NONLINEAR ADAPTIVE ROBUST CONTROL OF ONE-DOF ELECTRO-HYDRAULIC SERVO SYSTEMS *

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Abstract

This paper intends to provide a rigorous theoretic framework for the high performance robust control of electro-hydraulic servo-systems. This is achieved through applying the recently proposed adaptive robust control (ARC) while taking into account the particular nonlinearities and model uncertainties of the electro-hydraulic servo-systems. In particular, the robust motion control of a typical one degree of freedom (DOF) electro-hydraulic servo-system will be considered. The system consists of an inertia load driven by a double rod cylinder regulated by a two-stage servovalve. The paper will consider the effect of both parametric uncertainties coming from the inertia load and the cylinder and the uncertain nonlinearities such as friction forces. Non-differentiability of the inherent nonlinearities associated with hydraulic dynamics is carefully examined and strategies are provided for handling the non-differentiability of the control term due to the directional change of valve opening when doing backstepping design via ARC Lyapunov function. The resulting controller guarantees a prescribed transient performance and final tracking accuracy in the presence of both parametric uncertainties and uncertain nonlinearities while achieving asymptotic tracking in the presence of parameteric uncertainties.

Keywords

Electro-Hydraulic System, Motion Control, Adaptive Control, Robust Control, Servo Control

*Part of the paper was presented in the 1997 ASME International Mechanical Engineering Congress and Exposition (IMECE'97)

1 Introduction

Hydraulic systems have been used in industry in a wide number of applications by virtue of their small size-to-power ratios and the ability to apply very large forces and torques; examples like electro-hydraulic positioning systems [1], active suspension control [2], and hydraulic robot manipulators [3, 4]. However, hydraulic systems also have a number of characteristics which complicate the development of high performance closed-loop controllers, and advanced control techniques have not been developed to address those issues. Other drive technologies, such as electric servo-motors, although not being able to compete directly in terms of size and power, have succeeded in displacing hydraulic drives from many precision control applications (e.g., machine tools in machining). This leads to the urgent need for advancing hydraulics technologies by combining the high power of hydraulic actuation with the versatility of electronic control.

The dynamics of hydraulic systems are highly nonlinear [5]. Furthermore, the system may be subjected to *non-smooth and discontinuous nonlinearities* due to control input saturation, directional change of valve opening, friction, and valve overlap. Aside from the nonlinear nature of hydraulic dynamics which demands the use of nonlinear control, hydraulic servosystems also have large extent of model uncertainties. The uncertainties can be classified into two categories: *parametric uncertainties* and *uncertain nonlinearities*. Examples of parametric uncertainties include the large changes in load seen by the system in industrial use and the large variations in the parameters (e.g., bulk modulus) that characterize the system due to the change of temperature and component wear [6]. Other general uncertainties, such as the external disturbances, leakage, and friction, cannot be modeled exactly and the nonlinear functions that describe them are not known. These kinds of uncertainties are called uncertain nonlinearities. These model uncertainties may cause the controlled system, designed on the nominal model, to be unstable or have a much degraded performance. Nonlinear robust control techniques, which can deliver high performance in spite of both parametric uncertainties and uncertain nonlinearities, are essential for successful operations of high-performance hydraulic servosystems.

In the past, due to the difficulties and the lack of the systematic procedures in designing nonlinear robust controllers, much of the work in the control of hydraulic systems uses linear control theory [7, 8, 3, 4]. These schemes linearize the nonlinear hydraulic dynamics around a specific operation point and cannot handle the large operating range; quite normally hydraulic servosystems are commanded to move the load from one extreme position to the other. Recently, linear robust control method [1] and adaptive control techniques [9, 10] are also used to cope with the changes of the operation point over time. However, these schemes are still based on the linearized dynamics and cannot cope with the nonlinearities inherent in hydraulic systems. In [11, 2], Alleyne and Hedrick applied the nonlinear adaptive control to the force control of an active suspension driven by a double-rod cylinder. They demonstrated that nonlinear control schemes can achieve a much better performance than conventional linear controllers. They considered the parametric uncertainties of the cylinder only. Uncertain nonlinearities such as uncompensated friction force cannot be included in the framework due to the non-robustness of adaptive control to uncertain nonlinearities. Furthermore, transient performance is not clear.

Recently, Yao and Tomizuka proposed a new approach, adaptive robust control (ARC) [12, 13, 14, 15], for high performance robust control of uncertain nonlinear systems in the presence of both parametric uncertainties and uncertain nonlinearities. The approach effectively combines the design techniques of adaptive control (AC) [16, 17], and those of deterministic robust control (DRC) (e.g., sliding mode control, SMC) [18, 19] and improves performance by preserving the advantages of both AC and DRC. Specifically, through robust feedback as in DRC [18, 19], the proposed ARC attenuates the effect of model uncertainties coming from both parametric uncertainties and uncertain nonlinearities and, thus, a guaranteed performance can be achieved in terms of both the transient error and the final tracking accuracy. This result overcomes the drawbacks of possible non-robustness to uncertain nonlinearities of adaptive control (AC) [20, 16, 17], and makes the approach attractive from the view point of applications. Through parameter adaptation as in adaptive control [16, 17], the proposed ARC reduces model uncertainties and, thus, achieves asymptotic tracking (zero final tracking error) in the presence of parametric uncertainties without resorting to a discontinuous control law [18] or an infinite gain in the feedback loop [21]. In this sense, ARC has a better tracking performance than DRC and overcomes the conservativeness of DRC designs. Comparative experimental results for trajectory tracking control of robot manipulators [13] and high-speed/high-accuracy motion control of machine tools [22] have shown the advantages of the proposed ARC and the improvement of performance. A general formulation of the proposed ARC is presented in terms of adaptive robust control (ARC) Lyapunov functions [15]. Through the backstepping design, ARC Lyapunov functions have been successfully constructed for a large class of multi-input multi-output (MIMO) nonlinear systems transformable to semi-strict feedback forms [15].

