Sliding Mode Flow Rate Observer Design

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Abstract

Dynamic flow rate information is needed in a lot of applications; however it is normally not measurable. Calculation of the flow rate from fluid pressure dynamic equations usually results in poor estimates due to the very noisy pressure measurements and unavoidable modelling uncertainties. This paper proposes a sliding mode dynamic flow rate observer to estimate the meter-in and meter-out flow rates of a hydraulic cylinder. Theoretical convergence can be guaranteed and simulation and experimental results show the effectiveness of the proposed flow rate observer.

Keywords

Dynamic flow rate, sliding mode observer

1 Introduction

Dynamic flow rate information is needed in a lot of applications, such as automated modelling of valve flow mapping, fault detection, diagnostics and prognostics. However, with a conventional flow meter which is only capable of measuring static flow rate, the dynamic flow rate is not measurable. In order to obtain the flow rate information, one must seek other approaches to avoid the demand of direct measurement.

In hydraulic systems activated by hydraulic cylinders, the flow rate is directly related to the cylinder pressure dynamics and the motion of the cylinder rod. The cylinder pressure dynamics can be written as [1]

$$\begin{aligned} \frac{V_1(x)}{\beta_e}\dot{P}_1 &= -A_1\dot{x} + Q_1 + \Delta_1 \\ \frac{V_2(x)}{\beta_e}\dot{P}_2 &= A_2\dot{x} - Q_2 + \Delta_2 \end{aligned} \tag{1}$$

where x is the cylinder rod displacement, A_1 and A_2 are the ram areas of the cylinder, P_1 and P_2 represent pressures at the two chambers, Q_1 and Q_2 stand for the meter-in and meter-out flow rates respectively, β_e is the effective bulk modulus, and $V_1(x)$ and $V_2(x)$ are the total cylinder volumes of the head and rod ends including connecting hose volumes, Δ_1 and Δ_2 represent the lumped disturbance flows that are not associated with the control valves (i.e., the cylinder internal and external leakage flows, and so on).

It is assumed that:

a. the cylinder rod position x and the velocity $\dot{x}$ as well as the pressures P_1 and P_2 are available,
b. the effective bulk modulus β_e does not change or changes slowly, the actual value of β_e may not be known but the bounds β_{emin} and β_{emax} are known.
c. the lumped disturbance flows Δ_1 and Δ_2 are bounded by some constant known bounds δ_1 and δ_2, respectively.

From (1) one may think to calculate flow rates from the pressure dynamic equations, e.g.,

$$Q_1 = A_1\dot{x} + \frac{V_1(x)}{\beta_e}\dot{P}_1 - \Delta_1 \tag{2}$$

Though the pressure P_1 is measurable, it is very noisy. It is not practical to numerically differentiate P_1 unless a low pass filter is used, which would introduce phase lag. The accuracy of the calculation based on (2) also depends on the accuracy of the effective bulk modulus β_e, which is usually of large variance and very hard to be determined exactly. The appearance of the disturbance flow Δ_1 further complicates the problem.

A flow rate observer may be a solution to this problem. However, because the flow rate Q_i, i=1 or 2, is not a state of the system dynamics, Luenberger observer does not work in this case even when all parameters are known and the disturbance flow Δ_1 is negligible. In order to design the flow rate observer, one must overcome the following difficulties:

a. The system dynamics is not linear due to the change in total cylinder volumes $V_1(x)$ and $V_2(x)$.
b. Some system parameters, such as β_e may not be exactly known.
c. The flow rate Q_1 and Q_2 are inputs to the system instead of being system states.

This paper proposes a sliding mode flow rate observer to overcome the nonlinearity and model uncertainties

[2,3,4]. The proposed sliding mode flow rate observer has strong theoretical performance and robustness. It guarantees convergence in finite time when there is no model uncertainties and bounded estimation error in the presence of modelling error.

2 Sliding Mode Flow Rate Observer

Since the observers for Q_1 and Q_2 are basically same, only the one for Q_1 would be designed here, the other can be worked out in the same way.

