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INDIRECT ADAPTIVE ROBUST CONTROL OF SISO NONLINEAR SYSTEMS IN SEMI-STRICT FEEDBACK FORMS

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Abstract: In this paper, the modularized adaptive backstepping designs are incorporated into the recently proposed adaptive robust control framework to synthesize indirect adaptive robust controllers that achieve not only good output tracking performance but also better parameter estimation processes to obtain accurate parameter estimates for secondary purposes such as machine health monitoring and prognostics. Departing from the modularized adaptive backstepping designs, the proposed indirect adaptive robust control (IARC) uses available a priori knowledge on the physical bounds of unknown parameters, along with preset adaptation rate limits, to construct projection type parameter estimation algorithms with rate limits for a controlled estimation process. By doing so, regardless of the estimation algorithm to be used, a guaranteed transient performance and final tracking accuracy can be achieved even in the presence of disturbances and uncertain nonlinearities, a desirable feature in applications. In addition, the theoretical performance of the adaptive designs, asymptotic output tracking in the presence of parametric uncertainties only, is also preserved. The precision motion control of a linear motor drive system is used as an application example. Experimental results are obtained to show the improved parameter estimation process of the proposed IARC design.

Keywords: Adaptive Control, Robust Control, Nonlinear Systems, Uncertainties

1. INTRODUCTION

With the increasing demand for better control performance, it becomes necessary to explicitly consider the effect of nonlinearities and uncertainties associated with physical systems. As such, robust control of uncertain nonlinear systems has received significant attentions during the past twenty years. Two approaches have been popular: adaptive control (AC) (Krstic *et al.*, 1995; Landau, 1998) and deterministic robust control (DRC) (Utkin, 1992; Corless and Leitmann, 1981).

In (Yao and Tomizuka, 1994; Yao and Tomizuka, 2001; Yao, 1997), an adaptive robust control (ARC) approach is presented to systematically construct performance oriented control laws for nonlinear systems transformable to semi-strict feedback forms. The resulting ARC controllers enjoy the benefits of both AC and DRC methods while overcoming the practical performance limitations associated with AC and DRC. The approach has been applied to several applications and comparative experimental results have demonstrated the substantially improved performance of the ARC approach in implementation (Yao *et al.*, 1997; Yao *et al.*, 2000; Xu and Yao, 2000; Xu and Yao, 2001).

The underline parameter adaptation law in ARC controllers in (Yao, 1997) are based on the direct adaptive control designs including the tuning function based adaptive backstepping (Krstic *et al.*, 1995), in which the adaptive control law and parameter adaptation law are synthesized simultaneously to meet the sole objective of reducing the output tracking error. Such a design normally leads to a controller whose dynamic order is as low as the number of unknown parameters to be adapted while achieving excellent output tracking performance. However, the direct approach also has the drawback that the design of control law and the parameter estimation law cannot be separated and the choice of the

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parameter estimation law is limited to the gradient type with certain actual tracking errors as driving signals. It is well known that the gradient type of estimation law may not have as good parameter convergence properties as other types of parameter estimation laws (e.g., the least square method). Furthermore, for a well designed direct adaptive control law, the actual tracking errors in implementation are normally very small, and thus more prone to be corrupted by other factors such as the sampling delay and noise that have been neglected when synthesizing the parameter adaptation law. As a result, in implementation, the parameter estimates in the direct adaptive control are normally not accurate enough to be used for secondary purposes such as prognostics and machine component health monitoring, even when the desired trajectory is persistently exciting enough.

This paper concerns with applications that need accurate parameter estimates for other secondary purposes in addition to the good output tracking performance. An indirect adaptive robust control design will be presented to overcome the poor parameter estimates problem of the direct ARC designs (Yao and Tomizuka, 2001; Yao, 1997). Such an objective is achieved by separating the construction of parameter estimation law from the design of underline robust control law as in the indirect adaptive control designs (Krstic *et al.*, 1995; Landau, 1998). Two types of indirect adaptive robust controllers (IARC) are constructed and compared. The two methods are based on the modularized adaptive backstepping designs in (Krstic *et al.*, 1995) with z-swapping and x-swapping estimation algorithms respectively, but have a fundamental different view of the problem and the use of parameter adaptation. Specifically, the modularized adaptive backstepping designs (Krstic *et al.*, 1995) assume the system free of disturbances and uncertain nonlinearities. As a result, the theoretical boundedness of the parameter estimates and their derivatives can be achieved through the use of estimation algorithms with normalization and/or certain nonlinear damping, which may not be valid in applications due to the unavoidable disturbances and uncertain nonlinearities. The proposed methods, in contrast, use available a priori knowledge on the physical bounds of unknown parameters, along with preset adaptation rate limits, to construct projection type parameter estimation algorithms with rate limits for a controlled estimation process. By doing so, regardless of the specific adaptation law to be used (the gradient method or the least square method, with or without normalization), a guaranteed transient performance and final tracking accuracy is achieved even in the presence of disturbances and uncertain nonlinearities, which is very important in applications. In addition, the theoretical performance of the adaptive designs–asymptotic output tracking in the presence of parametric uncertainties only–is also preserved in the proposed IARC.