This paper intends to provide a rigorous theoretic framework for the high performance robust control of electro-hydraulic servo-systems, which will be achieved through generalizing the ARC approach to take into account the particular nonlinearities and model uncertainties of the electro-hydraulic servo-systems. In particular, the robust motion control of a typical electro-hydraulic servo-system shown in Fig.1 will be considered. The system consists of an inertia load driven by a double rod cylinder regulated by a two-stage servovalve. The paper will consider the effect of both parametric uncertainties coming from the inertia load and the cylinder and the uncertain nonlinearities such as friction forces. The unknown inertia load necessitates the backstepping design via ARC Lyapunov function. The *non-smoothness of nonlinearities* associated with hydraulic dynamics (e.g., the nonlinear function describing the relationship between the flow rate and the valve opening) prohibits the direct application of the ARC design in [15] since all nonlinearities studied are assumed to be sufficiently smooth as in adaptive backstepping design [16, 17]. In this paper, a novel strategy is provided to overcome this difficulty. The resulting controller guarantees a prescribed transient performance and final tracking accuracy in the presence of both parametric uncertainties and uncertain nonlinearities while achieving asymptotic tracking in the presence of parameteric uncertainties only.

2 Problem Formulation and Dynamic Models

The system under consideration is depicted in Fig. 1. The goal is to have the inertia load to track any specified motion trajectory as closely as possible; examples like a machine tool axis [22], an injection molding process, and a forming process.

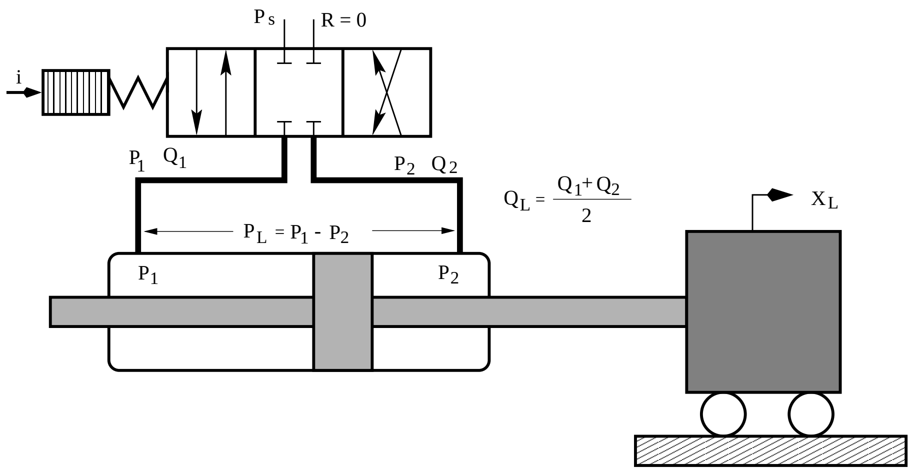

Figure 1: A One DOF Electro-Hydraulic Servo System

The dynamics of the inertia load can be described by

$$m\ddot{x}_L = P_L A - b\dot{x}_L - F_{fc}(\dot{x}_L) + \tilde{f}(t, x_L, \dot{x}_L) \tag{1}$$

where x_L and m represent the displacement and the mass of the load respectively, $P_L = P_1 - P_2$ is the load pressure of the cylinder, A is the ram area of the cylinder, b represents the combined coefficient of the modeled damping and viscous friction forces on the load and the cylinder rod, F_{fc} represents the modeled Coulomb friction force, and $\tilde{f}(t, x_L, \dot{x}_L)$ represents the external disturbances as well as terms like the unmodeled friction forces. Neglecting the effect of leakage flows in the cylinder and the servovalve, the actuator (or the cylinder) dynamics can be written as [5, 11]

$$\frac{V_t}{4\beta_e}\dot{P}_L = -A\dot{x}_L - C_{tm}P_L + Q_L \tag{2}$$

where V_t is the total volume of the cylinder and the hoses between the cylinder and the servovalve, β_e is the effective bulk modulus, C_{tm} is the coefficient of the total leakage of the cylinder due to pressure, and Q_L is the load flow. Q_L is related to the spool valve displacement of the servovalve, x_v, by [5, 11]

$$Q_L = C_d w x_v \sqrt{\frac{P_s - sgn(x_v)P_L}{\rho}} \tag{3}$$

where C_d is the discharge coefficient, w is the spool valve area gradient, and P_s is the supply pressure of the fluid. The servovalve is a typical two-stage type with force feedback and an current input. For simplicity, the same servovalve as in [11] will be used in this study. Basically, the spool valve displacement is related to the current input i by a third-order transfer function [11]:

$$\frac{x_v(s)}{i(s)} = \frac{1.31\times10^8}{s^3+4092s^2+(2.62\times10^7)s+4.03\times10^9}\frac{m}{A} \tag{4}$$

The system given by (4) has three poles; two complex poles at $-1761.6+4740.7j$ and one real pole at -157.3. Since the two complex poles are an order of magnitude faster than the real pole and are far beyond the normal operating frequency, they can be neglected in the controller design stage. The resulting reduced first-order model of the two- stage servovalve is thus given by [11]

$$\frac{x_v(s)}{i(s)} = \frac{5.1}{s+157.3} = \frac{K}{\tau s+1}\frac{m}{A} \tag{5}$$

where $\tau = 0.00636$ and $K = 0.0324$.

The entire system, Eqs. (1)-(3) and (5), can be represented in a state space form as

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \tfrac{1}{m}(Ax_3 - bx_2 - F_{fc}(x_2)) + d(t,x_1,x_2), \qquad\qquad d = \tfrac{1}{m}\tilde{f}(t,x_1,x_2) \\
\dot{x}_3 &= \tfrac{4\beta_e}{V_t}[-Ax_2 - C_{tm}x_3 + \tfrac{C_d w}{\sqrt{\rho}} g_{3o}(x_3,x_4)x_4] \\
\dot{x}_4 &= -\tfrac{1}{\tau}x_4 + \tfrac{K}{\tau}u
\end{aligned} \tag{6}$$

where $x = [x_1, x_2, x_3, x_4]^T$ is the state, which is defined by $x \triangleq [x_L,\ \dot{x}_L,\ P_L,\ x_v]^T$, $g_{3o}(x_3,x_4) = \sqrt{P_s - sgn(x_4)x_3}$, and $u = i$ is the control input.

