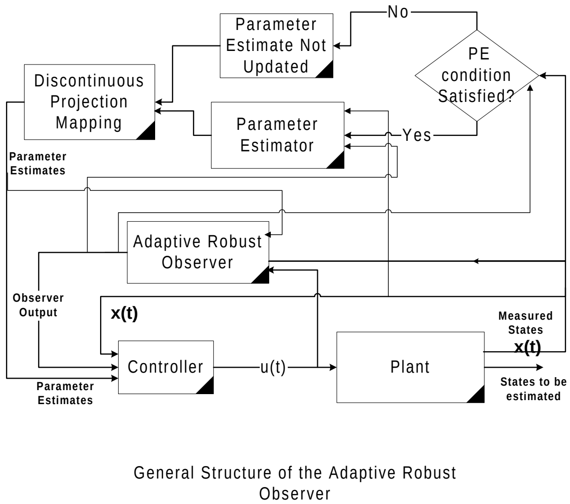


Figure 2. GENERAL STRUCTURE OF ADAPTIVE ROBUST OBSERVER

## 3.2 Assumptions and Definitions

**Assumption 1.** *The unknown but constant parameters* $\theta_i$ *lie in a known bounded region* $\Omega_{\theta_i}$*:*

$$\theta_i \in \Omega_{\theta_i} = \{\theta_i : \theta_{i_{min}} < \theta_i < \theta_{i_{max}}\} \tag{10}$$

**Assumption 2.** *The uncertain nonlinearities* $\Delta_i, i = 1,2,3$ *are bounded, i.e.,*

$$\Delta_i \in \Omega_{\Delta_i} = \{\Delta_i : \ \Delta_i(x,\eta,u,t)| \leq \delta_i\} \tag{11}$$

*where* $\delta_i$ *are some constants.*

Note that the above two assumptions are rather practical for the electro-hydraulic systems.

### 3.2.1 Projection Mapping and Definitions

Let $\hat{\theta}(t)$ denote the estimate of $\theta$ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta} = \hat{\theta}(t) - \theta$). Defining the discontinuous projection as:

**Definition 1.** *Let* $\Omega_\theta$ *be a convex set with the interior of the set denoted by* $\overset{\circ}{\Omega}_\theta$ *and its boundary by* $\partial\Omega_\theta$*. Since,* $\Omega_\theta$ *is a convex set, let* $n_{\hat{\theta}}$ *be the unit outward normal at* $\hat{\theta} \in \partial\Omega_\theta$*. The standard projection mapping (Krstic, 1995) is:*

$$Proj_{\hat{\theta}}(\zeta) = \begin{cases} \zeta, & if\ \ \hat{\theta} \in \overset{\circ}{\Omega}_\theta\ \ or\ n_{\hat{\theta}}^T \zeta \leq 0 \\ (I - \Gamma \frac{n_{\hat{\theta}} n_{\hat{\theta}}^T}{n_{\hat{\theta}}^T \Gamma n_{\hat{\theta}}})\zeta, & \hat{\theta} \in \partial\Omega_\theta\ and\ n_{\hat{\theta}}^T \zeta > 0 \end{cases} \tag{12}$$

*where* $\zeta \in R^{\,p}$ *is any function and* $\Gamma(t) \in R^{\,p\times p}$ *can be any time-varying positive definite symmetric matrix.*

**Lemma 1.** *By using the projection type adaptation law given by*

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\zeta), \quad \hat{\theta}(0) \in \Omega_\theta \tag{13}$$

*it can be shown (Sastry, 1989; Goodwin, 1989) that the projection mapping in equation (12) has the following desirable properties*

*P1. The parameter estimates are always within the known closed set* $\bar{\Omega}_\theta$*, i.e.,* $\hat{\theta}(t) \in \bar{\Omega}_\theta, \forall t$*.*

*P2. If the true parameters are within the known convex set* $\Omega_\theta$*, then for any adaptation function* $\tau$ *and* $\Gamma(t) > 0$*,*

$$\tilde{\theta}^T(\Gamma^{-1} Proj_{\hat{\theta}}(\Gamma\tau) - \tau) \leq 0, \forall\tau, \Gamma(t), and\ \theta \in \Omega_\theta, \tag{14}$$

**Definition 2.** *A system* $\dot{x} = f(x,u)$ *is Input to State Practically Stable (ISpS) if there exists a class* $KL$ *function* $\beta$*, a class* $K$ *function* $\gamma$*, and a non-negative constant* $d$ *such that, for any initial condition* $x(0)$ *and each input* $u \in L_\infty[0,t)$*, the corresponding solution* $x(t)$ *satisfies*

$$|x(t)| \leq \beta(|x(0)|,t) + \gamma(||u(t)||) + d, \quad \forall t \geq 0 \tag{15}$$

