



# Non-linear adaptive robust control of electro-hydraulic systems driven by double-rod actuators

BIN YAO†\*, FANPING BU† and GEORGE T. C. CHIU†

This paper studies the high performance robust motion control of electro-hydraulic servo-systems driven by double-rod hydraulic actuators. The dynamics of hydraulic systems are highly non-linear and the system may be subjected to non-smooth and discontinuous non-linearities due to directional change of valve opening, friction and valve overlap. Aside from the non-linear nature of hydraulic dynamics, hydraulic servosystems also have large extent of model uncertainties. To address these challenging issues, the recently proposed adaptive robust control (ARC) is applied and a discontinuous projection based ARC controller is constructed. The resulting controller is able to take into account the effect of the parameter variations of the inertia load and the cylinder hydraulic parameters as well as the uncertain non-linearities such as the uncompensated friction forces and external disturbances. Non-differentiability of the inherent non-linearities associated with hydraulic dynamics is carefully examined and addressing strategies are provided. Compared with previously proposed ARC controller, the controller in the paper has a more robust parameter adaptation process and may be more suitable for implementation. Finally, the controller guarantees a prescribed transient performance and final tracking accuracy in the presence of both parametric uncertainties and uncertain non-linearities while achieving asymptotic tracking in the presence of parametric uncertainties.

## 1. Introduction

Hydraulic systems have been used in industry in a wide number of applications by virtue of their small size-to-power ratios and the ability to apply very large forces and torques; examples like electro-hydraulic positioning systems (FitzSimons and Palazzolo 1996) and active suspension control (Alleyne and Hedrick 1995, Alleyne 1996). However, hydraulic systems also have a number of characteristics which complicate the development of high performance closed-loop controllers, and advanced control techniques have not been developed to address those issues. This leads to the urgent need for advancing hydraulics technologies by combining the high power of hydraulic actuation with the versatility of electronic control (Yao *et al.* 1998).

The dynamics of hydraulic systems are highly non-linear (Merritt 1967). Furthermore, the system may be subjected to non-smooth and discontinuous non-linearities due to control input saturation, directional change of valve opening, friction, and valve overlap. Aside from the non-linear nature of hydraulic dynamics which demands the use of non-linear control, hydraulic servosystems also have large extent of model uncertainties. The uncertainties can be classified into two categories: *parametric uncertainties* and *uncertain non-linearities*. Examples of parametric uncertainties include the large changes in load seen by the system in industrial use and the large variations in the hydraulic parameters (e.g. bulk modulus) due to the change of temperature

and component wear (Whatton 1989). Other general uncertainties, such as the external disturbances, leakage, and friction, cannot be modelled exactly and the non-linear functions that describe them are not known. These kinds of uncertainties are called uncertain non-linearities. These model uncertainties may cause the controlled system, designed on the nominal model, to be unstable or have a much degraded performance. Non-linear robust control techniques, which can deliver high performance in spite of both parametric uncertainties and uncertain non-linearities, are essential for successful operations of high-performance hydraulic systems.

In the past, much of the work in the control of hydraulic systems uses linear control theory (Tsao and Tomizuka 1994, Jeronimo and Muto 1996, Bobrow and Lum 1996, FitzSimons and Palazzolo 1996, Plummes and Vaughan 1996) and feedback linearization techniques (Re and Isidori 1995, Vossoughi and Donath 1995, Schl and Bobrow 1999). Alleyne and Hedrick (1995) applied the non-linear adaptive control to the force control of an active suspension driven by a double-rod cylinder. They demonstrated that non-linear control schemes can achieve a much better performance than conventional linear controllers. They considered the parametric uncertainties of the cylinder only. The results are also extended to the trajectory tracking in Alleyne (1998).

It is noted that none of the above adaptive electro-hydraulic algorithms address the effect of the unavoidable uncertain non-linearities well and the transient performance of these algorithms is normally unknown. Recently, an adaptive robust control (ARC) approach has been proposed in (Yao and Tomizuka 1994, 1997, 1999, Yao 1997) for high performance robust control of

Received 1 November 1999. Revised 1 December 2000.

\* Author for correspondence e-mail: byao@ecn.purdue.edu

† School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, USA.

uncertain non-linear systems in the presence of both parametric uncertainties and uncertain non-linearities. The approach effectively combines the design techniques of adaptive control (AC) (Slotine and Li 1988, Krsti *et al.* 1992, 1995) and those of deterministic robust control (DRC) (Corless and Leitmann 1981, Utkin 1992) (e.g. sliding mode control, SMC) and improves performance by preserving the advantages of both AC and DRC. In Yao *et al.* (1997b), the ARC approach was generalized to provide a rigorous theoretic framework for the high performance robust control of a one DOF electro-hydraulic servo-system by taking into account the particular non-linearities and model uncertainties of the electro-hydraulic servo-systems. A novel strategy was provided to overcome the difficulty in carrying out the backstepping design via ARC Lyapunov function (Yao and Tomizuka 1999) caused by the non-smooth non-linearities of the hydraulic dynamics.

This paper continues the work done in Yao *et al.* (1997b) and will construct a simpler but more robust ARC controller for electro-hydraulic servo-system. Specifically, in Yao *et al.* (1997b), smooth projections (Yao and Tomizuka 1997, 1999) were used to solve the design conflicts between adaptive control technique and robust control technique, which is technical and may not be convenient for practical implementation. Here, instead of using the smooth projection (Yao *et al.* 1997b), the widely used discontinuous projection method in adaptive systems (Goodwin and Mayne 1989, Sastry and Bodson 1989) will be used to solve the conflicts between the robust control design and adaptive control design. As a result, the resulting controller becomes simpler and the parameter adaptation process is more robust in the presence of uncertain non-linearities. The discontinuous projection method has been successfully implemented and tested in the motion control of robot manipulators (Yao and Tomizuka 1994) and the motion control of machine tools (Yao *et al.* 1997a), in which the design techniques for both systems are essentially for non-linear systems with 'relative degree' of one. For non-linear systems with 'relative degree' of more than one, the underlining parameter adaptation laws in the previously proposed ARC controllers (Yao and Tomizuka 1997, 1999) and the robust adaptive control designs (Polycarpou and Ioannou 1993, Freeman *et al.* 1996) are based on the tuning function based adaptive backstepping design (Krstic *et al.*, 1995), which needs to incorporate the adaptation law in the design of control functions at each step. As a result, either smooth projections (Yao and Tomizuka 1997, 1999) or smooth modifications of adaptation law (Freeman *et al.* 1996, Yao and Tomizuka 1996) are necessary since the control functions have to be smooth for backstepping design (Krstic *et al.* 1995, Lin 1997); either method is technical

and may be hard to implement. Only recently, Yao (1997) is able to construct simple ARC controllers for non-linear systems with 'relative degree' of more than one by using a discontinuous projection method. However, the scheme in Yao (1997) cannot be directly applied to the control of electro-hydraulic servo systems studied here since, as will be shown in the paper, parametric uncertainties will also appear in the input channel of each layer (Yao and Tomizuka 1999). Therefore, the paper not only constructs a practical ARC controller for electro-hydraulic servo systems but also extends the theoretical results in Yao (1997). Extensive simulation results are obtained to illustrate the effectiveness of the proposed method.

## 2. Problem formulation and dynamic models

The system under consideration is the same as that in Yao *et al.* (1997b), which is depicted in figure 1. The goal is to have the inertia load to track any specified motion trajectory as closely as possible; examples like a machine tool axis (Yao *et al.* 1997a).

The dynamics of the inertia load can be described by

$$m\ddot{x}_L = P_L A - b\dot{x}_L - F_{fc}(\dot{x}_L) + \tilde{f}(t, x_L, \dot{x}_L) \quad (1)$$

where  $x_L$  and  $m$  represent the displacement and the mass of the load respectively,  $P_L = P_1 - P_2$  is the load pressure of the cylinder,  $A$  is the ram area of the cylinder,  $b$  represents the combined coefficient of the modelled damping and viscous friction forces on the load and the cylinder rod,  $F_{fc}$  represents the modelled Coulomb friction force, and  $\tilde{f}(t, x_L, \dot{x}_L)$  represents the external disturbances as well as terms like the unmodelled friction forces. Neglecting the effect of leakage flows in the cylinder and the servovalve, the actuator (or the cylinder) dynamics can be written as (Merritt 1967)

$$\frac{V_t}{4\beta_e} \dot{P}_L = -A\dot{x}_L - C_{im}P_L + Q_L \quad (2)$$

where  $V_t$  is the total volume of the cylinder and the hoses between the cylinder and the servovalve,  $\beta_e$  is the effective bulk modulus,  $C_{im}$  is the coefficient of the total internal leakage of the cylinder due to pressure,

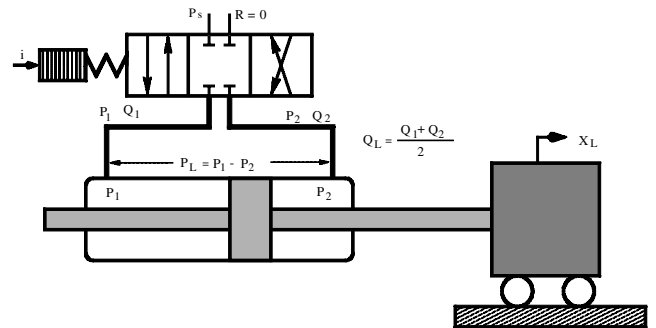


Figure 1. A one DOF electro-hydraulic servo system.

and  $Q_L$  is the load flow.  $Q_L$  is related to the spool valve displacement of the servovalve,  $x_v$ , by (Merritt 1967)

$$Q_L = C_d w x_v \sqrt{\frac{P_s - \text{sgn}(x_v) P_L}{\rho}} \quad (3)$$

where  $C_d$  is the discharge coefficient,  $w$  is the spool valve area gradient, and  $P_s$  is the supply pressure of the fluid.

For simplicity, the same servovalve as in Alleyne (1996) will be used in this study; the spool valve displacement  $x_v$  is related to the current input  $i$  by a first-order system given by

$$\tau_v \dot{x}_v = -x_v + K_v i \quad (4)$$

where  $\tau_v$  and  $K_v$  are the time constant and gain of the servo-valve respectively.