Given the desired motion trajectory $x_{Ld}(t)$, *the objective is to synthesize a control input u such that the output $y = x_1$ tracks $x_{Ld}(t)$ as closely as possible in spite of various model uncertainties.*

3 Adaptive Robust Control of Electro-Hydraulic Servo Systems

To begin the controller design, practical and reasonable assumptions on the system have to be made. In general, the system is subjected to parametric uncertainties due to the variations of m, b, F_{fc}, β_e, C_{tm}, C_d, ρ, τ and K. For simplicity, in this paper, we only consider the parametric uncertainties of important parameters like m, β_e, and the nominal value of the disturbance d, d_n. The importance of estimating m and d_n for the precision control of an inertia load can been partly seen from the experimental results obtained for machine tools [22]. Other parametric uncertainties can be dealt with in the same way if necessary. In order to use parameter adaptation to reduce parametric uncertainties to improve performance, it is necessary to linearly parametrize the state space equation (6) in terms of a set of unknown parameters. To achieve this, define the unknown parameter set $\theta = [\theta_1, \theta_2, \theta_3]^T$ as $\theta = [\frac{1}{m}, d_n, \frac{4\beta_e}{V_t}]^T$. The state space equation (6) can thus be linearly parametrized in terms of θ as

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \theta_1(Ax_3 - bx_2 - F_{fc}(x_2)) + \theta_2 + \tilde{d}(t,x_1,x_2), \qquad \tilde{d} = d(t,x_1,x_2) - d_n \\
\dot{x}_3 &= \theta_3[-Ax_2 - C_{tm}x_3 + g_3(x_3,x_4)x_4], \qquad g_3 = \tfrac{C_d w}{\sqrt{\rho}} g_{3o}(x_3,x_4) \\
\dot{x}_4 &= -\tfrac{1}{\tau}x_4 + \tfrac{K}{\tau}u
\end{aligned} \tag{7}$$

Normally, the extents of the parametric uncertainties and uncertain nonlinearities are known. Thus the following practical assumption is made

Assumption 1 *Parametric uncertainties and uncertain nonlinearities satisfy*

$$\begin{array}{rcl} \theta & \in & \Omega_\theta \triangleq \{\theta : \quad \theta_{min} < \theta < \theta_{max} \ \} \\ |\tilde{d}(t, x_1, x_2)| & \leq & \delta_d(x_1, x_2, t) \end{array} \tag{8}$$

where $\theta_{min} = [\theta_{1min}, \theta_{2min}, \theta_{3min}]^T$, $\theta_{max} = [\theta_{1max}, \theta_{2max}, \theta_{3max}]^T$ *and* $\delta_d(t, x_1, x_2)$ *are known. In general,* $\bullet_i$ *represents the i-th component of the vector* $\bullet$ *and the operation* $<$ *for two vectors is performed in terms of the corresponding elements of the vectors. Noting that physically* $\theta_1 > 0$ *and* $\theta_3 > 0$*, we assume that* $\theta_{1min} > 0$ *and* $\theta_{3min} > 0$. ◇

At this stage, it is ready to see that the main difficulties in controlling (8) are: (a) the system has unmatched model uncertainties since parametric uncertainties and uncertain nonlinearities appear in equations that do not contain control input u; this difficult can be overcome by employing backstepping design as done in the following; (b) the term g_3, which representing the nonlinear static gain between the flow rate Q_L and the valve opening x_4, is a function of x_4 also and is *non-smooth* since x_4 appears through a discontinuous sign function $sgn(x_4)$; this prohibits the direct application of the general results in [15] to obtain an ARC controller. The effect of the directional change of the valve opening, i.e., the term $sgn(x_4)$, has been neglected in all previous studies due to either technical requirements of the smoothness of all terms in the design (e.g., the conventional backstepping design in [2]) or the use of linearization techniques (either around a nominal operational point for linear controller design or feedback linearization), which need the differentiability of all terms. Here, we proceed the design in the following way to take into account of the particular nonlinearities of the hydraulic dynamics and the effect of the non-smoothness nonlinearities mentioned above.

Smooth Projection

Some notations have to be introduced first in order to begin the controller design as in [14]. Let $\hat{\theta}$ denote the estimate of θ. Viewing (8), a simple smooth projection map π can be defined for $\hat{\theta}$ and satisfies the following properties:

Property 1 .
*(**a**).* $\forall \hat{\theta} \in \Omega_\theta, \quad \pi(\hat{\theta}) = \hat{\theta}$*;*
*(**b**).* $\forall \hat{\theta}, \quad \pi(\hat{\theta}) \in \Omega_{\hat{\theta}} = \{\mu : \quad \theta_{min} - \varepsilon_\theta \leq \mu \leq \theta_{max} + \varepsilon_\theta\}$ *where* ε_θ *is a known vector of positive numbers that can be arbitrarily small;*
*(**c**).* $\pi_i(\hat{\theta}_i)$ *is a nondecreasing function of* $\hat{\theta}_i$*;*
*(**d**). The derivatives of the projection are bounded up to a sufficiently high-order.*

See [14, 15] for further details. For convenience, define $\hat{\theta}_\pi$ as $\hat{\theta}_\pi = \pi(\hat{\theta})$ and the projected estimation error as $\tilde{\theta}_\pi = \hat{\theta}_\pi - \theta$. Denote $\theta_{iM} = \theta_{imax} - \theta_{imin} + \varepsilon_{\theta i}$. From Property (b), $|\tilde{\theta}_{i\pi}| = |\hat{\theta}_{i\pi} - \theta_i| \leq \theta_{iM}$, which is an important relationship used in the design later. For simplicity, let $\hat{\dot{x}}_2$ and $\hat{\dot{x}}_3$ represent the calculable part of the $\dot{x}_2$ and $\dot{x}_3$ respectively, which are given by

$$\begin{aligned}\hat{\dot{x}}_2 &= \hat{\theta}_{1\pi}(Ax_3 - bx_2 - F_{fc}(x_2)) + \hat{\theta}_{2\pi} \\ \hat{\dot{x}}_3 &= \hat{\theta}_{3\pi}[-Ax_2 - C_{tm}x_3 + g_3(x_3, x_4)x_4]\end{aligned} \tag{9}$$

Controller Design

The design parallels the recursive backstepping design procedure via ARC Lyapunov functions in [15] as follows.

