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AN ADAPTIVE ROBUST SCHEME FOR MULTIPLE ACTUATOR FAULT ACCOMMODATION

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ABSTRACT

In this paper, we solve the problem of output tracking for linear systems in presence of unknown actuator failures using discontinuous projection based output feedback adaptive robust control (ARC) scheme. The faulty actuators are characterized as unknown inputs stuck within certain bounds at unknown instants of time. This problem is of prime importance for safety critical missions like flight control system. Many existing techniques to solve this problem use model reference adaptive control (MRAC), which is not well suited for handling various disturbances and modeling errors inherent to any realistic system model. In comparison, the backstepping based output feedback ARC approach used here can effectively deal with such uncertainties. Simulation studies are carried out on a linearized Boeing 747 model, which shows the effectiveness of the proposed scheme. Furthermore, we compare our simulation results with that of MRAC in presence of disturbances, which clearly illustrates the superior performance of the proposed ARC based actuator fault compensation scheme.

INTRODUCTION

For many safety critical missions like flight control systems, it is desirable to have a certain degree of fault tolerance with respect to various faults. In this work, we focus on the problem of fault accommodation for unknown actuator failures in a linear system with unknown parameters and subjected to bounded disturbances. The faults are modeled as actuators getting stuck at unknown instants of time, within certain bounds and are allowed to vary within the bound. Furthermore, we do not assume the knowledge of failed actuators, as fault isolation for systems with redundant actuators is a fairly difficult problem, if not impossible. Fortunately, adaptive schemes, by virtue of its on-line learning capability can bypass this problem. Consequently, many adaptive schemes have been developed to solve this problem.

A novel approach for solving the problem of unknown actuator failure compensation was posed and solved in [1] for linear systems. They further extended their technique to nonlinear systems in [2]. These approaches are inherently limited as they rely on conventional MRAC, which suffers from poor transients during the learning phase and offers difficulty in checking stability and robustness bounds in presence of exogenous disturbances. Robust schemes for actuator fault accommodation, which can handle such disturbances and unstructured uncertainties with guaranteed transient performance, include LMI based techniques [3] and sliding mode control based approaches [4]. But, in presence of large parametric uncertainties, the robust control laws can result in very high gain controllers and may not guarantee good final tracking accuracy. One approach which potentially alleviates these problems is multiple model adaptive control (MMAC), switching and tuning [5, 6]. It is worth noting that in multiple model based approaches, the problem of covering the state-space with a finite set of nominal models is not a trivial one, especially in presence of unstructured uncertainties. Furthermore, as pointed out in [7], MMAC based techniques are not intrinsically stable and a safe switching rule needs to be designed.

Given the need for stability in safety critical missions, the large parametric uncertainties introduced due to unknown actuator failures and the inherent limitations of conventional adaptive control, the idea of *safe adaptive control* is coming to forefront, which ensures certain stability properties even without adaptation [7, 8]. In this respect, we would like to point out that ARC based schemes have already resolved this issue [9, 10] and may be classified as the so-called safe adaptive control. Switching the adaptation off at any instant converts the adaptive robust controller into a deterministic robust controller with guaranteed transient performance. Moreover, the design procedure allows us to calculate explicit upper bound for tracking errors over the entire time history in terms of certain controller parameters and achieve prespecified final tracking accuracy. Thus, ARC based schemes are natural choices for safety sensitive systems over conventional adaptive and robust schemes.

In the present work, we develop an output feedback ARC based scheme for accommodation of unknown actuator faults. The technique used here [11] is a combination of adaptive backstepping [12] and discontinuous projection based ARC proposed in [10].

The paper is organized as follows. In the next section, we describe the problem we are trying to solve and certain assumptions that are needed to solve the problem. In the third section, we describe the output feedback based ARC approach to unknown actuator fault accommodation. In the fourth section, we present comparative simulation results to demonstrate the superior performance achievable using the proposed scheme and finally, we conclude the paper by summarizing the main contributions.

PROBLEM STATEMENT

In the present work, we consider systems which can be represented in the input-output form as follows,

$$y(t) = \sum_{j=1}^{k} \frac{B_j(s)}{A(s)} u_j(t) + \frac{D(s)}{A(s)} \Delta(y,t) + d_y(t) \tag{1}$$

where, $A(s) = s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0$, $B_j(s) = b_{jm}s^m + \ldots + b_{j1}s + b_{j0}$ and $D(s) = d_l s^l + \ldots + d_1 s + d_0$, and $m \leq l \leq n$. The plant parameters a_i and b_i are unknown constants. The coefficients d_i corresponding to the disturbance distribution are assumed to be known but, the results can be readily extended to the case where they are unknown constants. $d_y(t)$ represents the output disturbance, and $\Delta(y,t)$ represents any disturbance coming from the intermediate channels of the plant. An implicit assumption in the system representation (1) is,

A1: The relative degree $\rho = n - m$ is known and same for *any* input u_j.

