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ADAPTIVE ROBUST CONTROL FOR A CLASS OF NONLINEAR UNCERTAIN SYSTEM WITH UNKNOWN INPUT BACKLASH

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ABSTRACT

An adaptive robust control (ARC) algorithm is developed for a class of nonlinear dynamic system with unknown input backlash, parametric uncertainties and uncertain disturbances. Due to the non-smooth dynamic nonlinear nature of backlash, existing robust adaptive control methods mainly focus on using approximate inversion of backlash by on-line parameter adaptation. But experimental results show that a linear controller alone can perform better than a controller including the selected backlash inverter with a correctly estimated or overestimated backlash gap. Unlike many existing control schemes, the backlash inverse is not constructed in this paper. A new linearly parameterized model for backlash is presented. The backlash nonlinearity is linearly parameterized globally with bounded model error. The proposed adaptive robust control law ensure that all closed-loop signals are bounded and achieves the tracking within the desired precision. Simulations results illustrate the performance of the ARC.

INTRODUCTION

Backlash characteristics is common in control systems such as servomechanisms, electronic relay circuits and electromagnetic devices with hysteresis. It severely limits systems performance in such manners as giving rise to undesirable inaccuracies or oscillation, which can even lead to instability. Control of system with backlash nonlinearity is an important area of control system research and typically challenging. For systems with backlash, linear controllers have been investigated, including PID controllers, high-order linear controllers, state feedback controllers [1]. Recently, inverse compensations of backlash are designed widely. An adaptive inverse was construct to cancel the effect of backlash nonlinearity in [2][3] with the strict initial condition. A smooth inverse was also developed to compensation the effect of backlash with back-stepping approach in [4], where the derivation of the control was used to get the controller, which maybe unavailable. Backlash compensation using neural network [5][6] or fuzzy logic [7] has been used in feedback control system. For those compensations, neural networks or fuzzy logic were mainly used for cancellation of the inversion error by their excellent nonlinearity approximation ability. The common feature of these inverse schemes is that they rely on the construction of an inverse backlash to mitigate the effect of the backlash nonlinearity. Experiments by [8] show that a linear controller alone performed better than a controller including the selected backlash inverter with a correctly estimated or overestimated backlash gap, the reason being that measurement noise induced chattering in the inverter. It was noted that the linear controller alone also traverses the backlash gap rapidly since only the motor moment of inertia (and not the load) is driven inside the backlash gap.

In this paper, a new approach for adaptive robust control (ARC) of nonlinear systems with backlash is introduced without constructing the backlash inverse. Based only on the intuitive concept and piece-wise description of backlash, the unknown backlash nonlinearity is linearly parameterized globally with bounded model error. The ARC controller is consist of three parts: one is the usual model compensation with the physical parameter estimates; For the second part, a

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simple proportional feedback part is used to stabilize the nominal system; The last one is a robust feedback term used to attenuate the effect of various model uncertainties. The ARC control law ensures a global stability of the entire adaptive system and achieves the tracking within a desired precision. Computer simulations were carried out to illustrate the effectiveness of the approaches.

This paper is organized as follows: Section II states the problem of this note, where the linear model of backlash is introduced. In section III, the proposed ARC schemes is presented. In section IV, simulation results obtained on an uncertain nonlinear system are presented to illustrate the effectiveness of the proposed approach. In section V, conclusions are drawn.

PROBLEM STATEMENT

System Model

The following same class of nonlinear system as in [9][10] proceeded by unknown input backlash is considered:

$$x^{(n)} = \sum_{i=1}^{p} a_i Y_i(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) + bw(t) + \Delta(x,t)$$
$$y = x(t) \quad w(t) = B(u(t)) \tag{1}$$

where $Y_i, i = 1,2,\cdots,p$ are known continuous nonlinear functions, parameter a_i and control gain b presents unknown constants. It is a common assumption that the sign of b is known. From this point onward, without losing generality, we shall assume $b > 0$. $u(t)$ is the output from the controller, $w(t)$ is the actual input to the plant, and $y(t)$ is the output from the plant. The actuator nonlinearity $w = B(u)$ is described as a backlash characteristic. $\Delta(x,t)$ represents the lumped uncertain nonlinearity including external disturbances.