As seen in the simulation, scaling of state variables is also very important in minimizing the numerical error and facilitating the gain-tuning process. For this purpose, we introduced scaling factors to the load pressure and valve opening as  $\bar{P}_L = P_L/S_{c3}$  and  $\bar{x}_v = x_v/S_{c4}$ , where  $S_{c3}$  and  $S_{c4}$  are constant scaling factors. The entire system, equations (1)–(3) and (4), can be rewritten as

$$\left. \begin{aligned} \dot{x}_L &= \frac{AS_{c3}}{m} \left( \bar{P}_L - \frac{b}{AS_{c3}} \dot{x}_L - \frac{F_{fc}(x_L)}{AS_{c3}} \right) + d(t, x_L, \dot{x}_L) \\ d &= \frac{1}{m} \tilde{f}(t, x_L, \dot{x}_L) \\ \dot{\bar{P}}_L &= \frac{4\beta_e S_{c4} C_d w}{V_t \sqrt{S_{c3}} \sqrt{\rho}} \left[ -\frac{A}{S_{c4} \sqrt{S_{c3}} C_d w} \dot{x}_L \right. \\ &\quad \left. - \frac{\sqrt{S_{c3}} \sqrt{\rho}}{S_{c4} C_d w} C_{im} \bar{P}_L + g_3(\bar{P}_L, \bar{x}_v) \bar{x}_v \right] \\ \dot{\bar{x}}_v &= -\frac{1}{\tau_v} \bar{x}_v + \frac{K_v}{S_{c4} \tau_v} u \end{aligned} \right\} \quad (5)$$

where  $g_3(\bar{P}_L, \bar{x}_v) = \sqrt{\bar{P}_s - \text{sgn}(\bar{x}_v) \bar{P}_L}$ ,  $\bar{P}_s = P_s/S_{c3}$ , and  $u = i$  is the control input. Define the state variables  $x = [x_1, x_2, x_3, x_4]^T \triangleq [x_L, \dot{x}_L, \bar{P}_L, \bar{x}_v]^T$ . The system can be expressed in state space form as

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{AS_{c3}}{m} (x_3 - \bar{b}x_2 - \bar{F}_{fc}) + d(t, x_1, x_2) \\ \dot{x}_3 &= \frac{4\beta_e S_{c4} C_d w}{V_t \sqrt{S_{c3}} \sqrt{\rho}} [-\bar{A}x_2 - \bar{C}_{im}x_3 + g_3(x_3, x_4)x_4] \\ \dot{x}_4 &= -\frac{1}{\tau_v} x_4 + \frac{\bar{K}_v}{\tau_v} u \end{aligned} \right\} \quad (6)$$

where

$$\begin{aligned} \bar{b} &= \frac{1}{AS_{c3}} b, & \bar{F}_{fc} &= \frac{1}{AS_{c3}} F_{fc}, \\ \bar{A} &= \frac{1}{S_{c4} \sqrt{S_{c3}} C_d w} A, & \bar{C}_{im} &= \frac{\sqrt{S_{c3}} \sqrt{\rho}}{S_{c4} C_d w} C_{im}, \quad \text{and} \\ \bar{K}_v &= \frac{1}{S_{c4}} K_v \end{aligned}$$

Given the desired motion trajectory  $x_{Ld}(t)$ , the objective is to synthesize a control input  $u$  such that the output  $y = x_1$  tracks  $x_{Ld}(t)$  as closely as possible in spite of various model uncertainties.

### 3. Discontinuous projection based adaptive robust control of electro-hydraulic servo systems

#### 3.1. Design model and issues to be addressed

To begin the controller design, practical and reasonable assumptions on the system have to be made. In general, the system is subjected to parametric uncertainties due to the variations of  $m$ ,  $b$ ,  $F_{fc}$ ,  $\beta_e$ ,  $C_{im}$ ,  $C_d$ ,  $\rho$ ,  $\tau$  and  $K$ . For simplicity, in this paper we only consider the parametric uncertainties of important parameters like  $m$ ,  $\beta_e$ , and the nominal value of the disturbance  $d$ ,  $d_n$ ; the importance of estimating  $m$  and  $d_n$  for the precision control of an inertia load can be partly seen from the experimental results obtained for machine tools (Yao *et al.* 1997a). Other parametric uncertainties can be dealt with in the same way if necessary. In order to use parameter adaptation to reduce parametric uncertainties to improve performance, it is necessary to linearly parametrize the state space equation (6) in terms of a set of unknown parameters. To this end, define the unknown parameter set  $\theta = [\theta_1, \theta_2, \theta_3]^T$  as

$$\theta_1 = \frac{AS_{c3}}{m}, \quad \theta_2 = d_n, \quad \theta_3 = \frac{4\beta_e S_{c4} C_d w}{V_t \sqrt{S_{c3}} \sqrt{\rho}}$$

The state space equation (6) can thus be linearly parametrized in terms of  $\theta$  as

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \theta_1 (x_3 - \bar{b}x_2 - \bar{F}_{fc}(x_2)) + \theta_2 + \tilde{d}(t, x_1, x_2), \\ \tilde{d} &= d(t, x_1, x_2) - d_n \\ \dot{x}_3 &= \theta_3 [-\bar{A}x_2 - \bar{C}_{im}x_3 + g_3(x_3, x_4)x_4] \\ \dot{x}_4 &= -\frac{1}{\tau_v} x_4 + \frac{\bar{K}_v}{\tau_v} u \end{aligned} \right\} \quad (7)$$

For most applications, the extent of the parametric uncertainties and uncertain non-linearities are known. Thus, the following practical assumption is made.

**Assumption 1:** Parametric uncertainties and uncertain non-linearities satisfy

$$\left. \begin{aligned} \theta \in \Omega_\theta \triangleq \{ \theta : \theta_{\min} < \theta < \theta_{\max} \} \\ |\tilde{d}(t, x_1, x_2)| \leq \delta_d(x_1, x_2, t) \end{aligned} \right\} \quad (8)$$

where

$\theta_{\min} = [\theta_{1\min}, \theta_{2\min}, \theta_{3\min}]^T$ ,  $\theta_{\max} = [\theta_{1\max}, \theta_{2\max}, \theta_{3\max}]^T$  and  $\delta_d(t, x_1, x_2)$  are known.

In (8),  $\bullet_i$  represents the  $i$ th component of the vector  $\bullet$  and the operation  $<$  for two vectors is performed in terms of the corresponding elements of the vectors. Physically,  $\theta_1 > 0$  and  $\theta_3 > 0$ . So, without loss of generality, it is also assumed that  $\theta_{1\min} > 0$  and  $\theta_{3\min} > 0$ .

At this stage, it is easy to see that the main difficulties in controlling (7) are: (a) the system has unmatched model uncertainties since parametric uncertainties and uncertain non-linearities appear in equations that do not contain control input  $u$ ; this difficulty can be overcome by employing backstepping design as done in the following; (b) the term  $g_3$ , which representing the non-linear static gain between the flow rate  $Q_L$  and the valve opening  $x_4$ , is a function of  $x_4$  also and is *non-smooth* since  $x_4$  appears through a discontinuous sign function  $\text{sgn}(x_4)$ ; this prohibits the direct application of the general results in Yao and Tomizuka (1996) to obtain an ARC controller. The effect of the directional change of the valve opening, i.e. the term  $\text{sgn}(x_4)$ , has been neglected in previous studies due to either technical requirements of the smoothness of all terms in the design, e.g. the conventional backstepping design in Alleyne and Hedrick (1995) and Alleyne (1998), or the use of linearization techniques (either around a nominal operational point for linear controller design or feedback linearization), which need the differentiability of all terms.

In the following, a discontinuous projection based ARC controller will be presented to solve the above design difficulties.

### 3.2. Notations and discontinuous projection mapping

Let  $\hat{\theta}$  denote the estimate of  $\theta$  and  $\tilde{\theta}$  the estimation error (i.e.  $\tilde{\theta} = \hat{\theta} - \theta$ ). Viewing (8), a simple discontinuous projection can be defined (Goodwin and Mayne 1989, Sastry and Bodson 1989) as

$$\text{Proj}_{\hat{\theta}}(\bullet) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{i\max} \text{ and } \bullet_i > 0 \\ 0 & \text{if } \hat{\theta}_i = \theta_{i\min} \text{ and } \bullet_i < 0 \\ \bullet & \text{otherwise} \end{cases} \quad (9)$$

By using an adaptation law given by

$$\dot{\hat{\theta}} = \text{Proj}_{\hat{\theta}}(\Gamma\tau) \quad (10)$$

where  $\Gamma > 0$  is a diagonal matrix and  $\tau$  is an adaptation function to be synthesized later. It can be shown (Yao and Tomizuka 1994) that for any adaptation function  $\tau$ , the projection mapping used in (10) guarantees

$$\left. \begin{aligned} (\text{P1}) \quad \hat{\theta} \in \bar{\Omega}_\theta \triangleq \{ \hat{\theta} : \theta_{\min} \leq \hat{\theta} \leq \theta_{\max} \} \\ (\text{P2}) \quad \tilde{\theta}^T (\Gamma^{-1} \text{Proj}_{\hat{\theta}}(\Gamma\tau) - \tau) \leq 0, \quad \forall \tau \end{aligned} \right\} \quad (11)$$

In the following,  $\hat{x}_2$  and  $\hat{x}_3$  are used to represent the calculable part of the  $\dot{x}_2$  and  $\dot{x}_3$  respectively, which are given by

$$\left. \begin{aligned} \hat{x}_2 &= \hat{\theta}_1(x_3 - \bar{b}x_2 - \bar{F}_{fc}(x_2)) + \hat{\theta}_2 \\ \hat{x}_3 &= \hat{\theta}_3[-\bar{A}x_2 - \bar{C}_{m1}x_3 + g_3(x_3, x_4)x_4] \end{aligned} \right\} \quad (12)$$

### 3.3. Controller design

The design parallels the recursive backstepping design procedure via ARC Lyapunov functions in Yao and Tomizuka (1999) and Yao (1997) as follows.

*Step 1:* Noting that the first equation of (7) does not have any uncertainties, an ARC Lyapunov function can thus be constructed for the first two equations of (7) directly. Define a switching-function-like quantity as

$$z_2 = \dot{e}_1 + k_p e_1 = x_2 - x_{2\text{eq}}, \quad x_{2\text{eq}} \triangleq \dot{x}_{1d} - k_p e_1 \quad (13)$$

where  $e_1 = x_1 - x_{1d}(t)$  is the output tracking error,  $x_{1d}(t)$  is the desired trajectory to be tracked by  $x_1$ , and  $k_p$  is any positive feedback gain. Since  $\bar{G}_s(s) = e_1(s)/z_2(s) = 1/(s + k_p)$  is a stable transfer function, making  $e_1$  small or converging to zero is equivalent to making  $z_2$  small or converging to zero. So the rest of the design is to make  $z_2$  as small as possible with a guaranteed transient performance. Differentiating (13) and noting (7)

$$\dot{z}_2 = \dot{x}_2 - \dot{x}_{2\text{eq}} = \theta_1(x_3 - \bar{b}x_2 - \bar{F}_{fc}) + \theta_2 + \tilde{d} - \dot{x}_{2\text{eq}}, \quad \dot{x}_{2\text{eq}} \triangleq \dot{x}_{1d} - k_p \dot{e}_1 \quad (14)$$

In (14), if we treat  $x_3$  as the input, we can synthesize a virtual control law  $\alpha_2$  for  $x_3$  such that  $z_2$  is as small as possible. Since (14) has both parametric uncertainties  $\theta_1$  and  $\theta_2$  and uncertain non-linearity  $\tilde{d}$ , the ARC approach proposed in Yao (1997) will be generalized to accomplish the objective. The generalization comes from the fact that parametric uncertainties appear in the input channel of (14) also while the system studied in Yao (1997) assumes no parametric uncertainties in the input channel of each step.

