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ROBUST IMPEDANCE CONTROL OF ROBOT MANIPULATORS

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Abstract

A variable structure control scheme is developed for impedance control of robot manipulators in the presence of both parametric uncertainties and external disturbances. The specification of the impedance is given in terms of a desired motion trajectory, a desired interaction force trajectory, and a desired second-order impedance function between motion errors and interaction force errors. Using variable structure model reaching control, the desired impedance is achieved in the sliding mode with robust performance. Furthermore, the reaching transient response is guaranteed with prescribed quality. Force tracking control can also be achieved with some special environment constraints. To illustrate the method, simulation results are given.

Key Words

Variable structure control, mechanical impedance, manipulators

1. Introduction

When a robot interacts with an environment, interaction forces are generated that have to be accommodated rather than rejected. In addition to position control, force control is required to accomplish this task.

A number of force control schemes have been proposed in recent years. Whitney [1] surveyed the bulk of these approaches and gave a historical perspective of the field. Despite the diversity of approaches, there are two basic methods, hybrid position/force control [2–5] and impedance control [6–16].

Impedance control was first proposed by Hogan [6]. In this method, motion is commanded and controlled, and the response for deviation from that motion owing to interaction force is given in the form of an impedance. By proper choice of the desired impedance, dynamic interaction may be controlled to obtain the proper force response [6].

Robust implementation of impedance control by using adaptive control is discussed in [13–15]. As pointed out in [13], an assumption is made that the rate of change of the system parameters is much lower than the rate of adjustment of the controller gain. This limits the rate at which the arm can maneuver [13]. In [14], the adaptive motion control schemes of a rigid robot [15, 17] are extended to achieve robust impedance control against parametric uncertainties of the robot by introducing a compensation controller for motion control. Nevertheless, the transient analysis of adaptive systems is still in its infancy, and few significant results are available [18].

Variable structure control (VSC) as an alternative robust approach was first applied to trajectory control of robot manipulators by Young [19]. In [20], a variable structure model reaching control (VSMRC) strategy was proposed. Model-reaching is realized by employing a dynamic sliding mode in the framework of VSC strategy. This concept can be considered an extension of the dynamic compensation in linear control strategy. A dynamic compensator is introduced in the sliding mode such that its dimension is made equal to that of the system. The resultant sliding mode can then be identified with the desired model. By the use of VSC, the expanded system is maintained in the sliding mode and model reaching is realized [20, 21].

In this article, we present a novel robust impedance control of robot manipulators in the presence of both parametric uncertainties and external disturbancces by using VSMRC. The specification of the desired impedance differs from the one previously used [13, 14] in that the desired interaction force trajectory is included in the desired impedance in such a way that force tracking control is made available with some special environment constraints. Based on the VSMRC, the desired impedance is achieved in the sliding mode in finite time. Furthermore, the reaching transient response is also guaranteed with prescribed quality. Simulation results of the robot moving on a vertical surface are given to illustrate the proposed method.

This article is organized as follows. Section 2 gives the robot manipulator model and its main properties. The specification of the desired impedance and the proposed VSC impedance controller are given in Section 3. Simulation results are presented in Section 4, and a brief conclusion is given in Section 5.

2. Dynamic Equation of Robot Manipulator

The dynamic equation of a general rigid-link manipulator having n degrees of freedom can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + J^T(q)F + \tilde{f}(t) = \tau \qquad (1)$$

where q is the $n \times 1$ joint displacement vector, τ is the applied joint torque, $M(q)$ is the inertia matrix, $C(q,\dot{q})\dot{q}$ is the Coriolis and centrifugal vector, $G(q)$ is the gravitational vector, F is the interaction forces/ moments vec-

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tor on the environment exerted by the robot at the end-effector, $\tilde{f}(t)$ is the external disturbance, and $J(q)$ is the manipulator Jacobin matrix, which is assumed to be nonsingular in workspace Ω given by

$$J(q) = \frac{\partial x(q)}{\partial q} \tag{2}$$

where x is the position and orientation vector of the end-effector frame in the world space.

It is assumed that measurement of position, velocity, and force is available. Equation (1) has the following properties that we will use

Property 1 [17, 18]: For any finite workspace Ω, $M(q)$ is a symmetric positive definite matrix. Moreover, there exist $k' > 0$ and $k'' > 0$ such that

$$k'I \leq M(q) \leq k''I \quad \forall q \in \Omega \tag{3}$$

Property 2 [17, 18]: The matrix $N(q,\dot{q}) = \dot{M}(q) - 2C(q,\dot{q})$ is a skew-symmetric matrix.