2. PROBLEM FORMULATION

The system under consideration is described by

$$\begin{aligned}\dot{x}_1 &= b_1 x_2 + \varphi_1(x_1,t)^T\theta + \Delta_1(\bar{x}_n,t)\\ &\cdots\\ \dot{x}_n &= b_n u + \varphi_n(\bar{x}_n,t)^T\theta + \Delta_n(\bar{x}_n,t)\end{aligned} \tag{1}$$

where x_1 is the system output, $\bar{x}_i = [x_1, ..., x_i]^T$ is the vector of the first i states, b_i is the unknown input gain of the i-th channel, $\theta = [\theta_1, ..., \theta_{p_\theta}]^T$ represents the vector of other unknown parameters, $\Delta_i(\bar{x}_n,t)$ is the uncertain nonlinearity in the i^{th} channel, and u is the control input. For notation simplicity, let $\theta_b \in R^p$ be the vector of all unknown parameters, i.e., $\theta_b = [\theta^T, b_1, \ldots, b_n]^T$. The following nomenclature is used throughout this paper: $\hat{\bullet}$ is used to denote the estimate of $\bullet$, $\tilde{\bullet}$ is used to denote the parameter estimation error of $\bullet$, e.g., $\tilde{\theta} = \hat{\theta} - \theta$, $\bullet_i$ is the i^{th} component of the vector $\bullet$, $\bar{\bullet}_i$ is a column vector of the first i components of $\bullet$, e.g., $\bar{b}_i = [b_1, b_2, ..., b_i]^T$, $\bullet_{max}$ and $\bullet_{min}$ are the maximum and minimum value of $\bullet(t)$ for all t respectively. The following practical assumptions are made:

Assumption 1. The unknown parameter vector θ_b is within a known bounded convex set Ω_{θ_b}. Furthermore, within Ω_{θ_b}, the input gains $b_i, i = 1, \ldots, n$, are of known signs and do not converge to zero. Without loss of generality, it is assumed that $\forall \theta_b \in \Omega_{\theta_b},\ \theta_{imin} \leq \theta_i \leq \theta_{imax}$ and $0 < b_{imin} \leq b_i \leq b_{imax}$, where θ_{imin}, θ_{imax}, b_{imin}, and b_{imax} are some known constants.

Assumption 2. The uncertain nonlinearity $\Delta_i(\bar{x}_n,t)$ can be bounded by

$$|\Delta_i(\bar{x}_n,t)| \leq \delta_i(\bar{x}_i) d_i(t), \quad \forall i \tag{2}$$

where $\delta_i(\bar{x}_i)$ is a known positive function, and $d_i(t)$ is an unknown but bounded positive time-varying function.

3. IARC USING Z-SWAPPING IDENTIFIER

In this section, an indirect adaptive robust control (IARC) based on the modular adaptive backstepping design with z-swapping identifier (Krstic *et al.*, 1995) is developed. Due to the appearance of the uncertain nonlinearities Δ_i in (1), a fundamental different view of the problem has to be taken in achieving the separation of controller and identifier designs. Specifically, we will use available knowledge of the physical bounds of the system parameters, along with preset adaptation rate limits to construct projection-type parameter estimation algorithms with rate limits for a controlled estimation process. By doing so, parameter estimates and their derivatives are guaranteed to be within certain known bounds, regardless of the specific estimation algorithm to be used and the appearances of disturbances or uncertain nonlinearities. Additionally, the known ranges of the variations of the parameter estimates and their derivatives can be incorporated into the controller design to synthesize robust controllers that achieve a guaranteed transient performance and final tracking accuracy without the use of strong nonlinear damping terms in (Krstic *et al.*, 1995).