The control function  $\alpha_2$  consists of two parts given by

$$\left. \begin{aligned} \alpha_2(x_1, x_2, \hat{\theta}_1, \hat{\theta}_2, t) &= \alpha_{2a} + \alpha_{2s} \\ \alpha_{2a} &= \bar{b}x_2 + \bar{F}_{fc} + \frac{1}{\hat{\theta}_1}(\dot{x}_{2\text{eq}} - \hat{\theta}_2) \end{aligned} \right\} \quad (15)$$

in which  $\alpha_{2s}$  is a robust control law to be specified later, and  $\alpha_{2a}$  functions as an adjustable model compensation to reduce model uncertainties through on-line parameter adaptation given by (10). If  $x_3$  were the actual control input, then  $\tau$  in (10) would be

$$\tau_2 = w_2 \phi_2 z_2, \quad \phi_2 \triangleq \begin{bmatrix} \alpha_{2a} - \bar{b}x_2 - \bar{F}_{fc} \\ 1 \\ 0 \end{bmatrix} \quad (16)$$

In the tuning function based backstepping adaptive control (Krstic *et al.* 1995), one of the key points is to incorporate the adaptation function  $\tau$  (or tuning function) in the construction of control functions to compensate for the possible destabilizing effect of the time-varying adaptation law. Here, due to the use of discontinuous projection (9), the adaptation law (10) is discontinuous and thus *cannot* be used in the control law design at each step; backstepping design needs the control function synthesized at each step to be sufficiently smooth in order to obtain its partial derivatives. To compensate for this loss of information, the robust control law has to be strengthened; the robust control function  $\alpha_{2s}$  consists of two terms given by

$$\left. \begin{aligned} \alpha_{2s} &= \alpha_{2s1} + \alpha_{2s2} \\ \alpha_{2s1} &= -k_{2s1} z_2, \quad k_{2s1} \geq \frac{1}{\theta_{1\min}} (k_2 + \|C_{\phi 2} \Gamma \phi_2\|^2) \end{aligned} \right\} \quad (17)$$

where  $k_2$  is any positive feedback gain,  $C_{\phi 2}$  a positive definite constant diagonal matrix to be specified later, and  $\alpha_{2s2}$  is a robust control function designed as follows. Let  $z_3 = x_3 - \alpha_2$  denote the input discrepancy. Substituting (15) and (17) into (14) while noting (16)

$$\begin{aligned} \dot{z}_2 &= \theta_1 z_3 + \theta_1 \alpha_{2s} + \hat{\theta}_1 (\alpha_{2a} - \bar{b}x_2 - \bar{F}_{fc}) \\ &\quad - \tilde{\theta}_1 (\alpha_{2a} - \bar{b}x_2 - \bar{F}_{fc}) + \theta_2 + \tilde{d} - \dot{x}_{2eq} \\ &= \theta_1 z_3 - \theta_1 k_{2s1} z_2 + \theta_1 \alpha_{2s2} - \tilde{\theta}^T \phi_2 + \tilde{d} \end{aligned} \quad (18)$$

The robust control function  $\alpha_{2s2}$  is now chosen to satisfy the conditions

$$\left. \begin{aligned} \text{Condition i} \quad & z_2 [\theta_1 \alpha_{2s2} - \tilde{\theta}^T \phi_2 + \tilde{d}] \leq \varepsilon_2 \\ \text{Condition ii} \quad & z_2 \theta_1 \alpha_{2s2} \leq 0 \end{aligned} \right\} \quad (19)$$

where  $\varepsilon_2$  is a design parameter which can be arbitrarily small. Essentially, Condition i of (19) shows that  $\alpha_{2s2}$  is synthesized to dominate the model uncertainties coming from both parametric uncertainties  $\tilde{\theta}$  and uncertain nonlinearities  $\tilde{d}$ , and Condition ii is to make sure that  $\alpha_{2s2}$  is dissipating in nature so that it does not interfere with the functionality of the adaptive control part  $\alpha_{2a}$ . How to

choose  $\alpha_{2s2}$  to satisfy constraints like (19) can be found in Yao (1997) and Yao and Tomizuka (1997, 1999).

**Remark 1:** One example of a smooth  $\alpha_{2s2}$  satisfying (19) can be found in the following way. Let  $h_2$  be any smooth function satisfying

$$h_2 \geq \|\theta_M\|^2 \|\phi_2\|^2 + \delta_d^2 \quad (20)$$

where  $\theta_M = \theta_{\max} - \theta_{\min}$ . Then,  $\alpha_{2s2}$  can be chosen as

$$\alpha_{2s2} = -\frac{h_2}{2\theta_{1\min}\varepsilon_2} z_2 \quad (21)$$

It can be shown that (19) is satisfied (Yao and Tomizuka 1997). Other smooth or continuous examples of  $\alpha_{2s2}$  can be found in Yao (1997) and Yao and Tomizuka (1997, 1999).

Define a positive semi-definite (p.s.d.) function  $V_2$  as

$$V_2 = \frac{1}{2} w_2 z_2^2 \quad (22)$$

where  $w_2 > 0$  is a weighting factor. From (18), its time derivative is

$$\dot{V}_2 = w_2 \theta_1 z_2 z_3 + w_2 z_2 (\theta_1 \alpha_{2s2} - \tilde{\theta}^T \phi_2 + \tilde{d}) - w_2 \theta_1 k_{2s1} z_2^2 \quad (23)$$

*Step 2:* If we neglect the effect of the directional change of the valve opening (i.e.  $\text{sgn}(x_4)$ ) as in previous studies (Alleyne and Hedrick 1995),  $g_3$  in the third equation of (7) would be a function of  $x_3$  only. In that case, Step 2 would be to synthesize a control function  $\alpha_3$  for the virtual control  $x_4$  such that  $x_3$  tracks the desired control function  $\alpha_2$  synthesized in Step 1 with a guaranteed transient performance. Since most operations (especially the position or force regulation at the end of an operation) do involve the directional change of the valve opening, the effect of the discontinuous sign function  $\text{sgn}(x_4)$  will be carefully treated. Here, instead of defining  $x_4$  as the virtual control for the third equation of (7), we define the scaled actual load flow rate  $\bar{Q}_L = g_3(x_3, x_4)x_4$  as the virtual control, which makes physical sense since physically it is the flow rate that regulates the pressure inside the cylinder. Thus in this step, we will synthesize a control function  $\alpha_3$  for  $\bar{Q}_L$  such that  $x_3$  tracks the desired control function  $\alpha_2$  synthesized in Step 1 with a guaranteed transient performance.

Similar to (15), the control function  $\alpha_3$  consists of two parts given by

$$\alpha_3(\bar{x}_3, \hat{\theta}, t) = \alpha_{3a} + \alpha_{3s} \quad (24)$$

where  $\alpha_{3a}$  and  $\alpha_{3s}$  are synthesized as follows. From (15) and (7)

$$\begin{aligned}\dot{\alpha}_2 &= \frac{\partial \alpha_2}{\partial x_1} x_2 + \frac{\partial \alpha_2}{\partial x_2} \dot{x}_2 + \frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}} + \frac{\partial \alpha_2}{\partial t} \\ &= \dot{\alpha}_{2c} + \dot{\alpha}_{2u}\end{aligned}\quad (25)$$

where

$$\left. \begin{aligned}\dot{\alpha}_{2c} &= \frac{\partial \alpha_2}{\partial x_1} x_2 + \frac{\partial \alpha_2}{\partial x_2} \hat{x}_2 + \frac{\partial \alpha_2}{\partial t} \\ \dot{\alpha}_{2u} &= \frac{\partial \alpha_2}{\partial x_2} [-(x_3 - \bar{b}x_2 - \bar{F}_{fc})\tilde{\theta}_1 - \tilde{\theta}_2 + \tilde{d}] + \frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}}\end{aligned}\right\}\quad (26)$$

In (25),  $\dot{\alpha}_{2c}$  is calculable and can be used in the design of control functions but  $\dot{\alpha}_{2u}$  cannot due to various uncertainties. Therefore,  $\dot{\alpha}_{2u}$  has to be dealt with in this step design. Let  $z_4 = \bar{Q}_L - \alpha_3$ . From (7)

$$\dot{z}_3 = \theta_3 z_4 + \theta_3 [-\bar{A}x_2 - \bar{C}_{lm}x_3 + \alpha_3] - \dot{\alpha}_{2c} - \dot{\alpha}_{2u} \quad (27)$$

Consider the augmented p.s.d. function  $V_3$  given by

$$V_3 = V_2 + \frac{1}{2}w_3 z_3^2, \quad w_3 > 0 \quad (28)$$

Noting (23) and (26)

$$\begin{aligned}\dot{V}_3 &= \theta_1 w_2 z_2 z_3 + \dot{V}_2|_{\alpha_2} + w_3 z_3 \dot{z}_3 \\ &= \dot{V}_2|_{\alpha_2} + w_3 z_3 \theta_3 z_4 + w_3 z_3 \\ &\quad \times \left\{ \frac{w_2}{w_3} \theta_1 z_2 + \theta_3 [-\bar{A}x_2 - \bar{C}_{lm}x_3 + \alpha_3] - \dot{\alpha}_{2c} - \dot{\alpha}_{2u} \right\} \\ &= \dot{V}_2|_{\alpha_2} + \theta_3 w_3 z_3 z_4 + w_3 z_3 \theta_3 \alpha_{3s} + w_3 z_3 \\ &\quad \times \left\{ \hat{\theta}_3 \alpha_{3a} + \alpha_{3e} - \tilde{\theta}^T \phi_3 - \frac{\partial \alpha_2}{\partial x_2} \tilde{d} - \frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}} \right\}\end{aligned}\quad (29)$$

where  $\dot{V}_2|_{\alpha_2}$  denotes  $\dot{V}_2$  under the condition that  $x_3 = \alpha_2$  (or  $z_3 = 0$ ), and  $\alpha_{3e}$  and  $\phi_3$  are defined as

$$\left. \begin{aligned}\alpha_{3e} &= \frac{w_2}{w_3} z_2 \hat{\theta}_1 + \hat{\theta}_3 (-\bar{A}x_2 - \bar{C}_{lm}x_3) - \dot{\alpha}_{2c} \\ \phi_3 &= \begin{bmatrix} \frac{w_2}{w_3} z_2 - \frac{\partial \alpha_2}{\partial x_2} (x_3 - \bar{b}x_2 - \bar{F}_{fc}) \\ -\frac{\partial \alpha_2}{\partial x_2} \\ -\bar{A}x_2 - \bar{C}_{lm}x_3 + \alpha_{3a} \end{bmatrix}\end{aligned}\right\}\quad (30)$$