Assume that the available values of $M(q), C(q,\dot{q}), G(q)$ are $\hat{M}(q), \hat{C}(q,\dot{q}), \hat{G}(q)$, and the modeling errors are bounded as

$$\begin{aligned}
|\Delta M(q)| &\leq \delta M(q) \quad \Delta M(q) = M(q) - \hat{M}(q) \\
|\Delta C(q,\dot{q})| &\leq \delta C(q,\dot{q}) \quad \Delta C(q,\dot{q}) = C(q,\dot{q}) - \hat{C}(q,\dot{q}) \\
|\Delta G(q)| &\leq \delta G(q) \quad \Delta G(q) = G(q) - \hat{G}(q) \\
|\tilde{f}(t)| &\leq \delta \tilde{f}(t)
\end{aligned} \tag{4}$$

where $|A| \leq B$ is true in elements, i.e., $|A_{ij}| \leq B_{ij}$ (in the following, the operation of matrix is understood in the same way). The modeling errors account for parametric uncertainties of the robot and terms that are neglected owing to computation efficiency.

In the world space, the dynamic equation of the robot is given [3] by

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) + F + \tilde{F}(x,t) = T \tag{5}$$

where

$$\begin{aligned}
M(x) &= J^{-T}(q)M(q)J^{-1}(q) \\
C(x,\dot{x}) &= J^{-T}(q)C(q,\dot{q})J^{-1}(q) \\
&\quad - J^{-T}(q)M(q)J^{-1}(q)\dot{J}(q)J^{-1}(q) \\
G(x) &= J^{-T}(q)G(q) \quad \tilde{F}(x,t) = J^{-T}(q)\tilde{f}(t) \\
T &= J^{-T}(q)\tau
\end{aligned} \tag{6}$$

From (6), the estimated values of $M(x), C(x,\dot{x}), G(x)$ can be calculated by

$$\begin{aligned}
\hat{M}(x) &= J^{-T}\hat{M}(q)J^{-1} \\
\hat{C}(x,\dot{x}) &= J^{-T}\hat{C}(q,\dot{q})J^{-1} - J^{-T}\hat{M}J^{-1}\dot{J}J^{-1} \\
\hat{G}(x) &= J^{-T}\hat{G}(q)
\end{aligned} \tag{7}$$

and the modeling errors are bounded as

$$\begin{aligned}
|\Delta M(x)| &\leq \delta M(x) \quad \delta M(x) = |J^{-T}|\delta M(q)|J^{-1}| \\
|\Delta C(x,\dot{x})| &\leq \delta C(x,\dot{x}) \quad \delta C(x,\dot{x}) = |J^{-T}|\delta C(q,\dot{q})|J^{-1}| \\
&\qquad + |J^{-T}|\delta M(q)|J^{-1}\dot{J}J^{-1}| \\
|\Delta G(x)| &\leq \delta G(x) \quad \delta G(x) = |J^{-T}|\delta G(q) \\
|\tilde{F}(x,t)| &\leq \delta \tilde{F}(x,t) \quad \delta \tilde{F}(x,t) = |J^{-T}|\delta \tilde{f}(t)
\end{aligned} \tag{8}$$

Equation (5) has the following properties

Property 3: For any finite workspace Ω in which $J(q)$ is nonsingular, $M(x)$ is a symmetric positive definite matrix with

$$k'_x I \leq M(x) \leq k''_x I \quad \forall q \in \Omega \tag{9}$$

where

$$k'_x = \frac{k'}{c_1^2} \quad k''_x = \frac{k''}{c_2^2}$$

$$c_1 = \max_{q\in\Omega}[\sigma_{\max}(J(q))] \quad c_2 = \min_{q\in\Omega}[\sigma_{\min}(J(q))] \tag{10}$$

and $\sigma(J)$ means singular value of matrix J.

Proof: From (6), we have

$$y^T M(x) y = (J^{-1}y)^T M(q)(J^{-1}y) \quad \forall y \in R^n$$

From property 1, then

$$k' y^T (J^{-T}J^{-1})y \leq y^T M(x) y \leq k'' y^T (J^{-T}J^{-1})y$$

$$k' \sigma_{\min}^2(J^{-1}) y^T y \leq y^T M(x) y \leq k'' \sigma_{\max}^2(J^{-1}) y^T y$$

$$\frac{k'}{\sigma_{\max}^2(J)} y^T y \leq y^T M(x) y \leq \frac{k''}{\sigma_{\min}^2(J)} y^T y$$

This leads to (9).

Property 4 [17]: The matrix $N(x,\dot{x}) = \dot{M}(x) - 2C(x,\dot{x})$ is a skew-symmetric matrix.

3. VSC Impedance Controller

The desired impedance is specified as

$$M_m\ddot{e} + B_m\dot{e} + K_m e = -K_f e_f$$

$$e = x(t) - x_d(t) \quad e_f = F(t) - F_d(t) \tag{11}$$

where M_m, B_m, K_m are the desired inertia, damping, and stiffness; $x_d(t)$ is the desired motion trajectory in the world space; and $F_d(t)$ is the desired interaction force trajectory. Usually, M_m, B_m, K_m, K_f are chosen as diagonal matrices to obtain decoupled response.