Due to the space limit, only outline of the designs are presented below. The design details and the proofs of all lemmas and theorems are given in the full version of the paper (Yao and Palmer, 2002) and can be obtained from the first author.

3.1 *Projection Type Adaptation Law with Rate Limits*

As in (Yao, 1997), the widely used projection mapping $Proj_{\hat{\theta}_b}(\bullet)$ will be used to keep the parameter estimates within the known bounded set $\bar{\Omega}_{\theta_b}$, the closure of the set Ω_{θ_b}. The standard projection mapping is (Krstic *et al.*, 1995):

$$Proj_{\hat{\theta}_b}(\zeta) = \begin{cases} \zeta, & \text{if } \hat{\theta}_b \in \overset{\circ}{\Omega}_{\theta_b} \text{ or } n_{\hat{\theta}_b}^T \zeta \le 0 \\ \left(I - \Gamma \dfrac{n_{\hat{\theta}_b} n_{\hat{\theta}_b}^T}{n_{\hat{\theta}_b}^T \Gamma n_{\hat{\theta}_b}} \right) \zeta, & \\ \quad \text{if } \hat{\theta}_b \in \partial\Omega_{\theta_b} \text{ and } n_{\hat{\theta}_b}^T \zeta > 0 & \end{cases} \tag{3}$$

where $\zeta \in R^p$, $\Gamma(t) \in R^{p \times p}$, $\overset{\circ}{\Omega}_{\theta_b}$ and $\partial\Omega_{\theta_b}$ denote the interior and the boundary of Ω_{θ_b} respectively, and $n_{\hat{\theta}_b}$ represents the outward unit normal vector at $\hat{\theta}_b \in \partial\Omega_{\theta_b}$. For any $\zeta \in R^p$, define a saturation function as:

$$sat_{\dot{\theta}_M}(\zeta) = s_0 \zeta, \quad s_0 = \begin{cases} 1, & ||\zeta|| \le \dot{\theta}_M \\ \dfrac{\dot{\theta}_M}{||\zeta||}, & ||\zeta|| > \dot{\theta}_M \end{cases} \tag{4}$$

where $\dot{\theta}_M$ is a pre-set rate limit. Using the properties of the projection operator in Lemma E.1 in (Krstic *et al.*, 1995) and noting that s_0 is a positive scalar, it is easy to verify that the following lemma holds:

Lemma 3. Suppose that the parameter estimate $\hat{\theta}_b$ is updated using the following projection type adaptation law with a pre-set rate limit $\dot{\theta}_M$:

$$\dot{\hat{\theta}}_b = sat_{\dot{\theta}_M}\left(Proj_{\hat{\theta}_b}(\Gamma\tau) \right), \quad \hat{\theta}_b(0) \in \Omega_{\theta_b} \tag{5}$$

where τ is any adaptation function and $\Gamma(t) > 0$ is any continuously differentiable positive symmetric adaptation rate matrix. With this adaptation law, the following desirable properties hold:

P1. The parameter estimates are always within the known bounded set $\bar{\Omega}_{\theta_b}$, i.e., $\hat{\theta}_b(t) \in \bar{\Omega}_{\theta_b}$, $\forall t$. Thus, from Assumption 1, $\forall t$, $\theta_{imin} \le \hat{\theta}_i(t) \le \theta_{imax}$ and $0 < b_{imin} \le \hat{b}_i(t) \le b_{imax}$.

P2. $\tilde{\theta}_b^T \left(\Gamma^{-1} Proj_{\hat{\theta}_b}(\Gamma\tau)) - \tau \right) \le 0, \quad \forall \tau$

P3. The parameter update rate is uniformly bounded by $||\dot{\hat{\theta}}_b(t)|| \le \dot{\theta}_M$, $\forall t$ △

3.2 *Adaptive Robust Control Law*

With (5), the parameter estimates and their derivatives are bounded with known bounds, regardless of the estimation function τ to be used. Thus, similar to (Yao and Tomizuka, 2001), one can use backstepping to construct an adaptive robust control (ARC) law for the system (1) that achieves a guaranteed transient and final tracking accuracy, independent of the specific identifier to be used later. The resulting ARC controller is outlined below.