Step 1

Noting that the first equation of (7) does not have any uncertainties, an ARC Lyapunov function can thus be constructed for the first two equations of (7) directly. Define a switching-function-like quantity as

$$z_2 = \dot{e}_1 + k_p e_1 + k_I \int e_1 dt = x_2 - x_{2eq}, \qquad x_{2eq} \overset{\Delta}{=} \dot{x}_{1d} - (k_p e_1 + k_I \int e_1 dt) \tag{10}$$

where $e_1 = x_1 - x_{1d}(t)$, in which the desired trajectory $x_{1d}(t)$ will be specified later to achieve a guaranteed transient performance, k_p and k_I are any positive feedback gains. Note that integral feedback is introduced in the design. If z_2 is small or converges to zero, then the output tracking error e_1 will be small or converge to zero since $G_s(s) = \frac{e_1(s)}{z_2(s)} = \frac{s}{s^2 + k_p s + k_I}$ is a stable transfer function. So the rest of the design is to make z_2 as small as possible with a guaranteed transient performance. Differentiating (10) and noting (7)

$$\dot{z}_2 = \dot{x}_2 - \dot{x}_{2eq} = \theta_1(Ax_3 - bx_2 - F_{fc}) + \theta_2 + \tilde{d} - \dot{x}_{2eq}, \qquad \dot{x}_{2eq} \overset{\Delta}{=} \ddot{x}_{1d} - (k_p \dot{e}_1 + k_I e_1) \tag{11}$$

In (11), if we treat x_3 as the input, we can synthesize a virtual control law α_2 for x_3 such that z_2 is as small as possible; this can be done by the ARC approach proposed in [15] since (11) has parametric uncertainties θ_1 and θ_2 as well as uncertain nonlinearity $\tilde{d}$. α_2 consists of two parts given by

$$\begin{aligned}\alpha_2(x_1, x_2, \hat{\theta}_{1\pi}, \hat{\theta}_{2\pi}, t) &= \alpha_{2a} + \alpha_{2s} \\ \alpha_{2a} &= \frac{1}{A}\left\{bx_2 + F_{fc} + \frac{1}{\hat{\theta}_{1\pi}}(\dot{x}_{2eq} - \hat{\theta}_{2\pi} - k_2 z_2)\right\}\end{aligned} \tag{12}$$

in which α_{2a} is used to achieve an improved model compensation through adjusting the parameter estimate $\hat{\theta}(t)$ on-line, which functions as an adaptive control law. α_{2s} is a robust control law designed as follows. Let $z_3 = x_3 - \alpha_2$ denote the input discrepancy. Substituting (12) into (11),

$$\begin{aligned}\dot{z}_2 &= \theta_1 A z_3 + \hat{\theta}_{1\pi}(A\alpha_2 - bx_2 - F_{fc}) - \tilde{\theta}_{1\pi}(A\alpha_2 - bx_2 - F_{fc}) + \theta_2 + \tilde{d} - \dot{x}_{2eq} \\ &= \theta_1 A z_3 - k_2 z_2 + \hat{\theta}_{1\pi} A \alpha_{2s} - \tilde{\theta}_{1\pi}(A\alpha_2 - bx_2 - F_{fc}) - \tilde{\theta}_{2\pi} + \tilde{d}\end{aligned} \tag{13}$$

The robust control function α_{2s} is now chosen to satisfy the following conditions

$$\begin{aligned}&\text{condition i} \qquad z_2[\hat{\theta}_{1\pi} A \alpha_{2s} - \tilde{\theta}_{1\pi}(A\alpha_2 - bx_2 - F_{fc}) - \tilde{\theta}_{2\pi} + \tilde{d}] \leq \varepsilon_2 \\ &\text{condition ii} \qquad z_2 \hat{\theta}_{1\pi} A \alpha_{2s} \leq 0\end{aligned} \tag{14}$$

where ε_2 is a design parameter which can be arbitrarily small. Basically, condition i of (14) shows that α_{2s} is synthesized to dominate the model uncertainties coming from both parametric uncertainties $\tilde{\theta}_{1\pi}$ and $\tilde{\theta}_{2\pi}$ and uncertain nonlinearities $\tilde{d}$, and condition ii is to make sure that α_{2s} is dissipating in nature so that it does not interfere with nominal identification process of adaptive control part α_{2a}.

Remark 1 *The existence of a robust control function α_{2s} satisfying (14) is guaranteed by the use of the smooth projections of the parameter estimates in the design of control law only. The smooth projections lie in known bounded regions all the time no matter what type of adaptation law will be used to adjust the parameter estimates–a sufficient condition for (14) to be solvable. One example of α_{2s} can be found in the following way. Let h_2 be any smooth function satisfying*

$$h_2 \geq \frac{1}{\varepsilon_{2a}}\theta_{1M}^2(A\alpha_{2a} - bx_2 - F_{fc})^2 + \frac{1}{\varepsilon_{2b}}\theta_{2M}^2 + \frac{1}{\varepsilon_{2c}}\delta_d^2(x_1, x_2, t) \tag{15}$$

where $\varepsilon_{2a}, \varepsilon_{2b}$ and ε_{2c} are any positive scalars such that $\varepsilon_2 = \varepsilon_{2a} + \varepsilon_{2b} + \varepsilon_{2c}$, then, α_{2s} can be chosen as

$$\alpha_{2s} = -\frac{1}{4\theta_{1min}A}h_2 z_2 \tag{16}$$

By choosing $\varepsilon_{\theta 1} < \theta_{1min}$ in defining the smooth projections, it is guaranteed that $\hat{\theta}_{1\pi} > 0$, from which condition ii of (14) is satisfied by (16). Noting (8) and the definition of smooth projections,

$$\begin{array}{l} \textit{Left side (LS) of condition i of (14)} = z_2\theta_1 A\alpha_{2s} + z_2[-\tilde{\theta}_{1\pi}(A\alpha_{2a} - bx_2 - F_{fc}) - \tilde{\theta}_{2\pi} + \tilde{d}] \\ \quad \leq -\frac{\theta_1}{4\theta_{1min}}h_2 z_2^2 + |z_2|\left[\theta_{1M}^2|A\alpha_{2a} - bx_2 - F_{fc}| + \theta_{2M} + \delta_d(x_1, x_2, t)\right] \\ \quad \leq \varepsilon_{2a} + \varepsilon_{2b} + \varepsilon_{2c} \leq \varepsilon_2 \end{array} \tag{17}$$

in which the last two steps are proved by the completion of square as in [14]. This shows that condition ii is satisfied and α_{2s} given by (16) satisfies all the requirements. △

Lemma 1 *For the positive semi-definite (p.s.d.) function V_2 defined by*

$$V_2 = \frac{1}{2}w_2 z_2^2 \tag{18}$$

where $w_2 > 0$ is a weighting factor, if $x_3 = \alpha_2$ or $z_3 = 0$, then,

a. In general,

$$\dot{V}_2\,|_{\alpha_2} \;\; \leq -2k_2 V_2 + w_2\varepsilon_2 \tag{19}$$

where $\dot{V}_2\,|_{\alpha_2}$ denote $\dot{V}_2$ under the condition that $x_3 = \alpha_2$.