Remark 1: The specification of the desired impedance usually consists of a desired motion trajectory, and a desired dynamic relationship between the motion errors and the interaction forces [6, 13, 14]. The regulation of interaction force is indirectly achieved by proper choice of the desired impedance. Here, if we set $K_f = I$, $F_d(t) = 0$, the desired impedance will be the same as the one used in [13, 14]. The desired interaction force $F_d(t)$ is introduced such that interaction force can be directly controlled with some special environment constraints. The reason for introducing K_f is that when we set $K_f = 0$, robust motion control is achieved against bounded interaction force such as friction force.

The robust impedance control problem can be stated as that of designing a controller so that the desired impedance (11) is achieved under the modeling errors (4). For this purpose, we will use the VSMRC method. First, a dynamic compensator is introduced:

$$\dot{z} = Az + K_{pz}e + K_{vz}\dot{e} + K_{fz}e_f \tag{12}$$

where z is the n-dimensional state vector of the compensator, A is any semi-negative definite matrix, and K_{pz}, K_{vz}, K_{fz} are specified to shape the dynamic sliding mode so that the desired impedance is achieved. The compensator is employed in forming the switching function

$$s(e, \dot{e}, z) = \dot{e} + F_1 e + F_2 z \tag{13}$$

where F_2 is any nonsingular matrix. The resultant sliding mode $\{s = 0, \dot{s} = 0\}$ is described by

$$\begin{aligned} z &= -F_2^{-1}(\dot{e} + F_1 e) \\ \dot{z} &= -F_2^{-1}(\ddot{e} + F_1\dot{e}) \end{aligned} \tag{14}$$

Substituting (12) into the sliding mode equation (14), we have

$$\begin{aligned} \ddot{e} + (F_1 - F_2AF_2^{-1} + F_2K_{vz})\dot{e} + (F_2K_{pz} - F_2AF_2^{-1}F_1)e \\ = -F_2K_{fz}e_f \end{aligned} \tag{15}$$

Comparing (15) with (11), we see that, if K_{pz}, K_{vz}, K_{fz} are chosen as

$$\begin{aligned} K_{vz} &= F_2^{-1}(M_m^{-1}B_m - F_1 + F_2AF_2^{-1}) \\ K_{pz} &= F_2^{-1}(M_m^{-1}K_m + F_2AF_2^{-1}F_1) \\ K_{fz} &= F_2^{-1}M_m^{-1}K_f \end{aligned} \tag{16}$$

the sliding mode equation (15) will be identical with the desired impedance (11). That is to say, the desired impedance is achieved in the sliding mode.

The control torque can be determined so that the system reaches the sliding mode in finite time and has prescribed reaching transient response.

Theorem 1: For the robot manipulator (5) with the modeling errors (8), the system achieves the desired impedance (11) if the following control torque is applied:

$$T = \hat{M}(x)\ddot{x}_{eq} + \hat{C}(x, \dot{x})\dot{x}_{eq} + \hat{G}(x) - T_d - Ds - \epsilon sgn(s) + F \tag{17}$$

where

$$\begin{aligned} \dot{x}_{eq} &= \dot{x}_d(t) - F_1 e - F_2 z \\ \ddot{x}_{eq} &= \ddot{x}_d(t) - F_1\dot{e} - F_2(Az + K_{pz}e + K_{vz}\dot{e} + K_{fz}e_f) \\ T_d &= [(T_d)_1, \ldots, (T_d)_n]^T \\ (T_d)_i &= (\delta T)_i sgn(s_i) \quad i = 1, \ldots, n \\ \delta T &= \delta M(x)|\ddot{x}_{eq}| + \delta C(x, \dot{x})|\dot{x}_{eq}| + \delta G(x) + \delta\tilde{F}(x, t) \\ \epsilon &> 0 \quad sgn(s) = [sgn(s_1), \ldots, sgn(s_n)]^T \end{aligned} \tag{18}$$

D is any positive definite matrix and sgn(.) is the sign function. Moreover, the reaching time t_r in which the system reaches the sliding mode is

$$t_r \le t_{\max} \quad t_{\max} = \frac{2}{c_3} ln(1 + \frac{c_3}{c_4}\sqrt{V_0}) \tag{19}$$

where

$$c_3 = \frac{2\lambda_{\min}(D)}{k_x''} \quad c_4 = \epsilon\sqrt{\frac{2}{k_x''}}$$

$$V_0 = \frac{1}{2}s_0^T M(x_0)s_0 \quad s_0 = s(e_0, \dot{e}_0, z_0)$$

and the reaching transient response is shaped by

$$||s|| \le \sqrt{\frac{2}{k_x'}}[(\sqrt{V_0} + \frac{c_4}{c_3})\exp^{-\frac{c_3}{2}t} - \frac{c_4}{c_3}] \tag{20}$$

where $\lambda_{\min}(D)$ means minimum eigenvalue of the matrix D.