Following the standard backstepping design procedure, at each step $i = 1, \ldots, n$, a virtual control law α_i will be developed in order that x_i will track its desired virtual control law α_{i-1} that was synthesized in step $i-1$ with a desired transient performance. For this purpose, let $z_i = x_i - \alpha_{i-1}$ be the transformed tracking error at step i and define the following terms recursively for step i, $i = 2, ..., n$, from the previous steps

$$\phi_i(\bar{x}_i, \hat{\theta}, \hat{\bar{b}}_i, t) = \varphi_i(\bar{x}_i, t) - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l} \varphi_l \tag{6}$$

$$\breve{\Delta}_i(\bar{x}_n, t) = \Delta_i(\bar{x}_n, t) - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l} \Delta_l \tag{7}$$

in which $\alpha_0(t) = x_{1d}(t)$ is the desired output trajectory. From Assumption 2, $\breve{\Delta}_i(\bar{x}_n, t)$ is bounded by

$$|\breve{\Delta}_i(\bar{x}_n, t)| \le \breve{\delta}_i(\bar{x}_i, t) \breve{d}_i(t) \tag{8}$$

where $\breve{\delta}_i$ is any smooth function satisfying $\breve{\delta}_i \ge i \left[\max\left\{ \delta_i, max_{l=1,\ldots,i-1}\{ |\frac{\partial \alpha_{i-1}}{\partial x_l}| \delta_l \} \right\} \right]$ and $\breve{d}_i(t) = max_{l=1,\ldots,i}\{d_l(t)\}$. Similar to (Yao and Tomizuka, 2001), the following lemma and theorem can be proved.

Lemma 4. For each $i \le n$, choose the following virtual control function for x_{i+1}

$$\begin{aligned} &\alpha_i(\bar{x}_i, \hat{\theta}, \hat{\bar{b}}_i, t) = \alpha_{ia} + \alpha_{is}, \\ &\alpha_{ia} = \frac{1}{\hat{b}_i} \left[\sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l} \hat{b}_l x_{l+1} + \frac{\partial \alpha_{i-1}}{\partial t} - \phi_i^T \hat{\theta} - \hat{b}_{i-1} z_{i-1} \right] \\ &\alpha_{is} = \alpha_{is1} + \alpha_{is2}, \ \alpha_{is1} = -\frac{1}{\hat{b}_i} k_i z_i, \ \alpha_{is2} = -k_{is2} z_i \end{aligned} \tag{9}$$

where k_i is a positive constant and $k_{is2}(\bar{x}_i, t)$ is a nonlinear feedback gain chosen to be large enough so that the following robust performance condition is satisfied

$$\begin{aligned} z_i &\left[\hat{b}_i \alpha_{is2} + \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l} \tilde{b}_l x_{l+1} - \tilde{b}_{i-1} z_{i-1} - \phi_i^T \tilde{\theta} - \tilde{b}_i \alpha_i \right. \\ &\left. - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{b}_l} \dot{\hat{b}}_l - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} + \breve{\Delta}_i \right] \le \varepsilon_{ci} + \varepsilon_{di} \breve{d}_i^2 \end{aligned} \tag{10}$$

where ε_{ci} and ε_{di} are positive constant design parameters. With the virtual control function (9), the i^{th} error equation can be written as

$$\begin{aligned} \dot{z}_i = b_i z_{i+1} - k_i z_i + \hat{b}_i \alpha_{is2} + \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l} \tilde{b}_l x_{l+1} - \phi_i^T \tilde{\theta} \\ - \hat{b}_{i-1} z_{i-1} - \tilde{b}_i \alpha_i + \breve{\Delta}_i - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{b}_l} \dot{\hat{b}}_l - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \end{aligned} \tag{11}$$

and the derivative of the augmented positive definite function $V_i = V_{i-1} + \frac{1}{2} z_i^2$ is given by

$$\begin{aligned} \dot{V}_i = -\sum_{l=1}^{i} k_l z_l^2 + b_i z_i z_{i+1} + \sum_{l=1}^{i} z_l \Big\{ \hat{b}_l \alpha_{ls2} \\ + \sum_{j=1}^{l-1} \frac{\partial \alpha_{l-1}}{\partial x_j} \tilde{b}_j x_{j+1} - \phi_l^T \tilde{\theta} - \tilde{b}_l \alpha_l + \breve{\Delta}_l \\ - \sum_{j=1}^{l-1} \frac{\partial \alpha_{l-1}}{\partial \hat{b}_j} \dot{\hat{b}}_j - \frac{\partial \alpha_{l-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - \tilde{b}_{l-1} z_{l-1} \Big\} \end{aligned} \tag{12}$$