b. In addition, when $\tilde{d} = 0$,

$$\dot{V}_2\,|_{\alpha_2} \;\; \leq -w_2 k_2 z_2^2 + \tau_{2a}\tilde{\theta}_{1\pi} + \tau_{2b}\tilde{\theta}_{2\pi} \tag{20}$$

where

$$\begin{aligned} \tau_{2a} &= -w_2 z_2(A\alpha_2 - bx_2 - F_{fc}) \\ \tau_{2b} &= -w_2 z_2 \end{aligned} \tag{21}$$

◇

Remark 2 *Lemma 1 shows that V_2 is an ARC Lyapunov function [15] for (10) with the control function given by (12) and the adaptation function given by (21).*

Proof. Noting (13) and condition i of (14), A of Lemma 1 is obvious. Noting condition ii of (14), B of Lemma 1 is obvious. □

Step 2

If we neglect the effect of the directional change of the valve opening (i.e., $sgn(x_4)$) as in previous studies [2], g_3 in the third equation of (7) would be a function of x_3 only. In that case, Step 2 would be to synthesize a control function α_3 for the virtual control x_4 such that x_3 tracks the desired control function α_2 synthesized in Step 1 with a guaranteed transient performance, which could be accomplished by the backstepping design via ARC Lyapunov functions as in [15]. Since most operations (especially the position or force regulation at the end of an operation) involve the directional change of the valve opening, its effect will be carefully treated in this paper. Here, instead of defining x_4 as the virtual control for the third equation of (7), we define actual load flow rate $Q_L = g_3(x_3, x_4)x_4$ as the virtual control, which makes physical sense since physically it is the flow rate that controls the pressure inside the cylinder. Thus in this step, we will synthesize a control function α_3 for Q_L such that x_3 tracks the desired control function α_2 synthesized in Step 1 with a guaranteed transient performance.

From (12) and (7),

$$\begin{aligned} \dot{\alpha}_2 &= \frac{\partial \alpha_2}{\partial x_1} x_2 + \frac{\partial \alpha_2}{\partial x_2} \dot{x}_2 + \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 + \frac{\partial \alpha_2}{\partial \hat{\theta}_2} \dot{\hat{\theta}}_2 + \frac{\partial \alpha_2}{\partial t} \\ &= \dot{\alpha}_{2c} + \dot{\alpha}_{2u} \end{aligned} \tag{22}$$

where

$$\begin{aligned} \dot{\alpha}_{2c} &= \frac{\partial \alpha_2}{\partial x_1} x_2 + \frac{\partial \alpha_2}{\partial x_2} \hat{\dot{x}}_2 + \frac{\partial \alpha_2}{\partial t} \\ \dot{\alpha}_{2u} &= \frac{\partial \alpha_2}{\partial x_2}[-(Ax_3 - bx_2 - F_{fc})\tilde{\theta}_{1\pi} - \tilde{\theta}_{2\pi} + \tilde{d}] + \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 + \frac{\partial \alpha_2}{\partial \hat{\theta}_2} \dot{\hat{\theta}}_2 \end{aligned} \tag{23}$$

In (22), $\dot{\alpha}_{2c}$ is calculable and can be used in the design of control functions but $\dot{\alpha}_{2u}$ cannot due to various uncertainties. Therefore, $\dot{\alpha}_{2u}$ has to be dealt with in this step design. This greatly complicates the controller design as shown below. Let $z_4 = Q_L - \alpha_3$. From (7),

$$\dot{z}_3 = \theta_3 z_4 + \theta_3[-Ax_2 - C_{tm}x_3 + \alpha_3] - \dot{\alpha}_{2c} - \dot{\alpha}_{2u} \tag{24}$$

Consider the augmented p.s.d. function V_3 given by

$$V_3 = V_2 + \tfrac{1}{2} w_3 z_3^2, \qquad w_3 > 0 \tag{25}$$

Noting (13) and (24),

$$\begin{aligned} \dot{V}_3 &= \theta_1 w_2 A z_2 z_3 + \dot{V}_2 \mid_{\alpha_2} + w_3 z_3 \dot{z}_3 \\ &= \dot{V}_2 \mid_{\alpha_2} + w_3 z_3 \theta_3 z_4 + w_3 z_3 \left\{ \frac{w_2}{w_3} \theta_1 A z_2 + \theta_3[-Ax_2 - C_{tm}x_3 + \alpha_3] - \dot{\alpha}_{2c} - \dot{\alpha}_{2u} \right\} \\ &= \dot{V}_2 \mid_{\alpha_2} + \theta_3 w_3 z_3 z_4 + w_3 z_3 \left\{ \hat{\theta}_{3\pi} \alpha_3 + \alpha_{3e} - \phi_{3a} \tilde{\theta}_{1\pi} + \frac{\partial \alpha_2}{\partial x_2} \tilde{\theta}_{2\pi} \right. \\ &\quad \left. -[-Ax_2 - C_{tm}x_3 + \alpha_3]\tilde{\theta}_{3\pi} - \frac{\partial \alpha_2}{\partial x_2} \tilde{d} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 - \frac{\partial \alpha_2}{\partial \hat{\theta}_2} \dot{\hat{\theta}}_2 \right\} \end{aligned} \tag{26}$$

where α_{3e} and ϕ_{3a} are calculable at this stage and given by

$$\begin{array}{l} \alpha_{3e} = \frac{w_2}{w_3} A z_2 \hat{\theta}_{1\pi} + \hat{\theta}_{3\pi}(-Ax_2 - C_{tm}x_3) - \dot{\alpha}_{2c} \\ \phi_{3a} = \frac{w_2}{w_3} A z_2 - \frac{\partial \alpha_2}{\partial x_2}(Ax_3 - bx_2 - F_{fc}) \end{array} \tag{27}$$