Proof: For the robot manipulator (5), we choose a Lyapunov function as

$$V = \frac{1}{2}s^T M(x)s \tag{21}$$

From property 3, we have

$$\frac{1}{2}k_x'||s||^2 \le V \le \frac{1}{2}k_x''||s||^2 \tag{22}$$

Differentiating V with respect to time yields

$$\begin{aligned} \dot{V} &= s^T M\dot{s} + \frac{1}{2}s^T\dot{M}s \\ &= s^T M(\ddot{x} - \ddot{x}_{eq}) + s^T C(x, \dot{x})s \\ &= s^T[T - M\ddot{x}_{eq} - C\dot{x}_{eq} - G(x) - F - \tilde{F}(t)] \end{aligned} \tag{23}$$

where property 4 has been used to eliminate the term $1/2s^T\dot{M}s$ owing to the time nature of inertia matrix. Substituting control torque (17) into (23) and noticing (22), we have

$$\begin{aligned} \dot{V} = &- s^T Ds - \epsilon s^T sgn(s) - s^T T_d \\ &- s^T[\Delta M\ddot{x}_{eq} + \Delta C\dot{x}_{eq} + \Delta G + \tilde{F}] \\ \le &- s^T Ds - \epsilon\sum_{i=1}^{n}|s_i| - s^T T_d + |s|^T\delta T \\ \le &- \lambda_{\min}(D)||s||^2 - \epsilon||s|| \\ \le &- c_3 V - c_4\sqrt{V} \end{aligned} \tag{24}$$

So

$$\sqrt{V} \leq (\sqrt{V_0} + \frac{c_4}{c_3}) \exp^{-\frac{c_3}{2}t} - \frac{c_4}{c_3} \quad (25)$$

which means that in finite time $V = 0$, i.e., $s = 0$. Moreover, from (22), the reaching transient response is shaped by (20). The upper limit $t_{\max}$ of the reaching time t_r is solved by setting the right side of (25) equal to zero, which is given by (19). Hence, the theorem is proved. QED.

Remark 2: In the above theorem, the role of the discontinuous torque T_d is to overcome the modeling errors so that the system reaches the sliding mode in finite time. Introducing the discontinuous term $\epsilon sgn(s)$ enhances this effect and enables us to explicitly control the reaching transient. As can be seen from (19), the larger ϵ and $\lambda_{\min}(D)$ are, the smaller $t_{\max}$ will be, i.e., the reaching transient will be shorter. However, if ϵ is large, in practice a strong chattering would probably appear, owing to its discontinuity. Therefore, the better choice is to take small ϵ and large $\lambda_{\min}(D)$, so that the reaching transient is rapid enough and at the same time the chattering is relatively small [22].

Remark 3: In contrast with [13, 14], in which the system response asymptotically follows the desired impedance, here the desired impedance is achieved in finite time and the reaching transient response is also guaranteed.

Remark 4: The $2n$-dimensional robot manipulator equation (5) incorporating the n-dimensional dynamic compensator (12) forms a new $3n$-dimensional system. The dynamic sliding mode equation are $2n$-dimensional motion, which can be selected to be identified with the desired model (here, the desired impedance). This is the main characteristic of the VSMRC strategy.

For real-time implementation, the control torque should be converted into the joint space.

Theorem 2: For the robot manipulator (1) with the modeling errors (4), the system achieves the desired impedance (11) if the following control torque is implemented:

$$\tau = \hat{M}(q)\ddot{q}_{eq} + \hat{C}(q,\dot{q})\dot{q}_{eq} + \hat{G}(q)$$
$$-J^T(T_d + Ds + \epsilon sgn(s)) + J^T F \quad (26)$$

where

$$\dot{q}_{eq} = J^{-1}\dot{x}_{eq}$$
$$\ddot{q}_{eq} = J^{-1}(\ddot{x}_{eq} - \dot{J}\dot{q}_{eq}) \quad (27)$$

Remark 5: The control law (26) is discontinuous across sliding surface. Such a control law leads to control chattering in practice. Chattering is undesirable because it involves high control activity and may excite the high-frequency dynamics neglected in the course of modeling. We can use the concept of boundary layer [23] to eliminate the phenomenon, that is, replacing $sgn(s_i)$ by saturation function sat (s_i/Δ_i) where Δ_i is the boundary layer thickness. This leads to the system response within a guaranteed precision as shown in the simulation.

From the above theorem, by application of the control torque (26), the controlled robot will behave according to the desired impedance (11) in a finite time. The overall stability of the system is then equal to the closed-loop system stability of the desired impedance (11) and the interactive environment dynamics. It is assumed that the desired impedance has been specified in such a way that when it is combined with the environment dynamics, the resulted closed-loop system is stable. Thus the overall stability of the suggested method can be guaranteed.

Since the specification of the desired impedance to achieve the closed-loop system stability depends on the environment dynamics, we do not discuss it in general. We use one example to illustrate it. Without loss of generality, suppose that the environment has the following dynamics

$$M_e\ddot{x} + B_e\dot{x} + K_e(x - x_e) = F \quad (28)$$

where x_e is the equilibrium position, M_e, B_e, K_e are the inertia, damping, and stiffness of the environment respectively, and are assumed to be semi-positive definite matrices. Combining (28) with (11), the closed-loop system is expressed as

$$(M_m + K_f M_e)\ddot{x} + (B_m + K_f B_e)\dot{x} + (K_m + K_f K_e)x$$
$$= M_m\ddot{x}_d + B_m\dot{x}_d + K_m x_d + K_f F_d + K_f K_e x_e \quad (29)$$

Let $K_f = I$, and M_m, B_m, K_m in the desired impedance be positive definite matrices; the closed-loop system (29) is stable.