Theorem 5. Consider the ARC law given by

$$u = \alpha_n(\bar{x}_n, \hat{\theta}, \hat{\bar{b}}_n, t) \qquad (13)$$

where α_n is given by equations (9) with $i = n$ and the parameter estimates are updated through (5), in which τ could be any adaptation function. In general, all signals in the resulting closed loop system are bounded. In addition, the tracking errors are bounded by

$$\begin{aligned} \|\bar{z}_n(t)\|^2 &\le e^{-\lambda_v t}\|\bar{z}_n(0)\|^2 + 2\int_0^t e^{-\lambda_v(t-\tau)}\varepsilon_v(\tau)d\tau \\ &\le e^{-\lambda_v t}\|\bar{z}_n(0)\|^2 + \frac{2\varepsilon_{vmax}}{\lambda_v}\left[1 - e^{-\lambda_v t}\right] \end{aligned} \qquad (14)$$

where $\lambda_v = 2\left(\min_{i=1,\dots,n}\{k_i\}\right)$ and $\varepsilon_v(t) = \sum_{i=1}^n [\varepsilon_{ci} + \varepsilon_{di}\breve{d}_i^2(t)]$ △

Remark 6. P1 and P3 of Lemma 3 guarantee that the parameter estimation error $\tilde{\theta}_b$ and its derivative $\dot{\hat{\theta}}_b$ are bounded with known bounds. Thus, how to choose the nonlinear feedback gain $k_{is2}(\bar{x}_i, t), i = 1, \dots, n$, to satisfy the robust performance condition (10) can be worked out in the same way as in (Yao, 1997). Furthermore, $\hat{b}_i$ is guaranteed to be non-zero, which makes the control law (9) free of singularity. △

3.3 *Parameter Estimation Algorithm*

In the above subsection, an adaptive robust control law which can admit any estimation function τ has been constructed and a guaranteed transient and final tracking performance is achieved even in the presence of uncertain nonlinearities. Thus, the reminder of the paper is to construct suitable estimation functions τ so that an improved final tracking accuracy–asymptotic tracking or zero final tracking error in the presence of parametric uncertainties only–can be obtained with an emphasis on good parameter estimation process as well. As such, in this subsection, we assume the system is absence of uncertain nonlinearities, i.e., let $\Delta_i = 0, i = 1, ..., n$, in (1).

From (9) and (11), when $\Delta_i = 0$, as in (Krstic *et al.*, 1995), the error dynamics can be put in the following concise form:

$$\dot{\bar{z}}_n = A_n(\bar{z}_n, \hat{\theta}_b, t)\bar{z}_n + W(\bar{z}_n, \hat{\theta}_b, t)^T\tilde{\theta}_b + Q(\bar{z}_n, \hat{\theta}_b, t)\dot{\hat{\theta}}_b \qquad (15)$$

where the matrices A_n, W and Q are defined in the same way as in (Krstic *et al.*, 1995) with A_n being an exponentially stable matrix. Construct the following filters:

$$\dot{\Omega}^T = A_n\Omega^T + W^T \qquad (16)$$

$$\dot{\Omega}_0 = A_n\Omega_0 - W^T\hat{\theta}_b - Q\dot{\hat{\theta}}_b \qquad (17)$$

Define the prediction error as $\epsilon = \bar{z}_n + \Omega_0 + \Omega^T\hat{\theta}_b$, which is calculable. It is shown in (Krstic *et al.*, 1995) that ϵ can be written as:

$$\epsilon = \Omega^T\tilde{\theta}_b + \tilde{\epsilon} \qquad (18)$$

where $\tilde{\epsilon}$ is governed by $\dot{\tilde{\epsilon}} = A_n\tilde{\epsilon}$ which exponentially converges to zero. Thus we have a static model (18) that is linearly parameterized in terms of $\tilde{\theta}_b$, with an additional term $\tilde{\epsilon}$ that exponentially decays to zero. With this static model, various estimation algorithms can be used to identify unknown parameters, of which the gradient estimation algorithm and the least squares estimation algorithm (Krstic *et al.*, 1995) are given below.