The control functions are thus chosen as

$$\left. \begin{aligned}\alpha_{3a} &= -\frac{1}{\hat{\theta}_3} \alpha_{3e} \\ \alpha_{3s} &= \alpha_{3s1} + \alpha_{3s2} \\ \alpha_{3s1} &= -k_{3s1} z_3 \\ k_{3s1} &\geq \frac{1}{\theta_{3\min}} \left( k_3 + \left\| C_{\theta_3} \left( \frac{\partial \alpha_2}{\partial \hat{\theta}} \right)^T \right\|^2 + \|C_{\phi_3} \Gamma \phi_3\|^2 \right)\end{aligned}\right\}\quad (31)$$

where  $k_3 > 0$  is a constant,  $C_{\theta_3}$  and  $C_{\phi_3}$  are positive definite constant diagonal matrixes, and  $\alpha_{3s2}$  is a robust control function satisfying the two conditions

$$\left. \begin{aligned}\text{Condition i} \quad & z_3 \left[ \theta_3 \alpha_{3s2} - \tilde{\theta}_1^T \phi_3 - \frac{\partial \alpha_2}{\partial x_2} \tilde{d} \right] \leq \varepsilon_3 \\ \text{Condition ii} \quad & z_3 \theta_3 \alpha_{3s2} \leq 0\end{aligned}\right\}\quad (32)$$

where  $\varepsilon_3$  is a design parameter. As in Remark 1, one example of  $\alpha_{3s2}$  satisfying (32) is given by

$$\alpha_{3s2} = -\frac{1}{2\theta_{3\min}\varepsilon_3} h_3 z_3 \quad (33)$$

satisfying

$$h_3 \geq \|\theta_M\|^2 \|\phi_3\|^2 + \left\| \frac{\partial \alpha_2}{\partial x_2} \right\| \delta_d^2 \quad (34)$$

From (29) and (31)

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2|_{\alpha_2} + w_3 \theta_3 z_3 z_4 + w_3 z_3 \left( \theta_3 \alpha_{3s2} - \tilde{\theta}^T \phi_3 - \frac{\partial \alpha_2}{\partial x_2} \tilde{d} \right) \\ &\quad - w_3 \theta_3 k_{3s1} z_3^2 - w_3 z_3 \left( \frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}} \right)\end{aligned}\quad (35)$$

*Step 3:* Noting the last equation of (7), Step 3 is to synthesize an actual control law for  $u$  such that  $\bar{Q}_L$  tracks the desired control function  $\alpha_3$  synthesized in Step 2 with a guaranteed transient performance. This can be done by the same backstepping design via ARC Lyapunov functions as in Step 2 except that here  $\bar{Q}_L$  is not differentiable at  $x_4 = 0$  since it contains  $\text{sgn}(x_4)$ . Fortunately, since the actual control input  $u$  can have finite jumps and is the control law to be synthesized at this step, we can proceed with the design as follows by noting that  $\bar{Q}_L$  is differentiable anywhere except at the singular point of  $x_4 = 0$  and is continuous everywhere. By the definition of  $\bar{Q}_L$  and  $g_3$ , it can be checked out that the derivative of  $\bar{Q}_L$  is given by

$$\dot{\bar{Q}}_L = \frac{\partial g_3}{\partial x_3} \dot{x}_3 x_4 + g_3(x_3, x_4) \dot{x}_4, \quad \forall x_4 \neq 0 \quad (36)$$

where

$$\frac{\partial g_3}{\partial x_3} = -\frac{\text{sgn}(x_4)}{2\sqrt{\bar{P}_s - \text{sgn}(x_4)x_3}}$$

From (31) and (7)

$$\dot{\alpha}_3 = \dot{\alpha}_{3c} + \dot{\alpha}_{3u} \quad (37)$$

where the calculable part  $\dot{\alpha}_{3c}$  and the incalculable part  $\dot{\alpha}_{3u}$  are given by

$$\left. \begin{aligned} \dot{\alpha}_{3c} &= \frac{\partial \alpha_3}{\partial x_1} x_2 + \frac{\partial \alpha_3}{\partial x_2} \hat{x}_2 + \frac{\partial \alpha_3}{\partial x_3} \hat{x}_3 + \frac{\partial \alpha_3}{\partial t} \\ \dot{\alpha}_{3u} &= \frac{\partial \alpha_3}{\partial x_2} [-(x_3 - \bar{b}x_2 - \bar{F}_{fc})\tilde{\theta}_1 - \tilde{\theta}_2 + \tilde{d}] \\ &\quad - \frac{\partial \alpha_3}{\partial x_3} (-\bar{A}x_2 - \bar{C}_{im}x_3 + g_3x_4)\tilde{\theta}_3 + \frac{\partial \alpha_3}{\partial \hat{\theta}} \dot{\hat{\theta}} \end{aligned} \right\} \quad (38)$$

Consider the augmented p.s.d. function  $V_4$  given by

$$V_4 = V_3 + \frac{1}{2}w_4z_4^2, \quad w_4 > 0 \quad (39)$$

Noting (29), (7), and (39)

$$\begin{aligned} \dot{V}_4 &= \theta_3 w_3 z_3 z_4 + \dot{V}_3|_{\alpha_3} + w_4 z_4 [\dot{\bar{Q}}_L - \dot{\alpha}_3] \\ &= \dot{V}_3|_{\alpha_3} + w_4 z_4 \left( \frac{\bar{K}_v}{\tau_v} g_3 u + \alpha_{4e} - \tilde{\theta}^T \phi_4 - \frac{\partial \alpha_3}{\partial x_2} \tilde{d} - \frac{\partial \alpha_3}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) \end{aligned} \quad (40)$$

where

$$\left. \begin{aligned} \alpha_{4e} &= \hat{\theta}_3 \frac{w_3}{w_4} z_3 - \frac{1}{\tau_v} g_3 x_4 + \frac{\partial g_3}{\partial x_3} \hat{x}_3 x_4 - \dot{\alpha}_{3c} \\ \phi_4 &= \begin{bmatrix} -\frac{\partial \alpha_3}{\partial x_2} (x_3 - \bar{b}x_2 - \bar{F}_{fc}) \\ -\frac{\partial \alpha_3}{\partial x_2} \\ \frac{w_3}{w_4} z_3 + \left[ \frac{\partial g_3}{\partial x_3} x_4 - \frac{\partial \alpha_3}{\partial x_3} \right] (-\bar{A}x_2 - \bar{C}_{im}x_3 + \bar{Q}_L) \end{bmatrix} \end{aligned} \right\} \quad (41)$$

Similar to (31), the control law consists of two parts given by

$$\left. \begin{aligned} u &= u_a(x, \hat{\theta}, t) + u_s(x, \hat{\theta}, t) \\ u_a &= -\frac{\tau_v}{\bar{K}_v g_3} \alpha_{4e} \\ u_s &= u_{s1} + u_{s2} \\ u_{s1} &= -k_{4s1} z_4 \\ k_{4s1} &\geq \frac{\tau_v}{g_3 \bar{K}_v} \left( k_4 + \left\| C_{\theta 4} \left( \frac{\partial \alpha_3}{\partial \hat{\theta}} \right)^T \right\|^2 + \|C_{\phi 4} \Gamma \phi_4\|^2 \right) \end{aligned} \right\} \quad (42)$$

where  $C_{\theta 4}$  and  $C_{\phi 4}$  are constant positive definite diagonal matrices, and  $u_{s2}$  is a robust control function satisfying the two conditions

$$\left. \begin{aligned} \text{Condition i} \quad & z_4 \left[ \frac{\bar{K}_v}{\tau_v} g_3 u_{s2} - \tilde{\theta}^T \phi_4 - \frac{\partial \alpha_3}{\partial x_2} \tilde{d} \right] \leq \varepsilon_4 \\ \text{Condition ii} \quad & z_4 \frac{g_3 \bar{K}_v}{\tau_v} u_{s2} \leq 0 \end{aligned} \right\} \quad (43)$$

in which  $\varepsilon_4$  is a design parameter. As in Remark 1, one example of  $u_{s2}$  satisfying (43) is given by

$$u_{s2} = -\frac{\tau_v}{2\bar{K}_v g_3 \varepsilon_4} h_4 z_4 \quad (44)$$

in which  $h_4$  is any continuous function satisfying

$$h_4 \geq \|\theta_M\|^2 \|\phi_4\|^2 + \left\| \frac{\partial \alpha_3}{\partial x_2} \right\| \delta_d^2 \quad (45)$$

### 3.4. Main results

**Theorem 1:** Let the parameter estimates be updated by the adaptation law (10) in which  $\tau$  is chosen as

$$\tau = \sum_{j=2}^4 w_j z_j \phi_j \quad (46)$$

By choosing non-linear controller gains  $k_{2s1}$ ,  $k_{2s2}$  and  $k_{2s3}$  large enough such that the inequality conditions in (17), (31) and (42) are satisfied for a set of

$$C_{\theta j} = \text{diag}\{c_{\theta jl}, l = 1, 2, 3\}, j = 3, 4$$

and  $C_{\phi k} = \text{diag}\{c_{\phi kl}\}, k = 2, 3, 4$  with

$$c_{\phi kl}^2 \geq \frac{3w_k}{4} \left( \frac{w_3}{c_{\theta 3l}^2} + \frac{w_4}{c_{\theta 4l}^2} \right), \quad \forall k, l$$

then, the control law (42) with the adaptation law (10) guarantees that

**A.** In general, all signals are bounded. Furthermore,  $V_4$  given by (39) is bounded above by

$$V_4(t) \leq \exp(-\lambda_V t) V_4(0) + \frac{\varepsilon_V}{\lambda_V} [1 - \exp(-\lambda_V t)] \quad (47)$$

where  $\lambda_V = 2 \min\{k_2, k_3, k_4\}$  and

$$\varepsilon_V = w_2 \varepsilon_2 + w_3 \varepsilon_3 + w_4 \varepsilon_4$$

**B.** If after a finite time  $t_0$ ,  $\tilde{d} = 0$ , i.e. in the presence of parametric uncertainties only, then, in addition to results in A, asymptotic output tracking (or zero final tracking error) is also achieved.

**Remark 2:** Results in A of Theorem 1 indicate that the proposed controller has an exponentially converging transient performance with the exponentially converging rate  $\lambda_V$  and the final tracking error being able

to be adjusted via certain controller parameters freely in a known form; it is seen from (47) that  $\lambda_V$  can be made arbitrarily large, and  $\varepsilon_V/\lambda_V$ , the bound of  $V(\infty)$  (an index for the final tracking errors), can be made arbitrarily small by increasing feedback gains  $k = [k_2, k_3, k_4]^T$  and/or decreasing controller parameters  $\varepsilon = [\varepsilon_2, \varepsilon_3, \varepsilon_4]^T$ . Such a guaranteed transient performance is especially important for the control of electro-hydraulic systems since execute time of a run is short. Theoretically, this result is what a well-designed robust controller can achieve. In fact, when the parameter adaptation law (10) is switched off, the proposed ARC law becomes a deterministic robust control law and Results A of the Theorem remain valid (Yao and Tomizuka 1994, 1997).

**B** of Theorem 1 implies that the parametric uncertainties may be reduced through parameter adaptation and an improved performance is obtained. Theoretically, Result **B** is what a well-designed adaptive controller can achieve.