The control objective is to design a control law for $u(t)$ to ensure that the plant state vector $\mathbf{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T$ follows a specified desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, \cdots, x_d^{(n-1)}]^T$,i.e. $\mathbf{x} \to \mathbf{x}_d$ within a desired accuracy as $t \to \infty$.

Assumption 1: $\theta = [a_1, \cdots, a_p, b]^T$ lies in a known bounded set Ω_θ:

$$\theta \in \Omega_\theta \triangleq \{\theta : a_{j\min} \le \theta_j \le a_{j\max}, j = 1,\cdots,p$$
$$0 < b_{\min} \le \theta_{p+1} \le b_{\max}, \qquad \}$$

Assumption 2: The uncertain nonlinearity can be bounded by

$$|\Delta(x,t)| \le B_0$$

where B_0 is a known positive constant.

Backlash Characteristic

A backlash nonlinearity is shown as Fig.1

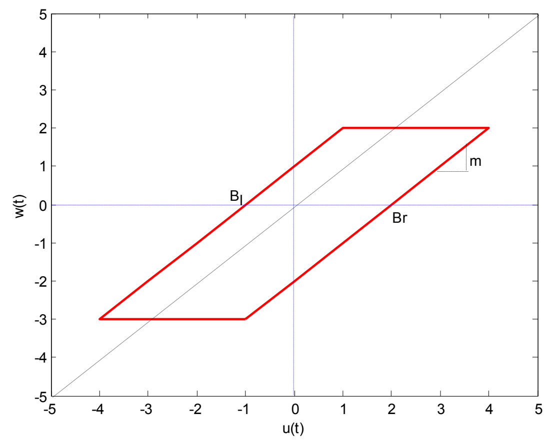

FIG 1. BACKLASH MODEL

Traditionally, it can be described by:

$$w(t) = B(u)$$
$$= \begin{cases} m(u(t) - B_r) & if\ \dot{u} > 0\ and\ w(t) = m(u(t) - B_r) \\ m(u(t) - B_l) & if\ \dot{u} < 0\ and\ w(t) = m(u(t) - B_l) \\ w(t_-) & otherwise \end{cases} \tag{2}$$

where m is the constant slope of the lines, B_r, B_l are constant parameters. $w(t_-)$ means no change occurs in $w(t)$.

For the development of control law, the following assumptions are made:

Assumption 3: The backlash parameters m, B_r, B_l are unknown, but their signs are known. In general, let $m > 0$, $mB_r > 0, mB_l < 0$.

Assumption 4: The backlash parameters are within the known bounds:

$$0 < m_{\min} \le m \le m_{\max}$$
$$0 < (mB_r)_{\min} \le mB_r \le (mB_r)_{\max}$$
$$(mB_l)_{\min} \le mB_l \le (mB_l)_{\max} < 0$$

Assumption 5: The backlash output $w(t)$ is not available for measurement.

From the above, we can rewrite the backlash model (2) as

$$w(t) = B(u) = mu(t) + d_b(u(t)) \tag{3}$$

Where $d_b(u(t))$ is model error from linearly parameterized of backlash and it can be calculated by (2) as

$$d_b(u(t)) = \begin{cases} -mB_r & if\ \dot{u} > 0\ and\ w(t) = m(u(t) - B_r) \\ -mB_l & if\ \dot{u} < 0\ and\ w(t) = m(u(t) - B_l) \\ w(t_-) - mu(t) & otherwise \end{cases} \tag{4}$$

Lemma 1: with (2) (4) and assumption 4, $d_b(u(t))$ is bounded, and satisfies

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$$|d_b(u(t))| \le \rho \tag{5}$$

where ρ is the upper-bound, which can be chosen as

$$\rho = \max\{(mB_r)_{\max}, -(mB_l)_{\min}\}$$

Combing (1) and (3), the system (1) can be rewritten as

$$\begin{aligned} x^{(n)} &= \sum_{i=1}^{p} a_i Y_i(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) \\ &\quad + mbu(t) + bd_b(u) + \Delta(x,t) \\ y &= x(t) \end{aligned} \tag{6}$$

