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AN OUTPUT FEEDBACK BASED ADAPTIVE ROBUST FAULT TOLERANT CONTROL SCHEME FOR A CLASS OF NONLINEAR SYSTEMS

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ABSTRACT

In this paper we present an output feedback based Adaptive Robust Fault Tolerant Control (ARFTC) strategy to solve the problem of output tracking in presence of actuator failures, disturbances and modeling uncertainties for a class of nonlinear systems. The class of faults addressed here include stuck actuators, actuator loss of efficiency or a combination of the two. We assume no a priori information regarding the instant of failure, failure pattern or fault size. The ARFTC combines the robustness of sliding mode controllers with the online learning capabilities of adaptive control to accommodate sudden changes in system parameters due to actuator faults. Comparative simulation studies are carried out on a nonlinear hypersonic aircraft model, which shows the effectiveness of the proposed scheme over backstepping based robust adaptive fault-tolerant control.

INTRODUCTION

Reliability and performance are twin objectives of many complex systems like chemical plants, nuclear plants, flight control system etc., and one cannot be sacrificed for the other. For such systems, it is desirable to have a certain degree of fault tolerance with respect to various faults. In this work, we focus on the problem of fault accommodation for unknown actuator failures for a class of nonlinear systems with unknown parameters and uncertain nonlinearities. We address two types of fault scenarios: actuator loss in efficiency and stuck actuators. We do not assume the knowledge of failed actuators or failure pattern in the present work. Fortunately, adaptive schemes, by virtue of its on-line learning capability can bypass this problem. Consequently, many adaptive schemes have been developed to solve this problem.

Tao et al. proposed a model reference adaptive control (MRAC) based direct scheme to solve the problem of actuator fault-accommodation for linear system in [1]. Such approaches are inherently limited as they rely on conventional MRAC, which suffers from poor transients during the learning phase and offers difficulty in checking stability and robustness bounds in presence of exogenous disturbances. They also addressed various classes of nonlinear systems using backstepping based adaptive control in [2] and [3]. Robust Adaptive backstepping based fault compensation scheme for a class of nonlinear systems was proposed in [4]. Note that even when robustness modifications are made to backstepping based adaptive control, there is still no transparent way to attenuate the effect of disturbances and modeling uncertainties on the transient response and steady-state tracking error. Robust control based schemes, on the other hand, can handle such disturbances and unstructured uncertainties with guaranteed transient performance and attenuate their effect on the steady-state error. LMI based fault-tolerant control was proposed in [5] and sliding mode control based approaches were used in [6]. But, in presence of large parametric uncertainties, the robust control based direct fault-accommodation schemes can result in input chattering or large steady-state errors when smoothing techniques are used. Thus, both adaptive and robust control based schemes can solve one part of the problem, but cannot address all the issues associated with actuator faults, viz., desired transient response and small steady-state tracking error in presence of parametric and non-parametric uncertainties.

In spite of the inherent limitations of adaptive control based techniques, it has been realized that adaptation is of key importance in dealing with large parametric uncertainties introduced due to actuator faults in safety-critical missions like flight control systems. Consequently, the idea of *safe adaptive control* is coming to forefront, which ensures certain stability properties even without adaptation [7, 8]. In this respect, we would like to point out that ARC based schemes have already resolved this issue [9, 10] and may be classified as the so-called safe adaptive control. Switching the adaptation off at any instant converts the adaptive robust controller into a deterministic robust controller with guaranteed transient performance. Moreover, the design procedure allows us to calculate explicit upper bound for tracking errors over the entire time history in terms of certain controller parameters and achieve prespecified final tracking accuracy. Thus, ARC based schemes are natural choices for safety sensitive systems over conventional adaptive and robust schemes.

In the present work, we develop an output feedback ARC based scheme for accommodation of unknown actuator faults. The technique used here is a combination of adaptive backstepping [11] and discontinuous projection based ARC proposed in [10] and differs significantly from the techniques presented in [4] and [2] which relies on backstepping based direct adaptive control. The fundamental difference between the two schemes is due to the fact that ARC uses robust filter structures as the baseline controller, and adaptation is used only as a means to reduce the extent of parametric uncertainties. This is the reason that switching the adaptation off at any instant converts the ARC controller to a deterministic robust controller with guaranteed performance. On the other hand, in direct adaptive designs, adaptation is the underlying mechanism which makes the controller work. Furthermore, in backstepping based adaptive designs, tuning functions are used to compensate the for parameter-estimation error dynamics. But, as discontinuous projection is used in our approach, tuning functions cannot be used. In order to compensate for the effects of parameter-estimation error dynamics, the robust component of the control law is strengthened in ARC. In order to show the superior performance of the proposed scheme, comparative studies are performed using a hypersonic aircraft model.

PROBLEM STATEMENT

We will consider systems in the following form

$$\begin{aligned}
\dot{x}_1 &= x_2 + \varphi_{0,1}(y) + \sum_{j=1}^{p} a_j \varphi_{1,j} + \Delta_1(x,t) \\
&\vdots \\
\dot{x}_{\rho-1} &= x_\rho + \varphi_{0,\rho-1}(y) + \sum_{j=1}^{p} a_j \varphi_{\rho-1,j} + \Delta_{\rho-1}(x,t) \\
\dot{x}_\rho &= x_{\rho+1} + \varphi_{0,\rho}(y) + \sum_{j=1}^{p} a_j \varphi_{\rho,j} \\
&\quad + \sum_{j=1}^{q} b_{m,j} \beta_j(y) u_j(t) + \Delta_\rho(x,t) \\
&\vdots \\
\dot{x}_n &= \varphi_{0,n}(y) + \sum_{j=1}^{p} a_j \varphi_{n,j} + \sum_{j=1}^{q} b_{0,j} \beta_j(y) u_j(t) + \Delta_n(x,t) \quad (1)
\end{aligned}$$

where $\rho = n - m$ is the relative degree, u_j is the control input, $y = x_1$ is the measured output, $\varphi_{0,i}(y)$ and $\beta_j(y)$ are known smooth functions of y and $\beta_j(y) \neq 0$ for any y. $\Delta_i(x,t)$ represents uncertain nonlinearities and a_i, $b_{i,j}$ are unknown constants such that sign of the high frequency gain ($sgn(b_{m,j})$) is known. We will make the following realistic assumptions regarding the uncertainties present in the system