In this work, we will consider actuator failures which can be modeled as [1],

$$\begin{aligned} &u_j(t) = \bar{u}_j(t), \quad t \geq t_j, \quad j \in \{1,2,\ldots,m\} \text{ and} \\ &\bar{u}_{j,min} \leq \bar{u}_j(t) \leq \bar{u}_{j,max} \end{aligned} \tag{2}$$

where, $\bar{u}_{j,min}$ and $\bar{u}_{j,max}$ are known bounds and t_j is the unknown instant of failure for each j. We will also describe in a remark how to deal with other fault scenarios where this bound in unknown. Without actuator redundancy, actuator faults cannot be accommodated and this is formally stated in the following assumption,

A2: System (1) can fulfill the desired control objective with up to $m-1$ failed actuators, when implemented with unknown parameters.

Thus, in presence of actuator failures, the input vector can be represented as,

$$u(t) = u^*(t) + \sigma(\bar{u}(t) - u^*(t)) \tag{3}$$

where $u^*(t)$ is the control input to be designed and,

$$\bar{u} = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_k]^T, \quad \sigma = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_k\} \tag{4}$$

$$\sigma_i = \begin{cases} 1 & \text{if the } i\text{th actuator fails} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Now, the problem we attempt to solve in this work can be stated precisely as follows. For the system described by (1), subjected to unknown actuator failures (3-5) and bounded disturbances, the goal is to design an output feedback control law such that the output tracking error converges exponentially to a prespecified bound and has a guaranteed transient performance.

In addition to actuator fault compensation, it is also desirable that the closed-loop system posses good disturbance rejection properties. In the present approach, such properties are achieved by explicitly taking into account $\Delta(y,t)$: we use prior information about the nature of disturbance to construct a nominal disturbance model $\Delta_n(y,t) = q(y,t)^T c$, where $q(y,t) = [q_p(y,t), \ldots, q_1(y,t)]^T \in \mathbb{R}^p$ represents the vector of known basis shape functions and $c = [c_p, \ldots, c_1]^T$ represents the vector of unknown magnitudes. Thus, the disturbance can be represented as, $\Delta = \Delta_n + \tilde{\Delta}$, where $\tilde{\Delta}$ is the modeling error. Adaptation will be used to compensate for the effect of Δ_n on the output tracking performance and $\tilde{\Delta}$ will be dealt with via certain robust feedback for robust performance.

OUTPUT FEEDBACK BASED ARC

Observer Canonical Form

In the present work, we will assume that control signals to all the actuators are same [1], i.e., $u_1^* = \ldots = u_k^* = u^*$. With this choice of control input, the system with $p \in \{1, \ldots, m-1\}$ failed actuators can be represented as,

$$\begin{aligned} y(t) &= \sum_{j \neq J_p} \frac{B_j(s)}{A(s)} u_j^*(t) + \sum_{j \in J_p} \frac{B_j(s)}{A(s)} \bar{u}_j(t) \\ &\quad + \frac{D(s)}{A(s)} \Delta(y,t) + d_y(t) \qquad (6) \\ &= \frac{b_m^p s^m + \ldots + b_1^p s + b_0^p}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} u^*(t) \\ &\quad + \sum_{j \in J_p} \frac{b_{jm}s^m + \ldots + b_{j1}s + b_{j0}}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} \bar{u}_j(t) \\ &\quad + \frac{d_l s^l + \ldots + d_1 s + d_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} \Delta(y,t) + d_y(t) \qquad (7) \end{aligned}$$

where, $J_p = \{j_1, \ldots, j_p\}$ is a set of subscripts such that $[\bar{u}_{j_1}, \ldots, \bar{u}_{j_p}]$ represents the set of unknown failed actuators.

Also, $b_i^p = \sum_{j \neq J_p} b_{ji}$ and the superscript p represents that this value corresponds to the J_p failure pattern. The following assumption will be made which is standard in adaptive control,

A3: The polynomial $\sum_{j \neq J_p} B_j(s)$ is Hurwitz and the sign of the high frequency gain (sign(b_m^p)) is known, irrespective of the failure patter J_p.