Define a constant d_c and time-varying $\Delta^*(t)$ such that

$$d_c + \Delta^*(t) = bd_b(t) + \Delta(x,t) \tag{7}$$

Conceptually, (7) divides the backlash modeling error d_b and the original system uncertain nonlinearity $\Delta(x,t)$ into the static component (or low frequency component in reality) d_c and the high frequency component $\Delta^*(t)$. By Assumption 1 and Assumption 2 and lemma 1, one obtains

$$\begin{aligned} d_c &\in \Omega_{dc} \triangleq \{d_c : |d_c| \le \delta_d\} \\ \Delta^*(t) &\in \Omega_\Delta \triangleq \{\Delta^* : |\Delta^*| \le \delta_\Delta\} \end{aligned} \tag{8}$$

where δ_d and δ_Δ are known.

DESIGN OF ADAPTIVE ROBUST CONTROLLER

In this section, the ARC strategy [11] will be designed for system (6). As in [12][13], the first step is to use a projection type adaptation law structure to achieve a controlled learning or adaptation process.

Projection Type Adaptation Law

Let $\hat{\theta}$ denote the estimation of θ and $\tilde{\theta}$ be the estimation error $\tilde{\theta} = \theta - \hat{\theta}$. The following projection-type parameter adaption law[14] is used

$$\begin{aligned} \dot{\hat{\theta}} &= \mathrm{Proj}_{\hat{\theta}}(\Gamma \tau), \quad \hat{\theta}(0) \in \Omega_\theta \\ \mathrm{Proj}_{\hat{\theta}}(\bullet_i) &= \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{i\max} \text{ and } \bullet_i > 0 \\ 0 & \text{if } \hat{\theta}_i = \theta_{i\min} \text{ and } \bullet_i < 0 \\ \bullet_i & \text{otherwise} \end{cases} \end{aligned} \tag{9}$$

where Γ is a diagonal matrix of adaption rates and τ is an adaption function to be synthesized further on. Such a parameter adaption law has the following desirable properties. At any time instant, i.e., $\forall t$:

(P1) $\hat{\theta}(t) \in \Omega_\theta$

(P2) $\tilde{\theta}^T(\tau - \Gamma^{-1}\mathrm{Proj}_{\hat{\theta}}(\Gamma\tau)) \le 0$

ARC Law

With the use of the above projection type adaptation law, the parameter estimates are bounded. In the following, this property will be used to synthesize an adaptive robust control for the system (6) which achieves a guaranteed transient and steady-state output tracking accuracy.

Define

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} x_e(t) \qquad \text{with } \lambda > 0 \tag{10}$$

which can be rewritten as

$$s(t) = \Lambda^T X_e(t)$$

where

$$\begin{aligned} \Lambda^T &= [\lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots, 1] \\ X &= [x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)]^T \\ X_d &= [x_d(t), \dot{x}_d(t), \cdots, x_d^{(n-1)}(t)]^T \\ X_e &= X - X_d \end{aligned}$$

Combing (6) (7) and (10), we get

$$\begin{aligned} \dot{s}(t) &= \Lambda_v^T X_e(t) + x_e^{(n)}(t) \\ &= \Lambda_v^T X_e(t) + x^{(n)}(t) - x_d^{(n)}(t) \\ &= \Lambda_v^T X_e(t) + \sum_{i=1}^{p} a_i Y_i(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) \\ &\quad + mbu(t) + d_c + \Delta^*(t) - x_d^{(n)}(t) \end{aligned} \tag{11}$$

where: $\Lambda_v^T = [0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots, (n-1)\lambda]$.