A1 The extent of parametric uncertainties and uncertain nonlinearities satisfy

$$\begin{aligned}
a_i &\in \Omega_a \triangleq \{a_i : (a_i)_{min} < a_i < (a_i)_{max}\} \\
b_{i,j} &\in \Omega_b \triangleq \{b_{i,j} : (b_{i,j})_{min} < b_{i,j} < (b_{i,j})_{max}\} \\
\Delta_i &\in \Omega_\Delta \triangleq \{\Delta_i : |\Delta_i(x,t)| \leq \delta_i(t)\} \qquad (2)
\end{aligned}$$

where $(a_i)_{min}$, $(a_i)_{max}$, $(b_{i,j})_{min}$, $(b_{i,j})_{max}$ are known and $\delta_i(t)$ is a bounded but unknown function.

In this work, we will consider faults which can be modeled as

$$u_j(t) = \begin{cases} \bar{u}_j & \text{if stuck actuator and } t \geq T_f \\ \eta_{jj} u_j(t) & \text{if loss of efficiency and } t \geq T_f \end{cases} \qquad (3)$$

where T_f is the unknown instant of failure, $\bar{u}_j$ is an unknown constant value at which the actuator gets stuck, and $\eta_{jj} \in [(\eta_{jj})_{min}, 1]$ represents actuator loss in efficiency. Without actuator redundancy and sufficient control authority, actuator faults cannot be accommodated and the same is stated formally in the following assumption

A2 System (1) is such that the desired control objective can be fulfilled with up to $m-1$ stuck actuators and any number of actuators with loss in efficiency.

We will make another standard assumption which guarantees stability of the zero dynamics

A3 The polynomials $B_j(s) = b_{m,j}s^m + b_{m-1,j}s^{m-1} + \ldots + b_{0,j}$, $j = 1, 2, .., q$ are stable.

The problem we attempt to solve in this paper can now be stated as follows. For the uncertain nonlinear system (1), subjected to faults (3) the goal is to design an output feedback based control strategy such that the output tracking error converges exponentially to a prespecified bound and has a guaranteed transient performance.

OUTPUT FEEDBACK BASED ARFTC

As in [3], we will design a control law such that $\beta_j(y)u_j(t) = u^*(t)$. With fault model (3) and the chosen actuation scheme, we can rewrite the control inputs as follows,

$$u_j(t) = \frac{\eta_{jj}}{\beta_j(y)}(1 - \sigma_{jj})u^*(t) + \sigma_{jj}\bar{u}_j, \quad j = 0, 1, \ldots, m \qquad (4)$$

where $\sigma_{jj}=1$ corresponds to stuck actuators, $\sigma_{jj}=0$, $(\eta_{jj})_{min} \geq \eta_{jj} \leq 1$ represents actuator loss of efficiency and $\sigma_{jj}=0$, $\eta_{jj}=1$ corresponds to healthy actuators.

With this, we can rewrite the system as follows

$$\begin{aligned}
\dot{x}_i &= x_{i+1} + \varphi_{0,i}(y) + \sum_{j=1}^{p} a_j \varphi_{i,j} + \Delta_i(x,t), \;\; i = 1,2,..,\rho-1 \\
\dot{x}_\rho &= x_{\rho+1} + \varphi_{0,\rho}(y) + \sum_{j=1}^{p} a_j \varphi_{\rho,j} \\
&\quad + \sum_{j=1}^{q} \kappa_m u^*(t) + \sum_{j=1}^{q} \mu_{m,j}\beta_j(y) + \Delta_\rho(x,t) \\
&\;\vdots \\
\dot{x}_n &= \varphi_{0,n}(y) + \sum_{j=1}^{p} a_j \varphi_{n,j} \\
&\quad + \sum_{j=1}^{q} \kappa_0 u^*(t) + \sum_{j=1}^{q} \mu_{0,j}\beta_j(y) + \Delta_n(x,t)
\end{aligned} \tag{5}$$

where $\kappa_i = \sum_{j=1}^{q} \eta_{jj}(1-\sigma_{jj})b_{i,j}$ and $\mu_{i,j} = \sigma_{jj}\bar{u}_j b_{i,j}$, $i = 0,1,..,m$, $j = 1,2,..,q$. Note that assumption A3 implies $\kappa_m s^m + \kappa_{m-1}s^{m-1} + \ldots + \kappa_0$ is a stable polynomial, irrespective of the failure pattern.