3.3.1. *Gradient Estimator* With the gradient type estimation algorithm, the resulting adaptation law is given by (5), in which Γ can be chosen as a constant positive diagonal matrix, i.e., $\Gamma = diag[\gamma_1, ..., \gamma_p]$, and τ is defined as

$$\tau = -\frac{\Omega\epsilon}{1 + \nu\|\Omega\|_F^2}, \quad \nu \ge 0 \qquad (19)$$

where by allowing $\nu = 0$, we encompass unnormalized adaptation function, and $\|\Omega\|_F$ represents the Frobenius norm of Ω, given by $\|\Omega\|_F^2 = tr\{\Omega^T\Omega\}$, in which $tr\{\bullet\}$ is the trace operation.

3.3.2. *Least Squares Estimator* When the least squares type estimation algorithm with co-variance re-setting and exponential forgetting is used, the resulting adaptation law is given by (5), in which $\Gamma(t)$ is updated by

$$\dot{\Gamma} = \alpha\Gamma - \Gamma\frac{\Omega\Omega^T}{1 + \nu tr\{\Omega^T\Gamma\Omega\}}\Gamma, \ \Gamma(t_r^+) = \rho_0 I, \ \nu \ge 0 \qquad (20)$$

where $\Gamma(0) = \Gamma^T(0) > 0$, $\nu = 0$ leads to the unnormalized algorithm, and τ is defined as

$$\tau = -\frac{\Omega\epsilon}{1 + \nu tr\{\Omega^T\Gamma\Omega\}} \qquad (21)$$

In (20), $\alpha \ge 0$ is the forgetting factor, t_r is the covariance resetting time, i.e., the time when $\lambda_{min}(\Gamma(t)) = \rho_1$ where ρ_1 is a pre-set lower limit for $\Gamma(t)$ satisfying $0 < \rho_1 < \rho_0$. In practice, the above least square estimator may lead to estimator windup (i.e., $\lambda_{max}(\Gamma(t)) \longrightarrow \infty$) when the regressor is not persistently exciting. To prevent this estimator windup problem and take into account the effect of the rate-limited adaptation law (5), (20) is modified to

$$\dot{\Gamma} = \begin{cases} \alpha\Gamma - \dfrac{\Gamma\Omega\Omega^T\Gamma}{1 + \nu tr\{\Omega^T\Gamma\Omega\}}, & \text{if} \quad \lambda_{max}(\Gamma(t)) \le \rho_M \\ & \text{and} \quad \|Proj_{\hat{\theta}_b}(\Gamma\tau)\| \le \dot{\theta}_M \\ 0 & \text{otherwise} \end{cases} \qquad (22)$$

where ρ_M is the pre-set upper bound for $\|\Gamma(t)\|$ with $\rho_M > \rho_0$. With these practical modifications, $\rho_1 I \le \Gamma(t) \le \rho_M I$, $\forall t$. It can thus be shown that the following lemma and theorem hold when these estimation algorithms are used:

Lemma 7. When the rate-limited projection type adaptation law (5) with either the gradient estimator (19) or the least squares estimator (21) is used, the following results hold:

$$\epsilon \in \mathcal{L}_2[0, \infty) \cap \mathcal{L}_\infty[0, \infty) \qquad (23)$$

$$\dot{\hat{\theta}}_b \in \mathcal{L}_2[0, \infty) \cap \mathcal{L}_\infty[0, \infty) \qquad (24)$$

Theorem 8. In the presence of parametric uncertainties only, i.e., $\Delta_i = 0$, $i = 1, ..., n$, by using the control law (13), filter (16)-(17), adaptation law (5) with either the gradient type estimation function (19) or the least squares type estimation function (21), in addition to the robust performance results stated in Theorem 1, an improved final tracking performance, asymptotic output tracking, is also achieved, i.e., $\bar{z}_n \to 0$ as $t \to \infty$. △

Remark 9. In (Krstic *et al.*, 1995), although only the ideal case of no disturbances and uncertain nonlinearities is considered, for closed loop stability, it is necessary to use the normalized estimation algorithms ($\nu > 0$ in (19) or (21)) and/or strong nonlinear damping. In the proposed IARC, the design is the same for normalized and un-normalized estimation functions. $\triangle$

4. IARC USING X-SWAPPING IDENTIFIER

The prediction error model (18) is obtained from the transformed tracking error dynamics, which are prone to be corrupted by neglected factors such as the measurement noises. As a result, the quality of parameter estimates is usually poor, even with estimation algorithms having better parameter convergency property as observed in experiments (e.g., the least square estimator (20). To overcome this problem, in this section, the x-swapping adaptive backstepping technique in (Krstic *et al.*, 1995) will be used to construct an IARC controller whose prediction error model is based on the system physical model rather than the transformed tracking error dynamics. Such an IARC design totally decouples the estimator construction from the robust control law design. Furthermore, as only the measured system states and actual system model are used in implementing the estimator, the effect of neglected factors such as the measurement noises is reduced, which makes the accurate parameter estimation possible in practice.