Thus:

$$\begin{aligned} &\frac{1}{mb}\dot{s}(t) \\ &= \frac{1}{mb}\Lambda_v^T X_e(t) + \sum_{i=1}^{p} \frac{a_i}{mb} Y_i(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) \\ &\quad + u(t) + \frac{1}{mb} d_c + \frac{1}{mb}\Delta^*(t) - \frac{1}{mb} x_d^{(n)}(t) \\ &= u(t) + \vartheta^T \varphi + \bar{\Delta}(t) \end{aligned} \tag{12}$$

where:

$$\begin{aligned} \vartheta &= \left[\frac{1}{mb}, \frac{a_1}{mb}, \cdots, \frac{a_p}{mb}, \frac{d_c}{mb}\right]^T \\ \varphi &= [\Lambda_v^T X_e(t) - x_d^{(n)}(t), Y_1, \cdots, Y_p, 1] \\ \bar{\Delta}(x,t) &= \frac{1}{mb}\Delta^*(t) \end{aligned}$$

The following ARC is proposed to design u as follows:

$$u = u_a + u_s$$

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$$u_s = u_{s1} + u_{s2}$$
$$u_a = -\hat{\vartheta}^T \varphi \quad (13)$$
$$u_{s1} = -k_{s1} s$$
$$\dot{\hat{\vartheta}} = \mathrm{Pr}\,oj(\Gamma_\vartheta \varphi s) \quad (14)$$

In (13), u_a represents the usual model compensation with the physical parameter estimates $\hat{\vartheta}$,which is updated by using an on-line adaptation projection algorithm. u_s represents the robust control term in which u_{s1} is a simple proportional feedback to stabilize the nominal system and u_{s2} is a robust feedback term used to attenuate the effect of various model uncertainties for guaranteed robust control performance in general.

Substituting (13) into (12), one obtains

$$\frac{1}{mb}\dot{s}(t) = -k_{s1}s + u_{s2} + \tilde{\vartheta}^T\varphi + \overline{\Delta}(t) \quad (15)$$

Noting Assumption 1-4, and P(1), there exists a u_{s2} such that the following two conditions are satisfied：

(i)

$$s \cdot u_{s2} \le 0 \quad (16)$$

(ii)

$$s(u_{s2} + \tilde{\vartheta}^T\varphi + \overline{\Delta}(t)) \le \varepsilon \quad (17)$$

where ε is design parameters which can be arbitrarily small. Essential,(17) shows that u_{s2} is synthesized to dominate the model uncertainties coming from both parametric uncertainties and uncertain nonlinearities. And (16) is to make sure that u_{s2} is dissipative in nature so that it doesn't interfere with the functionality of the adaptive control part u_a.

Remark 1: One example of u_{s2} satisfying (16) and (17) can be chosen as:

$$u_{s2} = -\frac{1}{4\varepsilon}[\|\tilde{\vartheta}_{\max}\|\|\varphi\| + \overline{\Delta}_{\max}]^2 \cdot s \quad (18)$$

where

$$\tilde{\vartheta}_{\max} = \vartheta_{\max} - \vartheta_{\min}, \qquad \overline{\Delta}_{\max} = \frac{\delta_\Delta}{(mb)_{\min}}$$

$$\vartheta_{\max} = [\frac{1}{(mb)_{\min}}, \frac{a_{1\max}}{(mb)_{\min}}, \cdots, \frac{a_{p\max}}{(mb)_{\min}}, \frac{\delta_d}{(mb)_{\min}}]^T$$

$$\vartheta_{\min} = [\frac{1}{(mb)_{\max}}, \frac{a_{1\min}}{(mb)_{\max}}, \cdots, \frac{a_{p\min}}{(mb)_{\max}}, \frac{-\delta_d}{(mb)_{\max}}]^T$$

It can be easily to show that the above choice of u_{s2} does satisfy (16) and (17).