*where* $u(t)$ *is the truncated function of* $u$ *at* $t$ *and* $||\cdot||$ *represents the* $L_\infty$ *supremum norm.*

## 3.3 Adaptive Robust Observer Design

Let $\eta = x_1$ and $x = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} \in R^{\,2}$ represent the unmeasured state and the vector of the measured signals. The system given by the equations in (8) can be written in a canonical form as

$$\begin{aligned}\dot{\eta} &= F_\eta(x,u)\theta + \Delta_\eta \\ \dot{x} &= F_x(x,u)\theta + \phi(x,u,\theta)\eta + \Delta_x \end{aligned} \tag{16}$$

where, $F_\eta(x,u) = [x_2 - \bar{A}x_3 \quad 1 \quad 0]$, $F_x(x,u) = \begin{bmatrix} 0 & 0 & r_1(x_2, sign(u))u \\ 0 & 0 & -r_2(x_3, sign(u))u \end{bmatrix}$, $\phi(x,u,\theta) = \theta_3 \begin{bmatrix} -l_1 \\ l_2 \end{bmatrix}$, $\Delta_\eta = \Delta_1$ and $\Delta_x = [\Delta_2,\ \Delta_3]^T$.

Since information about the unmeasurable state $\eta$ is contained in the dynamics of $x$ (e.g., the second equation in (16)), let us define a co-ordinate transformation of the form:

$$\xi = \eta - \omega(x,\theta_\omega), \;\; \omega(x,\theta_\omega) = \sigma(x_2,x_3)\theta_\omega, \;\; \theta_\omega = 1/\theta_3 \tag{17}$$

where $\sigma(x_2,x_3)$ could be any smooth functions as long as the resulting function $A_\xi$ given by

$$A_\xi(x,u) = -\frac{\partial\omega}{\partial x}\phi(x,u,\theta) = \frac{\partial\sigma}{\partial x_2}l_1 - \frac{\partial\sigma}{\partial x_3}l_2 \tag{18}$$

is a negative function, i.e., there exists a positive constant $c_\xi > 0$ such that $A_\xi < -c_\xi$. Such a coordinate transformation always exists. For example, let $\sigma(x_2,x_3) = -k_1x_2 + k_2x_3$, where $k_1 > 0$ and $k_2 > 0$ are constant gains. Then,

$$A_\xi = -(k_1l_1 + k_2l_2) = -c_\xi < 0 \tag{19}$$

With the coordinate transformation (17), the dynamics of the transformed state are given by:

$$\begin{aligned}\dot{\xi} &= \dot{\eta} - \dot{\omega}(x,\theta_\omega) \\ &= (F_\eta(x,u)\theta + \Delta_\eta) - \frac{\partial\omega}{\partial x}(F_x(x,u)\theta + \phi(x,u,\theta)\eta + \Delta_x) && (20) \\ &= F_\eta(x,u)\theta + A_\xi(x,u)\eta - \frac{\partial\omega}{\partial x}F_x(x,u)\theta + \Delta_\xi && (21) \\ &= A_\xi(x,u)\xi + F_\psi(x,u,\theta_1,\theta_2) + A_\xi(x,u)\sigma(x)\theta_\omega + \Delta_\xi && (22)\end{aligned}$$

where, for simplicity,

$$A_\xi(x,u) = -\frac{\partial\omega}{\partial x}\phi(x,u,\theta) \tag{23}$$

$$F_\psi = \theta_1(x_2 - \bar{A}x_3) + \theta_2 - \frac{\partial\sigma}{\partial x_2}r_1u + \frac{\partial\sigma}{\partial x_3}r_2u \tag{24}$$

$$\Delta_\xi = \Delta_\eta - \frac{\partial\omega}{\partial x}\Delta_x \tag{25}$$

As $A_\xi(x,u)$ is a negative function, if $\theta$ and $\theta_\omega$ were known, a nonlinear observer could be designed as,

$$\dot{\hat{\xi}} = A_\xi(x,u)\hat{\xi} + F_\psi(x,u,\theta_1,\theta_2) + A_\xi(x,u)\sigma(x)\theta_\omega \tag{26}$$

