Performance Improvement of Proportional Directional Control Valves: Methods and Experiments

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Abstract

This paper studies local valve control of the electro-hydraulic system. The sluggish response of hydraulic control valve usually becomes the bottleneck of whole system performance. Although fast valves (e.g. high-bandwidth servovalves) are available, they are far more expensive than slow valves (e.g. proportional directional control valves). To improve the performance of proportional directional control valves, three different types of controllers are synthesized. Firstly, based on the pole zero cancellation technique, an open loop compensator is designed which requires the accurate valve dynamic model information; Secondly, a full state feedback adaptive robust controller (ARC) is synthesized, which effectively takes into account the effect of parametric uncertainties and uncertain nonlinearities such as friction force and flow force. Finally, an output feedback ARC controller is synthesized to address the problem of unmeasurable states. Theoretically, the proposed ARC controllers guarantee a prescribed output tracking transient performance and final tracking accuracy while achieving asymptotic output tracking in the presence of parametric uncertainties. Comparative experimental results are obtained to show the advantages and limitations of each method.

1 Introduction

Valve plays an important role in the modern electro-hydraulic systems, which integrates the versatility of advanced electronic control with the high power density of hydraulic actuation. However, the sluggish valve response often becomes the bottleneck of whole system performance and even causes instability of the resulting closed-loop system if not proper addressed. Although fast valves like high-bandwidth servovalves are available, they are far more expensive than slow valves such as the proportional directional control (PDC) valves due to the much more stringent manufacturing tolerance requirements. It is thus of practical significance to see if one can improve the performance of cheaper PDC valves through using advanced nonlinear control algorithms to reduce cost while without sacrificing the achievable performance much.

In the past, much of the work in the control of hydraulic systems uses linear control theory [1, 2, 3, 4] and feedback linearization techniques [5, 6], the valve dynamics is either ignored or considered in the linearized model. In [7, 8], the nonlinear control strategy is applied to the force control of hydraulic system and a better performance is achieved compared with linear controller. In [7], Alleyne and Hedrick considered the parametric uncertainties associated with the system, in which a first order approximation of the valve dynamics is used. In [8], Sohl and Bobrow adds the friction compensation, in which the valve dynamics is ignored.

In [9, 10], the adaptive robust control (ARC) approach proposed by Yao and Tomizuka in [11, 12, 13] was generalized to provide a rigorous theoretical framework for the motion control of electro-hydraulic systems by taking into account various parametric uncertainties and uncertain nonlinearities. To deal with the non-smoothness of the flow equation (i.e. the flow gains depend on the sign of the spool position), only a first order valve dynamic model is considered. Because of the complexity of the resulting controller associated with the backstepping designs [14], in implementation, the actual valve dynamics are neglected for simplicity. The experimental results obtained show that the limited valve bandwidth has a significant effect on the achievable system performance. In [15], the nonlinear characteristics associated with proportional directional control valves are addressed, in which valve dynamics are neglected for simplicity.

If integrating the valve dynamics into the system level controller design may not be possible or realistic due to the complexity of the resulting nonlinear controllers, a pragmatic approach would be to use a cascade controller structure, i.e., independent of system level controllers to be used, a local valve controller is firstly synthesized to raise the bandwidth of the resulting valve dynamics high enough to be neglected in the system level control design, which is the focus of the paper. Three different types of controllers will be proposed to improve the performance of a slow proportional directional control valve. Specifically, a simple open loop pole zero cancellation compensator is constructed first; the controller requires accurate valve model information and is sensitive to model uncertainties. To handle the effect of various model uncertainties, a full state feedback ARC controller is then synthesized. The ARC controller effectively addresses the effect of both parametric uncertainties and uncertain nonlinearities associated with the valve, but needs the feedback of valve spool velocity, which may be unavailable or noisy. To deal with this sensing problem of full-state feedback ARC control, an output feedback ARC controller [16] is also constructed, which needs the feedback of spool position only. Theoretically, all the proposed ARC controllers guarantees a prescribed transient performance and final tracking accuracy while achieving asymptotic tracking in the presence of parametric uncertainties only.

2 Dynamic Model and System Identification

The proportional directional control (PDC) valve used in the experiment is driven by solenoid. A quite realistic second-order dynamic model of the valve is given by

$$\ddot{x}_v + a_1\dot{x}_v + a_0 x_v = b_0(u + d(t, x_v, \dot{x}_v)) \tag{1}$$

where x_v represents the spool displacement, a_1, a_0 and b_0 are model parameters which could be unknown, u represents the control input, and $d(t, x_v, \dot{x}_v)$ represents the lumped external disturbances acted on the valve spool (e.g. the flow forces and friction forces).

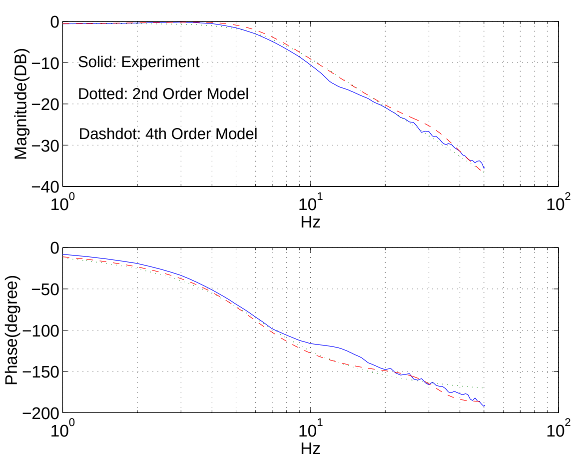

Figure 1: Frequency Response of a PDC Valve and Approximation Models

The PDC valve tested in this paper is manufactured by Parker Hannifan (Parker D3FXE01HCNBJ0011) Inc. The valve is equipped with LVDT sensors for spool position feedback. The standard frequency response test shown in Fig.1 reveals that the valve bandwidth is around $6.5Hz$. As shown in the figure, the frequency response could be approximated well by a second order system with a transfer function given by

$$G_{valve}(s) = \frac{X_v(s)}{U(s)} = \frac{1502.057}{s^2 + 58.771s + 1657.206} \tag{2}$$

and better approximated by a 4th order system given by

$$G_{valve}(s) = \frac{1206s^2 + 229256s + 46882988}{s^4 + 163s^3 + 47859s^2 + 1969807s + 56553725} \tag{3}$$

Given the desired spool position trajectory $x_{vLd}(t)$, the control objective is to synthesize a control input u such that the spool position x_v tracks $x_{vLd}(t)$ as closely as possible in spite of various model uncertainties.