The construction of underline ARC law is the same as in section 3. So as in subsection 3.3, the reminder of the section is to construct suitable estimation functions τ that is based on the original system model (1), rather than the transformed tracking error dynamics (11), to achieve asymptotic output tracking. For this purpose, note that, when $\Delta_i = 0$, the system dynamics (1) can be re-written as

$$\dot{\bar{x}}_n = f_0(\bar{x}_n, u) + F^T(\bar{x}_n, u)\theta_b \tag{25}$$

where the matrix F is defined as

$$F^T(\bar{x}_n, u) = \begin{bmatrix} \varphi_1^T & x_2 & 0 & \ldots\ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \varphi_n^T & 0 & \ldots & \ldots & 0 & u \end{bmatrix} \tag{26}$$

and the vector of known functions $f_0 \in \Re^n$ is added for generality and represents the lumped effect of all known nonlinearities, which is zero for (1) for simplicity of presentation. Construct the following filters:

$$\begin{aligned} \dot{\Omega}^T &= A\Omega^T + F^T \\ \dot{\Omega}_0 &= A(\Omega_0 + \bar{x}_n) - f_0 \end{aligned} \tag{27}$$

where A is an exponentially stable matrix. Let $y = \bar{x}_n + \Omega_0$. From (25) and (27),

$$\begin{aligned} \dot{y} &= f_0 + F^T\theta_b + A(\Omega_0 + \bar{x}_n) - f_0 \\ &= F^T\theta_b + A(\Omega_0 + \bar{x}_n) \end{aligned} \tag{28}$$

Let $\tilde{\epsilon} = \bar{x}_n + \Omega_0 - \Omega^T\theta_b$. As in (Krstic *et al.*, 1995), it is easy to verify that y can be written as

$$y = \Omega^T\theta_b + \tilde{\epsilon} \tag{29}$$

where $\tilde{\epsilon}$ exponentially decays to zero and is governed by $\dot{\tilde{\epsilon}} = A\tilde{\epsilon}$. Now define the estimate of y as

$$\hat{y} = \Omega^T\hat{\theta}_b \tag{30}$$

and define the prediction error as $\epsilon = \hat{y} - y = \Omega^T\hat{\theta}_b - \bar{x}_n - \Omega_0$. The resulting prediction error model is

$$\epsilon = \Omega^T\tilde{\theta}_b - \tilde{\epsilon} \tag{31}$$

Thus, we have a static model (29) that is linearly parameterized in terms of $\tilde{\theta}_b$, with an additional term $\tilde{\epsilon}$ that exponentially decays to zero. With this static model, various estimation algorithms can be used to identify unknown parameters. For example, the gradient type estimation algorithm would be given by (19) but with the prediction error ϵ calculated from (31) instead. Similarly, the least squares type estimation algorithm would be (22) but with the prediction error ϵ from (31).

With the above estimators, it can be shown that the same results as in Lemma 7 and Theorem 8 hold.

Remark 10. In (Krstic *et al.*, 1995), although only the ideal case of no disturbances and uncertain nonlinearities is considered, for closed loop stability, it is necessary to use the normalized estimation algorithms ($\nu > 0$ in (19) or (21)) or complicated nonlinear damping in the filter matrix A in (27). In the above proposed IARC, the filter matrix A can be any exponentially stable constant matrix and the design is the same for normalized and un-normalized estimation functions. $\triangle$

5. IARC PRECISION MOTION CONTROL OF LINEAR MOTOR DRIVE SYSTEMS

The proposed two IARC designs have been applied to the precision motion control of a linear motor drive system (Xu and Yao, 2000). The details on how the IARC control laws are implemented are given in (Yao and Palmer, 2002). This section only gives some typical experimental results for illustration purpose.