State Estimation

We need to construct state-estimator for

$$\begin{aligned}
\dot{x} &= A_0 x + \bar{k}y + \varphi_0(y) + \Phi(y)a \\
&\quad + \sum_{i=0}^{m} e_{n-i}\kappa_i u^* + \sum_{j=1}^{q}\sum_{i=0}^{m} e_{n-i}\mu_{i,j}\beta_j(y) + \Delta
\end{aligned} \tag{6}$$

where k is the observer gain and A_0 is the observer matrix such that $A = A_0 + kc^T$ and satisfies

$$PA_0 + A_0^T P = -I, \;\; P = P^T > 0$$

We will define the following set of filters for the purpose of state-estimation,

$$\begin{aligned}
\dot{\xi}_0 &= A_0\xi_0 + ky + \varphi_0(y), \;\; \xi_0 \in \mathbb{R}^{n\times 1} \\
\dot{\xi} &= A_0\xi + \Phi(y), \;\; \xi \in \mathbb{R}^{n\times p} \\
\dot{\vartheta}_i &= A_0\vartheta_i + e_{n-i}u^*, \;\; \vartheta_i \in \mathbb{R}^{n\times 1} \\
\dot{\psi}_{i,j} &= A_0\psi_{i,j} + e_{n-i}\beta_j(y), \;\; \psi_{i,j} \in \mathbb{R}^{n\times 1}
\end{aligned} \tag{7}$$

where $i = 0,1,..,m$ and $j = 1,2,..,q$. Due to the special structure of A_0, the order of K-filters can be reduced by using the following two filters

$$\begin{aligned}
\dot{\lambda} &= A_0\lambda + e_n u^* \\
\dot{\zeta}_j &= A_0\zeta_j + e_n\beta_j, \;\; j = 1,2,..,q
\end{aligned} \tag{8}$$

and the following algebraic equations

$$\begin{aligned}
\vartheta_i &= A_0^i\lambda \\
\psi_{i,j} &= A_0^i\zeta_j, \;\; i = 0,1,..,m
\end{aligned} \tag{9}$$

Also, it can be easily verified from the algebraic equations that

$$\begin{aligned}
\vartheta_{i,j} &= [*,*,....,*,1]\begin{bmatrix}\lambda_1 \\ \vdots \\ \lambda_{i+j}\end{bmatrix} \\
\psi_{i,j,k} &= [*,*,....,*,1]\begin{bmatrix}\zeta_{j,1} \\ \vdots \\ \zeta_{j,i+k}\end{bmatrix}
\end{aligned} \tag{10}$$

The estimated state can be written as

$$\hat{x} = \xi_0 + \xi a + \sum_{i=0}^{m}\kappa_i\vartheta_i + \sum_{j=1}^{q}\sum_{i=0}^{m}\mu_{i,j}\psi_{i,j} \tag{11}$$

Let $\varepsilon = x - \hat{x}$ be the estimation error. Then, the state-estimation error dynamics is given by

$$\dot{\varepsilon} = A_0\varepsilon + \Delta \tag{12}$$

Now, noting the fact from assumption A1 that $|\Delta_i(x,t)| \leq \delta_i(t)$ and that A_0 is stable, we can conclude that the estimation remains in a residual ball whose radius depends on $\delta_i(t)$ and the gain matrix i.e.,

$$\varepsilon \in \Omega_\varepsilon \triangleq \{\varepsilon : |\varepsilon(t)| \leq \delta_\varepsilon(t)\} \tag{13}$$

Parameter Projection

Let $\hat{\theta}$ denote the estimate of θ and $\tilde{\theta} = \hat{\theta} - \theta$ denote the estimation error. It is well known fact that gradient based parameter estimation algorithms suffer from *parameter drift* in presence of disturbances, and can result in system states growing unboundedly. We use discontinuous parameter projection to deal with this problem. The update law and the projection mapping used here have the following form,

$$\dot{\hat{\theta}} = \text{Proj}_{\hat{\theta}}(\Gamma\tau) \tag{14}$$

$$\text{Proj}_{\hat{\theta}_i} = \begin{cases} 0 \;\; \text{if } \hat{\theta}_i = \theta_{i,max} \text{ and } \bullet_i > 0 \\ 0 \;\; \text{if } \hat{\theta}_i = \theta_{i,min} \text{ and } \bullet_i < 0 \\ \bullet_i \;\; \text{otherwise} \end{cases} \tag{15}$$

where $\Gamma > 0$ is a diagonal matrix, and τ is any adaptation function. The projection mapping guarantees that the following two properties are always satisfied,

$$\textbf{P1} \quad \hat{\theta} \in \Omega_\theta = \{\hat{\theta} : \theta_{min} \le \hat{\theta} \le \theta_{max}\} \tag{16}$$
$$\textbf{P2} \quad \tilde{\theta}^T(\Gamma^{-1}\text{Proj}_{\hat{\theta}}(\Gamma\tau) - \tau) \le 0, \quad \forall\tau \tag{17}$$

Controller Design

The controller design presented here combines the adaptive backstepping [11] and discontinuous projection based ARC proposed in [10]. The main idea is to synthesize a virtual control law which will drive the error to a small residual ball. But, as in this case only a single state is available for measurement, the synthesized virtual control law will replace the reconstructed state at each step, and the state estimation error will be dealt with via robust feedback. Also, it should be noted that the use of discontinuous projection implies a tuning function based backstepping cannot be used, and hence a stronger robust control law is needed to negate the effects of parameter estimation transients. For advantages of discontinuous projection based technique over smooth modifications of adaptive law like smooth projection, and other details regarding the controller design presented here, the reader is referred to [10].

Step 1: The derivative of the output tracking error $z_1 = y - y_d$ is given by,

$$\begin{aligned}\dot{z}_1 &= \dot{y} - \dot{y}_d \\ &= x_2 + \varphi_{0,1}(y) + \sum_{j=1}^{p} a_j\varphi_{1,j} - \dot{y}_d + \Delta_1(x,t) \\ &= \hat{x}_2 + \varphi_{0,1}(y) + \sum_{j=1}^{p} a_j\varphi_{1,j} - \dot{y}_d + \bar{\Delta}_1(x,t) \\ &= \omega_0 + \omega^T\theta + \bar{\Delta}_1(x,t) \\ &= \kappa_m\vartheta_{m,2} + \omega_0 + \bar{\omega}^T\theta + \bar{\Delta}_1(x,t)\end{aligned} \tag{18}$$