**Remark 3:** In the above design, the intermediate control functions  $\alpha_i$  given by (15) and (31) have to be differentiable. Consequently, the Coulomb friction compensation term  $F_{fc}(x_2)$  in the control functions has to be a differential function of  $x_2$ . This requirement can be easily accommodated in the proposed ARC framework since  $F_{fc}(x_2)$  can be chosen as any differentiable function which approximates the actual discontinuous Coulomb friction (e.g. replacing  $\text{sgn}(x_2)$  in the conventional Coulomb friction modelling by the smooth  $\tanh(x_2)$ ). The approximation error can be lumped into the uncertain non-linearity term  $\tilde{d}$ .

### 3.5. Trajectory initialization

It is seen from (47) that transient tracking error is affected by the initial value  $V_4(0)$ , which may depend on the controller parameters also. To further reduce transient tracking error, the desired trajectory initialization can be used as follows. Namely, instead of simply letting the desired trajectory for the controller be the actual desired trajectory or position (i.e.  $x_{1d}(t) = x_{Ld}(t)$ ), we can generate  $x_{1d}(t)$  using a filter. For example,  $x_{1d}(t)$  can be generated by the 4th order stable system

$$x_{1d}^{(4)} + \beta_1 x_{1d}^{(3)} + \cdots + \beta_4 x_{1d} = x_{Ld}^{(4)} + \beta_1 x_{Ld}^{(3)} + \cdots + \beta_4 x_{Ld} \quad (48)$$

The initial conditions of the system (48) can be chosen to render  $V_4(0) = 0$  to reduce transient tracking error.

**Lemma 1:** *If the initials  $x_{1d}(0), \dots, x_{1d}^{(3)}(0)$  are chosen as*

$$\left. \begin{aligned} x_{1d}(0) &= x_1(0) \\ \dot{x}_{1d}(0) &= x_2(0) \\ \ddot{x}_{1d}(0) &= \hat{x}_2(0) \\ x_{1d}^{(3)}(0) &= \hat{\theta}_1(0) \left[ \hat{x}_3(0) - \left( b + \frac{\partial F_{fc}}{\partial x_2}(0) \right) \hat{x}_2(0) \right] \end{aligned} \right\} \quad (49)$$

then,  $e_1(0) = 0, z_i(0) = 0, i = 2, 3, 4$  and  $V_4(0) = 0$ .

**Proof:** The lemma can be proved in the same way as in Yao and Tomizuka (1997).  $\square$

**Remark 4:** It is seen from (49) that the above trajectory initialization is independent from the choice of controller parameters such as  $k = [k_2, k_3, k_4]^T$  and  $\varepsilon = [\varepsilon_2, \varepsilon_3, \varepsilon_4]^T$ . Thus, once the initial position of the servosystem is determined, the above trajectory initialization can be performed off-line.

**Remark 5:** By using the trajectory initialization (49),  $V_4(0) = 0$ . Thus, from (47), the controller output tracking error  $e_1 = x_1 - x_{1d}$  is within a ball whose size can be made arbitrarily small by increasing  $k$  and/or decreasing  $\varepsilon$  in a known form. From (48) and (49), the trajectory planning error,  $e_d(t) = x_{1d}(t) - x_{Ld}(t)$ , can be guaranteed to possess any good transient behaviour by suitably choosing the Hurwitz polynomial  $G_d(s) = s^4 + \beta_1 s^3 + \beta_2 s^2 + \beta_3 s + \beta_4$  and is known in advance. Therefore, in principle, any prescribed transient performance of the actual output tracking error  $e_y = e_1(t) + e_d(t)$  can be achieved by the choice of controller parameters.

## 4. Simulation results

To illustrate the above designs, simulation results are obtained for a system showing in figure 1 having the following actual parameters:  $m = 100$  kg,  $A = 3.35 \times 10^{-4}$  m<sup>2</sup>,  $b = 20$  N/(m/s),  $4\beta_e/V_t = 4.52 \times 10^{13}$  N/(m<sup>5</sup>),  $C_{mm} = 2.21 \times 10^{-14}$  m<sup>5</sup>/Ns,  $C_d w/\sqrt{\rho} = 3.42 \times 10^{-5}$  m<sup>3</sup> $\sqrt{\text{N}}$ s,  $P_s = 10\,342\,500$  Pa,  $K_v = 0.0324$ ,  $\tau = 0.00636$ . By using scaling factors of  $S_{c3} = 5.97 \times 10^5$  and  $S_{c4} = 4.99 \times 10^{-7}$ , the actual value of  $\theta$  is  $\theta = [2, 0, 1]^T$  for no external disturbances. The bounds of uncertain ranges are given by  $\theta_{\min} = [1, -1, 0.442]^T$ ,  $\theta_{\max} = [4, 1, 1.549]^T$ , and  $\delta_d = 2$ . The initial estimate of  $\theta$  is chosen as  $\hat{\theta}(0) = [1.5, 0.5, 0.7]^T$ , which satisfies (8) but differs significantly from its actual value  $\theta$  to test the effect of parametric uncertainties. A sampling period of 1 ms is used in all simulation. The following three controllers are compared.

**ARC(d):** the discontinuous projection based ARC law proposed in this paper. The controller parameters are:  $k_p = 220$ ,  $w_2 = 1$ ,  $k_2 = 220$ ,  $C_{\phi 2} = \text{diag}\{166, 22.4, 86.6\}$ ,  $\varepsilon_2 = 0.1$ ,



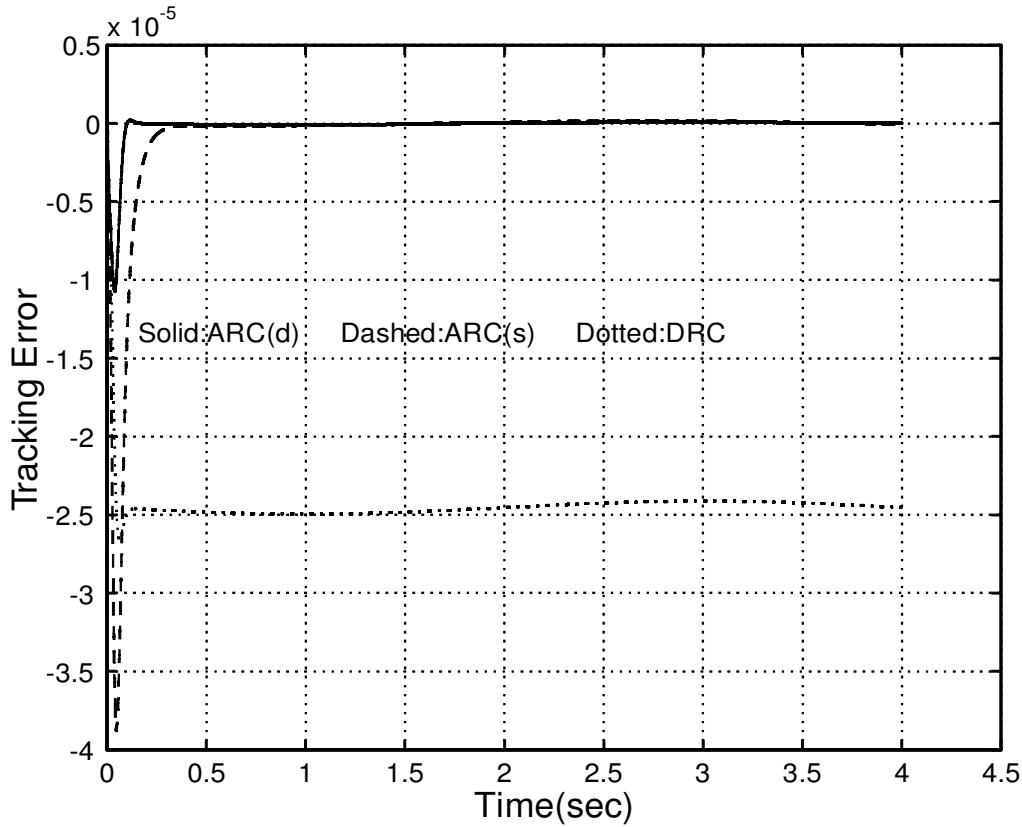


Figure 2. Tracking errors in the presence of parametric uncertainty only.

$w_3 = 1$ ,  $k_3 = 220$ ,  $C_{\phi 3} = \text{diag}\{166, 22.4, 86.6\}$ ,  $C_{\theta 3} = \text{diag}\{1.732 \times 10^{-2}, 5.477 \times 10^{-2}, 1.732 \times 10^{-2}\}$ ,  $\varepsilon_3 = 1 \times 10^6$ ,  $w_4 = 1$ ,  $k_4 = 220$ ,  $C_{\phi 4} = \text{diag}\{166, 22.4, 86.6\}$ ,  $C_{\theta 4} = \text{diag}\{5.477 \times 10^{-3}, 5.477 \times 10^{-2}, 1.225 \times 10^{-2}\}$ ,  $\varepsilon_4 = 1 \times 10^{10}$ , and adaptive gains of  $\Gamma = \text{diag}\{5 \times 10^{-8}, 3.5 \times 10^{-7}, 1 \times 10^{-6}\}$ . It can be checked out that the conditions in Theorem 1 are satisfied for the chosen controller parameters. Trajectory planning parameters in (48) are  $\beta_1 = 400$ ,  $\beta_2 = 60000$ ,  $\beta_3 = 4 \times 10^6$  and  $\beta_4 = 1 \times 10^8$ .

**ARC(s):** the smooth projection based ARC law proposed in Yao *et al.* (1997b). The same smooth projection as in Yao and Tomizuka (1997) is used where  $\varepsilon_\theta = 0.001$ , and the remaining controller parameters are the same as in ARC(d).

**DRC:** the deterministic robust control law, which is obtained by using the same control law as in ARC(d) but without parameter adaptation.

To test the nominal tracking performance of each controller, simulations are first run for the ideal case

of parametric uncertainties only (i.e.  $\tilde{d} = 0$ ). The desired trajectory is a sinusoidal curve given by

$$x_{Ld} = 0.05 \sin\left(\frac{\pi}{2}t\right)$$

Tracking errors are shown in figure 2. As shown, all three controllers have very small tracking errors, which verifies the excellent tracking capability of the proposed algorithms. Furthermore, the tracking errors of both ARC(d) and ARC(s) converge to zero quickly as in contrast to the non-zero tracking error of DRC; this verifies the effectiveness of introducing parameter adaptation. ARC(d) also has the smallest transient tracking error.