Similar to (12), the control function α_3 consists of two parts given by

$$\begin{array}{l} \alpha_3(\bar{x}_3, \hat{\theta}, t) = \alpha_{3a} + \alpha_{3s} \\ \alpha_{3a} = \frac{1}{\hat{\theta}_{3\pi}}[-k_3 z_3 - \alpha_{3e} - \frac{\partial \alpha_2}{\partial \hat{\theta}_1}\gamma_1 \tau_{3a} - \frac{\partial \alpha_2}{\partial \hat{\theta}_2}\gamma_2 \tau_{3b}] \end{array} \tag{28}$$

where α_{3s} is a robust control function satisfying the following two conditions

$$\begin{array}{ll} \text{condition i} & z_3 \left[\hat{\theta}_{3\pi}\alpha_{3s} - \phi_{3a}\tilde{\theta}_{1\pi} + \frac{\partial \alpha_2}{\partial x_2}\tilde{\theta}_{2\pi} - (-Ax_2 - C_{tm}x_3 + \alpha_{3a} + \alpha_{3s})\tilde{\theta}_{3\pi} - \frac{\partial \alpha_2}{\partial x_2}\tilde{d}\right] \leq \varepsilon_3 \\ \text{condition ii} & z_3 \hat{\theta}_{3\pi}\alpha_{3s} \leq 0 \end{array} \tag{29}$$

where ε_3 is a design parameter. α_{3s} can be found in the same way as in Remark 1. One example is given by

$$\alpha_{3s} = -\frac{1}{4\theta_{3min}} h_3 z_3 \tag{30}$$

in which h_3 is any differentiable function satisfying

$$h_3 \geq \frac{1}{\varepsilon_{3a}}\theta_{1M}^2 \phi_{3a}^2 + \frac{1}{\varepsilon_{3b}}\theta_{2M}^2 \left(\frac{\partial \alpha_2}{\partial x_2}\right)^2 + \frac{1}{\varepsilon_{3c}}\theta_{3M}^2 \left(-Ax_2 - C_{tm}x_3 + \alpha_{3a}\right)^2 + \frac{1}{\varepsilon_{3d}}\left(\frac{\partial \alpha_2}{\partial x_2}\right)^2 \delta_d^2 \tag{31}$$

where ε_{3a}, ε_{3b}, ε_{3c}, and ε_{3d} are any positive scalars such that $\varepsilon_3 = \varepsilon_{3a} + \varepsilon_{3b} + \varepsilon_{3c} + \varepsilon_{3d}$.

In (28), the last two terms in α_{3a} are used to compensate for the effect of time-varying $\hat{\theta}(t)$ since parameter adaptation will be specified later to adjust $\hat{\theta}(t)$, in which $-\gamma_1\tau_{3a}$ and $-\gamma_2\tau_{3b}$ would be the updating laws for $\hat{\theta}_1$ and $\hat{\theta}_2$ respectively if Q_L is the actual input. The expressions for τ_{3a} and τ_{3b} are

$$\begin{array}{l} \tau_{3a} = \tau_{2a} - w_3 z_3 \phi_{3a} \\ \tau_{3b} = \tau_{2b} + w_3 z_3 \frac{\partial \alpha_2}{\partial x_2} \end{array} \tag{32}$$

Lemma 2 *If $Q_L = \alpha_3$ or $z_4 = 0$, then,*

a. *In general,*

$$\dot{V}_3 \mid_{\alpha_3} \quad \leq -\lambda_{V3} V_3 + w_2\varepsilon_2 + w_3\varepsilon_3 - w_3 z_3 [\frac{\partial \alpha_2}{\partial \hat{\theta}_1}(\dot{\hat{\theta}}_1 + \gamma_1 \tau_{3a}) + \frac{\partial \alpha_2}{\partial \hat{\theta}_2}(\dot{\hat{\theta}}_2 + \gamma_2 \tau_{3b})] \tag{33}$$

where $\lambda_{V3} = 2min\{k_2, k_3\}$ and $\dot{V}_3 \mid_{\alpha_3}$ denote $\dot{V}_3$ under the condition that $v = \alpha_3$.

b. *In addition, when $\tilde{d} = 0$,*

$$\dot{V}_3 \mid_{\alpha_3} \quad \leq -w_2 k_2 z_2^2 - w_3 k_3 z_3^2 + \tau_{3a}\tilde{\theta}_{1\pi} + \tau_{3b}\tilde{\theta}_{2\pi} + \tau_{3c}\tilde{\theta}_{3\pi} - w_3 z_3 [\frac{\partial \alpha_2}{\partial \hat{\theta}_1}(\dot{\hat{\theta}}_1 + \gamma_1 \tau_{3a}) + \frac{\partial \alpha_2}{\partial \hat{\theta}_2}(\dot{\hat{\theta}}_2 + \gamma_2 \tau_{3b})] \tag{34}$$

where

$$\tau_{3c} \quad = -w_3 z_3(-Ax_2 - C_{tm}x_3 + \alpha_3) \tag{35}$$

$\diamondsuit$

Proof. Noting (19) and condition i of (29), A of Lemma 2 can be proved by substituting (28) into (26). Noting (20) and condition ii of (29), B of Lemma 2 can be proved by substituting (28) into (26). □

Step 3

Noting the last equation of (7), Step 3 is to synthesize an actual control law for u such that Q_L tracks the desired control function α_3 synthesized in Step 2 with a guaranteed transient performance. This can be done by the same backstepping design via ARC Lyapunov functions as in Step 2 except that here Q_L is not differentiable at $x_4 = 0$ due to the appearance of $sgn(x_4)$. Fortunately, since the actual control input u can have finite jumps and is the control law to be synthesized in this step and, we can proceed the design as follows by noting that Q_L is differentiable anywhere except at the singular point of $x_4 = 0$ and is *continuous everywhere*. By the definition of Q_L and g_3, it can be checked out that the derivative of Q_L is given by

$$\dot{Q}_L = \frac{\partial g_3}{\partial x_3}\dot{x}_3 + g_3(x_3, x_4)\dot{x}_4, \qquad \forall x_4 \neq 0 \tag{36}$$

where $\frac{\partial g_3}{\partial x_3} = -\frac{C_d w}{\sqrt{\rho}} \frac{sgn(x_4)}{2\sqrt{P_s - sgn(x_4)x_3}}$. From (28) and (7),

$$\dot{\alpha}_3 = \dot{\alpha}_{3c} + \dot{\alpha}_{3u} \tag{37}$$

where the calculable part $\dot{\alpha}_{3c}$ and the incalculable part $\dot{\alpha}_{3u}$ are given by

$$\begin{aligned}
\dot{\alpha}_{3c} &= \frac{\partial \alpha_3}{\partial x_1}x_2 + \frac{\partial \alpha_3}{\partial x_2}\hat{\dot{x}}_2 + \frac{\partial \alpha_3}{\partial x_3}\hat{\dot{x}}_3 + \frac{\partial \alpha_3}{\partial t} \\
\dot{\alpha}_{3u} &= \frac{\partial \alpha_3}{\partial x_2}[-(Ax_3 - bx_2 - F_{fc})\tilde{\theta}_{1\pi} - \tilde{\theta}_{2\pi} + \tilde{d}] - \frac{\partial \alpha_3}{\partial x_3}(-Ax_2 - C_{tm}x_3 + g_3x_4)\tilde{\theta}_{3\pi} + \frac{\partial \alpha_3}{\partial \hat{\theta}_1}\dot{\hat{\theta}}_1 + \frac{\partial \alpha_3}{\partial \hat{\theta}_2}\dot{\hat{\theta}}_2 + \frac{\partial \alpha_3}{\partial \hat{\theta}_3}\dot{\hat{\theta}}_3
\end{aligned} \tag{38}$$