We now show that the proposed controller can achieve force tracking control with some special environment constraints.

3.1 Robust Hybrid Position and Force Control

When the robot comes in contact with the environment, in the normal of the contact surfaces, contact force needs to be controlled, while along the tangent of the contact surfaces, motion control is needed [2, 3].

Here, we will consider a special case of this problem. The normal directions are assumed along some coordinate axes that are denoted as the x_f subspace, and the tangent directions are along other coordinate axes that are denoted as the x_p subspace. Suppose that in the constrained subspace x_f, the environment is assumed to be an elastic model with known stiffness K_e (either from force sensor or from the environment). Then, the interaction force F is

$$F = \begin{bmatrix} F_f \\ F_p \end{bmatrix} \quad F_f = K_e x_f \quad (30)$$

where F_p is tangential friction force. The desired impedance (11) is now specified as

$$\begin{bmatrix} M_{m1} & \\ & M_{m2} \end{bmatrix}\ddot{e} + \begin{bmatrix} B_{m1} & \\ & B_{m2} \end{bmatrix}\dot{e} + \begin{bmatrix} K_{m1} & \\ & K_{m2} \end{bmatrix}e = -\begin{bmatrix} I & \\ & 0 \end{bmatrix}e_f \quad (31)$$

$$e = \begin{bmatrix} e_{x_f} \\ e_{x_p} \end{bmatrix} \quad e_f = \begin{bmatrix} e_{F_f} \\ e_{F_p} \end{bmatrix}$$

$$x_d(t) = \begin{bmatrix} x_{fd}(t) \\ x_{pd}(t) \end{bmatrix} \quad F_d(t) = \begin{bmatrix} F_{fd}(t) \\ 0 \end{bmatrix}$$

where M_{mi}, B_{mi}, K_{mi} are any positive definite matrix, $x_{pd}(t)$ is the desired motion trajectory in the tangent, and $F_{fd}(t)$ is the desired normal contact force. The desired trajectory $x_{fd}(t)$ is specified as

$$x_{fd}(t) = K_e^{-1} F_{fd}(t) \tag{32}$$

The compensator (12) is calculated by (16) and the control torque is given by (26) so that the desired impedance (31) is achieved. Then, substituting (30) and (32) into (31), the closed-loop equation is given by

$$M_{m1}\ddot{e}_{x_f} + B_{m1}\dot{e}_{x_f} + (K_{m1} + K_e)e_{x_f} = 0$$

$$M_{m2}\ddot{e}_{x_p} + B_{m2}\dot{e}_{x_p} + K_{m2}e_{x_p} = 0 \tag{33}$$

from which we obtain

$$e_{x_f} \to 0 \quad e_{x_p} \to 0 \quad e_{F_f} = K_e \quad e_{x_f} \to 0$$

Hence, in the constrained subspace, the system exerts the desired contact force $F_{fd}(t)$ on the environment, while in the unconstrained subspace x_p, the system follows the desired motion $x_{pd}(t)$ against friction force F_p of the environment.

3.2 Robust Constrained Motion Control

When the robot is in contact with rigid surfaces, kinematic constraints are imposed on the manipulator motion [4, 5]. Supposing that the constraint surfaces are described as

$$\Phi(x) = 0 \quad \Phi(x) = [\phi_1(x), \ldots, \phi_m(x)]^T \tag{34}$$

Then in the absence of friction force, the generalized constrained force [4, 5] is given by

$$F = \left(\frac{\partial \Phi}{\partial x}\right)^T F_f \quad F_f \in R^m \tag{35}$$

where F_f is a vector of generalized multipliers associated with the constraints. In this case, the motion on the constraint surface (34) and the generalized constrained force F, or the generalized multipliers vector F_f, have to be controlled. The problem has been extensively studied in recent years as a singular system of differential equation [4, 5].

We now consider a particular case of the problem where the constraint surfaces are assumed along some coordinate axes, that is,

$$x_f = 0 \quad x_f = [x_1, \ldots, x_m]^T \tag{36}$$

The motion on the constraint surface (34) is then completely described by the $x_p = [x_{m+1}, \ldots, x_n]$ subspace. The interaction force F is given by

$$F = \begin{bmatrix} F_f \\ F_p \end{bmatrix}$$

where friction force F_p is also considered.