3 Controller Design

3.1 Design Model and Problems to Be Addressed

Define the unknown parameter vector as $\theta = [a_1, a_0, b_0, b_0 d_n]^T$. The design model can be written as:

$$\ddot{x}_v + \theta_1\dot{x}_v + \theta_0 x_v = \theta_3 u + \theta_4 + \tilde{d}(x_v, \dot{x}_v, t) \tag{4}$$

where $b_0 d(x_v, \dot{x}_v, t) = b_0 d_n + \tilde{d}(x_v, \dot{x}_v, t)$, d_n is the nominal part of external disturbance and $\tilde{d}(x_v, \dot{x}_v, t)$ represents the lumped modeling error. The following practical assumptions are made

Assumption 1 *Parametric uncertainties and uncertain nonlinearities satisfy*

$$\begin{array}{rcl} \theta & \in & \Omega_\theta \overset{\Delta}{=} \{\theta : \ \theta_{min} < \theta < \theta_{max}\} \\ |\tilde{d}(t, x_v, \dot{x}_v)| & \leq & \delta_d(t, x_v, \dot{x}_v) \end{array} \tag{5}$$

where $\theta_{min} = [\theta_{1min}, \ldots, \theta_{4min}]^T$, $\theta_{max} = [\theta_{1max}, \ldots, \theta_{4max}]^T$, $\delta_d(t, x_v, \dot{x}_v)$ *and the sign of* b_0 *are known.* ◇

Some of the issues that we would like to address are:

(i) Since most PDC valves do not have spool position sensor, it is of practical interest to see if one can find ways to increase the valve bandwidth without feedback. For this purpose, an open loop pole zero cancellation compensator will be constructed.

(ii) In reality, PDC valves usually have significant parametric uncertainties due to the variation of the system parameters over various operating conditions and uncertain nonlinearities such as flow forces and friction forces. It is thus of practical significance to apply performance oriented nonlinear robust control strategies to obtain a consistent valve performance. For this purpose, the adaptive robust control (ARC) approach [11, 13] will be used - certain robust feedback will be employed to attenuate the effect of various model uncertainties as much as possible while parameter adaptation will be utilized to reduce model uncertainties for an improved performance.

(iii) For high performance, it is necessary to incorporate as much prior information on the desired trajectory into the controller design as possible [13]. As such, the higher order derivatives of the desired trajectory may be needed for a better model compensation [11, 13]. Here, since a cascade overall control architecture is used, the desired trajectory for the local valve controller is the spool position command synthesized by upper level controller. As such, the derivatives of the desired trajectory is not available for the design of local valve controller. To address this issue, desired trajectory reshaping will be used.

(iv) As presented later, the proposed full state feedback ARC controller design is simple and easy to implement. However, it requires the full state feedback including spool velocity. Since only the spool position measurement is available for feedback in most PDC valves, a pragmatic approach of using numerical difference of spool position feedback plus filter is used in implementation. Although this appears solving the problem, it is still of practical and theoretical interests to construct output feedback ARC controllers that uses spool position feedback only, and to compare the achievable performance of various ARC controllers, which will be carried out in later subsections.

3.2 Trajectory Reshaping

The purpose of trajectory reshaping is to provide a feasible reference motion trajectory that the valve can track. This will help to reduce the transient response of the tracking controller. As seen from (1), the maximal allowable reference acceleration $\ddot{x}_{vM}$ can be roughly estimated by

$$\ddot{x}_{vM} \le b_0 u_{max} \tag{6}$$

where u_{max} denotes the maximal control input that can be supplied to the system. For the valve used in this paper, $u_{max} = 10V$. Similarly, the maximal allowable reference velocity $\dot{x}_{vM}$ can be roughly estimated by

$$\dot{x}_{vM} \le b_0 u_{max}/a_1 \tag{7}$$

A reference motion trajectory $x_{vd}(t)$ which satisfies the constraints $|\dot{x}_{vd}| \le \dot{x}_{vM}$ and $|\ddot{x}_{vd}| \le \ddot{x}_{vM}$ can be generated using a simple second-order filter with some rate limiting modifications as in [17]. The detailed procedure is as follows:

Step 1 At sampling instance $t = k\Delta T$ where ΔT is the sampling period, $\ddot{x}_{vd}$ is generated by passing the desired valve spool trajectory x_{vLd} through a second-order filter,

$$\ddot{x}_{vd}(k) = -2\xi_n\omega_n\dot{x}_{vd}(k) - \omega_n^2(x_{vd}(k) - x_{vLd}(k)) \tag{8}$$

where ξ_n and ω_n represent the damping ratio and the natural frequency of the second-order filter.

Step 2 Check if $\ddot{x}_{vd}(k)$ generated by (8) satisfies the constraints. If not, i.e. $|\ddot{x}_{vd}(k)| > \ddot{x}_{vM}$, reset $\ddot{x}_{vd}(k)$ to $\ddot{x}_{vd}(k) = \ddot{x}_{vM} sign(\ddot{x}_{vd}(k))$ so that the constraint can be satisfied.