The condition (i) is easily to satisfied by the control law (16). For condition (ii), we can get

$$\begin{aligned}
&\varepsilon - su_{s2} \\
&= [\frac{1}{2\sqrt{\varepsilon}}(\|\tilde{\vartheta}_{\max}\|\|\varphi\| + \overline{\Delta}_{\max})]^2 s^2 + (\sqrt{\varepsilon})^2 \\
&\ge 2 \cdot \frac{1}{2\sqrt{\varepsilon}}(\|\tilde{\vartheta}_{\max}\|\|\varphi\| + \overline{\Delta}_{\max})|s| \cdot \sqrt{\varepsilon} \\
&= |s|\|\tilde{\vartheta}_{\max}\|\|\varphi\| + |s|\overline{\Delta}_{\max} \\
&\ge s(\tilde{\vartheta}^T\varphi + \overline{\Delta}(t))
\end{aligned}$$

Thus:

$$s(u_{s2} + \tilde{\vartheta}^T\varphi + \overline{\Delta}(t)) \le \varepsilon$$

So the condition (ii) is also satisfied.

Theorem1: Consider the system (1) consisting of the adaptive robust controller given by (13) and the projection type parameters adaptation law (14), all signals in the resulting closed loop system are bounded, and the output tracking is guaranteed to have a prescribed transient performance and final tracking accuracy in the sense that the tracking error index *s* is bounded by:

$$s^2(t) \le e^{-2mbk_{s1}t}s^2(0) + \frac{\varepsilon}{k_{s1}}[1 - e^{-2mbk_{s1}t}] \quad (19)$$

Proof:

Consider the following Lyapunov function:

$$V = \frac{1}{2mb}s^2 \quad (20)$$

then by (15) and (17), the derivation of *V*

$$\begin{aligned}
\dot{V} &= \frac{1}{mb}s\dot{s} \\
&= s(-k_{s1}s + u_{s2} + \tilde{\vartheta}^T\varphi + \overline{\Delta}(t)) \\
&= -k_{s1}s^2 + s(u_{s2} + \tilde{\vartheta}^T\varphi + \overline{\Delta}(t)) \\
&\le -k_{s1}s^2 + \varepsilon
\end{aligned}$$

Thus:

$$\dot{V} \le -2mbk_{s1}V + \varepsilon \quad (21)$$

Then:

$$V(t) \le e^{-2mbk_{s1}t}V(0) + \frac{\varepsilon}{2mbk_{s1}}[1 - e^{-2mbk_{s1}t}] \quad (22)$$

Then (19) is true. The theorem shows that the output tracking precision is guaranteed by setting ε and k_{s1} respectively.

Theorem2:Consider the system (1) consisting of the parameters and the adaptive robust controller given (13)(14), In the absence of uncertain nonlinearities(i.e., assuming $\overline{\Delta}(t) = 0$), asymptotic position tracking is also achieved.

Proof: Consider the following Lyapunov function:

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$$V = \frac{1}{2mb} s^2 + \frac{1}{2} \tilde{\vartheta}^T \Gamma_\vartheta^{-1} \tilde{\vartheta}$$

Noting (14) – (16) and the P2 of the projection law, then the derivation of V

$$\begin{aligned} \dot{V} &= \frac{1}{mb} s\dot{s} + \tilde{\vartheta}^T \Gamma_\vartheta^{-1} \dot{\tilde{\vartheta}} \\ &= s(u + \vartheta^T \varphi) - \tilde{\vartheta}^T \Gamma_\vartheta^{-1} \dot{\hat{\vartheta}} \\ &= -k_{s1} s^2 + s(u_{s2} + \tilde{\vartheta}^T \varphi) - \tilde{\vartheta}^T \Gamma_\vartheta^{-1} \operatorname{Pr}oj(\Gamma_\vartheta \varphi s) \\ &\le -k_{s1} s^2 + s u_{s2} + \tilde{\vartheta}^T [s\varphi - \Gamma_\vartheta^{-1} \operatorname{Pr}oj(\Gamma_\vartheta \varphi s)] \\ &\le -k_{s1} s^2 \end{aligned}$$

By the Barbalat's lemma, it follows that $\lim_{t\to\infty} s(t) = 0$, which implies that $\lim_{t\to\infty} x_e(t) = 0$.

Theorem 2 indicates that the asymptotic position tracking can be achieved if the system works only on the single slope side of the backlash (i.e., $d_b(t)$ is constant as $d_b(t) = mb_r$ or $d_b(t) = mb_l$).