The desired impedance is specified as

$$\begin{bmatrix} M_{m1} & \\ & M_{m2} \end{bmatrix}\ddot{e} + \begin{bmatrix} 0 & \\ & B_{m2} \end{bmatrix}\dot{e} + \begin{bmatrix} 0 & \\ & K_{m2} \end{bmatrix} e = -\begin{bmatrix} I & \\ & 0 \end{bmatrix} e_f \tag{37}$$

$$x_d(t) = \begin{bmatrix} 0 \\ x_{pd}(t) \end{bmatrix} \quad F_d(t) = \begin{bmatrix} F_{fd}(t) \\ 0 \end{bmatrix}$$

where M_{m1}, B_{m2}, K_{m2} are any positive definite matrix. The compensator (12) is calculated by (16) and the control torque is given by (26) so that the desired impedance (37) is achieved. On the constraint surfaces (36), from (37), we have

$$F_f - F_{fd}(t) = 0$$

$$M_{m2}\ddot{e}_{x_p} + B_{m2}\dot{e}_{x_p} + K_{m2}e_{x_p} = 0 \tag{38}$$

That is, the robot exerts the desired generalized constrained force $F_{fd}(t)$ on the environment. In the unconstrained subspace x_p, clearly, the robot follows the desired motion $x_{pd}(t)$.

4. Simulation

Figure 1 shows a two-link robot manipulator moving on a vertical surface S. The dynamic equation of the robot is given by

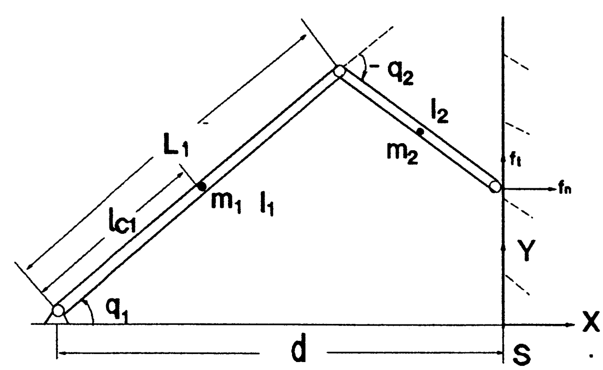

Figure 1. Configuration of the robot.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + J^T(q)F = \tau \tag{39}$$

where

$$M = \begin{bmatrix} m_1 l_{c1}^2 + m_2(L_1^2 + l_{c2}^2) + I_1 + I_2 + 2m_2 L_1 l_{c2}\cos q_2 & m_2 L_1 l_{c2}\cos q_2 + m_2 l_{c2}^2 + I_2 \\ m_2 L_1 l_{c2}\cos q_2 + m_2 l_{c2}^2 + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -m_2 L_1 l_{c2}\dot{q}_2 \sin q_2 & -m_2 L_1 l_{c2}(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ m_2 L_1 l_{c2}\dot{q}_1 \sin q_2 & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} (m_1 l_{c1} + m_2 L_1) g \cos q_1 + m_2 l_{c2} g \cos(q_1+q_2) \\ m_2 l_{c2} g \cos(q_1 + q_2) \end{bmatrix}$$

$$J(q) = \begin{bmatrix} -L_1 \sin q_1 - L_2 \sin(q_1+q_2) & -L_2 \sin(q_1+q_2) \\ L_1 \cos q_1 + L_2 \cos(q_1+q_2) & L_2 \cos(q_1+q_2) \end{bmatrix}$$

Actual parameter values of the robot are

$$L_1 = 1m \quad L_2 = 0.5m \quad l_{c1} = 0.5m \quad l_{c2} = 0.25m$$

$$I_1 = 2kgm^2 \quad I_2 = 2kgm^2 \quad m_1 = 10kg \quad m_2 = 10kg \quad d = 1m$$

The exact values of m_2, I_2 are assumed to be unknown with their estimated values and parameter boundaries given as

$$\hat{m}_2 = 5kg \quad \hat{I}_2 = 1kgm^2 \quad \delta m_2 = 5kg \quad \delta I_2 = 1kgm^2$$

The boundary of the modeling errors (4) is now given by

$$\delta M = \begin{bmatrix} \delta m_2(L_1^2 + l_{c2}^2 + 2L_1 l_{c2}|\cos q_2|) + \delta I_2 & \delta m_2(L_1 l_{c2}|\cos q_2| + l_{c2}^2) + \delta I_2 \\ \delta m_2(L_1 l_{c2}|\cos q_2| + l_{c2}^2) + \delta I_2 & \delta m_2 l_{c2}^2 + \delta I_2 \end{bmatrix}$$

$$\delta C = \begin{bmatrix} \delta m_2 L_1 l_{c2}|\dot{q}_2 \sin q_2| & \delta m_2 L_1 l_{c2}|(\dot{q}_1+\dot{q}_2)\sin q_2| \\ \delta m_2 L_1 l_{c2}|\dot{q}_1 \sin q_2| & 0 \end{bmatrix}$$

$$\delta G = \begin{bmatrix} \delta m_2 g|L_1 \cos q_1 + l_{c2}\cos(q_1+q_2)| \\ \delta m_2 l_{c2} g|\cos(q_1+q_2)| \end{bmatrix}$$