A typical high-speed/high-acceleration motion trajectory for the pick-and-place operations in industry is used in all experiments. The desired trajectory has a moving distance of $0.4m$ with a maximum speed of $1m/s$ and an acceleration more than $12m/sec^2$. The following controllers are implemented: (1) **DCIARCzg**, desired compensation IARC (DCIARC) with z-swapping identifier and gradient type update law, (2) **DCIARCzls**, DCIARC with z-swapping identifier and least squares update law, (3) **DCIARCxg**, DCIARC with x-swapping identifier and gradient update law, and (4) **DCIARCxls**, DCIARC with x-swapping identifier and least squares update law.

The experimental results for no-load situation are shown in Figs.1 and 2, which has an off-line estimated actual value of $\theta_1 = 10.11, \theta_2 = 3.33$, and $b = 37.04$. The tracking errors of all four DCIARC controllers are comparable and within $70\mu m$ over the entire run as shown in Fig.1. However, as seen from Fig.2, the parameter estimates of both DCIARCzg and DCIARCzls do not converge at all. In fact they drift away to wrong values. In contrast, the parameter estimates of both DCIARCxg and DCIARCxls approach their true values. These results verify the claim that the x-swapping identifier has a more robust parameter estimation process than the z-swapping identifier.

6. CONCLUSIONS

In this paper, indirect adaptive robust control (IARC) designs have been developed to synthesize nonlinear controllers that achieve not only good output tracking performance but also accurate parameter estimates for other secondary purposes such as machine health monitoring. Experimental results have been obtained to show the improved parameter estimation process of the proposed IARC designs.

7. REFERENCES

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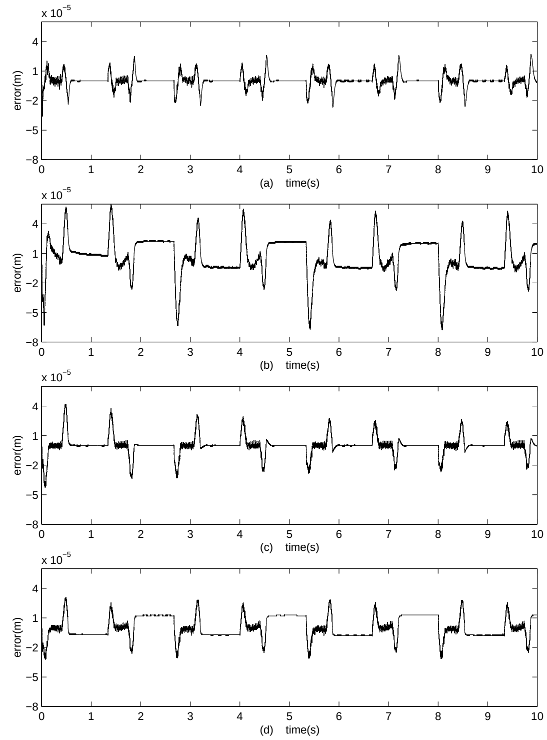

Fig. 1. Tracking error for (a)DCIARCzg, (b)DCIARCzls, (c)DCIARCxg, and (d)DCIARCxls with no load.

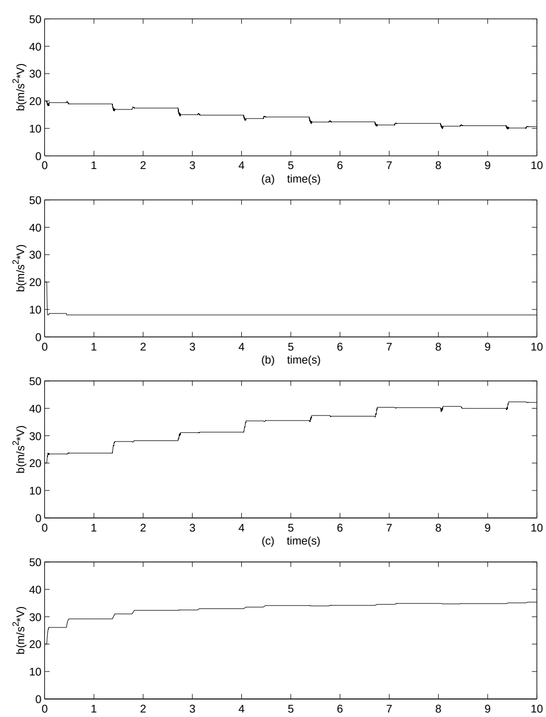

Fig. 2. $\hat{b}$ for (a)DCIARCzg, (b)DCIARCzls, (c)DCIARCxg, and (d)DCIARCxls with no load.