Step 3 By using the simple Euler approximation method and the modification $\ddot{x}_{vd}$ in step 2, the reference velocity at next sample instance $\dot{x}_{vd}(k+1)$ can be generated by

$$\dot{x}_{vd}(k+1) = \dot{x}_{vd}(k) + \ddot{x}_{vd}(k)\Delta T \tag{9}$$

Check if $\dot{x}_{vd}(k+1)$ satisfies the constraint. If not, i.e., $|\dot{x}_{vd}(k+1)| > \dot{x}_{vM}$, the trajectory is not feasible. Reset $\ddot{x}_{vd}(k)$ to

$$\ddot{x}_{vd}(k) = \frac{1}{\Delta T}(\dot{x}_{vM} sign(\dot{x}_{vd}(k+1)) - \dot{x}_{vd}(k)) \tag{10}$$

This guarantees that the reference valve spool displacement and velocity updated by the simple Euler approximation

$$\begin{aligned} x_{vd}(k+1) &= x_{vd}(k) + \dot{x}_{vd}(k)\Delta T \\ \dot{x}_{vd}(k+1) &= \dot{x}_{vd}(k) + \ddot{x}_{vd}(k)\Delta T \end{aligned} \tag{11}$$

will satisfy the constraints that $|\dot{x}_{vd}| \le \dot{x}_{vM}$ and $|\ddot{x}_{vd}| \le \ddot{x}_{vM}$.

3.3 Pole-Zero Cancellation Compensator Design

The structure of pole zero cancellation compensator is shown in the Fig. 2. Essentially, if the dynamic model of the valve

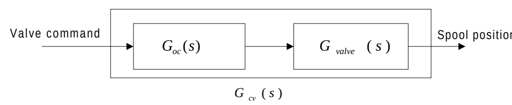

Figure 2: Pole-zero Cancellation Compensator

obtained from the system identification is accurate enough, we can simply use zeros in the compensator to cancel the slow poles in the valve dynamics to raise the bandwidth of the compensated valve dynamics. For example, a compensator based on the 2nd order valve model (2) is given by

$$G_{oc} = \frac{6168503}{1455.97}\frac{s^2+54.05s+1561.22}{s^3+470s^2+79652s+6168503} \tag{12}$$

and a compensator based on the 4th order valve model (3) is

$$G_{oc} = \frac{2880s^4+469440s^3+137833920s^2+5673044160s+162874733760}{s^5+753s^4+237999s^3+40483517s^2+3703487601s+151893300250} \tag{13}$$

With the above compensators, the dominant poles of the compensated valve dynamics have a break frequency around 25Hz.

3.4 Notations and Discontinuous Projection Mapping

For the subsequent ARC design, for simplicity of notations, let $\hat{\theta}$ denote the estimate of θ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta} = \hat{\theta} - \theta$). Viewing (5), a simple discontinuous projection can be defined [18, 19] as

$$Proj_{\hat{\theta}_i}(\bullet_i) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{imax} \text{ and } \bullet_i > 0 \\ 0 & \text{if } \hat{\theta}_i = \theta_{imin} \text{ and } \bullet_i < 0 \\ \bullet_i & \text{otherwise} \end{cases} \tag{14}$$

where $i = 1, \cdots, 4$. By using an adaptation law given by

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\tau) \tag{15}$$

where $Proj_{\hat{\theta}}(\bullet) = [Proj_{\hat{\theta}_1}(\bullet_1), \cdots, Proj_{\hat{\theta}_4}(\bullet_4)]^T$, $\Gamma > 0$ is a diagonal adaptation rate matrix, and τ is an adaptation function to be synthesized later. It can be shown [11] that for any adaptation function τ, the projection mapping used in (15) guarantees

$$\begin{aligned} &\textbf{(P1)} \qquad \hat{\theta} \in \bar{\Omega}_\theta \triangleq \{\hat{\theta} : \theta_{min} \le \hat{\theta} \le \theta_{max}\} \\ &\textbf{(P2)} \qquad \tilde{\theta}^T(\Gamma^{-1} Proj_{\hat{\theta}}(\Gamma\tau) - \tau) \le 0, \ \ \forall\tau \end{aligned} \tag{16}$$

3.5 Full State Feedback ARC Design

Letting $x_1 = x_v$ and $x_2 = \dot{x}_v$, a state space representation for (4) could be

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\theta_1 x_2 - \theta_2 x_1 + \theta_3 u + \theta_4 + \tilde{d} \end{aligned} \tag{17}$$

The design parallels the recursive backstepping design procedure via ARC Lyapunov functions in [13, 20] as follows.

Define a switching-function-like quantity as

$$z_2 = \dot{z}_1 + k_1 z_1 = x_2 - x_{2eq}, \quad x_{2eq} = \dot{x}_{vd} - k_1 z_1 \tag{18}$$

where $z_1 = x_1 - x_{vd}$, x_{vd} is the desired trajectory after the trajectory reshaping, and k_1 is any positive feedback gain. Since $G(s) = \frac{z_1(s)}{z_2(s)} = \frac{1}{s+k_1}$ is a stable transfer function, making z_1 small or converging to zero is equivalent to making z_2 small or converging to zero. Thus the remaining of the controller design is to synthesize a control law u such that z_2 approaches to a small value with guaranteed transient performance and final tracking accuracy. The derivative of z_2 can be written as

$$\dot{z}_2 = \dot{x}_2 - \dot{x}_{2eq} = -\theta_1 x_2 - \theta_2 x_1 + \theta_3 u + \theta_4 + \tilde{d} - \dot{x}_{2eq} \tag{19}$$

where $\dot{x}_{2eq} = \ddot{x}_{vd} - k_1(x_2 - \dot{x}_{vd})$. Noting that(19) has both parametric uncertainties θ_1, θ_2, θ_3, θ_4 and uncertain nonlinearity $\tilde{d}$, the ARC approach proposed in [13] will be generalized to accomplish the objective. From the Assumption 1, $sign(\theta_3)$(or $sign(b_0)$) is known. Without losing generality, let us assume $\theta_3 > \theta_{3min} > 0$. Then the control function u consists of two parts given by