SIMULATIONS

The proposed algorithms is applied to the following nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = bw(t) + a_1 \dfrac{1 - e^{-x_1}}{1 + e^{-x_1}} + a_2 x_2 \sin(\dot{x}_1) + \Delta(x,t) \\ y = x_1 \end{cases}$$

where $w(t)$ is the output of a backlash described by:

$$w(t) = \begin{cases} 1.2[u(t) - 0.5] & \dot{u}(t) > 0 \text{ and } w(t) = 1.2[u(t) - 0.5] \\ 1.2[u(t) - (-0.8)] & \dot{u}(t) < 0 \text{ and } w(t) = 1.2[u(t) + 0.8] \\ w(t_-) & \text{otherwise} \end{cases}$$

The actual values of physical parameters are set as $a_1 = 1$, $a_2 = -2$, $b = 1$ and the bounds of them are chosen as $a_1 \in [0.5, 1.5]$, $a_2 \in [-3, -1]$, $b \in [0.5, 1.5]$. The actual backlash parameters of $m = 1.2$, $mB_r = 0.6$ and $mB_l = -0.96$ are assumed not known but within the known range of $m \in [0.5, 1.8]$, $mB_r \in [0.1, 1.2]$, and $mB_l \in [-1.5, -0.5]$ respectively. The disturbance is chosen as $\Delta(x,t) = 0.05\sin(t)$, which is within [-0.05,0.05]. The control objective is to let the system follow the desired trajectory:

$$y_d(t) = 0.5[\sin(t) + \sin(1.6t) + \sin(2.7t) + \sin(3.5t)]$$

The initial values are chosen as $x(0) = [0.1, 1]^T$, $u(0) = 0$, $\hat{m}(0) = 1$, $\Gamma_\vartheta = 10 I_3$, $k_{s1} = 8$, $\varepsilon = 0.01$.

Assuming a sampling period T=0.001*sec*, the simulation results are shown in figures 2-4. Fig.2 shows the position tracking performance of ARC. Fig.3 shows the corresponding tracking error and Fig.4 shows the output of the ARC controller.