Then, the state estimation error $\tilde{\xi} = \hat{\xi} - \xi$ would be governed by the following dynamic system

$$\dot{\tilde{\xi}} = A_\xi(x,u)\tilde{\xi} - \Delta_\xi \tag{27}$$

which is exponentially stable with a bounded disturbance $\Delta_\xi$. Since the parameters $\theta$ and $\theta_\omega$ are not known, the observer as defined in equation (26) is not implementable but it provides motivation for the design of the following nonlinear filters

$$\begin{aligned}\dot{\zeta}_0(x,u) &= A_\xi(x,u)\zeta_0(x,u) - \frac{\partial\sigma}{\partial x_2}r_1u + \frac{\partial\sigma}{\partial x_3}r_2u \\ \dot{\zeta}_\theta(x,u) &= A_\xi(x,u)\zeta_\theta(x,u) + A_\psi(x,u) \\ \dot{\zeta}_\omega(x,u) &= A_\xi(x,u)\zeta_\omega(x,u) + A_\xi(x,u)\sigma(x)\end{aligned} \tag{28}$$

where $\zeta_0 \in R$, $\zeta_\theta = [\zeta_{\theta1},\ \zeta_{\theta2}] \in R^{1\times2}$, $A_\psi = [x_2 - \bar{A}x_3,\ 1]$, and $\zeta_\omega \in R$. The state estimate can then be represented as,

$$\hat{\xi} = \zeta_0(x,u) + \zeta_{\theta1}(x,u)\theta_1 + \zeta_{\theta2}(x,u)\theta_2 + \zeta_\omega(x,u)\theta_\omega \tag{29}$$

From (28) and (29), it can be verified that the observer error dynamics is still represented by (27). Therefore, the equivalent expression for the unmeasurable state $\eta$ is

$$\begin{aligned}\eta &= \xi + \omega(x,\theta_\omega) \\ &= \zeta_0 + \zeta_{\theta1}\theta_1 + \zeta_{\theta2}\theta_2 + \zeta_\omega\theta_\omega + \sigma(x)\theta_\omega - \tilde{\xi} && (30) \\ &= \zeta_0 + \Upsilon(x,u)\theta_\Upsilon - \tilde{\xi} && (31)\end{aligned}$$

where $\Upsilon(x,u) = [\zeta_\theta(x,u),\ (\zeta_\omega(x,u) + \sigma(x))]$ and $\theta_\Upsilon = [\theta_1,\theta_2,\theta_\omega]^T \in R^3$.

Even now the estimation of the $\eta$ subsystem is not implementable because of lack of knowledge of the system parameters. Hence, the values of the estimated parameters are used in the implementation of the observer. This results in the following observer equation

$$\hat{\eta}(x,u,\hat{\theta},\hat{\theta}_\omega) = \zeta_0 + \Upsilon(x,u)\hat{\theta}_\Upsilon \tag{32}$$

where $\hat{\theta}_\Upsilon = [\hat{\theta}_1,\hat{\theta}_2,\hat{\theta}_\omega]^T \in R^3$ represents the estimate of $\theta_\Upsilon$.

Now an adaptation law needs to be designed to estimate the system parameters so that these estimates can be used in the implementation of the ARO in equation (32). Consider the dynamics of the $x$ subsystem in (16), using (30), the equation for the $x$ subsystem dynamics can be rewritten in the form

$$\dot{x} = F_x(x,u)\theta + \phi(x,u,\theta)(\zeta_0 + \Upsilon(x,u)\theta_\Upsilon - \tilde{\xi}) + \Delta_x \tag{33}$$

Utilizing the fact that $\phi(x,u,\theta)$ is linear in terms of $\theta_3$ and $\theta_\omega = 1/\theta_3$, it is easy to check that equation (33) can be linearly parameterized in terms of $\theta_0 = [\theta_1\theta_3,\ \theta_2\theta_3,\ \theta_3]^T \in R^3$ as

$$\dot{x} = f_0(x,u) + \Theta(x,u)\theta_0 - \phi(x,u,\theta)\tilde{\xi} + \Delta_x \tag{34}$$

where $f_0(x,u) = \begin{bmatrix} -(\zeta_\omega(x,u)+\sigma(x))l_1 \\ (\zeta_\omega(x,u)+\sigma(x))l_2 \end{bmatrix}$, $\Theta(x,u) = \begin{bmatrix} -\zeta_{\theta_1}l_1, & -\zeta_{\theta_2}l_1, & (-\zeta_0 l_1 + r_1 u) \\ \zeta_{\theta_1}l_2, & \zeta_{\theta_2}l_2, & (\zeta_0 l_2 - r_2 u) \end{bmatrix}$.