$$\begin{aligned} u &= u_a + u_s \\ u_a &= \frac{1}{\hat{\theta}_3}(\hat{\theta}_1 x_2 + \hat{\theta}_2 x_1 - \hat{\theta}_4 + \dot{x}_{2eq} - z_1) \\ u_s &= u_{s1} + u_{s2}, \quad u_{s1} = -\frac{1}{\theta_{3min}} k_2 z_2 \end{aligned} \tag{20}$$

where k_2 is a positive feedback gain, u_a functions as an adaptive control law used to achieve an improved model compensation through on-line parameter adaptation given by (15). The τ in (15) would be

$$\tau = \phi_2 z_2, \qquad \phi_2 \stackrel{\Delta}{=} [-x_2, -x_1, u_a, 1]^T \tag{21}$$

Substituting (20) into (19)

$$\dot{z}_2 = -z_1 - \frac{\theta_3}{\theta_{3min}} k_2 z_2 + \theta_3 u_{s2} - \tilde{\theta}^T \phi_2 + \tilde{d} \tag{22}$$

The robust control function u_{s2} is now chosen to satisfy the following robust control conditions

$$\begin{aligned} &\text{condition i} \qquad z_2(\theta_3 u_{s2} - \tilde{\theta}^T \phi_2 + \tilde{d}) \le \varepsilon_2 \\ &\text{condition ii} \qquad z_2 \theta_3 u_{s2} \le 0 \end{aligned} \tag{23}$$

where ε_2 is a positive design parameter which can be arbitrarily small. Essentially, condition i of (23) shows that u_{s2} is synthesized to dominate the model uncertainties coming from both parametric uncertainties and uncertain nonlinearities, and condition ii is to make sure that u_{s2} is dissipating in nature so that it does not interfere with the functionality of the adaptive control part u_a. How to choose u_{s2} to satisfy constraints like (23) can be found in [12, 20, 13].

For the positive semi-definite (p.s.d.) function V_2 defined by $V_2 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$, from(22), its time derivative is given by

$$\dot{V}_2 = -k_1 z_1^2 - \frac{\theta_3}{\theta_{3min}} k_2 z_2^2 + z_2(\theta_3 u_{s2} - \tilde{\theta}^T \phi_2 + \tilde{d}) \tag{24}$$

Theorem 1 *Let the parameter estimates be updated by the adaptation law (15) in which τ is defined by (21),the control law (20) guarantees that*

A. *In general, all signals are bounded. Furthermore, the p.s.d. function V_2, an index of tracking errors, is bounded above by*

$$V_2(t) \le exp(-\lambda_V t)V_2(0) + \frac{\varepsilon_V}{\lambda_V}[1 - exp(-\lambda_V t)] \tag{25}$$

where $\lambda_V = 2min\{k_1, k_2\}$ and $\varepsilon_V = \varepsilon_2$

B. *If after a finite time t_0, there exist parametric uncertainties only (i.e., $\tilde{d} = 0$, $\forall t \ge t_0$), then, in addition to result A, zero final tracking error is achieved, i.e, $z_1 \to 0$ and $z_2 \to 0$ as $t \to \infty$*

The proof of Theorem 1 can be worked out in the same way as in [20, 13].

3.6 Output Feedback ARC Design

The full state feedback ARC controller synthesized in subsection 3.5 needs the feedback of the spool velocity, which may not be available or too noisy. In this subsection, an output feedback ARC controller which needs the measurement of spool position only is presented.

Observer design

A state space representation of (4) is given by:

$$\begin{aligned} \dot{x}_1 &= x_2 - \theta_1 x_1 \\ \dot{x}_2 &= -\theta_2 x_1 + \theta_3 u + \theta_4 + \tilde{d} \\ y &= x_1 = x_v \end{aligned} \tag{26}$$

Since only output measurement is available, an observer is firstly designed to provide exponentially convergent estimates of the unmeasurable states. Rewriting (26) in the form:

$$\begin{aligned} \dot{x} &= A_0 x + (\bar{k} - a)y + bu + e_2\theta_4 + e_2\tilde{d} \\ y &= x_1 \end{aligned} \tag{27}$$

where $b = [0, \theta_3]^T$, $e_2 = [0, 1]^T$ and

$$A_0 = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix}, \quad \bar{k} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}, \quad a = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{28}$$

By suitably choosing $\bar{k}$, the observer matrix A_0 will be stable. Following the design procedure of [14], the filters are defined by:

$$\begin{aligned} &\dot{\xi}_2 = A_0\xi_2 + \bar{k}y \quad \dot{\xi}_1 = A_0\xi_1 + e_1 y \quad \dot{\xi}_0 = A_0\xi_0 + e_2 y \\ &\dot{v}_0 = A_0 v_0 + e_2 u \quad \dot{\psi}_1 = A_0\psi_1 + e_2 \end{aligned} \tag{29}$$

where $e_1 = [1,0]^T$. The state estimates $\hat{x}$ can thus be represented by

$$\hat{x} = \xi_2 - \theta_1\xi_1 - \theta_2\xi_0 + \theta_3 v_0 + \theta_4\psi_1 \tag{30}$$

Let $\varepsilon_x = x - \hat{x}$ be the estimation error, from (27), (29) and (30), the observer dynamics is given by

$$\dot{\varepsilon}_x = A_0\varepsilon_x + e_2\tilde{d} \tag{31}$$

The solution of this equation can be written as

$$\varepsilon_x = \varepsilon + \varepsilon_u \tag{32}$$

where ε is the zero input response satisfying $\dot{\varepsilon} = A_0\varepsilon$ and

$$\varepsilon_u = \int_0^t e^{A_0(t-\tau)} e_2\tilde{d}d\tau, \qquad t > 0 \tag{33}$$

is the zero state response. Noting the Assumption 1 and the fact that matrix A_0 is stable, ε_u is bounded, i.e.,

$$\varepsilon_u \in \Omega_\varepsilon \stackrel{\Delta}{=} \{\varepsilon : |\varepsilon_u \leq \delta_\varepsilon(t)\} \tag{34}$$

where $\delta_\varepsilon(t)$ is known. In the following controller design, ε and ε_u will be treated as disturbance and be attenuated by different robust control functions in each design step.