Consider the augmented p.s.d. function V_4 given by

$$V_4 = V_3 + \tfrac{1}{2}w_4 z_4^2, \qquad w_4 > 0 \tag{39}$$

Noting (26), (7), and (36),

$$\begin{aligned}
\dot{V}_4 &= \theta_3 w_3 z_3 z_4 + \dot{V}_3 \mid_{\alpha_3} + w_4 z_4[\dot{Q}_L - \dot{\alpha}_3] \\
&= \dot{V}_3 \mid_{\alpha_3} + w_4 z_4 \left\{ \frac{K}{\tau}g_3 u + \alpha_{4e} - \phi_{4a}\tilde{\theta}_{1\pi} + \frac{\partial \alpha_3}{\partial x_2}\tilde{\theta}_{2\pi} - \phi_{4c}\tilde{\theta}_{3\pi} - \frac{\partial \alpha_3}{\partial x_2}\tilde{d} - \frac{\partial \alpha_3}{\partial \hat{\theta}_1}\dot{\hat{\theta}}_1 - \frac{\partial \alpha_3}{\partial \hat{\theta}_2}\dot{\hat{\theta}}_2 - \frac{\partial \alpha_3}{\partial \hat{\theta}_3}\dot{\hat{\theta}}_3 \right\}
\end{aligned} \tag{40}$$

where

$$\begin{aligned}
\alpha_{4e} &= \hat{\theta}_{3\pi}\frac{w_3}{w_4}z_3 - \frac{1}{\tau}g_3x_4 + \frac{\partial g_3}{\partial x_3}\hat{\dot{x}}_3 - \dot{\alpha}_{3c} \\
\phi_{4a} &= -\frac{\partial \alpha_3}{\partial x_2}(Ax_3 - bx_2 - F_{fc}) \\
\phi_{4c} &= \frac{w_3}{w_4}z_3 + \left[\frac{\partial g_3}{\partial x_3} - \frac{\partial \alpha_3}{\partial x_3}\right](-Ax_2 - C_{tm}x_3 + v)
\end{aligned} \tag{41}$$

Similar to (28), the control law consists of two parts given by

$$\begin{aligned}
u &= u_a(x, \hat{\theta}, t) + u_s(x, \hat{\theta}, t) \\
u_a &= \frac{\tau}{Kg_3}\left[-k_4 z_4 - \alpha_{4e} - \frac{\partial \alpha_3}{\partial \hat{\theta}_1}\gamma_1\tau_{4a} - \frac{\partial \alpha_3}{\partial \hat{\theta}_2}\gamma_2\tau_{4b} - \frac{\partial \alpha_3}{\partial \hat{\theta}_3}\gamma_3\tau_{4c} + \frac{\partial \alpha_2}{\partial \hat{\theta}_1}\gamma_1 w_3 z_3 \phi_{4a} - \frac{\partial \alpha_2}{\partial \hat{\theta}_2}\gamma_2 w_3 z_3 \frac{\partial \alpha_3}{\partial x_2}\right]
\end{aligned} \tag{42}$$

where u_s is a robust function satisfying the following two conditions

$$\begin{array}{ll} \text{condition i} & z_4[\frac{K}{\tau}g_3u_s - \phi_{4a}\tilde{\theta}_{1\pi} + \frac{\partial\alpha_3}{\partial x_2}\tilde{\theta}_{2\pi} - \phi_{4c}\tilde{\theta}_{3\pi} - \frac{\partial\alpha_3}{\partial x_2}\tilde{d}] \le \varepsilon_4 \\ \text{condition ii} & z_4g_3u_s \le 0 \end{array} \tag{43}$$

where ε_4 is a design parameter. One example of u_s is given by

$$u_s = -\frac{\tau}{4Kg_3}h_4z_4 \tag{44}$$

in which h_4 is any differentiable function satisfying

$$h_4 \ge \frac{1}{\varepsilon_{4a}}\theta_{1M}^2\phi_{4a}^2 + \frac{1}{\varepsilon_{4b}}\theta_{2M}^2\left(\frac{\partial\alpha_3}{\partial x_2}\right)^2 + \frac{1}{\varepsilon_{4c}}\theta_{3M}^2\phi_{4c}^2 + \frac{1}{\varepsilon_{4d}}\left(\frac{\partial\alpha_3}{\partial x_2}\right)^2\delta_d^2 \tag{45}$$

In (42), the adaptation functions in u_a are defined by

$$\begin{aligned} \tau_{4a} &= \tau_{3a} - w_4z_4\phi_{4a} \\ \tau_{4b} &= \tau_{3b} + w_4z_4\frac{\partial\alpha_3}{\partial x_2} \\ \tau_{4c} &= \tau_{3c} - w_4z_4\phi_{4c} \end{aligned} \tag{46}$$

Lemma 3 *If the control law (42) with the estimated parameters updated by the following adaptation law*

$$\begin{aligned} \dot{\hat{\theta}}_1 &= -\gamma_1\tau_{4a} \\ \dot{\hat{\theta}}_2 &= -\gamma_2\tau_{4b} \\ \dot{\hat{\theta}}_3 &= -\gamma_3\tau_{4c} \end{aligned} \tag{47}$$

is used, then,

a. *In general, the system is exponentially stable and*

$$V_4(t) \le exp(-\lambda_V t)V_4(0) + \frac{\varepsilon_V}{\lambda_V}[1 - exp(-\lambda_V t)] \tag{48}$$

where $\lambda_V = 2min\{k_2, k_3, k_4\}$ and $\varepsilon_V = w_2\varepsilon_2 + w_3\varepsilon_3 + w_4\varepsilon_4$.

b. *In addition, when $\tilde{d} = 0$,*

$$\dot{V}_4 \quad \le -w_2k_2z_2^2 - w_3k_3z_3^2 - w_4k_4z_4^2 + \tau_{4a}\tilde{\theta}_{1\pi} + \tau_{4b}\tilde{\theta}_{2\pi} + \tau_{4c}\tilde{\theta}_{3\pi} \tag{49}$$