To test the performance robustness of the proposed schemes, a large constant disturbance  $\tilde{f}$  with an amplitude of 1960 N (corresponds to  $\tilde{d} = 2$ ) is added to the system during the period of  $0 < t < 1$  s. As shown in figure 3, all three controllers still have very small tracking errors in spite of the added large disturbance. Furthermore, the tracking errors of ARC(d) and ARC(s) converge to zero after the disturbance is removed at  $t = 1$  s. Comparing ARC(d) with ARC(s), it is seen that ARC(d) has a much shorter recovery period and a smaller transient tracking error. This is due to the qualitatively different parameter adaptation transient of the two schemes when the system is subjected to

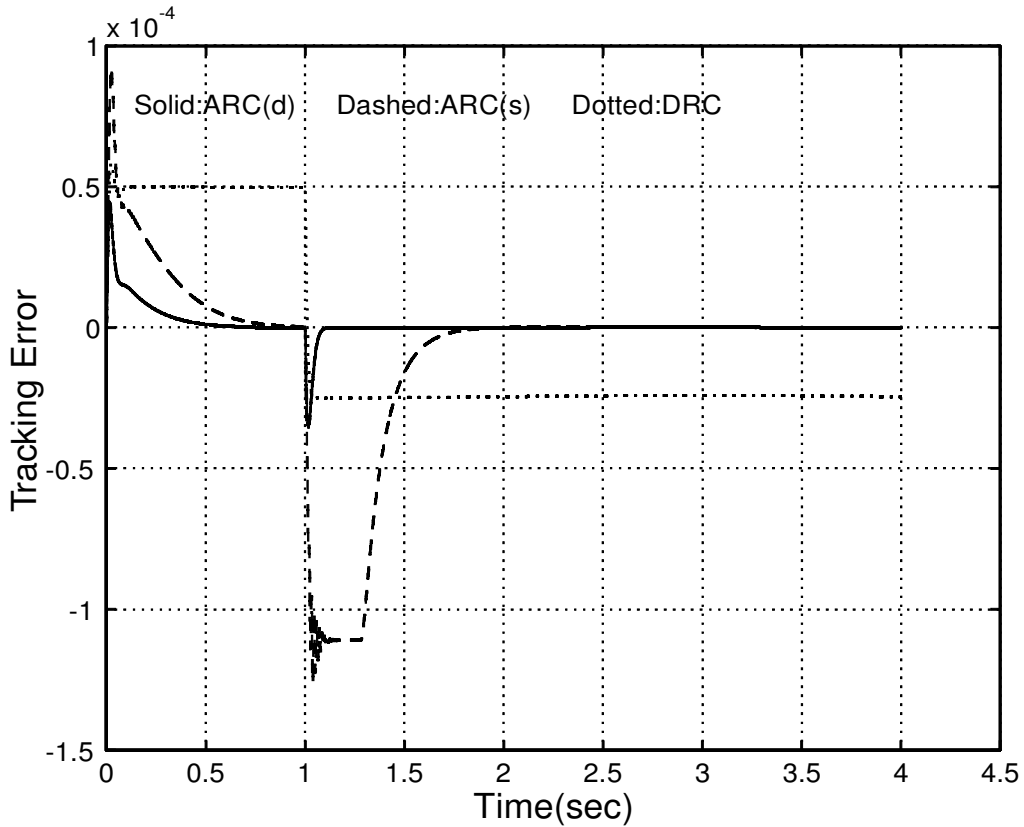


Figure 3. Tracking errors in the presence of parametric uncertainty and uncertain non-linearities.

large disturbances; as shown in figure 4, the discontinuous projection based ARC(d) guarantees that the parameter estimates stay within the known bounded range all the time, while the parameter estimates in the smooth projection based ARC(s) can become very large due to the wrong parameter adaptation process caused by the presence of large disturbance  $\tilde{f}$ . As a result, after the disturbance disappears at  $t = 1$  s, parameter estimates in ARC(d) converge to their correct values much faster than that in ARC(s), which leads to an improved tracking performance. This verifies that the discontinuous projection based ARC has a more robust parameter adaptation process in general. Consequently, a better performance is expected.

The simulation is also run for fast changing desired trajectory and similar results have been obtained. For example, for a 1 Hz desired trajectory given by  $x_{Ld} = 0.05 \sin(2\pi t)$ , tracking errors of three controllers shown in figure 5 have similar trends as in figure 2 for parametric uncertainties only.

Finally, simulation is run for point-to-point movement of the servosystem. Given the start and the final position of the system, a desired trajectory  $x_{Ld}(t)$  with a continuous velocity and acceleration is first planned. For a travel distance of 0.3 m, the planned  $x_{Ld}(t)$  and  $\dot{x}_{Ld}(t)$  are shown in figure 6, which has a maximum

speed of 0.432 m/s and has a maximum acceleration of  $60 \text{ m/s}^2$ . Tracking errors are shown in figure 7. As seen, during the start and the end when the system experiences large acceleration and deceleration, transient tracking errors become a little bit larger. Overall, tracking errors of all three controllers are still very small. Again, ARC(d) has the best tracking performance. The control inputs and load pressures of all three controllers are similar as shown in figures 8 and 9 respectively, which exhibit satisfactory transient period and are well within their physical limits. All these results verify the effectiveness of the proposed algorithm.

## 5. Conclusions

In this paper, instead of using smooth projection, a discontinuous projection based ARC controller is constructed for the high performance robust motion control of a typical one DOF electro-hydraulic servosystem driven by a double-rod hydraulic cylinder. The controller takes into account the particular non-linearities associated with hydraulic dynamics and allows parametric uncertainties due to variations of inertia load and hydraulic parameters (e.g. bulk modulus) as well as uncertain non-linearities coming from external disturbances, uncompensated friction forces, etc. Strategies are also developed to deal with the design difficulties caused

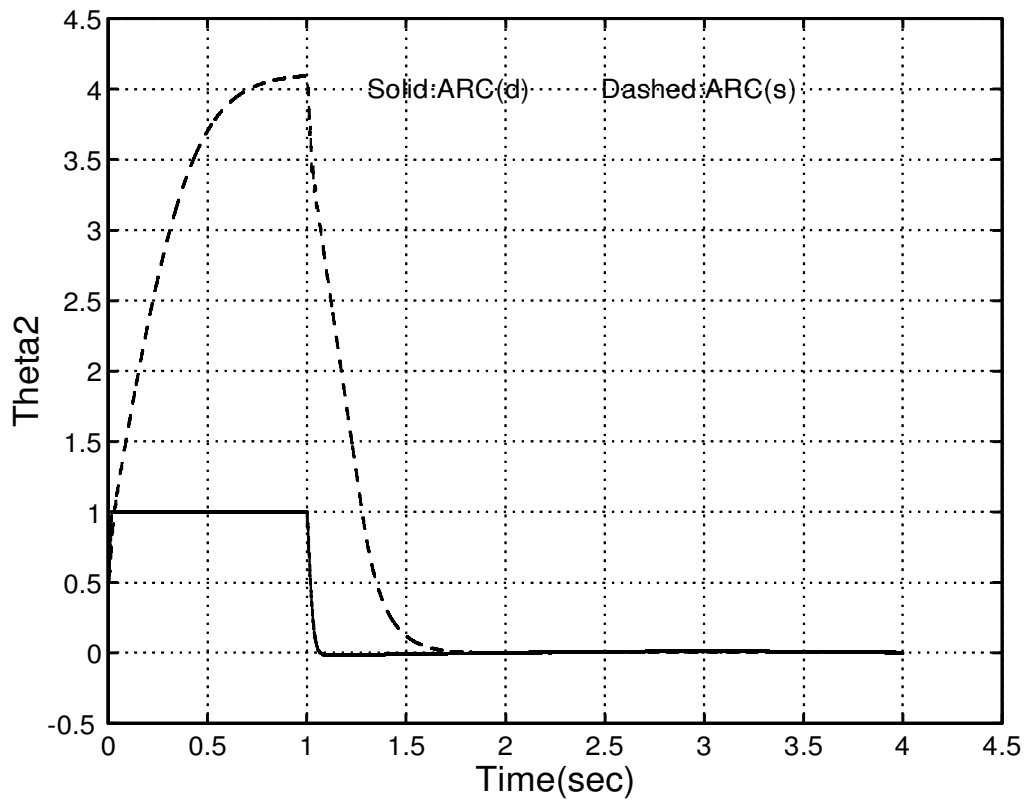


Figure 4. Parameter adaptation in the presence of uncertain non-linearities.

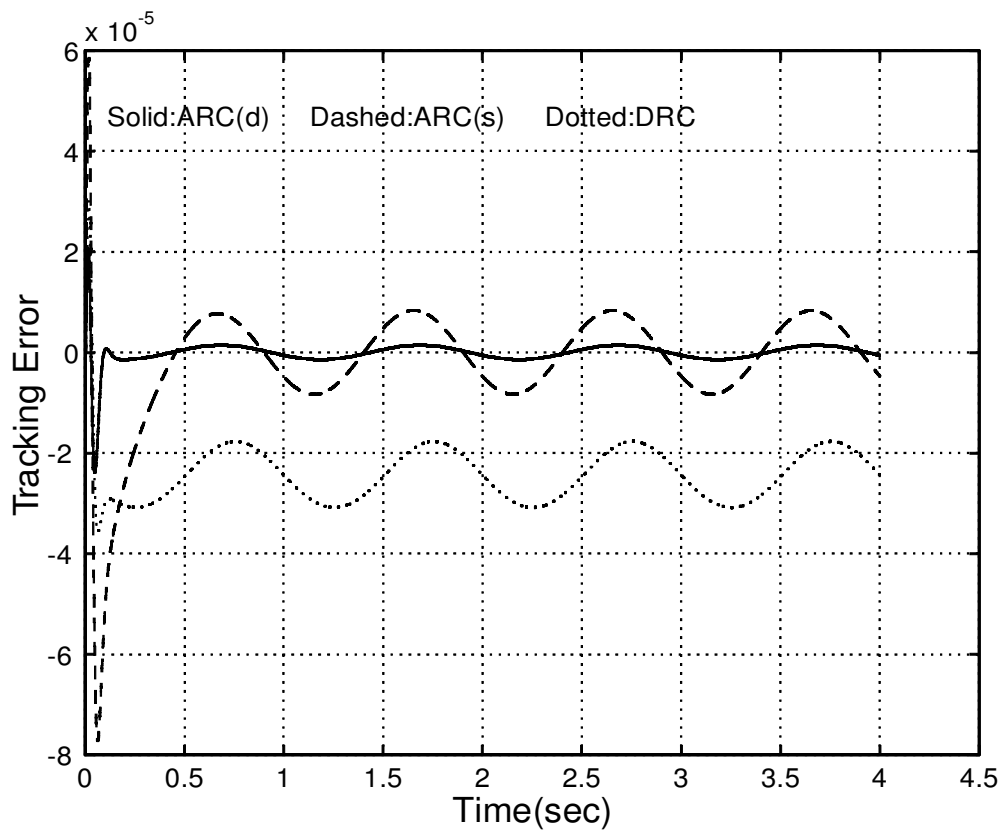


Figure 5. Tracking errors for a fast sine curve.