where

$$\begin{aligned}\omega_0 &= [\xi_{0,2} + \varphi_{0,1}], \\ \omega &= [\xi_{(2)} + \Phi_{(1)}, \vartheta_{m,2}, \vartheta_{m-1,2}, .., \vartheta_{0,2}, \\ &\quad \psi_{m,1(2)}, .., \psi_{m,q(2)}, .., \psi_{0,1(2)}, .., \psi_{0,q(2)}]^T \\ \bar{\omega} &= \omega - e^*_{p+1}\vartheta_{m,2} \\ \theta &= [a_1, a_2, .., a_p, \kappa_m, .., \kappa_0, \\ &\quad \mu_{m,1}, .., \mu_{m,q}, .., \mu_{0,1}, .., \mu_{0,q}]^T\end{aligned} \tag{19}$$

and e^*_k is the kth basis vector in $\mathbb{R}^{p+m+qm+1}$. If $\vartheta_{m,2}$ were the input, we would synthesize a virtual control law α_1 to make z_1 as small as possible

$$\begin{aligned}\alpha_1(y, \xi_0, \xi, \bar{\lambda}_{m+1}, \psi_{i,j,2}, \hat{\theta}, t) &= \alpha_{1a} + \alpha_{1s} \\ \alpha_{1a} &= -\frac{1}{\hat{\kappa}_m}\{\omega_0 + \bar{\omega}^T\theta - \dot{y}_d\}\end{aligned} \tag{20}$$

where α_{1a} represents the model compensation component of the control law. Substituting (20) into (18), we get

$$\dot{z}_1 = \kappa_m(z_1 + \alpha_{1s}) - \tilde{\theta}\phi_1 + \bar{\Delta}_1 \tag{21}$$

The robust component is designed to compensate for the potential destabilizing effect of the uncertainties on the right hand side of (21) as follows

$$\alpha_{1s} = \alpha_{1s1} + \alpha_{1s2} + \alpha_{1s3}, \quad \alpha_{1s1} = -\frac{1}{\kappa_{m,min}}k_{1s}z_1 \tag{22}$$

where k_{1s} is a nonlinear gain, such that

$$k_{1s} = g_1 + ||C_{\phi_1}\Gamma\phi_1||^2, \quad g_1 \ge 0 \tag{23}$$

in which C_{ϕ_1} is a positive definite constant diagonal matrix do be specified later. Substituting (23) in (21), we obtain

$$\dot{z}_1 = \kappa_m z_2 - \frac{\kappa_m}{\kappa_{m,min}}k_{1s}z_1 + \kappa_m(\alpha_{1s2} + \alpha_{1s3}) - \tilde{\theta}^T\phi_1 + \bar{\Delta}_1 \tag{24}$$

Define a positive semi-definite (p.s.d) function $V_1 = \frac{1}{2}z_1^2$. Its derivative is given by

$$\begin{aligned}\dot{V}_1 \le\; &\kappa_m z_1 z_2 - k_{1s}z_1^2 + z_1(\kappa_m\alpha_{1s2} - \tilde{\theta}^T\phi_1) \\ &+ z_1(\kappa_m\alpha_{1s3} + \bar{\Delta}_1)\end{aligned} \tag{25}$$

From assumption A1,

$$\| \tilde{\theta}^T\phi_1 \| \le \| \theta_M \| \| \phi_1 \|, \quad \theta_M = \theta_{max} - \theta_{min} \tag{26}$$

As $\| \tilde{\theta}^T\phi_1 \|$ is bounded by a known function, there exists a robust control function satisfying the following conditions

$$\begin{aligned}&(a) \quad z_1\{\kappa_m\alpha_{1s2} - \tilde{\theta}^T\phi_1\} \le \varepsilon_{11} \\ &(b) \quad z_1\alpha_{1s2} \le 0\end{aligned} \tag{27}$$

Similarly, from assumption A1 and (13), we have

$$|\bar{\Delta}_1| \le |\varepsilon_2| + |\Delta_1| = \delta_{\varepsilon2}(t) + \delta_1(t) \le \bar{\delta}_1(t) \tag{28}$$

Now, we can follow the same strategy as in (27) to design a robust control law. But, as $\bar{\delta}_1(t)$ is unknown, we cannot prespecify the level of control accuracy. Hence, we seek to achieve the following relaxed conditions

$$\begin{aligned}&(a) \quad z_1\{\kappa_m\alpha_{1s3} + \bar{\Delta}_1(t)\} \le \varepsilon_{12}\bar{\delta}_1^2 \\ &(b) \quad z_1\alpha_{1s2} \le 0\end{aligned} \tag{29}$$

Remark 1. One smooth example of α_{1s2} satisfying (27) is

$$\alpha_{1s2} = -\frac{h_1}{4\kappa_{m,min}\varepsilon_{11}} z_1, \;\; h_1 \geq \|\theta_M\|^2 \|\phi_1\|^2 \tag{30}$$

Similarly, an example of α_{1s3} satisfying (29) is given by

$$\alpha_{1s3} = -\frac{1}{4\kappa_{m,min}\varepsilon_{12}} z_1 \tag{31}$$

Step 2: From (8-11) and (18-20), we can obtain the derivative of α_1 as follows

$$\begin{aligned}
\dot{\alpha}_1 &= \dot{\alpha}_{1c} + \dot{\alpha}_{1u} \\
\dot{\alpha}_{1c} &= \frac{\partial \alpha_1}{\partial y}\{\omega_0 + \omega^T \hat{\theta}\} + \frac{\partial \alpha_1}{\partial \xi_0}\{A_0 \xi_0 + ky + \varphi_0(y)\} \\
&+ \frac{\partial \alpha_1}{\partial \xi}\{A_0 \xi + \Phi(y)\} + \sum_{i=1}^{m+1} \frac{\partial \alpha_1}{\partial \lambda_i}\dot{\lambda}_i \\
&+ \sum_{j=1}^{q}\sum_{i=1}^{m+1} \frac{\partial \alpha_1}{\partial \zeta_{i,j}}\dot{\zeta}_{i,j} + \frac{\partial \alpha_1}{\partial t} && (32)\\
\dot{\alpha}_{1u} &= \frac{\partial \alpha_1}{\partial y}(-\tilde{\theta}^T \omega + \bar{\Delta}_1) + \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} && (33)
\end{aligned}$$