2.1 Sliding mode flow rate observer design

Let $\theta_1 = \frac{1}{\beta_e}$, the pressure dynamics (1) can be rewritten as

$$V_1(x)\theta_1\dot{P}_1 = -A_1\dot{x} + Q_1 + \Delta_1 \tag{3}$$

The flow rate observer, based on the cylinder pressure dynamics (3), is given by

$$V_1(x)\hat{\theta}_1\dot{\hat{P}}_1 = -A_1\dot{x} + \hat{Q}_1 - K(\hat{P}_1 - P_1) \tag{4}$$

where $\hat{P}_1$, $\hat{Q}_1$ and $\hat{\theta}_1$ represent the estimates of P_1, Q_1 and θ_1, respectively, K is a positive observer gain. Subtract (3) from (4), one can obtain:

$$V_1(x)\hat{\theta}_1\left(\dot{\hat{P}}_1 - \dot{P}_1\right) + V_1(x)\left(\hat{\theta}_1 - \theta_1\right)\dot{P}_1 = \hat{Q}_1 - Q_1 - K(\hat{P}_1 - P_1) - \Delta_1 \tag{5}$$

Define $e = \hat{P}_1 - P_1$ as the pressure estimation error and $\tilde{\theta}_1 = \hat{\theta}_1 - \theta_1$ as the parameter estimation error, rewrite (5) in term of e,

$$V_1(x)\hat{\theta}_1\dot{e} + Ke = \hat{Q}_1 - Q_1 - V_1(x)\tilde{\theta}_1\dot{P}_1 - \Delta_1 \tag{6}$$

When parameters like the bulk modulus β_e is known and the disturbance flow Δ_1 is negligible (i.e., assume $\Delta_1 = 0$), Eq. (6) can be simplified into:

$$V_1(x)\hat{\theta}_1\dot{e} + Ke = \hat{Q}_1 - Q_1 \tag{7}$$

Since the metered flow rate Q_1 must go through the valve orifice, Q_1 is always limited by the valve or pump capability. Mathematically, Q_1 is bounded by a known bound, i.e., $|Q_1| \le Q_{\max}$.

In (7), $\hat{Q}_1$ can be considered as an input to the pressure estimation error dynamics. Therefore, one may want to choose a proper $\hat{Q}_1$ to make the estimation error *e(t)* converge to zero. A discontinuous $\hat{Q}_1$ is chosen as

$$\hat{Q}_1 = -Q_{\max} \cdot sign(e) \tag{8}$$

Note that during the sliding mode when $e = 0$, mathematically, (8) is defined by the so-called equivalent control [2,3] and can be approximated obtained by applying a low pass filter to the discontinuous $\hat{Q}_1$:

$$\ddot{\hat{Q}}_{1eq} + 2\xi\omega_n\dot{\hat{Q}}_{1eq} + \omega_n^2\hat{Q}_{1eq} = \hat{Q}_1 \tag{9}$$

where ξ and ω_n are the damping ratio and natural frequency of the filter.

Theorem: *Assuming parameters are known and the disturbance flow is negligible, the flow rate observer (4) and (8) guarantees that the pressure estimation error e(t) reaches zero in finite time and, subsequently, the equivalent flow estimate* $\hat{Q}_{1eq}$ *converges to the actual valve flow* Q_1.

Proof: Define a positive definite function $V = \frac{1}{2}V_1(x)\hat{\theta}_1 e^2$, where $V_1(x) = V_{10} + A_1 x$, V_{10} is the cylinder volume including hose and fitting volumes when the cylinder rod is fully retracted, i.e., $x = 0$. Differentiating V and noting (7), one can obtain:

$$\begin{aligned}\dot{V} &= \frac{1}{2}\dot{V}_1(x)\hat{\theta}_1 e^2 + V_1(x)\hat{\theta}_1 e\dot{e} \\ &= \frac{1}{2}A_1\dot{x}\hat{\theta}_1 e^2 - Ke^2 + \left(\hat{Q}_1 - Q_1\right)e \\ &= -(K - \frac{1}{2}A_1\dot{x}\hat{\theta}_1)e^2 - (Q_{\max}sign(e) + Q_1)e\end{aligned} \tag{10}$$

Since the cylinder rod velocity is always bounded due to finite flow capacity of any pump, it is easy to choose a K greater than $\left\|\frac{1}{2}A_1\dot{x}\hat{\theta}_1\right\|_\infty$. Therefore the derivative of V satisfies:

$$\dot{V} < -(Q_{\max} sign(e) + Q_1)e < -\eta |e| \tag{11}$$

for some positive η. Therefore, *e(t)* reaches zero in finite time and stays at zero.