Now we present an observer canonical realization of the above input-output model which is more suitable for the controller design technique presented here,

$$
\begin{aligned}
\dot{x}_1 &= x_2 - a_{n-1}x_1 \\
&\vdots \\
\dot{x}_{n-l-1} &= x_{n-l} - a_{l+1}x_1 \\
\dot{x}_{n-l} &= x_{n-l+1} - a_l x_1 + d_l q^T(y,t)c + d_l \tilde{\Delta} \\
&\vdots \\
\dot{x}_{\rho-1} &= x_\rho - a_{m+1}x_1 + d_{m+1}q^T(y,t)c + d_{m+1}\tilde{\Delta} \\
\dot{x}_\rho &= x_{\rho+1} - a_m x_1 + b_m^p u^*(t) + \sum_{j \in J_p} b_{jm}\bar{u}_j(t) \\
&\quad + d_m q^T(y,t)c + d_m \tilde{\Delta} \\
&\vdots \\
\dot{x}_n &= -a_0 x_1 + b_0^p u^*(t) + \sum_{j \in J_p} b_{j0}\bar{u}_j(t) + d_0 q^T(y,t)c + d_0\tilde{\Delta} \\
y &= x_1 + d_y(t)
\end{aligned}
\tag{8}
$$

In addition to the assumptions made previously, we will make the following realistic assumptions regrading the unknown parameters,

A4: The extent of parametric uncertainties, modeling error $\tilde{\Delta}(t)$, output disturbance $d_y(t)$ as well as derivative $\dot{d}_y(t)$ remain in a known bounded region,

$$
\begin{aligned}
\theta &\in \Omega_\theta \triangleq \{\theta : \theta_{min} < \theta < \theta_{max}\} \\
\tilde{\Delta} &\in \Omega_\Delta \triangleq \{\tilde{\Delta} : |\tilde{\Delta}(y,t)| \leq \delta(t)\} \\
d_y &\in \Omega_d \triangleq \{d_y : |d_y(t)| \leq \delta_d(t)\} \\
\dot{d}_y &\in \Omega_f \triangleq \{\dot{d}_y : |\dot{d}_y(t)| \leq \delta_f(t)\}
\end{aligned}
\tag{9}
$$

where $\theta = [-a_{n-1}, \ldots, -a_0, b_m^p, \ldots, b_0^p, c_p, \ldots, c_1]^T \in \mathbb{R}^{m+n+p+1}$

State Estimation

In this section, we will describe the design of K-filters [12] for state estimation. The state-space equations 8 can be rewritten as,

$$
\dot{x} = A_0 x + (\bar{k} - a)x_1 + dq^T(y,t)c + bu^* + B_f\bar{u} + d\tilde{\Delta} \tag{10}
$$

where,

$$
A_0 = \begin{bmatrix} -k_1 & \\ \vdots & I_{n-1} \\ -k_n & 0 \ldots 0 \end{bmatrix} \quad \bar{k} = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix} \quad a = \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \end{bmatrix} \quad b = \begin{bmatrix} 0_{(\rho-1)\times 1} \\ \bar{b} \end{bmatrix}
$$

$$
\bar{b} = \begin{bmatrix} b_m^p \\ \vdots \\ b_0^p \end{bmatrix} \quad d = \begin{bmatrix} 0_{(n-l-1)\times 1} \\ \bar{d} \end{bmatrix} \quad \bar{d} = \begin{bmatrix} d_l \\ \vdots \\ d_0 \end{bmatrix} \quad B_f = \begin{bmatrix} 0_{(\rho-1)\times p} \\ \bar{B}_f \end{bmatrix}
$$

$$
\bar{B}_f = \begin{bmatrix} b_{j_1 m} & \ldots & b_{j_p m} \\ \vdots & & \vdots \\ b_{j_1 0} & \ldots & b_{j_p 0} \end{bmatrix} \tag{11}
$$

The observer matrix A_0 can be made stable by a suitable choice of $\bar{k}$. Thus, there exists a symmetric positive definite matrix P such that,

$$
PA_0 + A_0^T P = -I, \;\; P = P^T > 0 \tag{12}
$$

For the purpose of state-estimation, the following set of K-filters is defined,

$$
\begin{aligned}
\dot{\xi}_n &= A_0\xi_n + \bar{k}y \\
\dot{\xi}_i &= A_0\xi_i + e_{n-i}y \quad 0 \leq i \leq n-1 \\
\dot{\upsilon}_i &= A_0\upsilon + e_{n-i}u \quad 0 \leq i \leq m \\
\dot{\psi}_i &= A_0\psi + dq_i(y,t) \quad 1 \leq i \leq p
\end{aligned}
\tag{13}
$$

where e_i denotes the ith standard basis vector in $\mathbb{R}^n$.

Remark 1: The ψ_i filter states are introduced for estimating the unknown parameters c_i in the disturbance function $\Delta(y,t)$. This results in improved disturbance rejection properties of the controller and better state-estimates.