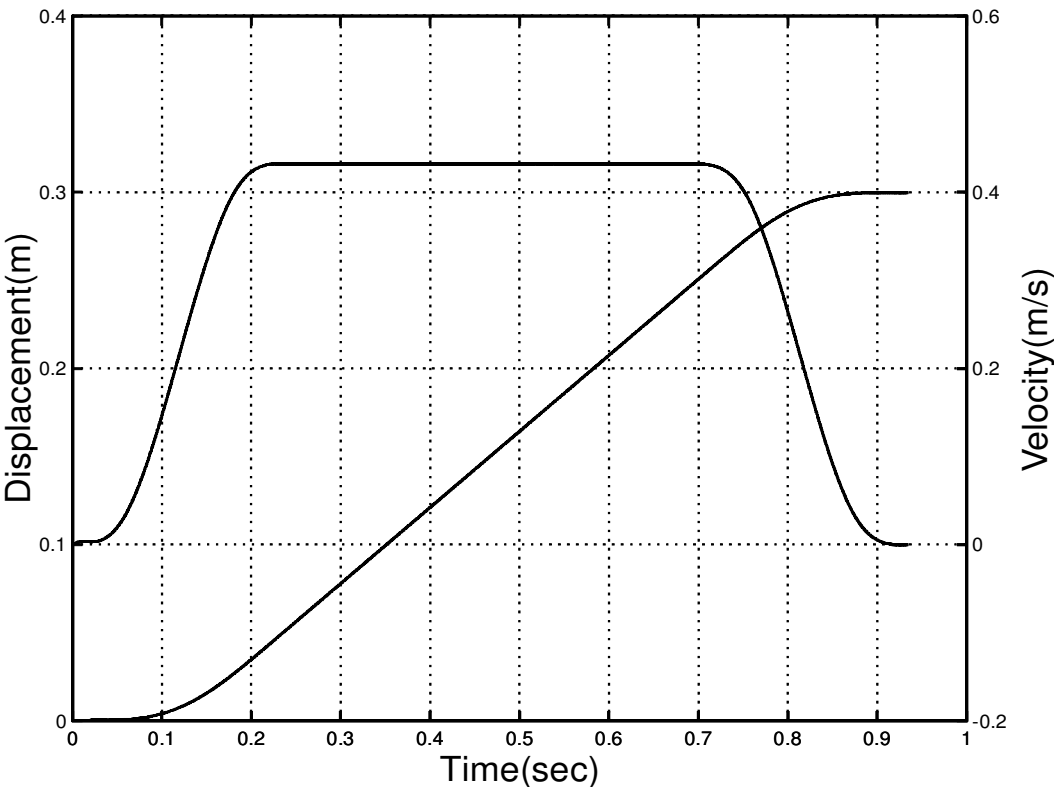


Figure 6. Point to point motion trajectory profile.

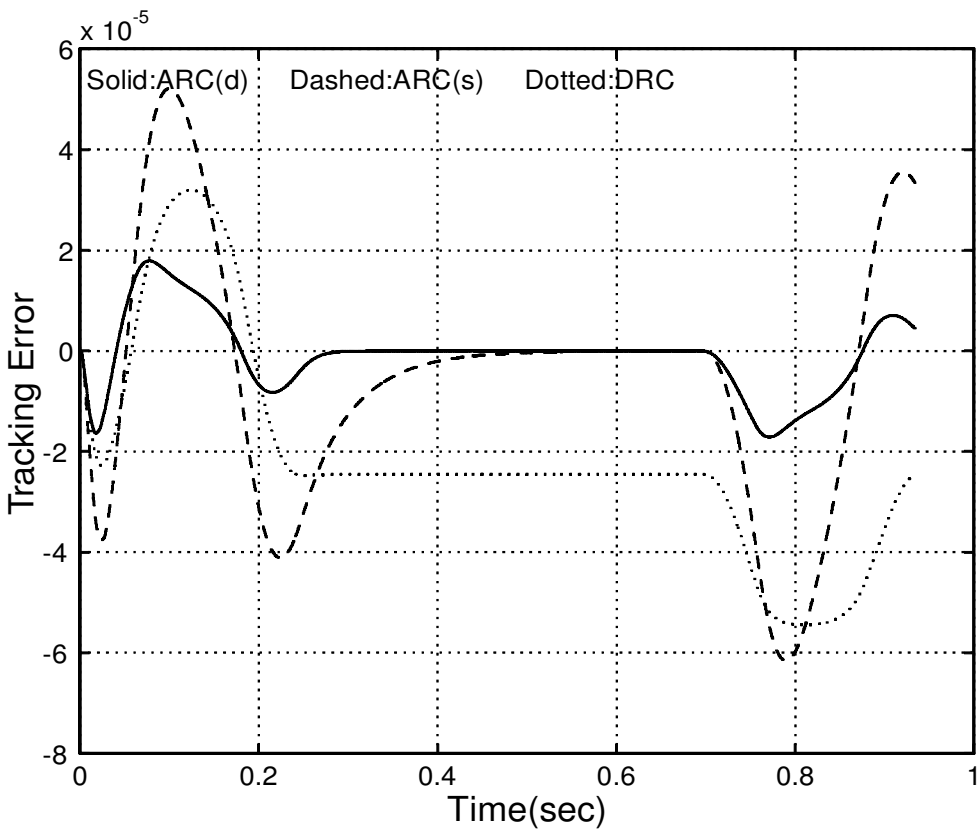


Figure 7. Tracking errors in point to point motion.

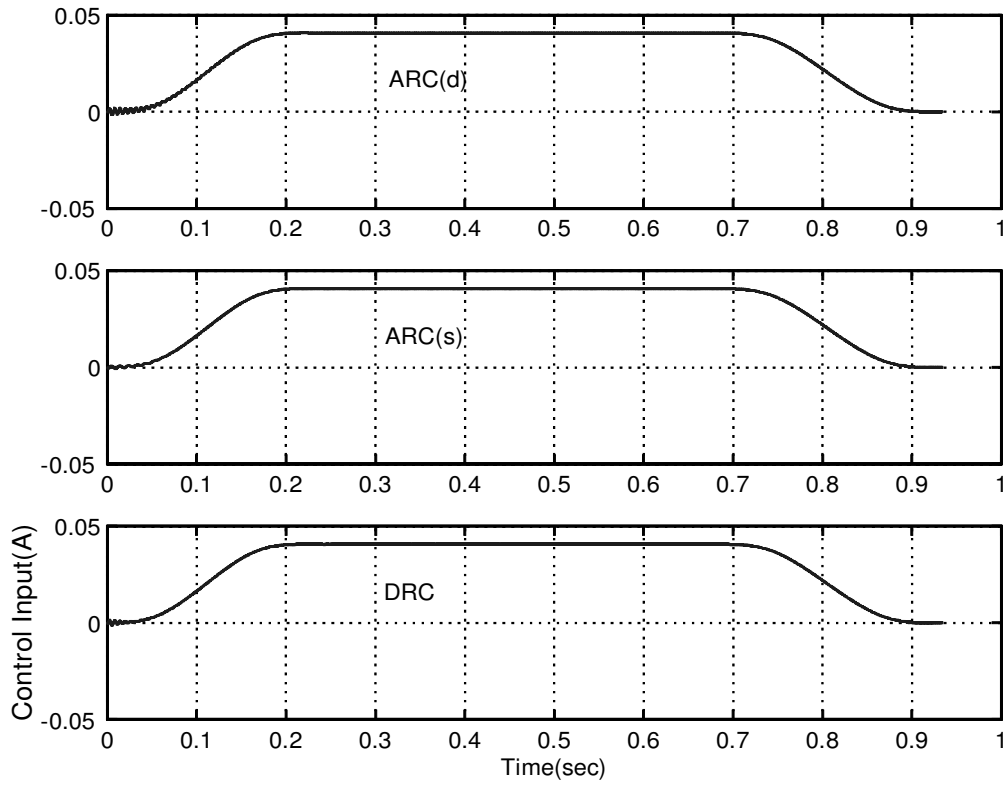


Figure 8. Control inputs in point to point motion.

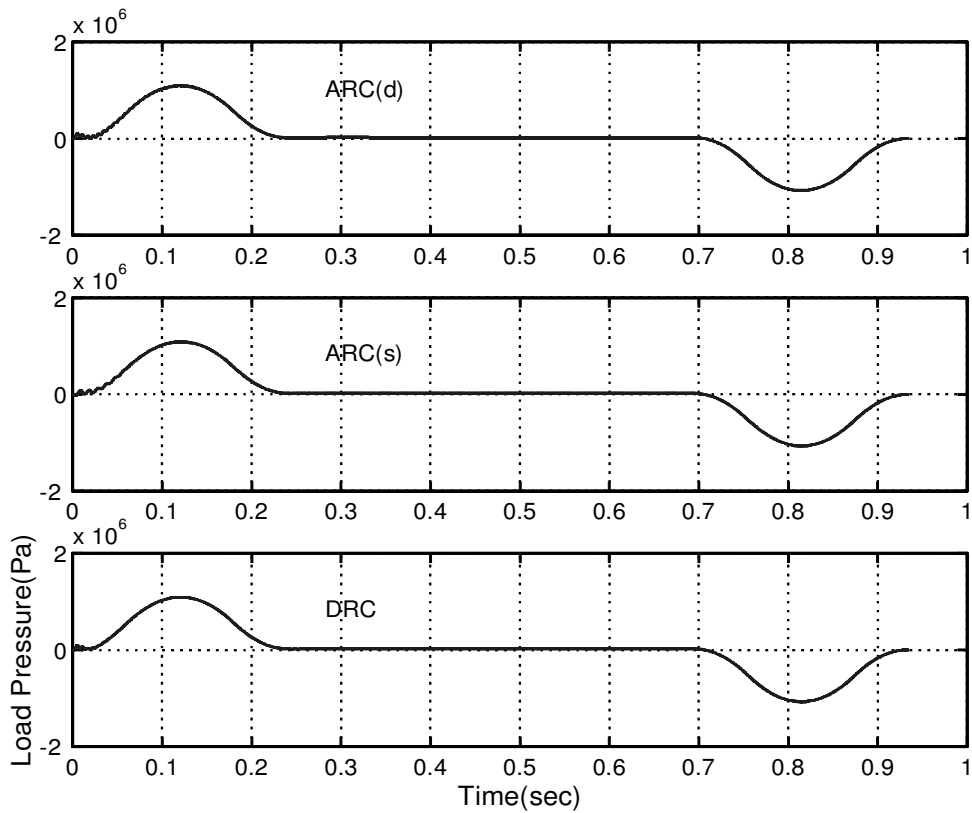


Figure 9. Load pressures in point to point motion.

by the non-differentiability of certain non-linearities inherited in hydraulic systems. Compared with our previously proposed ARC controller (Yao *et al.*, 1997b), the present scheme has a more robust parameter adaptation process and is more suitable for implementation. Extensive simulation results are obtained to illustrate the difference and the effectiveness of the proposed scheme.

### Acknowledgements

This work is supported in part by the National Science Foundation under the CAREER grant CMS-9734345 and a grant from Purdue Research Foundation.