Once *e(t)* is equal to zero, $\hat{P}_1 - P_1$ as well as $\dot{\hat{P}}_1 - \dot{P}_1$ would be zero. Hence Eq. (7) indicates the equivalent flow estimate $\hat{Q}_{1eq}$ would converge to Q_1. This ends the proof.

2.2 Robust performance analysis

When there is parametric uncertainty $\tilde{\theta}_1$ or the disturbance flow Δ_1 in Eq. (6), the derivative of V would change correspondingly from (10) to the following:

$$\dot{V} = -(K - \frac{1}{2} A_1 \dot{x} \hat{\theta}_1) e^2 - (Q_{\max} sign(e) + Q_1 + V_1(x)\tilde{\theta}_1 \dot{P}_1 + \Delta_1)e \tag{12}$$

Since both $V_1(x)\tilde{\theta}_1 \dot{P}_1$ and Δ_1 are bounded, as long as $Q_{\max}$ is chosen large enough to dominate Q_1 as well as $V_1(x)\tilde{\theta}_1 \dot{P}_1$ and Δ_1, the derivative of V still satisfies:

$$\begin{aligned} \dot{V} &< -(Q_{\max} sign(e) + Q_1 + V_1(x)\tilde{\theta}_1 \dot{P}_1 + \Delta_1)e \\ &< -\mu |e| \end{aligned} \tag{13}$$

for some positive μ. It is obvious that the pressure estimation error still reaches zero in finite time. After that, during the sliding mode when $e = 0$ and $\dot{e} = 0$, from (6), the equivalent flow estimate is

$$\hat{Q}_{1eq} = Q_1 + V_1(x)\tilde{\theta}_1 \dot{P}_1 + \Delta_1 \tag{14}$$

Thus, there will be a bounded valve flow estimation error of $\hat{Q}_{1eq} - Q_1 = V_1(x)\tilde{\theta}_1 \dot{P}_1 + \Delta_1$ that depends on the level of parametric uncertainties and the disturbance flow. However, the converging rate of the valve flow estimation is similar to the previous case when no model uncertainties exist as seen from (13) when compared to (11).

2.3 Remarks

The theoretically excellent robustness of sliding mode control is preserved in the sliding mode observer. The two main drawbacks of sliding mode control, i.e., large control authority and control chattering, do not limit the sliding observer's practical application. This is due to the fact that the chattering and large control authority issues in the sliding mode observer design are only linked to numerical implementation rather than "hard" mechanical limitations [3].

The equivalent flow estimate $\hat{Q}_{1eq}$ is obtained by sending $\hat{Q}_1$ through a low pass filter (LPF) to remove the high frequency chattering components and keep the effective low frequency components. The LPF used in the sliding mode observer has a different effect from the one used to filter pressure measurements in (2). The LPF used in flow rate calculation is to smooth the measurement noise, which is almost white and covers the entire frequency range from DC to Nyquist frequency. The cut off frequency of that LPF must be chosen very low to make the pressure differentiation possible.

On the other hand, the LPF used to obtain equivalent $\hat{Q}_{1eq}$ is to remove the chattering in $\hat{Q}_1$, whose frequency mainly depends on the observer gain K, the magnitude of the discontinuous term $Q_{\max}$ and the frequency of digital implementation. The larger K, $Q_{\max}$ and digital implementation frequency are, the higher the chattering frequency is. Practically, the observer can run at a higher sampling frequency than the control system sampling frequency, which would enable the chatting at a frequency even higher than the control system sampling frequency. Hence LPF in the observer design can have a much higher cutoff frequency and does not introduce severe phase lag to the flow rate estimation.