With the dynamics as given in equation (34), a set of filters need to be designed to create a static equation for the prediction error based on the model of the system. For this purpose, consider the following filters

$$\dot{\Omega}^T = A\Omega^T + \Theta(x,u) \tag{35}$$

$$\dot{\Omega}_0 = A(\Omega_0 + x) - f_0 \tag{36}$$

where $A$ is any exponentially stable matrix, $\Omega \in R^{2\times 3}$ and $\Omega_0 \in R^2$. Now define

$$z = x + \Omega_0 \tag{37}$$

which is calculable. By substituting equations (34) and (36) into the derivative of (37),

$$\dot{z} = Az + \Theta(x,u)\theta_0 - \phi(x,u,\theta)\tilde{\xi} + \Delta_x \tag{38}$$

Let $\varepsilon = x + \Omega_0 - \Omega^T\theta_0$, then $z$ can be written as

$$z = \Omega^T\theta_0 + \varepsilon \tag{39}$$

where $\varepsilon$ is governed by

$$\dot{\varepsilon} = A\varepsilon - \phi(x,u,\theta)\tilde{\xi} + \Delta_x \tag{40}$$

which has a stable dynamics with bounded disturbances. Now define the estimate of $z$ as

$$\hat{z} = \Omega^T\hat{\theta}_0 \tag{41}$$

and define the prediction error as $e = \hat{z} - z$. By doing so,

$$e = \Omega^T\tilde{\theta}_0 - \varepsilon \tag{42}$$

which is linearly parameterized in terms of the parameter estimation error $\tilde{\theta}_0$ with an additional term that exponentially converges to zero in the absence of disturbances (i.e., $\Delta_i = 0$). As such, various standard estimation algorithm can be used. For example, with the least squares type estimation algorithm, the resulting adaptation law is given by

$$\dot{\hat{\theta}}_0 = Proj_{\hat{\theta}_0}\left(-\Gamma\frac{\Omega e}{1+\nu Trace(\Omega^T\Gamma\Omega)}\right) \tag{43}$$

where $\Gamma(t)$ is the adaptation rate matrix updated by

$$\dot{\Gamma} = \frac{\alpha\Gamma - \Gamma\Omega\Omega^T\Gamma}{1+\nu\Omega^T\Gamma\Omega}, \quad \Gamma(0) = \Gamma^T(0) > 0 \tag{44}$$

in which the normalization factor $\nu$ and the forgetting factor $\alpha$ are non-negative constants, with $\nu = 0$ leading to un-normalized algorithm.

With the above ARO design, the observer estimation error of $\eta$ is given by

$$\tilde{\eta} = \hat{\eta} - \eta = \Upsilon\tilde{\theta}_\Upsilon + \tilde{\xi} \tag{45}$$

### 3.4 Main Performance Results

Given the desired velocity profile of the hydraulic system, the following qualitative results will hold for the ARO defined by equations (32) and (43).

1. In the presence of uncertain nonlinearities, the signals from the parameter estimator of the ARO given by equation (43) and the state estimator given by equation (32) are bounded. The ARO given by equations (32) and (43) is ISpS.
2. In the absence of uncertain nonlinearities, i.e., $\Delta_x = \Delta_\eta = 0$, if the parameters are updated only when certain persistence of excitation conditions are satisfied, then the parameter estimates converge to their true values.

These results are formally summarized in the following theorems and lemmas.

**Lemma 2.** *With the observer in (32) and the projection type adaptation law in (12), the parameter estimation error and the estimation error $\tilde{\xi}$ are always bounded, i.e., $\tilde{\theta}_0 \in L_\infty[0,\infty)$, and $\tilde{\xi} \in L_\infty[0,\infty)$.*

**Proof:** From the properties of the projection mapping, it can be seen that $\hat{\theta}_0$ is bounded, therefore $\tilde{\theta}_0$ is bounded i.e., $\tilde{\theta}_0 \in L_\infty$.

Consider a positive function $V_{\tilde{\xi}}(\tilde{\xi}) = 1/2|\tilde{\xi}|^2$. In viewing (29), the derivative of $V_{\tilde{\xi}}$ satisfies

$$\dot{V}_{\tilde{\xi}} = \tilde{\xi}\dot{\tilde{\xi}} = A_\xi|\tilde{\xi}|^2 - \tilde{\xi}\Delta_\xi$$

Noting assumption 2 and (18),

$$\begin{aligned}\dot{V}_{\tilde{\xi}} &\leq -c_\xi|\tilde{\xi}|^2 + |\tilde{\xi}||\Delta_\xi| \\ &\leq -c_\xi|\tilde{\xi}|^2 + |\tilde{\xi}|(\delta_1 + k_1\delta_2\theta_\omega + k_3\delta_3\theta_\omega) \qquad (46) \\ &\leq -(c_\xi - c_0)|\tilde{\xi}|^2 + \frac{1}{4c_0}(\delta_1 + k_1\delta_2\theta_\omega + k_3\delta_3\theta_\omega)^2 \qquad (47)\end{aligned}$$

where $c_0 > 0$ is any positive scalar satisfying $c_0 < c_\xi$. It is thus clear that $\tilde{\xi}$ is bounded. □