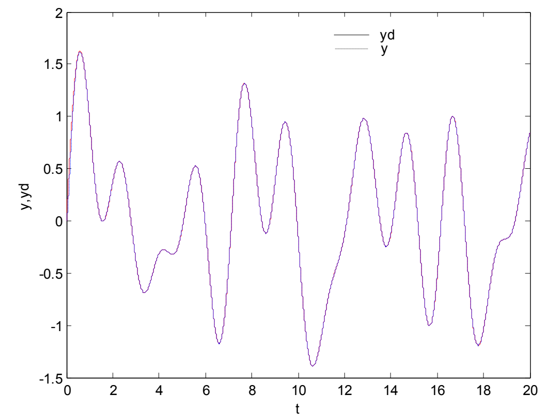

FIG 2. TRACKING PERFORMANCE

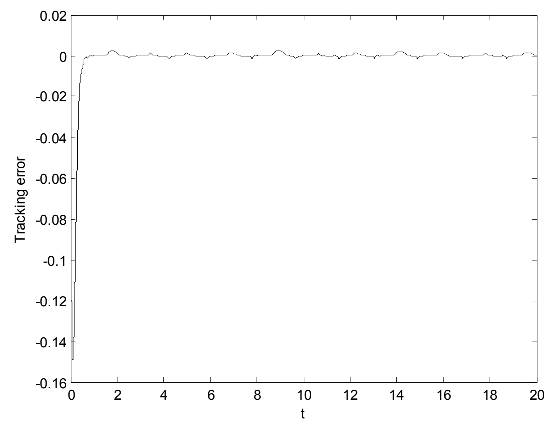

FIG 3. OUTPUT TRACKING ERROR

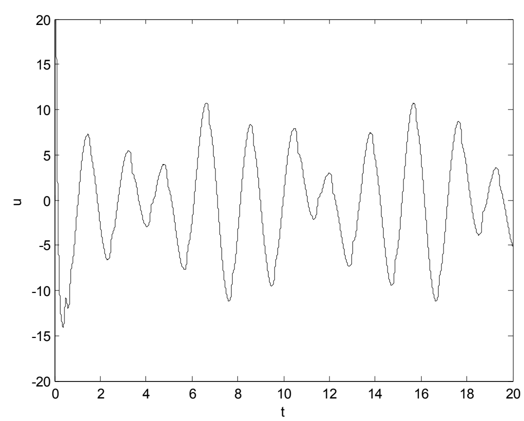

FIG. 4. CONTROL SIGNAL $u(t)$

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CONCLUSIONS

In this paper, an adaptive robust control (ARC) scheme has been developed for a class of uncertain nonlinear system preceded by non-smooth backlash nonlinearity. By using a new linear description of backlash, this adaptive robust control scheme is developed without constructing a backlash inverse. The control law ensures all closed-loop signals are bounded and the tracking precision within any desired precision.

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REFERENCES

[1] Mattias Nordin, Per-Olof Gutman, "Controlling mechanical systems with backlash—a survey," *Automatica*, vol.38, no.10, pp. 1633-1649, 2002.

[2] Tao, G., Kokotovic, P, "Adaptive control of systems with backlash." *Automatica*, vol.29, no.2, pp. 323–335, 1993.

[3] Tao, G., Kokotovic, P, "Continuous-time adaptive control of system with unknown backlash." *IEEE Trans. Automatic Control*. vol.40, no.6, pp.1083–1087,1995..

[4] Jing Zhou, Chengjin Zhang, and Changyun Wen, "Robust adaptive output control of uncertain nonlinear plants with unknown backlash Nonlinearity," *IEEE Trans. Automatic Control*., vol.52, no.3, pp.503-509, March 2007.

[5] Rastko R. Selmic, and Frank L. Lewis. "Backlash Compensation in Nonlinear Systems Using Dynamic Inversion by Neural Networks." *In Proc. of the IEEE International Conference on Control Applications*, Hawai'i, pp. 22-27. 1999.

[6] J. O. Jang, M. K. Son, and H. T. Chung, "Friction and output backlash compensation of systems using neural network and fuzzy logic," *in Proc. Amer. Control Conference*., Boston, pp.1758–1763, 2004.

[7] Chun-Yi Su, Masahiro Oya, and Henry Hong, "Stable adaptive fuzzy control of nonlinear systems preceded by unknown backlash-like hysteresis," *IEEE Trans. Fuzzy Systems*., vol.11, no.1, pp.1-8, February 2003

[8] Dean, S. R. H., Surgenor, B. W., and Iordanou, H. N. "Experimental evaluation of a backlash inverter as applied to a servomotor with gear train." *in Proc. 4th IEEE conference control applications*, pp. 580–585. 1995,

[9] Chun-Yi Su, Yury Stepanenko, jaroslav Svoboda, and T.P.Leung, "Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis," *IEEE Trans. Automatic Control.*, vol.45, no.12, pp.2427-2432, December 2000

[10] Jing Zhou, Changyun Wen, and Ying Zhang. "Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis". *IEEE Trans. Automatic Control.*, vol.49, no.10, pp.1751 -1757, October 2004

[11] B. Yao and M. Tomizuka, "Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form," *Automatica*, vol. 33, no. 5, pp. 893–900, 1997.

[12] B. Yao and M. Tomizuka, "Smooth robust adaptive sliding mode control of manipulators with guaranteed transient performance," *Trans. of ASME, Journal of Dynamic Systems, Measurement and Control*, vol. 118, no. 4, pp. 764–775, 1996.

[13] B. Yao, "High performance adaptive robust control of nonlinear systems: a general framework and new schemes," *in Proc. of IEEE Conference on Decision and Control*, pp. 2489–2494, 1997.

[14] Goodwin, G. C. , D. Q. Mayne, "A parameter estimation perspective of continuous time model reference adaptive control". *Automatica*, vol. 23, no.1, pp.57-70, 1989.

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