3 Simulations and Experiments

Simulations are done in Simulink. A simple PI controller is used to control a single-rod double-acting hydraulic cylinder with a servo valve, whose dynamics is neglected. The cylinder rod is controlled to track a point-to-point reference trajectory. White noise is added to pressure measurements.

The cylinder ram areas A_1 and A_2 are 0.002m^2 and 0.001m^2, respectively. The supply and tank pressures are set as 6900KPa (1000PSI) and 0KPa, respectively. The effective bulk modulus β_e is equal to 4.24e8. The cylinder internal leakage is added as the disturbance flow:

$$\Delta = K_{leak}\sqrt{|P_1 - P_2|}\cdot sign(P_1 - P_2) \quad (15)$$

where K_{leak} is chosen to be 1e-8.

The flow observer parameters K and $Q_{\max}$ are set as K=2e-11 and $Q_{\max}$=2e-4m^3/s. A second order LPF with cut off frequency equal to 40Hz is used to remove the chattering in $\hat{Q}_1$.

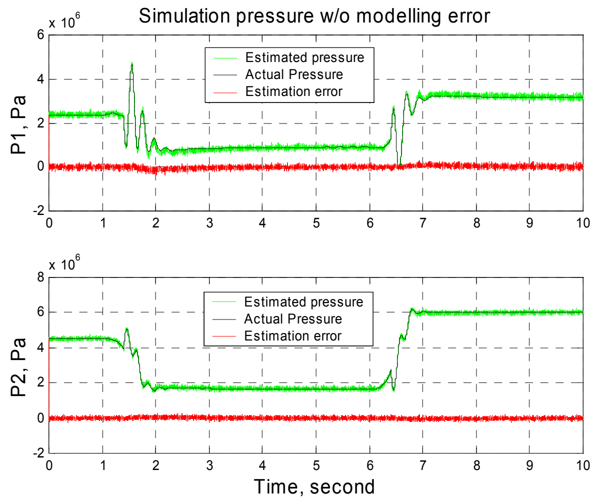

Fig. 1 *Simulation pressures w/o modelling error*

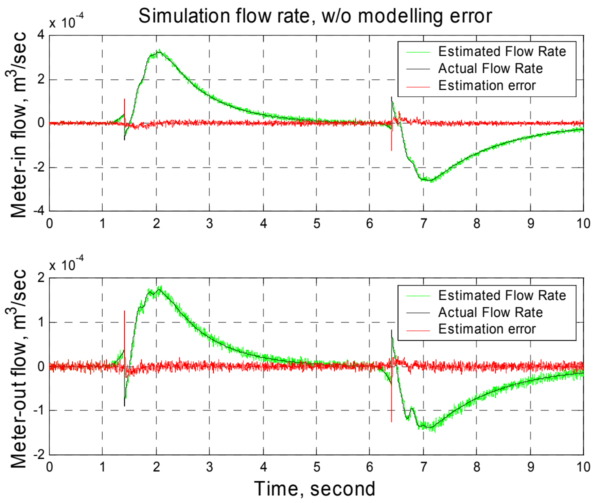

Fig. 2 *Simulation flow rates w/o modelling error*

Figures 1 and 2 show the pressure and flow rate estimates in the absence of disturbance flows. It is obvious that the pressure estimates for both chambers converge to the actual values very well without any phase lag, as predicted by the theorem. The flow rate estimates also converge very well except for two spikes where actual flow rates have discontinuous changes. However, this may not be a problem because flow rate would not change discontinuously in practice due to the valve and flow dynamics.