As the filter matrix $A$ is stable, it is easy to prove the following lemma and theorem:

**Lemma 3.** *The system of filters (35) and (36) is ISpS with the inputs being $x$ and $u$ and the state being the filer output $\Omega$ and $\Omega_0$.*

**Theorem 1.** *The Adaptive Robust Observer (ARO) given by equation (32) is ISpS with the measured states $x$ and $u$ as the inputs and the observer estimation error $\tilde{\eta}$ given by (45) as states.*

**Theorem 2.** *In the absence of uncertain nonlinearities, i.e., $\Delta_x = \Delta_\eta = 0$, the parameter estimation error $\tilde{\theta}_0 \to 0$ if the following persistent excitation (PE) condition is satisfied:*

$$\exists T, \alpha > 0,\ s.t.\ \int_t^{t+T} \Omega(\tau)\Omega^T(\tau)d\tau \geq \alpha I,\ \forall t \qquad (48)$$

**Proof:** In the absence of uncertain nonlinearities i.e., $\Delta_x = \Delta_\eta = 0$, from (27) and (40), we have

$$\dot{\tilde{\xi}} = A_\xi(x,u)\tilde{\xi}\dot{\varepsilon} = A\varepsilon - \phi(x,u,\theta)\tilde{\xi} \qquad (49)$$

It is thus clear that both $\tilde{\xi}$ and $\varepsilon$ exponentially converge to zero. Thus, using the standard adaptive deign techniques, it is easy to show that the least square type projection adaptation law guarantees that the prediction error $e \to 0$ as $t \to \infty$. From (43), $\dot{\hat{\theta}}_0 \to 0$ as $t \to \infty$.

As $e = \Omega^T\tilde{\theta}_0 - \varepsilon$, one has that $\Omega^T\tilde{\theta}_0 \to 0$ as $t \to \infty$. Since $\Omega^T\tilde{\theta}_0$ is a scalar quantity, $\tilde{\theta}_0^T\Omega\Omega^T\tilde{\theta}_0 \to 0$ as $t \to \infty$. Hence, for any $T$,

$$\int_t^{t+T} \tilde{\theta}_0^T\Omega\Omega^T\tilde{\theta}_0 d\tau \to 0, \quad \text{as} \quad t \to \infty \qquad (50)$$

Using the mean value theorem,

$$\tilde{\theta}_0(\tau) = \tilde{\theta}_0(t) + \dot{\tilde{\theta}}_0(\mu)(\tau - t) \qquad (51)$$

Thus, noting $\dot{\tilde{\theta}}_0 \to 0$ as $t \to \infty$, the facts (50) and (51) lead to

$$\tilde{\theta}_0^T(t)[\int_t^{t+T} \Omega\Omega^T d\tau]\tilde{\theta}_0(t) \to 0 \qquad (52)$$

Thus, when the PE condition (48) is satisfied,

$$\alpha||\tilde{\theta}_0(t)||^2 \leq \tilde{\theta}_0^T(t)[\int_t^{t+T} \Omega\Omega^T d\tau]\tilde{\theta}_0(t) \to 0 \qquad (53)$$

Hence, $\tilde{\theta}_0 \to 0$. □

## 4 Simulation and Experimental Results

### 4.1 Experiment Setup

The proposed ARO algorithm is implemented on the swing circuit of a three-link robotic arm (a scaled down version of industrial backhoe loader arm) using a DS1003 Controller board from dSPACE Inc. and a Pentium III 700 MHz computer. The detailed experimental set-up can be found in (1). The effective measurement resolution of the experimental system is $0.5mm$ for position and $1.035 \times 10^5 Pa$ for pressure.

### 4.2 Simulation Results

The schematic of the swing circuit of the electro-hydraulic robot arm is shown in Figure 3.