$\dot{\alpha}_{1c}$ is calculable and will be used for control function design. $\dot{\alpha}_{1u}$, however, is not calculable and will be dealt with via certain robust terms. From (7), the derivative of $z_2 = \vartheta_{m,2} - \alpha_1$ is given by

$$\dot{z}_2 = \vartheta_{m,3} - k_2 \vartheta_{m,1} - \dot{\alpha}_{1c} - \dot{\alpha}_{1u} \tag{34}$$

Define a p.s.d function $V_2 = V_1 + \frac{1}{2}z_2^2$. Then, derivative of V_2 using (25) and (34) is given by

$$\dot{V}_2 \leq \dot{V}_1|_{\alpha_1} + z_2\{\kappa_m z_1 + \vartheta_{m,3} - k_2\vartheta_{m,1} - \dot{\alpha}_{1c} - \dot{\alpha}_{1u}\} \tag{35}$$

where $\dot{V}_1|_{\alpha_1} = -k_{1s}z_1^2 + z_1(\kappa_m \alpha_{1s2} - \tilde{\theta}^T \phi_1) + z_1(\kappa_m \alpha_{1s3} + \bar{\Delta}_1)$. Similar to (20), we can now define α_2 for $\vartheta_{m,3}$ as follows

$$\begin{aligned}
&\alpha_2(y, \xi_0, \xi, \bar{\lambda}_{m+2}, \psi_{i,j,3}, \hat{\theta}, t) = \alpha_{2a} + \alpha_{2s} \\
&\alpha_{2a} = -\hat{\kappa}_m z_1 + k_2 \vartheta_{m,1} + \dot{\alpha}_{1c} \\
&\alpha_{2s} = \alpha_{2s1} + \alpha_{2s2} + \alpha_{2s3}, \;\; \alpha_{2s1} = -k_{2s} z_2 \\
&k_{2s} \geq g_2 + \|\frac{\partial \alpha_1}{\partial \hat{\theta}} C_{\theta 2}\| + \|C_{\phi 2}\Gamma \phi_2\|^2 && (36)
\end{aligned}$$

where $g_2 \geq 0$ is a constant, $C_{\theta 2}$ and $C_{\phi 2}$ are positive definite constant diagonal matrices, α_{2s2} and α_{2s3} are robust control functions to be synthesized later. Substituting (36) in (35), we obtain

$$\begin{aligned}
\dot{V}_2 \leq \dot{V}_1|_{\alpha_1} + z_2 z_3 - k_{2s} z_2^2 + z_2(\alpha_{2s2} - \tilde{\theta}^T \phi_2) \\
z_2(\alpha_{2s3} + \bar{\Delta}_2) - z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} && (37)
\end{aligned}$$

where $z_3 = \vartheta_{m,3} - \alpha_2$ represents the input discrepancy and

$$\phi_2 = e_{n+1}^* z_1 - \frac{\partial \alpha_1}{\partial y}\omega, \;\; \bar{\Delta}_2 = -\frac{\partial \alpha_1}{\partial y}\bar{\Delta}_1 \tag{38}$$

From (28), it follows that $\bar{\Delta}_2 \leq |\partial \alpha_1/\partial y|\bar{\delta}_1$. Similar to (29) and (35), the robust control functions α_{2s2} and α_{2s3} are chosen to satisfy

$$\begin{aligned}
&(a)\; z_2(\alpha_{2s2} - \tilde{\theta}^T \phi_2) \leq \varepsilon_{21} \\
&(b)\; z_2(\alpha_{2s3} + \bar{\Delta}_2) \leq \varepsilon_{22}\bar{\delta}_1^2 \\
&(c)\; z_2\alpha_{2s2} \leq 0 \;\;, \;\; z_2\alpha_{2s3} \leq 0 && (39)
\end{aligned}$$

where ε_{21} and ε_{22} are positive design parameters. As in (30) and (31), α_{2s2} and α_{2s3} can be chosen as,

$$\alpha_{2s2} = -\frac{h_2}{4\varepsilon_{21}} z_2 \;\;, \;\; \alpha_{2s3} = -\frac{1}{4\varepsilon_{21}}\left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 \tag{40}$$

where h_2 is any smooth function satisfying $h_2 \geq \| \theta_M \|^2 \| \phi_2 \|^2$. From (25) and h_2 defined above, the derivative of V_2 satisfies

$$\begin{aligned}
\dot{V}_2 \leq\; & z_2 z_3 - \sum_{j=1}^{2} k_{js} z_j^2 + z_1(\kappa_m \alpha_{1s2} - \tilde{\theta}_1 \phi_1) + z_1(\kappa_m \alpha_{1s3} + \bar{\Delta}_1) \\
& + z_2(\alpha_{2s2} - \tilde{\theta}^T \phi_2) + z_2(\alpha_{2s3} + \bar{\Delta}_2) - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} z_2 && (41)
\end{aligned}$$

Step i ($3 \leq i < \rho$)*:* Mathematical induction will be used to prove the general result for all the intermediate steps. At each step i, the ARC control function α_i will be constructed for virtual control input $\vartheta_{m,i+1}$. For any $j \in [3, i-1]$, let $z_j = \vartheta_{m,j} - \alpha_{j-1}$ and recursively design

$$\phi_j = -\frac{\partial \alpha_{j-1}}{\partial y}\omega \;\;, \;\; \bar{\Delta}_j = -\frac{\partial \alpha_{j-1}}{\partial y}\bar{\Delta}_1 \tag{42}$$