Remark 1 *The filter states ξ_i can be obtained from the algebraic expressions*

$$\begin{array}{l} \xi_2 = -A_0^2\eta \\ \xi_i = A_0^i\eta, \quad i = 0,1 \end{array} \tag{35}$$

where η is the state vector of dimension 2 of the filter defined by $\dot{\eta} = A_0\eta + e_2 y$ $\diamond$

Step 1: Define the tracking error $z_1 = y - x_{vd}$, then its derivative can be written as

$$\dot{z}_1 = x_2 - \theta_1 x_1 - \dot{x}_{vd} \tag{36}$$

Since x_2 is not measurable, we replace it by its estimation from (30)

$$x_2 = \xi_{2,2} - \theta_1\xi_{1,2} - \theta_2\xi_{0,2} + \theta_3 v_{0,2} + \theta_4\psi_{1,2} + \varepsilon_{x,2} \tag{37}$$

where $\bullet_{i,j}$ represents jth element of $\bullet_i$. Substituting (37) into (36)

$$\begin{array}{rl} \dot{z}_1 = & \xi_{2,2} - \theta_1(\xi_{1,2} + x_1) - \theta_2\xi_{0,2} + \theta_3 v_{0,2} + \theta_4\psi_{1,2} \\ & +\varepsilon_2 + \varepsilon_{u,2} - \dot{x}_{vd} \end{array} \tag{38}$$

If we treat $v_{0,2}$ as the control input, we can design a virtual control law α_1 for $v_{0,2}$ such that z_1 will approach to a small value with guaranteed transient performance. Since both parametric uncertainties and uncertain nonlinearities exist in (38), the ARC approach will be generalized to achieve the goal. The control function α_1 consists two parts given by

$$\begin{array}{l} \alpha_1 = \alpha_{1a} + \alpha_{1s} \\ \quad \alpha_{1a} = \frac{1}{\hat{\theta}_3}[-\xi_{2,2} + \hat{\theta}_1(\xi_{1,2} + x_1) + \hat{\theta}_2\xi_{0,2} - \hat{\theta}_4\psi_{1,2} + \dot{x}_{vd}] \\ \quad \alpha_{1s} = \alpha_{1s1} + \alpha_{1s2} + \alpha_{1s3}, \quad \alpha_{1s1} = -\frac{1}{\theta_{3min}} k_{s1} z_1 \end{array} \tag{39}$$

where k_{s1} is a positive feedback gain.

Let $z_2 = v_{0,2} - \alpha_1$ denote the input discrepancy. Substituting (39) into (38), we will have

$$\dot{z}_1 = -\frac{\theta_3}{\theta_{3min}} k_1 z_1 + \theta_3(\alpha_{1s2} + \alpha_{1s3}) - \tilde{\theta}^T\phi_1 + \varepsilon_2 + \varepsilon_{u,2} \tag{40}$$

where $\phi_1 = [-(\xi_{1,2} + x_1), -\xi_{0,2}, \alpha_{1a}, \psi_{1,2}]^T$

Define a positive semi-definite (p.s.d) function as $V_1 = \frac{1}{2}z_1^2$. Its derivative is given by

$$\begin{array}{rl} \dot{V}_1 = & \theta_3 z_1 z_2 - \frac{\theta_3}{\theta_{3min}} k_{s1} z_1^2 + z_1[(\theta_3\alpha_{1s2} - \tilde{\theta}^T\phi_1 + \varepsilon_{u,2}) \\ & +(\theta_3\alpha_{1s3} + \varepsilon_2)] \end{array} \tag{41}$$

The robust control function α_{1s2} is chosen to satisfy following conditions:

$$\begin{array}{ll} \text{condition i} & z_1(\theta_3\alpha_{1s2} - \tilde{\theta}^T\phi_1 + \varepsilon_{u,2}) \leq \varepsilon_{11} \\ \text{condition ii} & z_1\theta_3\alpha_{1s2} \leq 0 \end{array} \tag{42}$$

where ε_{11} is a positive design parameter which can be arbitrary small. In principle, the same strategy can be used to design a robust control function α_{1s3} to handle the effect of ε_2. However, since the bound of ε_2 is unknown, we cannot pre-specify the level of control accuracy. So we relax the condition to

$$z_1(\theta_3\alpha_{1s3} + \varepsilon_2) < \varepsilon_{12}\varepsilon_2^2 \tag{43}$$

with the level of attenuation ε_{12} being a design parameter [21].