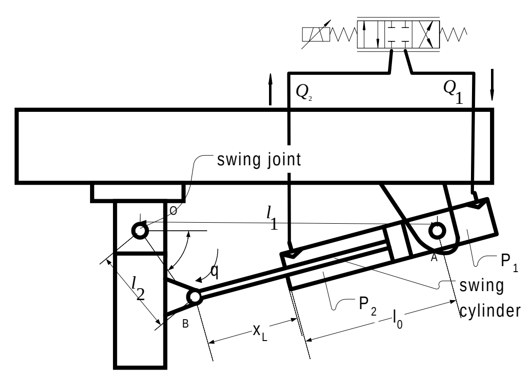


Figure 3. SWING CIRCUIT OF A HYDRAULIC ARM

The physical parameters of the swing cylinder are $A_1 = 2.027 \times 10^{-3}m^2$, $A_2 = 1.069 \times 10^{-3}m^2$, $v_{h1} = 4.995 \times 10^{-4}m^3$ and $v_{h2} = 9.068 \times 10^{-4}m^3$. The flow gain coefficients are $k_{q1} =$

$3.59 \times 10^{-8} m^3/\sqrt{Pa}sV$ and $k_{q2} = 3.721 \times 10^{-8} m^3/\sqrt{Pa}sV$. The swing inertia with just the arm is $J_0 = 78.69 kg\,m^2$. The effective bulk modulus of the system is $\beta_e = 2.71 \times 10^8 Pa$. During the simulation studies, the value of the external disturbance is set at $0N$. In the design of the observer, the gains $k_2$ and $k_3$ are related as $k_2 = \bar{A}k_3$, with $k_3 = 100$. The true values of the parameters are $\theta_1 = 9.2127 \times 10^{-7} m^2/kg$, $\theta_2 = 0N$ and $\theta_3 = 2.71 \times 10^8 Pa$. In simulations, a PID controller with the gains set to $k_p = 175$, $k_i = 150$ and $k_d = 1$ is used. The initial parameter estimates are chosen as $\hat{\theta}_1(0) = 1.1034 \times 10^{-6} m^2/kg$, $\hat{\theta}_2(0) = 0.001N$ and $\hat{\theta}_3(0) = 2.2 \times 10^8 Pa$. The smooth velocity profile of a point-to-point trajectory is chosen as the test trajectory for the simulations. The actual velocity profile and the estimated velocity profile of the observer along with the velocity estimation error are shown in Figure 4. The parameter estimation results are shown in Figures 5, 6 and 7 respectively. As can be seen from these plots, after the parameter estimates converge at around 12 seconds, the observer estimation error converges to zero.

Persistence of excitation condition checks the richness of input signals so that parameter estimates are updated only if the input is rich enough. The comparative simulation results have shown the improvement in the quality of the parameter estimates through this practical modification. In order to check for persistent excitation of the input signal, input signal time data for a period of 12 seconds is used in the simulations, which is the reason for the parameter adaptation to start after a period of around 12 seconds.