Lemma 1: At step i, choose the desired ARC control function α_i as

$$\begin{aligned}
&\alpha_i(y, \xi_0, \xi, \bar{\lambda}_{m+i}, \psi_{k,j,i+1}, \hat{\theta}, t) = \alpha_{ia} + \alpha_{is} \\
&\alpha_{ia} = -z_i + k_i \vartheta_{m,i} + \dot{\alpha}_{(i-1)c} \\
&\alpha_{is} = \alpha_{is1} + \alpha_{is2} + \alpha_{is3} \quad \alpha_{is1} = -k_{is} z_i \\
&k_{is} \geq g_i + \| \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} C_{\theta i} \| + \| C_{\phi i}\Gamma \phi_i \|^2 && (43)
\end{aligned}$$

where $g_i > 0$ is a constant, and $C_{\theta i}$ and $C_{\phi i}$ are positive definite constant diagonal matrices, α_{is2} and α_{is3} are robust control functions satisfying,

$$\begin{aligned}
&(a)\; z_i(\alpha_{is2} - \tilde{\theta}^T \phi_i) \leq \varepsilon_{i1} \\
&(b)\; z_i(\alpha_{is3} + \bar{\Delta}_i) \leq \varepsilon_{i2}\bar{\delta}_i^2 \\
&(c)\; z_i\alpha_{is2} \leq 0 \;\;, \;\; z_i\alpha_{is3} \leq 0 && (44)
\end{aligned}$$

and

$$\dot{\alpha}_{(i-1)c} = \frac{\partial \alpha_1}{\partial y}\{\omega_0 + \omega^T \hat{\theta}\} + \frac{\partial \alpha_1}{\partial \xi_0}\{A_0\xi_0 + ky + \varphi_0(y)\} + \frac{\partial \alpha_1}{\partial \xi}\{A_0\xi + \Phi(y)\} + \sum_{i=1}^{m+1}\frac{\partial \alpha_1}{\partial \lambda_i}\dot{\lambda}_i + \sum_{j=1}^{q}\sum_{i=1}^{m+1}\frac{\partial \alpha_1}{\partial \zeta_{i,j}}\dot{\zeta}_{i,j} + \frac{\partial \alpha_1}{\partial t} \quad (45)$$

Then, the ith error subsystem is

$$\dot{z}_i = z_{i+1} - z_{i-1} - k_{is}z_i + (\alpha_{is2} - \tilde{\theta}^T\phi_i) + (\alpha_{is3} + \bar{\Delta}_i) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \quad (46)$$

and the derivative of the augmented p.s.d function $V_i = V_{i-1} + 1/2z_i^2$ satisfies,

$$\dot{V}_i \le z_i z_{i+1} - \sum_{j=1}^{i} k_{js}z_j^2 + z_1(\kappa_m\alpha_{1s2} - \tilde{\theta}^T\phi_1) + \sum_{j=2}^{i} z_j(\kappa_m\alpha_{js2} - \tilde{\theta}^T\phi_j) + z_1(\kappa_m\alpha_{1s3} + \bar{\Delta}_1) + \sum_{j=2}^{i} z_j(\alpha_{js3} + \bar{\Delta}_j) - \sum_{j=2}^{i}\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}z_j \quad (47)$$

The lemma can be easily verified by recursively writing the various expressions and substituting the expressions obtained in step 1 and 2.

Step ρ*:* In this final step, the actual control law u^* will be synthesized such that $\vartheta_{m,\rho}$ tracks the desired ARC control function $\alpha_{\rho-1}$. The derivative of z_ρ can be obtained as

$$\dot{z}_\rho = \vartheta_{m,\rho+1} + u^* - k_\rho\vartheta_{m,1} - \dot{\alpha}_{(\rho-1)c} - \frac{\partial \alpha_{\rho-1}}{\partial y}(-\tilde{\theta}^T\omega + \bar{\Delta}_1) - \frac{\partial \alpha_{\rho-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \quad (48)$$

If $\vartheta_{m,\rho+1} + u^*$ were the virtual input, (48) would have the same form as the intermediate step i. Therefore, the general form, (42-48) applies to step ρ. Since u^* is the actual control input, it can be chosen as,

$$u^* = \alpha_\rho - \vartheta_{m,\rho+1} \quad (49)$$

where α_ρ is given by (47). Then, $z_{\rho+1} = u^* + \vartheta_{m,\rho+1} - \alpha_\rho = 0$.

Theorem 1: Let the parameter estimates be updated using adaptation law (14) in which τ is chosen as

$$\tau = \sum_{j=1}^{\rho}\phi_j z_j \quad (50)$$

If diagonal controller gain matrices $C_{\theta j}$ and $C_{\phi k}$ are chosen such that $c_{\phi kr}^2 \ge \frac{\rho}{4}\sum_{j=1}^{\rho} 1/c_{\theta jr}^2$, where $c_{\theta jr}$ and $c_{\phi kr}$ are the rth diagonal element of $C_{\theta j}$ and $C_{\phi k}$ respectively. Then, the control law (49) guarantees that,

1. In general the control input and all internal signals are bounded. Furthermore, V_ρ is bounded above by,

$$V_\rho(t) \le \exp(-\lambda_\rho t)V_\rho(0) + \frac{\bar{\varepsilon}_{\rho 1} + \bar{\varepsilon}_{\rho 2}\parallel \bar{\delta}_1 \parallel_\infty^2}{\lambda_\rho}[1 - \exp(-\lambda_\rho t)] \quad (51)$$

where $\lambda_\rho = 2\min\{g_1, \ldots, g_\rho\}$, $\bar{\varepsilon}_{\rho 1} = \sum_{j=1}^{\rho}\varepsilon_{j1}$, $\bar{\varepsilon}_{\rho 2} = \sum_{j=1}^{\rho}\varepsilon_{j2}$ and $\parallel \bar{\delta}_1 \parallel_\infty^2$ stands for the infinity norm of $\bar{\delta}_1$.
2. If after a finite time t_0, $\tilde{\Delta} = 0$ (i.e., in the presence of parametric uncertainties only) then, in addition to results in (51), asymptotic output tracking control is also achieved.