$\diamondsuit$

Proof. Substituting (42) into (40) and noting (33) and condition i of (43),

$$\begin{aligned} \dot{V}_4 \quad \le & -\lambda_V V_4 + \varepsilon_V - [w_3z_3\frac{\partial\alpha_2}{\partial\hat{\theta}_1} + w_4z_4\frac{\partial\alpha_3}{\partial\hat{\theta}_1}](\dot{\hat{\theta}}_1 + \gamma_1\tau_{4a}) \\ & [w_3z_3\frac{\partial\alpha_2}{\partial\hat{\theta}_2} + w_4z_4\frac{\partial\alpha_3}{\partial\hat{\theta}_2}](\dot{\hat{\theta}}_2 + \gamma_2\tau_{4b}) - w_4z_4\frac{\partial\alpha_3}{\partial\hat{\theta}_3}(\dot{\hat{\theta}}_3 + \gamma_3\tau_{4c}) \end{aligned} \tag{50}$$

Noting the adaptation law (47), (50) becomes

$$\dot{V}_4 \quad \leq -\lambda_V V_4 + \varepsilon_V \tag{51}$$

which leads to A of Lemma 3.

When $\tilde{d} = 0$, substituting (42) into (40) and noting (34), condition ii of (43), and the adaptation law (47), B of Lemma 3 can be proved. □

Trajectory Initialization

Let $x_{1d}(t)$ be the trajectory created by the following 4-th order stable system

$$x_{1d}^{(4)} + \beta_1 x_{1d}^{(3)} + \ldots + \beta_4 x_{1d} = x_{Ld}^{(4)} + \beta_1 x_{Ld}^{(3)} + \ldots + \beta_4 x_{Ld} \tag{52}$$

The initial conditions of the system (52) will be specified in the following to render $V_4(0) = 0$.

Lemma 4 *If the initials $x_{1d}(0), \ldots, x_{1d}^{(3)}(0)$ are chosen as*

$$\begin{aligned} x_{1d}(0) &= x_1(0) \\ \dot{x}_{1d}(0) &= x_2(0) \\ \ddot{x}_{1d}(0) &= \hat{\dot{x}}_2(0) \\ x_{1d}^{(3)}(0) &= \hat{\theta}_{1\pi}(0)\left[A\hat{\dot{x}}_3(0) - \left(b + \frac{\partial F_{fc}}{\partial x_2}(0)\right)\hat{\dot{x}}_2(0)\right] \end{aligned} \tag{53}$$

then, $e_1(0) = 0, z_i(0) = 0, i = 2, 3, 4$ and $V_4(0) = 0$. □

Proof: The Lemma can be proved in the same way as in [14]. □

Remark 3 *The same as in [14, 16], the above trajectory initialization (53) actually places the initial condition $x_{1d}^{(i)}(0)$ at the best initial estimate of $x_1^{(i)}(0)$ by substituting $\hat{\theta}_\pi(0)$ for θ. This implication implies that trajectory initialization can be performed independently from the choice of controller parameters such as $k = [k_2, k_3, k_4]^T$ and $\varepsilon = [\varepsilon_2, \varepsilon_3, \varepsilon_4]^T$ as seen from (53).* ♢

Theorem 1 *Given the desired trajectory $x_{1d}(t)$ generated by (52) with initial conditions (53), the following results hold if the control law (42) with the adaptation law (47) is applied:*

A. *In general, the control input is bounded. Furthermore,*

$$V_4(t) \quad \leq \frac{\varepsilon_V}{\lambda_V}[1 - exp(-\lambda_V t)] \tag{54}$$

and the output tracking error $e_y = x_1 - x_{Ld}(t)$ is guaranteed to have any prescribed transient performance by increasing k and/or decreasing ε.

B *If after a finite time t_0, $\tilde{d} = 0$, i.e., in the presence of parametric uncertainties only, in addition to results in A, asymptotic output tracking is also obtained for any gains k and ε.* $\triangle$

Remark 4 *In the above design, the intermediate control functions α_i given by (12) and (28) have to be differentiable. In return, the Coulomb friction compensation term $F_{fc}(x_2)$ used in them has to be a differential function of x_2. This requirement can be easily accommodated in the proposed ARC framework since $F_{fc}(x_2)$ can be chosen as any differentiable function which approximates the actual discontinuous Coulomb friction (e.g., replacing $sgn(x_2)$ in the conventional Coulomb friction modeling by the smooth* $\tanh(x_2)$ *). The approximation error can be lumped into the uncertain nonlinearity term $\tilde{d}$.* $\triangle$

Proof : From Lemma 4, $V_4(0) = 0$. From Lemma 3, (54) is true and thus $z = [z_2, z_3, z_4]^T$ is bounded. Since $\hat{\theta}$ appears in all formula in the form of $\bar{\theta}_\pi^{(j)}$ and $\bar{\theta}_\pi^{(j)}$ is bounded for any $\hat{\theta}$ and j, α_i, $\forall i$, is bounded. Thus the control input u and the state x are bounded. Furthermore, from (54) and (10), $e_1(t)$ is within a ball whose size can be made arbitrarily small by increasing k and/or decreasing ε in a known form. From (52) and (53), the trajectory planning error, $e_d(t) = x_{1d}(t) - x_{Ld}(t)$, can be guaranteed to possess any good transient behavior by suitably choosing the Hurwitz polynomial $G_d(s) = s^4 + \beta_1 s^3 + \ldots + \beta_4$ without being affected by k and ε. Therefore, any good transient performance of the output tracking error $e_y = e_1(t) + e_d(t)$ can be guaranteed by the choice of k and ε in a known form. This proves A of Theorem 1.

In the presence of parametric uncertainties only, i.e., $\tilde{d} = 0$, noting (49), B of the Theorem can be proved by using a p.s.d. function $V_a = V_4 + V_\theta(\tilde{\theta}, \theta)$, where V_θ is a non-qudratic positive function defined in [14]. The details are tedious and can be worked out in the same way as in [14, 15]. $\square$

4 Conclusions

In this paper, a nonlinear ARC controller is developed for the high performance robust motion control of a typical one DOF electro-hydraulic servosystem. The controller takes into account the particular nonlinearities associated with hydraulic dynamics and allows parametric uncertainties such as variations of inertia load and hydraulic parameters (e.g. bulk modulus) as well as uncertain nonlinearities coming from external disturbances, uncompensated friction forces, etc. A novel strategy is also developed to deal with the non-differentiability of the inherent nonlinearities in hydraulic systems. The controller achieves a prescribed transient performance and final tracking accuracy in general. In addition, zero output tracking is achieved in the presence of parameteric uncertainties.

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