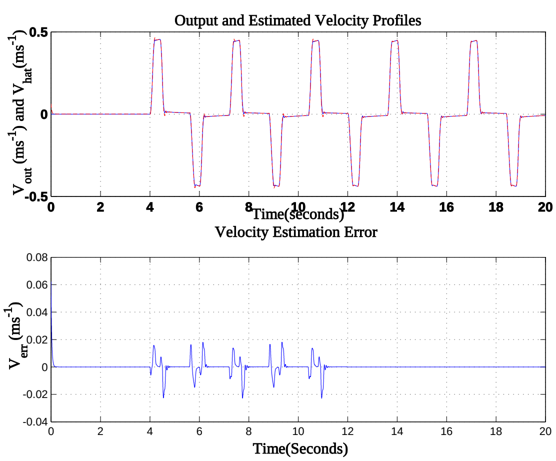


Figure 4. ACTUAL AND ESTIMATED VELOCITY TRAJECTORIES

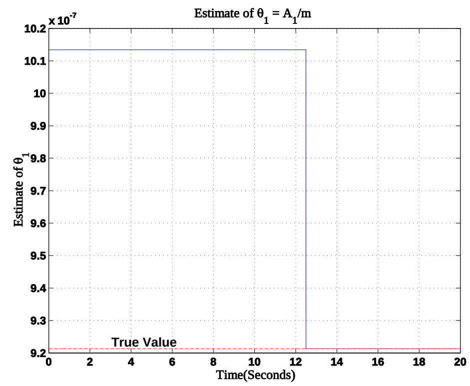


Figure 5. ESTIMATE OF $\theta_1 = A_1/m$

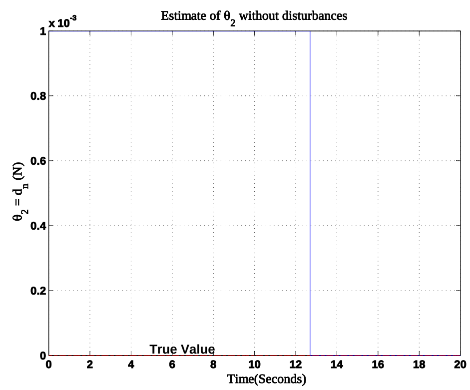


Figure 6. ESTIMATE OF $\theta_2 = d_n$

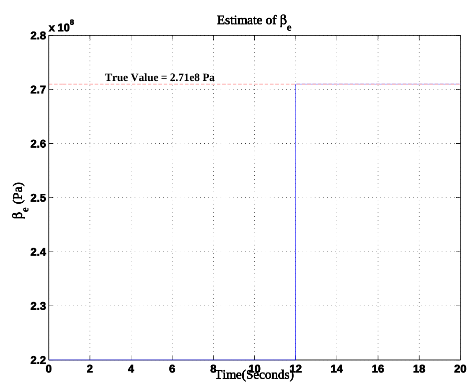


Figure 7. ESTIMATE OF $\theta_3 = \beta_e$

### 4.3 Experimental Results

In order to demonstrate the practical nature of the proposed ARO design, the observer was implemented on the swing circuit of the hydraulic robot arm. These experiments were conducted with no load at the end of the arm and with a load at the end of the arm. The results for the velocity and parameter estimation with the ARO without a load at the end of the arm are shown in Figures 8, 9, 10 and 11. The experimental results from the velocity estimation with a load at the end of the arm are given in figures 12, 13, 14 and 15. In these experiments a load of 45 Kgs is added to the end effector of the arm. As can be seen from these figures, the proposed ARO achieves reasonable good velocity estimate, especially after the parameter estimates approach to their true values after the initial adaptation period. It is also noted that the results for the loaded situation are better than those in unloaded situation. This difference might be caused by the stronger inertia effect of the loaded case, which makes the parameter estimation less sensitive to the modelling errors like the stiction that has been neglected in the ARO design.

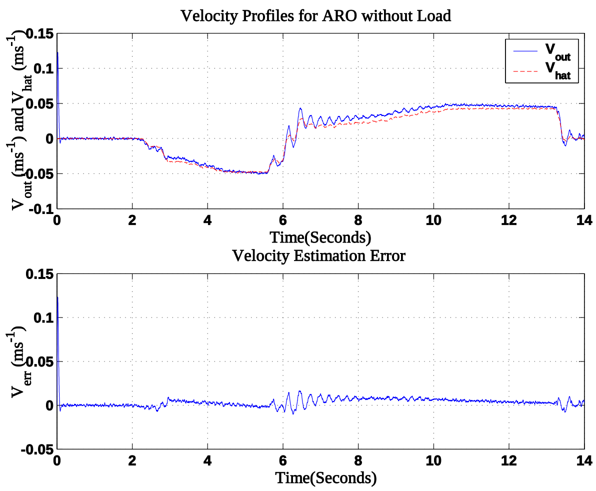


Figure 8. ACTUAL AND ESTIMATED VELOCITY PROFILES OF ARO WITH NO LOAD

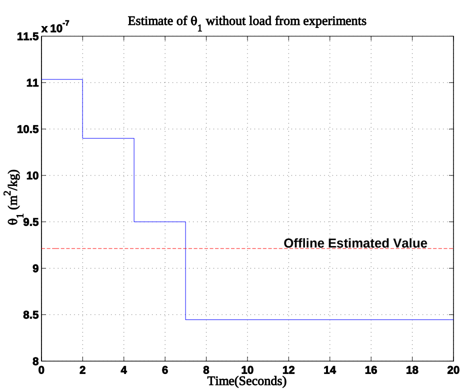


Figure 9. ESTIMATE OF $\theta_1$ FROM EXPERIMENTS (UNLOADED)

## 5 Conclusions

In this paper, an adaptive robust observer (ARO) has been developed for the velocity estimate of hydraulic cylinders using pressure measurements only. The design is based on the actual nonlinear hydraulic dynamics and directly addresses the effect of typical nonlinearities of hydraulic systems. Effective parameter estimation algorithm has been constructed to handle the large parameter variation range of hydraulic systems, and certain robust filter structures are employed to to attenuate the effect of various modelling errors. Through the use of projection type adap-

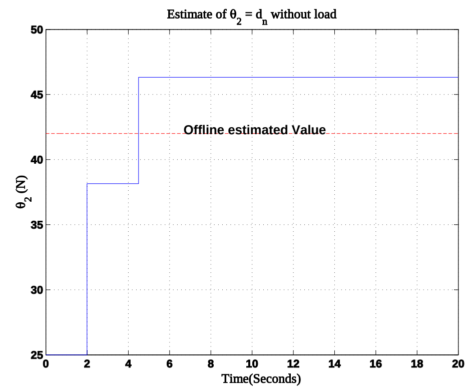


Figure 10. ESTIMATE OF $\theta_2$ FROM EXPERIMENTS (UNLOADED)