Proof of the theorem has been omitted due to space restrictions, but can be obtained from the authors upon request and is similar to one presented in [12].

In context of actuator fault compensation, (51) guarantees that the jump in parameter values due to failed actuator does not interfere with the desired transient performance. Furthermore, the accuracy can be improved by choosing suitable values of ε_{j1} and ε_{j2}. It may appear that we have neglected the $\rho+1$ to n states in the present analysis. But, due to the assumption of stable zero dynamics (A3) and bounded uncertainties (A1), it can be easily proved using standard adaptive control arguments that all internal signals remain bounded and do not interfere with the tracking performance.

SIMULATION

The proposed scheme is implemented on the reduced order nonlinear longitudinal model of a hypersonic aircraft, which was also used in [4] and thus, will provide a uniform platform to compare the performance of ARFTC and robust adaptive control (RAC) based fault-tolerant schemes.

$$\begin{aligned}\dot{x}_1 &= x_2 + a_1 y + a_2 sin(y) + a_3 y^2\sin(y) + a_4 cos(x_3) + \Delta_1(x,t)\\ \dot{x}_2 &= a_5 y^2 + a_6 y + (a_7 + a_8 y + a_9 y^2)x_2\\ &\quad + b_1 u_1 + b_2 u_2 + \Delta_2(x,t)\\ \dot{x}_3 &= a_{10}cos(x_3) - a_1 y - a_2 sin(y) + \Delta_3(x,t)\end{aligned} \quad (52)$$

where $y = x_1$ is the angle of attack, x_2 is the pitch rate, x_3 is the flight-path angle, a_i and b_i are unknown parameters, and Δ_i were introduced to capture the effect of non-parametric uncertainties. Further details of the model can be found in [13]. The nominal parameter values are given by

$$
\begin{aligned}
&a_1 = -0.0427, a_2 = -3.4496\times10^{-4}, a_3 = 5\times10^{-5},\\
&a_4 = 0.0014, a_5 = -4.2006, a_6 = 1.0821, a_7 = -3.6896,\\
&a_8 = 0.1637, a_9 = -0.1242, a_{10} = 0.0014, b_1 = 0.8, b_2 = 0.8
\end{aligned}
$$

The reference command is given by $y_d(t) = 0.01\sin(0.1t)$. Two faults are introduced: at $t = 50$ seconds the first actuator loses 40% efficiency and at $t = 75$ seconds, the second actuator gets stuck at $\bar{u}_2 = 0.1$ radians.

In the ARFTC scheme, the unknown parameter vector θ is

$$\theta = [a_1, a_2, a_3, a_5, a_6, \kappa_0, \mu_0]^T$$

and the initial values and bounds for the parameter estimates are

$$
\begin{aligned}
\hat{\theta}(0) &= [-0.05, -4\times10^{-4}, 0, -4.0, 0.9, 1.5, 0]\\
\hat{\theta}_{\min} &= [-0.06, -5\times10^{-4}, 4\times10^{-4}, -5, 0.8, 0.2, -1]\\
\hat{\theta}_{\max} &= [-0.03, -2\times10^{-4}, 7\times10^{-4}, -3.5, 1.2, 2, 1]
\end{aligned}
$$

The gain matrix for parameter estimation is given by $\Gamma = \mathrm{diag}\{1,1,1,1,1,5,0.1\}$. The observer gain matrix is chosen to be $\bar{k} = [2,1]^T$. Although, α_{1s2} and α_{1s3} can be designed explicitly in terms of known functions and parameter bounds, the controller implementation is simplified by choosing a sufficiently high value of k_{1s}, such that it is robust against $\tilde{\theta}$ and $\bar{\Delta}_1$. The same logic is used in the second step of the design as well. The controller gains chosen for simulation are $k_{1s} = -60,\ k_2 = 1,\ k_{2s} = -90$. Details of the RAC based scheme can be obtained from [4].

The proposed scheme is applied to the system in absence of disturbances and in presence of disturbances. From figure (1), we see that when the actuators fails i.e., at $t = 50$ seconds and $t = 75$ seconds, the tracking error increases, but quickly settles down to the desired level. The first set of simulations demonstrate the effectiveness of the proposed scheme in suppressing the undesired effects of jump in parameter values on transient response. The improved transient response of the ARFTC over RAC based scheme can be attributed to the underlying robust controller. The robust component of the control law incorporates the jump in parameter values (α_{is2} term) and ensures guaranteed transient response. Furthermore, the adaptation scheme learns the change in system parameter over time to ensure good model compensation, leading to small steady-state tracking errors.

In the second case, we add disturbances to the first and second channel - $\Delta_1(y,t) = 0.01\sin(2t)$ and $\Delta_2(y,t) = 0.01\sin(3t)$. Note that in addition to increasing the tracking error, modeling errors and disturbances can also cause the adaptive scheme to go unbounded. But, from figure (2), we see that the order of tracking error remains the same in spite of disturbances. This can be explained as follows. First, the robust component of the control law suppresses the effect of disturbances and modeling errors and second, the discontinuous projection guarantees the boundedness of all parameter estimates. On the other hand, the RAC based scheme can guarantee the boundedness of all signals, but may not be able achieve good transient response and small steady-state error, as can be seen from figure (2). Thus, the proposed scheme has desired transient properties and good final tracking accuracy even in presence of external disturbances and modeling errors.