Step 2: From (39), (30) and (26)

$$\begin{array}{rl} \dot{\alpha}_1 = & \frac{\partial\alpha_1}{\partial x_1}[\xi_{2,2} - \theta_1(\xi_{1,2} + x_1) - \theta_2\xi_{0,2} + \theta_3 v_{0,2} \\ & +\theta_4\psi_{1,2} + \varepsilon_2 + \varepsilon_{u2}] + \frac{\partial\alpha_1}{\partial\eta}\dot{\eta} + \frac{\partial\alpha_1}{\partial\psi_{1,2}}\dot{\psi}_{1,2} + \frac{\partial\alpha_1}{\partial t} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} \\ = & \dot{\alpha}_{1c} + \dot{\alpha}_{1u} \end{array} \tag{44}$$

where

$$\begin{array}{rl} \dot{\alpha}_{1c} = & \frac{\partial\alpha_1}{\partial x_1}[\xi_{2,2} - \hat{\theta}_1(\xi_{1,2} + x_1) - \hat{\theta}_2\xi_{0,2} + \hat{\theta}_3 v_{0,2} \\ & +\hat{\theta}_4\psi_{1,2}] + \frac{\partial\alpha_1}{\partial\eta}\dot{\eta} + \frac{\partial\alpha_1}{\psi_{1,2}}\dot{\psi}_{1,2} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\alpha_1}{\partial t} \\ \dot{\alpha}_{1u} = & \frac{\partial\alpha_1}{\partial x_1}(\tilde{\theta}_1(\xi_{1,2} + x_1) + \tilde{\theta}_2\xi_{0,2} - \tilde{\theta}_3 v_{0,2} - \tilde{\theta}_4\psi_{1,2} + \varepsilon_{u,2} + \varepsilon_2) \end{array} \tag{45}$$

In (45), $\dot{\alpha}_{1c}$ is calculable and can be used in the design of control functions, but $\dot{\alpha}_{1u}$ cannot due to various uncertainties. Therefore, $\dot{\alpha}_{1u}$ has to be dealt with robust feedback in this step. From (29), the derivative of z_2 can be written as:

$$\dot{z}_2 \quad = \dot{v}_{0,2} - \dot{\alpha}_1 = u - k_2 v_{0,1} - \dot{\alpha}_{1c} - \dot{\alpha}_{1u} \tag{46}$$

Consider the augmented p.s.d function $V_2 = V_1 + \frac{1}{2}z_2^2$, noting (41) and (46), the derivative of V_2 can be written as

$$\begin{array}{rl} \dot{V}_2 & = \dot{V}_1|_{z_2=0} + \theta_3 z_1 z_2 + z_2\dot{z}_2 \\ & = \dot{V}_1|_{z_2=0} + z_2(u - k_2 v_{0,1} + \theta_3 z_1 - \dot{\alpha}_{1c} - \dot{\alpha}_{1u}) \end{array} \tag{47}$$

where $\dot{V}_1|_{z_2=0}$ denotes $\dot{V}_1$ under condition that $z_2 = 0$. Similar to (39), control input u consists of two parts give by

$$\begin{aligned} u &= u_a + u_s \\ u_a &= k_2 v_{0,1} - \hat{\theta}_3 z_1 + \dot{\alpha}_{1c} \\ u_s &= u_{s1} + u_{s2} + u_{s3} \\ u_{s1} &= -k_{s2} z_2 \end{aligned} \tag{48}$$

where k_{s2} is a positive feedback gain. Substituting (48) to (47), we have

$$\dot{V}_2 = \dot{V}_1|_{z_2=0} + z_2(u_{s2} - \tilde{\theta}^T \phi_2 - \frac{\partial \alpha_1}{\partial x_1} \varepsilon_{u,2}) + z_2(u_{s3} - \frac{\partial \alpha_1}{\partial x_1} \varepsilon_2) \tag{49}$$

where $\phi_2 = [\frac{\partial \alpha_1}{\partial x_1}(\xi_{1,2} + x_1), \frac{\partial \alpha_1}{\partial x_1} \xi_{0,2}, z_1 - v_{0,2}, -\psi_{1,2}]^T$. The robust feedback part u_{s2} and u_{s3} can be chosen to satisfy following conditions

$$\begin{array}{ll} \text{condition i} & z_2(u_{s2} - \tilde{\theta}^T \phi_2 \frac{\partial \alpha_1}{\partial x_1} \varepsilon_{u2}) \leq \varepsilon_{21} \\ \text{condition ii} & z_2 u_{s2} \leq 0 \\ \text{condition iii} & z_2(u_{s3} - \frac{\partial \alpha_1}{\partial x_1} \varepsilon_2) \leq \varepsilon_{22} \varepsilon_2^2 \end{array} \tag{50}$$

where ε_{21} and ε_{22} are design parameters.

Theorem 2 *Let the parameter estimates be updated by the adaptation law (15) in which τ is given by*

$$\tau = \phi_1 z_1 + \phi_2 z_2 \tag{51}$$

Then the control law (48) guarantees that

A . *The control input and all internal signals are bounded. Furthermore, V_2 is bounded above by*

$$V_2(t) \leq exp(-\lambda_2 t) V_2(0) + \frac{\bar{\varepsilon}_{21} + \bar{\varepsilon}_{22} \|\varepsilon_2\|_\infty^2}{\lambda_2} [1 - exp(-\lambda_2 t)] \tag{52}$$

where $\lambda_2 = 2min\{k_{s1}, k_{s2}\}$, $\bar{\varepsilon}_{21} = \varepsilon_{11} + \varepsilon_{21}$, $\bar{\varepsilon}_{22} = \varepsilon_{12} + \varepsilon_{22}$, and $\|\varepsilon_2\|_\infty$ stands for the infinity norm of $\varepsilon_2(t)$. Noting that $\varepsilon_2(t)$ exponentially converges to zero, $V_2(\infty)$ is ultimately bounded by

$$V_2(\infty) \leq \frac{\bar{\varepsilon}_{21}}{\lambda_2} \tag{53}$$

B . *If after a finite time t_0, there exist parametric uncertainties only (i.e., $\tilde{d} = 0$, $\forall t \geq t_0$), then, in addition to result A, zero final tracking error is achieved, i.e, $z_1 \to 0$ and $z_2 \to 0$ as $t \to \infty$.* $\triangle$

Proof of the Theorem can be worked out in the same way as in [16].