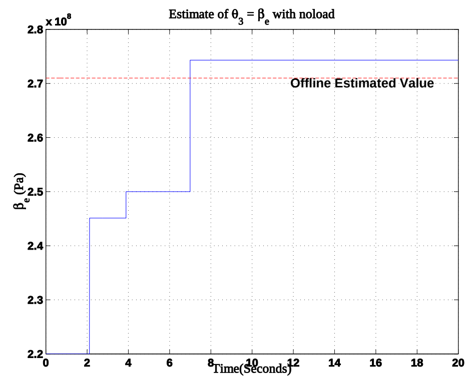


Figure 11. ESTIMATE OF $\theta_3$ FROM EXPERIMENTS (UNLOADED)

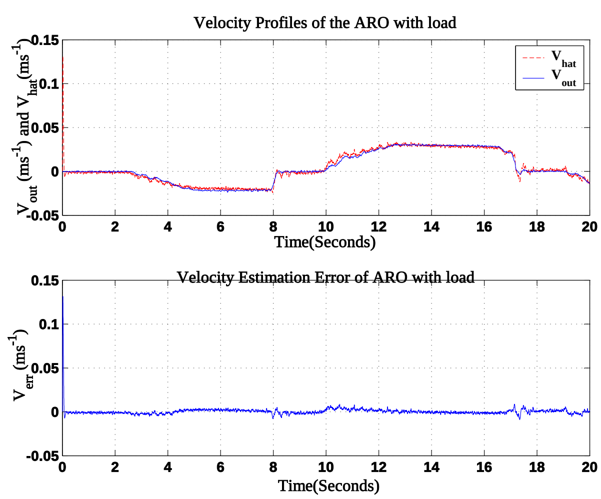


Figure 12. ACTUAL AND ESTIMATED VELOCITY PROFILES OF ARO WITH LOAD

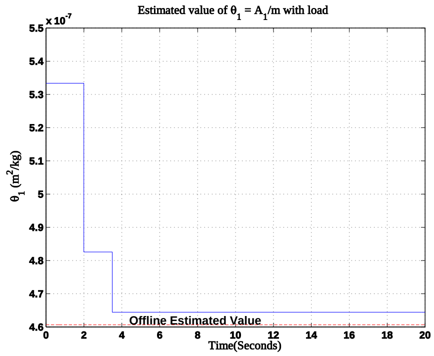


Figure 13. ESTIMATE OF $\theta_1$ FROM EXPERIMENTS (LOADED)

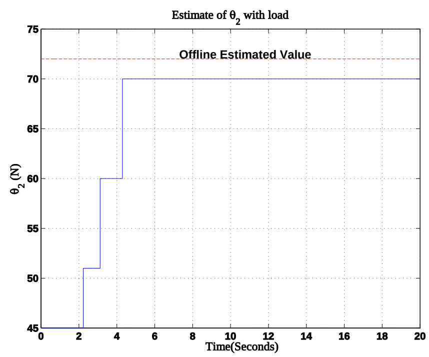


Figure 14. ESTIMATE OF $\theta_2$ FROM EXPERIMENTS (LOADED)

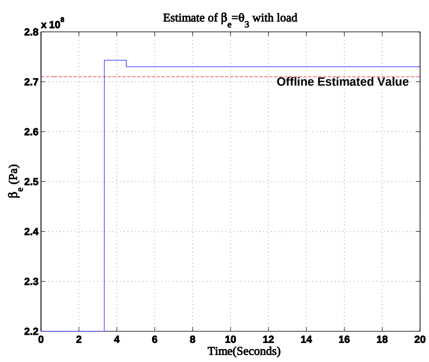


Figure 15. ESTIMATE OF $\theta_3$ FROM EXPERIMENTS (LOADED)

tation law structure, a controlled parameter adaptation process is achieved, even in the presence of large disturbances. As such, the design can be safely implemented. Aside from these theoretical advantages of the proposed ARO design, certain practical measures such as the explicit monitoring of signal excitation level and the persistence of excitation condition are used to further improve the quality of the parameter estimates in implementation, which subsequently leads to better observer performances. The simulation and experimental results obtained on the swing motion control of a hydraulic arm have demonstrated the effectiveness of the proposed ARO algorithm for both the velocity and the parameter estimation.

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