1 CONCLUSIONS

In this paper, an adaptive robust output feedback based scheme is presented for unknown actuator fault accommodation for a class of uncertain nonlinear system. A fault-tolerant scheme based on adaptive robust control can address the two principle issues associated with failing actuators - first, undesirable transients following jump in parameter values and second, large steady-state tracking error. A nonlinear model of hypersonic aircraft is used for comparative simulation studies. The results prove the effectiveness of the proposed scheme over backstepping based robust adaptive control in attenuating the effect of modeling uncertainties and unknown actuator faults on desired transient response and steady-state tracking error. The salient features of the fault accommodation scheme presented in this paper are,

1. capability to handle large parametric uncertainties due to unknown actuator failures like stuck actuators and actuator loss in efficiency with guaranteed transient performance
2. guaranteed robust performance when adaptation is switched off
3. calculable upper bound for tracking error based on controller parameters and ability to achieve prespecified final tracking accuracy

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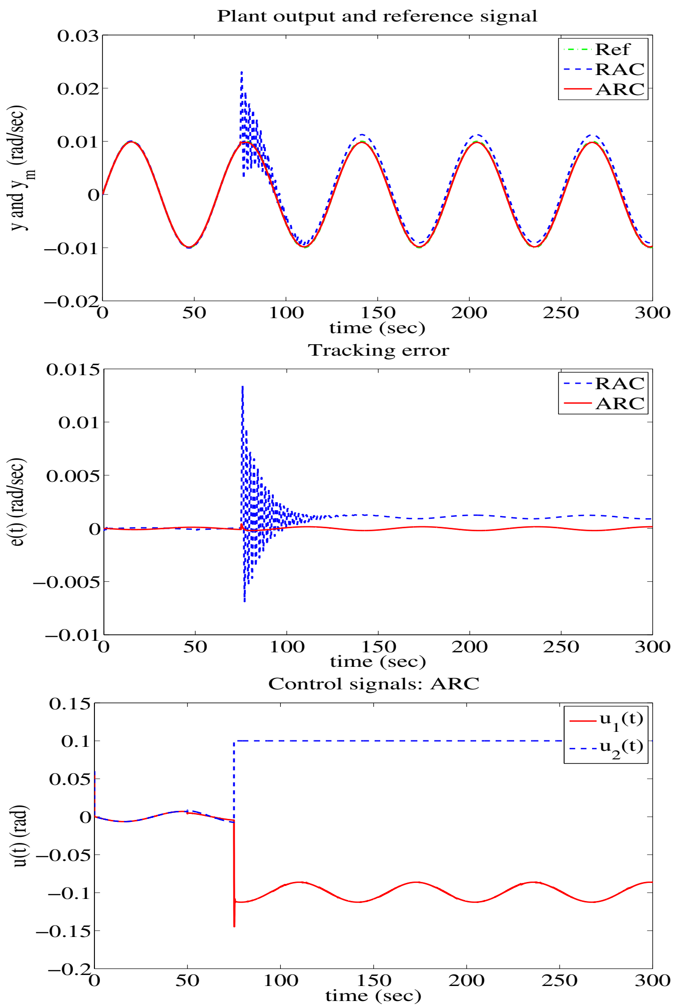

Figure 1. REFERENCE TRACKING, TRACKING ERROR AND CONTROL SIGNALS IN ABSENCE OF DISTURBANCES

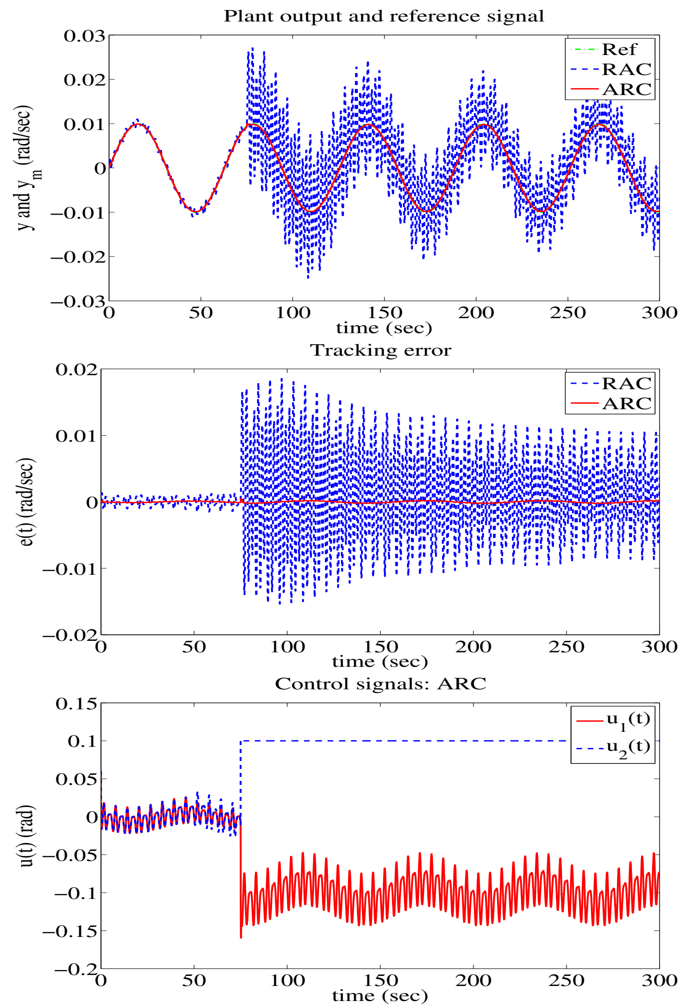

Figure 2. REFERENCE TRACKING, TRACKING ERROR AND CONTROL SIGNALS IN PRESENCE OF DISTURBANCES

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