4 Comparative Experiments

4.1 Performance Index

As in [22, 23], letting $e = x_v - x_{vLd}$ denote the tracking error, the following performance indexes will be used to measure the tracking quality of each control algorithm: (I1) $\|e\|_{rms} = (\frac{1}{T}\int_0^T e(t)^2 \mathrm{d}t)^{1/2}$, the *rms value* of the tracking error, is used to measure *average tracking performance*, where T represents the total running time; (I2) $e_M = \max_t\{|e(t)|\}$, the maximum absolute value of the tracking error, is used to measure *transient performance*; (I3) $e_F = \max_{T-2 \leq t \leq T}\{|e(t)|\}$, the maximum absolute value of the tracking error during the last 2 seconds, is used to measure *final tracking accuracy*; (I4) $\|u\|_{rms} = (\frac{1}{T}\int_0^T u(t)^2 \mathrm{d}t)^{1/2}$, the average control input, is used to evaluate the amount of *control effort*; and (I5)$c_u = \frac{\|\Delta u\|_{rms}}{\|u\|_{rms}}$, the normalized control variations, is used to measure the *degree of control chattering*, where $\|\Delta u\|_{rms} = \sqrt{\frac{1}{N}\sum_{j=1}^N |u(j\Delta T) - u((j-1)\Delta T)|^2}$ is the average of control input increments.

4.2 Comparative Experimental Results

To verify the effectiveness of the proposed algorithms, experiments are conducted on the proportional directional control valve mentioned before. For a fair comparison, the controllers are divided into two groups: the first group is of non-feedback type which includes the pole zero cancellation compensators and the open loop controller, and the other is of feedback type which includes (1) ARC with full state feedback; (2) output feedback ARC; and (3) a PID controller with feedforward compensation.

Pole-Zero Cancellation Compensators

Frequency response tests are performed on the PDC valve with the pole-zero cancellation compensators proposed in subsection 3.3. The results are shown in Fig.3 along with the system with no compensator. For the convenience of comparison, a gain is added to the no compensator case to make the static gain equal to 1. It can be seen from the figure that the pole-zero cancellation compensators are able to push the bandwidth of the PDC valve a few Hz higher and an improved phase plot. It is also observed that a more accurate model based pole-zero cancellation compensator achieves a better performance (comparing the results of the 4th order valve model based pole-zero compensator with those of the 2nd order valve model based compensator). Fig. 4 shows the responses of the valve to a 6Hz sine wave reference trajectory in three cases. As shown, the proposed compensators improve the valve's tracking capability. However, some quite large lags still exist. Overall, it can be observed that the achievable performance of pole-zero cancellation compensators heavily relies on the accuracy of the available system model. Although implementation of such compensators is easy and does not require spool position measurement, the performance improvement is limited.

ARC Controllers

The following three feedback controllers are implemented and compared:

PID with feedforward compensation: Supposing that all the parameters in (17) are known, the control objective can be achieved by using the following PID control law:

$$u = \frac{1}{\theta_3}(\ddot{x}_{vd} + \theta_1\dot{x}_v + \theta_2 x_v) - k_p z_1 - k_i \int z_1 dt - k_d \dot{z}_1 \quad (54)$$

where $z_1 = x_v - x_{vd}$. With such a control law, it is easy to verify that the closed-loop characteristic equation is given by

$$s^3 + (\theta_1 + k_d\theta_3)s^2 + (\theta_2 + k_p\theta_3)s + k_i\theta_3 = 0 \quad (55)$$

By placing the closed-loop poles at the desired locations, the controller gain k_p, k_i and k_d can be determined. Since θ_1, θ_2 and θ_3 are unknown parameters, instead of using (54), the following control law is used

$$\begin{aligned} u = &\frac{1}{\theta_3(0)}(\ddot{x}_{vd} + \theta_1(0)\dot{x}_v + \theta_2(0)x_v) - k_p z_1 \\ &- k_i \int z_1 dt - k_d \dot{z}_1 \end{aligned} \quad (56)$$

where $\theta_1(0)$, $\theta_2(0)$ and $\theta_3(0)$ are the fixed parameter estimates chosen as 20, 1200 and 1200 respectively. By placing all the closed-loop poles at the locations with a break frequency of $7Hz$, the control gains can be solved as $k_p = 3.837$, $k_i = 70.901$ and $k_d = 0.093$.

Full state feedback ARC: The controller parameters for the ARC controller presented in subsection 3.5 are given by: $k_1 = 120$, $k_2 = 120$, $\Gamma = diag\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\} = diag\{2, 110, 120, 500\}$, and $\theta(0) = [20, 1200, 1200, 0]^T$.

Output feedback ARC (OARC): The controller parameters for the output feedback ARC in subsection 3.6 are: $k_1 = 308$, $k_2 = 48400$, $k_{s1} = k_{s2} = 120$, $\Gamma = diag\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\} = diag\{2, 110, 120, 4000\}$, and $\theta(0) = [20, 1200, 1200, 0]^T$.

The experiments are run for a smooth point to point trajectory shown in Fig.5, which has a maximal acceleration of $64V/sec^2$ and a maximal velocity of $8V/sec$. The parameters used for the trajectory reshaping are $\omega_n = 314$ and $\xi_n = 0.8$. The experimental results in terms of performance indexes are given in Table 1. It can be seen from the Table that both ARC controllers perform better than PID with lesser degree of control input chattering. The ARC with full state feedback achieves the best tracking performance in terms of e_M, $||e||_{rms}$, and e_F. The output feedback ARC uses almost same level control effort as PID and has a lesser degree of control input chattering compared with PID and the full state feedback ARC. It is noted that the $7Hz$ break frequency of the closed-loop poles of the PID controller is almost the highest that we can push for; when we push the bandwidth of closed-loop system further, the control input of PID controller becomes more chattering and leads to control saturation.