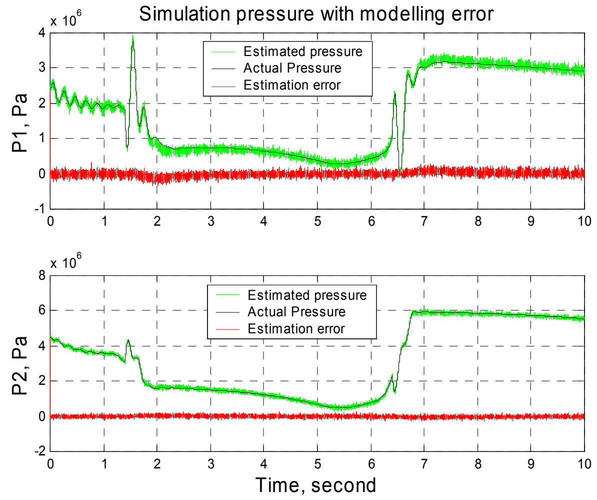

Fig. 3. *Simulation pressures with modelling error*

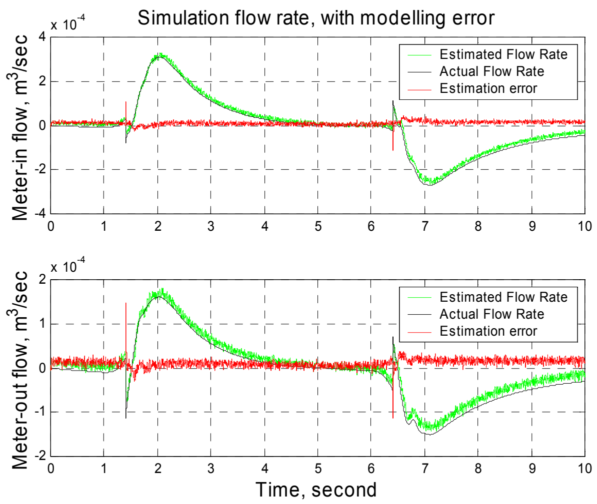

Fig. 4: *Simulation flow rates with modelling error*

Figures 3 and 4 illustrate the pressure and flow rate estimates in the presence of disturbance flows. The pressure estimates, as predicted by the theorem, are not affected by the disturbance flows and converge to the actual values very well. The flow rate estimates, though affected by the cylinder internal leakage, still show very good convergence.

Experiments are done with the cylinder used to activate the swing motion of a three degree-of-freedom electro-hydraulic robot arm. A nonlinear controller is used to control the swing motion to track an angular trajectory. Since the cylinder rod motion in the experiment is different from the one in simulation, the experiment flow rates look different from the simulation. Since there is no way to know the actual flow rate, we can not compare the estimates with their actual value. The flow rate estimates are shown in Fig. 5.

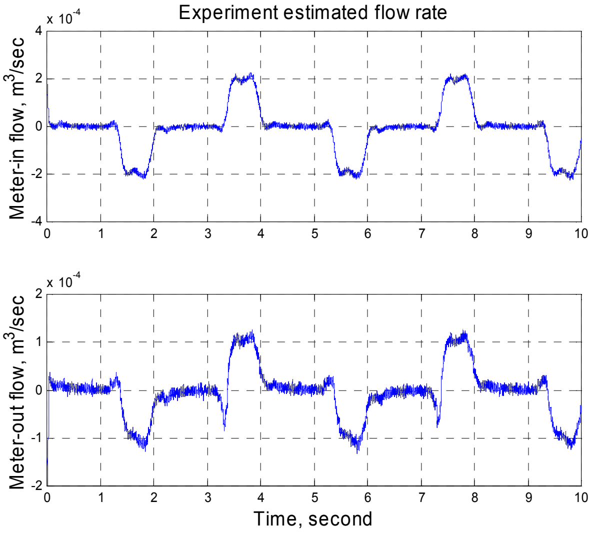

Fig. 5: *Experiment flow rate estimates*

4 Conclusions and Future Works

The proposed sliding mode dynamic flow rate observer takes advantage of the use of the equivalent control for the estimation of flow rates with discontinuous term. It successfully overcomes the nonlinear dynamics and uncertain modelling error, and theoretically guarantees that the valve flow rate estimates converge to their true values in the absence of the disturbance flow and the parameter estimation error, or a bounded estimation error in the presence of the disturbance flow and parameter estimation error. Simulation and experimental results are obtained to illustrate the performance of the proposed flow rate observer.

The flow rate observer is the first step toward an automated modelling mechanism of cartridge valve flow mapping. Future works include adopting adaptive robust observer (ARO) technique to deal with the changing parameters in the pressure dynamics, such as effective bulk modulus β_e, and to reduce effects of parameter variation and the disturbance flow estimation errors.

Acknowledgement

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