### Appendix

**Proof of Theorem 1:** Substituting (42) into (40) and noting (23) and (35)

$$\begin{aligned} \dot{V}_4 = & w_2 z_2 (\theta_1 \alpha_{2s2} - \tilde{\theta}^T \phi_2 + \tilde{d}) - w_2 \theta_1 k_{2s1} z_2^2 \\ & + w_3 z_3 \left( \theta_3 \alpha_{3s2} - \tilde{\theta}^T \phi_3 - \frac{\partial \alpha_2}{\partial x_2} \tilde{d} \right) - w_3 \theta_3 k_{3s1} z_3^2 \\ & + w_4 z_4 \left( \frac{g_3 \bar{K}_v}{\tau_v} u_{s2} - \tilde{\theta}^T \phi_4 - \frac{\partial \alpha_3}{\partial x_2} \tilde{d} \right) - w_4 k_{4s1} z_4^2 \\ & - \left( w_3 z_3 \frac{\partial \alpha_2}{\partial \hat{\theta}} + w_4 z_4 \frac{\partial \alpha_3}{\partial \hat{\theta}} \right) \dot{\hat{\theta}} \end{aligned} \quad (50)$$

Noting (46) and by completion of square

$$\begin{aligned} & \left| \sum_{j=3}^4 w_j z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right| \\ & = \left| \sum_{j=3}^4 \sum_{k=2}^4 w_j z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \frac{1}{\sqrt{3} w_j} C_{\theta j} \sqrt{3} w_j C_{\theta j}^{-1} \Gamma \phi_k w_k z_k \right| \\ & \leq \sum_{j=3}^4 \sum_{k=2}^4 \left( \frac{1}{\sqrt{3}} \sqrt{w_j} |z_j| \left\| C_{\theta j} \left( \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right)^T \right\| \right) \\ & \quad \times (\sqrt{3} w_j w_k \|C_{\theta j}^{-1} \Gamma \phi_k\| \sqrt{w_k} |z_k|) \\ & \leq \sum_{j=3}^4 \sum_{k=2}^4 \left[ \frac{1}{3} w_j z_j^2 \left\| C_{\theta j} \left( \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right)^T \right\|^2 \right. \\ & \quad \left. + \frac{3}{4} w_j w_k \|C_{\theta j}^{-1} \Gamma \phi_k\|^2 w_k z_k^2 \right] \end{aligned}$$

$$\begin{aligned} & \leq \sum_{j=3}^4 w_j \left\| C_{\theta j} \left( \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right)^T \right\|^2 z_j^2 \\ & \quad + \sum_{k=2}^4 w_k \left( \sum_{j=3}^4 \frac{3}{4} w_k w_j \|C_{\theta j}^{-1} \Gamma \phi_k\|^2 \right) z_k^2 \end{aligned} \quad (51)$$

If  $C_{\phi j}$  and  $C_{\theta j}$  satisfy the condition stated in the theorem, then, from (51)

$$\begin{aligned} & \left| \sum_{j=3}^4 w_j z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right| \leq \sum_{j=3}^4 w_j \left\| C_{\theta j} \left( \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right)^T \right\|^2 z_j^2 \\ & \quad + \sum_{k=2}^4 w_k \|C_{\phi k} \Gamma \phi_k\|^2 z_k^2 \end{aligned} \quad (52)$$

Thus, noting the formula for  $k_{2s1}$ ,  $k_{3s1}$  and  $k_{4s1}$ , (50) becomes

$$\begin{aligned} \dot{V}_4 \leq & - \sum_{j=2}^4 w_j k_j z_j^2 + w_2 z_2 (\theta_1 \alpha_{2s2} - \tilde{\theta}^T \phi_2 + \tilde{d}) \\ & + w_3 z_3 \left( \theta_3 \alpha_{3s2} - \tilde{\theta}^T \phi_3 - \frac{\partial \alpha_2}{\partial x_2} \tilde{d} \right) \\ & + w_4 z_4 \left( \frac{g_3 \bar{K}_v}{\tau_v} u_{s2} - \tilde{\theta}^T \phi_4 - \frac{\partial \alpha_3}{\partial x_2} \tilde{d} \right) \end{aligned} \quad (53)$$

From the condition i of (19), (32) and (43), we have

$$\dot{V}_4 \leq - \sum_{j=2}^4 (w_j k_j z_j^2 + w_j \varepsilon_j) \leq -2\lambda_V V_4 + \varepsilon_v \quad (54)$$

which leads to (47) and the results in **A** of Theorem 1.

When  $\tilde{d} = 0$ , noting conditions ii of (19), (32) and (43), (53) becomes

$$\dot{V}_4 \leq - \sum_{j=2}^4 [w_j k_j z_j^2 + w_j z_j \phi_j^T \tilde{\theta}] = - \sum_{j=2}^4 w_j k_j z_j^2 - \tau^T \tilde{\theta} \quad (55)$$

Define a new p.d. function  $V_\theta$  as

$$V_\theta = V_4 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad (56)$$

Noting (55) and P2 of (11), the derivative of  $V_\theta$  is

$$\dot{V}_\theta \leq - \sum_{j=2}^4 w_j k_j z_j^2 - \tilde{\theta}^T \tau + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \leq - \sum_{j=2}^4 w_j k_j z_j^2 \quad (57)$$

Therefore,  $z = [z_1, z_2, z_3]^T \in L_2^3$ . It is also easy to check that  $\dot{z}$  is bounded. So,  $z \rightarrow 0$  as  $t \rightarrow \infty$  by Barbalat's lemma, which leads to **B** of Theorem 1.  $\square$

## References

- ALLEYNE, A., 1996, Nonlinear force control of an electro-hydraulic actuator. *Proceedings of the Japan/USA Symposium on Flexible Automation*, Boston, USA, pp. 193–200.
- ALLEYNE, A., 1998, A systematic approach to the control of electrohydraulic servosystems. *Proceedings of the American Control Conference*, Philadelphia, USA, pp. 833–837.
- ALLEYNE, A., and HEDRICK, J. K., 1995, Nonlinear adaptive control of active suspension. *IEEE Transactions on Control Systems Technology*, **3**, 94–101.
- BOBROW, J. E., and LUM, K., 1996, Adaptive, high bandwidth control of a hydraulic actuator. *ASME J. Dynamic Systems, Measurement, and Control*, **118**, 714–720.
- CORLESS, M. J., and LEITMANN, G., 1981, Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems. *IEEE Transactions on Automatic Control*, **26**, 11139–1144.
- FITZSIMONS, P. M., and PALAZZOLO, J. J., 1996, Part i: Modeling of a one-degree-of-freedom active hydraulic mount; part ii: Control. *ASME Journal of Dynamic Systems, Measurement, and Control*, **118**, 439–448.
- FREEMAN, R. A., KRSTIC, M., and KOKOTOVIC, P. V., 1996, Robustness of adaptive nonlinear control to bounded uncertainties. *IFAC World Congress, Vol. F*, pp. 329–334.
- GOODWIN, G. C., and MAYNE, D. Q., 1989, A parameter estimation perspective of continuous time model reference adaptive control. *Automatica*, **23**, 57–70.
- JERONYMO, C. E., and MUTO, T., 1995, Application of unified predictive control (upc) for an electro-hydraulic servo system. *JSME International Journal, Series C*, **38**, 727–734.
- KRSTIC, M., KANELAKOPOULOS, I., and KOKOTOVIC, P. V., 1992, Adaptive nonlinear control without overparametrization. *Systems and Control Letters*, **19**, 177–185.
- KRSTIC, M., KANELAKOPOULOS, I., and KOKOTOVIC, P. V., 1995, *Nonlinear and Adaptive Control Design* (New York: Wiley).
- LIN, W., 1997, Global robust stabilization of minimum-phase nonlinear systems. *Automatica*, **33**, 453–462.
- MERRITT, H. E., 1967, *Hydraulic Control Systems* (New York: Wiley).
- PLUMMER, A. R., and VAUGHAN, N. D., 1996, Robust adaptive control for hydraulic servosystems. *ASME J. Dynamic System, Measurement and Control*, **118**, 237–244.
- POLYCARPOU, M. M., and IOANNOU, P. A., 1993, A robust adaptive nonlinear control design. *proceedings of American Control Conference*, pp. 1365–1369.
- RE, L. D., and ISIDORI, A., 1995, Performance enhancement of nonlinear drives by feedback linearization of linear-bilinear cascade models. *IEEE Transactions on Control Systems Technology*, **3**, 299–308.
- SASTRY, S., and BODSON, M., 1989, *Adaptive Control: Stability, Convergence and Robustness* (Englewood Cliffs, NJ: Prentice Hall).
- SLOTINE, J. J. E., and LI, W., 1988, Adaptive manipulator control: a case study. *IEEE Transactions on Automatic Control*, **33**, 995–1003.
- SOHL, G. A., and BOBROW, J. E., 1999, Experiments and simulations on the nonlinear control of a hydraulic system. *IEEE Transactions on Control System Technology*, **7**, 238–247.
- TAO, T. C., and TOMIZUKA, M., 1994, Robust adaptive and repetitive digital control and application to hydraulic servo for noncircular machining. *ASME Journal of Dynamic Systems, Measurement, and Control*, **116**, 24–32.
- UTKIN, V. I., 1992, *Sliding Modes in Control Optimization* (Springer Verlag).
- VOSSOUGH, R., and DONATH, M., 1995, Dynamic feedback linearization for electro-hydraulically actuated control systems. *ASME Journal of Dynamic Systems, Measurement, and Control*, **117**, 468–477.
- WHATTON, J., 1989, *Fluid Power Systems* (Prentice Hall).
- YAO, B., 1997, High performance adaptive robust control of nonlinear systems: a general framework and new schemes. *Proceedings of the IEEE Conference on Decision and Control*, pp. 2489–2494.
- YAO, B., AL-MAJED, M., and TOMIZUKA, M., 1997 a, High performance robust motion control of machine tools: An adaptive robust control approach and comparative experiments. *IEEE/ASME Transactions on Mechatronics*, **2**, 63–76. (Part of the paper also appeared in *Proceedings of the 1997 American Control Conference*.)
- YAO, B., CHIU, G. T. C., and REEDY, J. T., 1997 b, Nonlinear adaptive robust control of one-dof electro-hydraulic servo systems. *ASME International Mechanical Engineering Congress and Exposition (IMECE'97)*, FPST-Vol 4, Dallas, TX, USA, pp. 191–197.
- YAO, B., and TOMIZUKA, M., 1994, Smooth robust adaptive sliding mode control of robot manipulators with guaranteed transient performance. *Proceedings of the American Control Conference*, pp. 1176–1180. The full paper appeared in *ASME Journal of Dynamic Systems, Measurement and Control*, **118**, 764–775, 1996.
- YAO, B., and TOMIZUKA, M., 1997, Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form. *Automatica*, **33**, 893–900. (Part of the paper appeared in *Proceedings of the 1995 American Control Conference*, pp. 2500–2505.)
- YAO, B., and TOMIZUKA, M., 1999, Adaptive robust control of MIMO nonlinear systems in semi-strict feedback forms, 1999. Submitted to *Automatica* (revised in 1999 and accepted). Parts of the paper were presented in the *IEEE Conference on Decision and Control*, pp. 2346–2351, 1995, and the *IFAC World Congress*, Vol. F, pp. 335–340, 1996.
- YAO, B., ZHANG, J., KOEHLER, D., and LITHERLAND, J., 1998, High performance swing velocity tracking control of hydraulic excavators. *Proceedings of the American Control Conference*, pp. 818–822.