In the first simulation, the surface S is assumed to be elastic with the equilibrium position at $x = 0$. The interaction force F on the environment is

$$F = \begin{bmatrix} f_n \\ f_t \end{bmatrix} \quad f_n = k_e x \quad f_t = \mu |f_n| sgn(\dot{y})$$

$$k_e = 4000N/m \quad \mu = 0.2 \quad x > 0$$

where μ is the coefficient of dry friction between the end-effector and the surface S. The desired impedance (31) is chosen as

$$M_{m1}(\ddot{e}_x + b_1\dot{e}_x + k_1 e_x) = -e_{f_n}$$
$$\ddot{e}_y + b_2\dot{e}_y + k_2 e_y = 0 \tag{40}$$

where

$$M_{m1} = 10 \quad b_1 = 40 \quad k_1 = 0 \quad b_2 = 30 \quad k_2 = 225$$

which ensure the closed-loop system response (33) is decoupled and critically damped. The switching function is chosen as

$$s(e,\dot{e},z) = \dot{e} + z \tag{41}$$

and the dynamic compensator determined by (12) and (16) is

$$\dot{z} = Az + K_{pz}e + K_{vz}\dot{e} + K_{fz}e_f$$

where

$$A = \begin{bmatrix} -10 & \\ & -10 \end{bmatrix} \quad K_{pz} = \begin{bmatrix} 0 & \\ & 225 \end{bmatrix}$$

$$K_{vz} = \begin{bmatrix} 30 & \\ & 20 \end{bmatrix} \quad K_{fz} = \begin{bmatrix} 0.1 & \\ & 0 \end{bmatrix}$$

The control torque is calculatedby (26) where sgn (s_i) is replaced by sat (s_i/Δ_i) and the parameter values of the controller used are

$$D = \begin{bmatrix} 1000 & \\ & 1000 \end{bmatrix} \quad \epsilon = 1 \quad \Delta = \begin{bmatrix} 0.03 \\ 0.05 \end{bmatrix}$$

Simulation results are shown in Figures 2–6. Figures 2 and 4 show the time responses of motion and contact force, which verify the motion and force tracking control of the proposed VSC impedance controller. The time response of the switching function in Figure 5 implies that the desired impedance is achieved. Robot joint torques of Figure 6 demonstrate the elimination of chattering by use of the boundary layer technique.

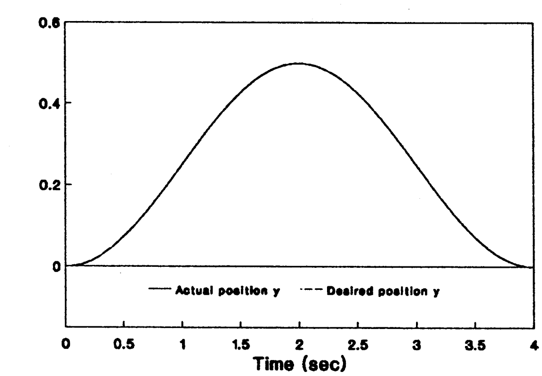

Figure 2. Time response of position y.

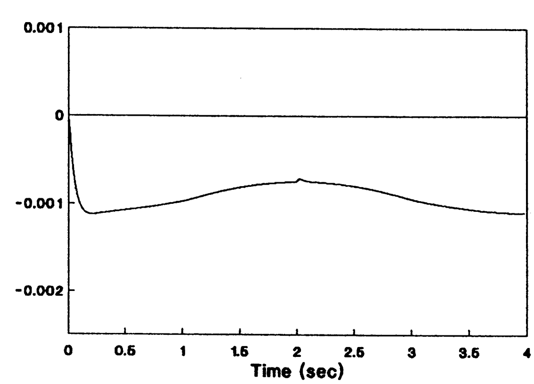

Figure 3. Tracking error of position y.

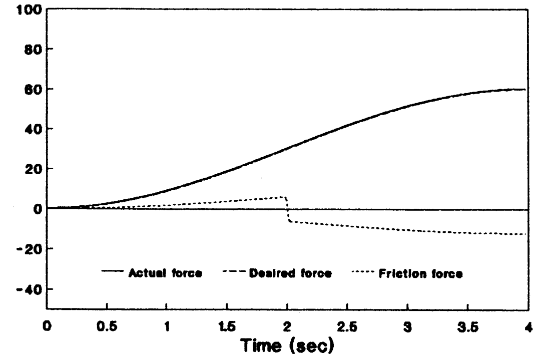

Figure 4. Time response of interaction force.

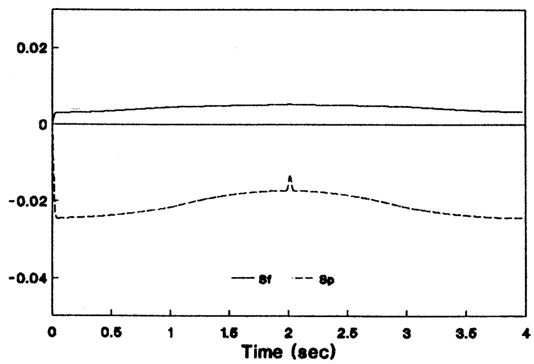

Figure 5. Time response of sliding surface.