Index	e_M(V)	e_F(V)	$\|\|e\|\|_{rms}$(V)	$\|\|u\|\|_{rms}$(V)	c_u
PID	0.207	0.147	0.038	2.313	0.423
ARC	0.145	0.112	0.032	2.362	0.416
OARC	0.158	0.120	0.036	2.314	0.341

Table 1. Performance Indexes of Feedback Controllers

5 Conclusions

In this paper, three different types of controllers are proposed to improve the performance of proportional directional control valves. Firstly, an open loop pole-zero cancellation compensator is introduced. The method is very simple but heavily relies on the accuracy of the identified valve model. To deal with the effect of model uncertainties and external disturbances, a full state feedback ARC controller is synthesized, which takes into account the effect of both parametric uncertainties and uncertain nonlinearities. Theoretically, the controller achieves a guaranteed transient and final tracking accuracy. Finally, an output feedback ARC controller is presented to address the problem of unmeasureable state of spool velocity. All controllers along with a PID controller with feedforward compensation are implemented. Comparative experimental results are obtained to show the advantages and the limitations of various algorithms.

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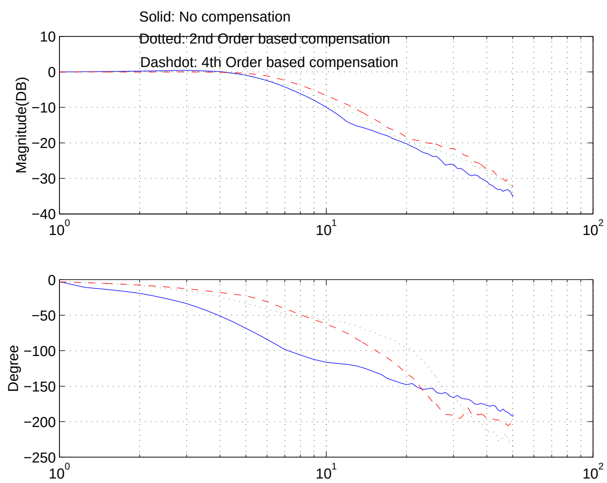

Figure 3: Frequency Responses of Open-loop Compensators

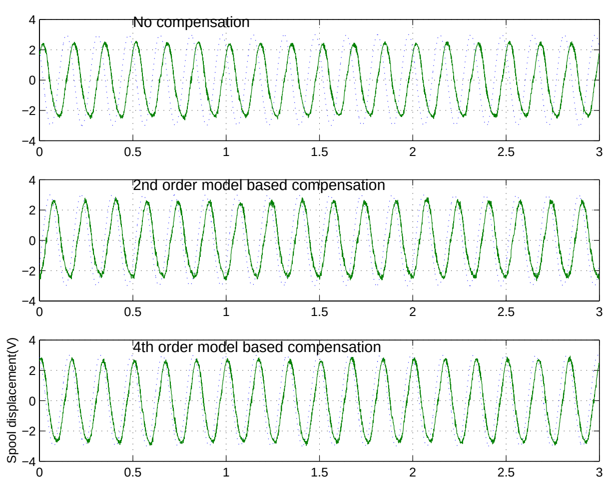

Figure 4: Tracking a 6Hz Reference Trajectory with Open-loop Compensators

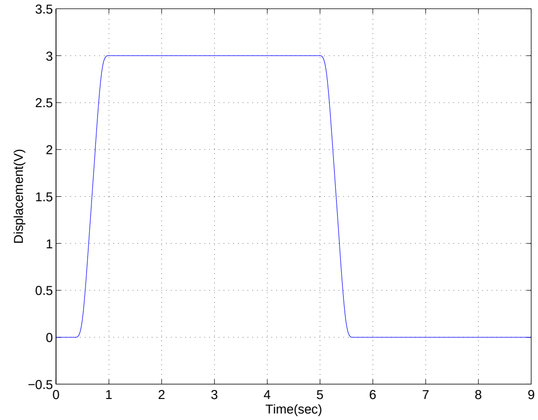

Figure 5: Reference Trajectory for Feedback Controllers

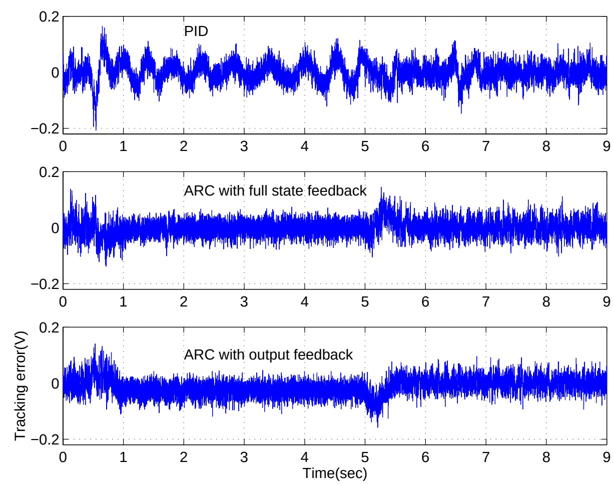

Figure 6: Tracking Errors of Feedback Controllers

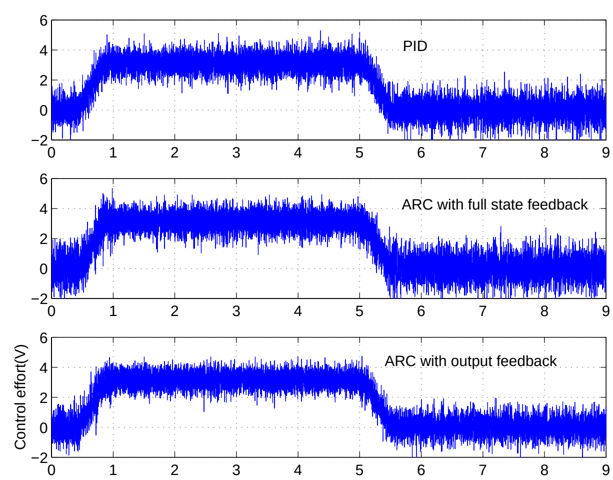

Figure 7: Control Inputs of Feedback Controllers