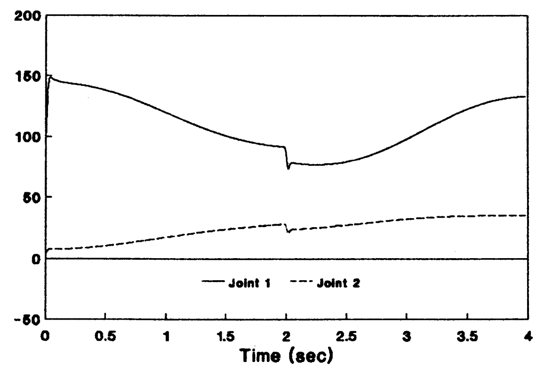

Figure 6. Joint torque of the robot.

In the second simulation, the surface S is assumed to be rigid. The constraint function (34) is given as

$$x(t) = 0$$

The interaction force is given by

$$F = \begin{bmatrix} f_n \\ f_t \end{bmatrix} \quad f_t = \mu |f_n| sgn(\dot{y}) \quad \mu = 0.2$$

The switching function is the same as (41). The parameter values of the desired impedance (40) and the controller are chosen as

$$M_{m1} = 100 \quad b_1 = 0 \quad k_1 = 0 \quad b_2 = 30 \quad k_2 = 225$$

$$A = \begin{bmatrix} 0 & \\ & -10 \end{bmatrix} \quad D = \begin{bmatrix} 1000 & \\ & 1000 \end{bmatrix} \quad \epsilon = 1 \quad \Delta = \begin{bmatrix} 0.03 \\ 0.05 \end{bmatrix}$$

The simulation results of Figures 7–10 verify the motion and force tracking control of the proposed impedance controller in constrained motion control.

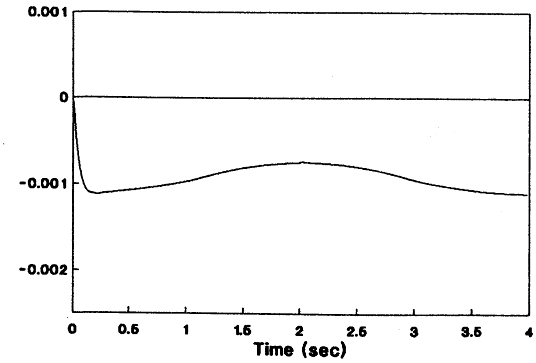

Figure 7. Tracking error of position y.

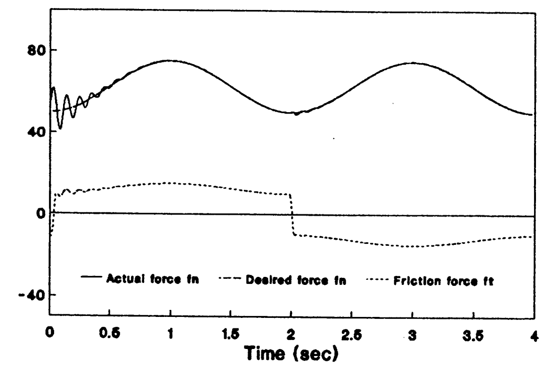

Figure 8. Time response of interaction force.

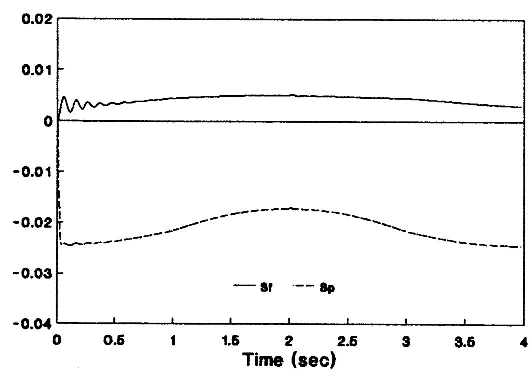

Figure 9. Time response of sliding surface.

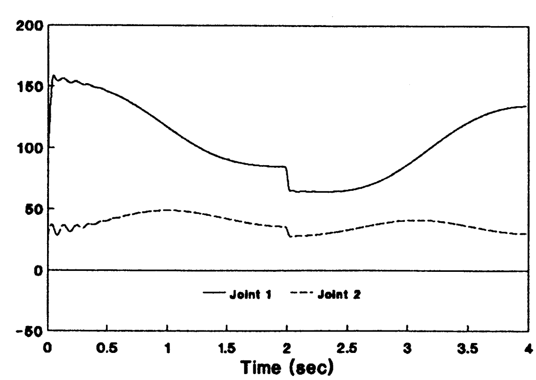

Figure 10. Joint torque of the robot.

5. Conclusion

In this article, we have considered the applications of VSC to the impedance control problem of robot manipulators. The specification of the desired impedance [13, 14] is generalized to include the desired interaction force so that force tracking control can be achieved with some special environment constraints. The motion error and the interaction force error are related by a second-order impedance function. Based on VSMRC, the desired impedance is achieved in the sliding mode in finite time and is robust against both parametric uncertainties and external disturbances. The reaching transient can also be guaranteed with prescribed quality. Simulation results verify the performance of the proposed controller.

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