Note that, due to the special structure of A_0, the order of the K-filters described above can be reduced by using the following two filters and certain algebraic expressions,

$$
\begin{aligned}
\dot{\eta} &= A_0\eta + e_n y \\
\dot{\lambda} &= A_0\lambda + e_n u
\end{aligned}
\tag{14}
$$

Now, the ξ_i and υ_i filter states can be obtained using the following expression,

$$
\begin{aligned}
\xi_n &= -A_0^n\eta \\
\xi_i &= A_0^i\eta \quad 0 \leq i \leq n-1 \\
\upsilon_i &= A_0^i\lambda \quad 0 \leq i \leq m
\end{aligned}
\tag{15}
$$

Using the above filters, the state estimates are given by,

$$
\hat{x} = \xi_n - \sum_{i=0}^{n-1} a_i\xi_i + \sum_{i=0}^{m} b_i^p\upsilon_i + \sum_{i=1}^{p} c_i\psi_i \tag{16}
$$

Let $\varepsilon_x = x - \hat{x}$ be the estimation error. Then, using (10), (16) and the filters described above, the estimation error dynamics is given by,

$$
\dot{\varepsilon}_x = A_0\varepsilon_x + (a - \bar{k})d_y + B_f\bar{u} + d\tilde{\Delta} \tag{17}
$$

The estimation error can be written as,

$$
\varepsilon_x = \varepsilon + \varepsilon_u \tag{18}
$$

where ε is the zero input response satisfying $\dot{\varepsilon} = A_0\varepsilon$ and

$$\varepsilon_u = \int_0^t e^{A_0(t-\tau)}[(a-\bar{k})d_y(\tau) + B_f\bar{u}(\tau) + d\tilde{\Delta}(y,\tau)]d\tau \qquad (19)$$

is the zero state response. Now, from assumption 2 and the fact A_0 is stable, it is easy to see that,

$$\varepsilon_u \in \Omega_\varepsilon \triangleq \{\varepsilon_u : |\varepsilon_u| \leq \delta_\varepsilon(t)\} \qquad (20)$$

where $\delta_\varepsilon(t)$ is a vector of unknown but bounded functions. In the present approach, ε and ε_u will be treated as disturbances and robust control functions will be used to achieve guaranteed robust performance.

Parameter Projection

Let $\hat{\theta}$ denote the estimate of θ and $\tilde{\theta} = \hat{\theta} - \theta$ denote the estimation error. It is well known fact that gradient based parameter estimation algorithms suffer from *parameter drift* in presence of disturbances, and can result in system states growing unboundedly. We use discontinuous parameter projection to deal with this problem. The update law and the projection mapping used here have the following form,

$$\dot{\hat{\theta}} = \text{Proj}_{\hat{\theta}}(\Gamma\tau) \qquad (21)$$

$$\text{Proj}_{\hat{\theta}_i}(\bullet_i) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{i,max} \text{ and } \bullet_i > 0 \\ 0 & \text{if } \hat{\theta}_i = \theta_{i,min} \text{ and } \bullet_i < 0 \\ \bullet_i & \text{otherwise} \end{cases} \qquad (22)$$

where $\Gamma > 0$ is a diagonal matrix, and τ is any adaptation function. The projection mapping guarantees that the following two properties are always satisfied,

$$\textbf{P1} \quad \hat{\theta} \in \Omega_\theta = \{\hat{\theta} : \theta_{min} \leq \hat{\theta} \leq \theta_{max}\} \qquad (23)$$

$$\textbf{P2} \quad \tilde{\theta}^T(\Gamma^{-1}\text{Proj}_{\hat{\theta}}(\Gamma\tau) - \tau) \leq 0, \quad \forall\tau \qquad (24)$$

Controller Design

Now, we can apply the controller design technique developed in [11]. The design combines the adaptive backstepping [12] and discontinuous projection based ARC proposed in [10]. The main idea is to synthesize a virtual control law which will drive the error to a small residual ball. But, as in this case only a single state is available for measurement, the synthesized virtual control law will replace the reconstructed state at each step, and the state estimation error will be dealt with via robust feedback. Also, it should be noted that the use of discontinuous projection implies a tuning function based backstepping cannot be used, and hence a stronger robust control law is needed to negate the effects of parameter estimation transients. For advantages of discontinuous projection based technique over smooth modifications of adaptive law like smooth projection, and other details regarding the controller design presented here, the reader is referred to [10].

Step 1: The derivative of the output tracking error $z_1 = y - y_r$ is given by,

$$\dot{z}_1 = x_2 - a_{n-1}y + a_{n-1}d_y + \dot{d}_y - \dot{y}_r \qquad (25)$$

But, x_2 is not measured and is replaced by its estimate,

$$x_2 = \xi_{n,2} - \xi_{(2)}a + \upsilon_{(2)}\bar{b} + \psi_{(2)}c + \varepsilon_{x2} \qquad (26)$$

where $\varepsilon_{x2} = \varepsilon_2 + \varepsilon_{u2}$ is the estimation error of x_2, and

$$\xi_{(2)} = [\xi_{n-1,2}, \ldots, \xi_{0,2}] \quad \upsilon_{(2)} = [\upsilon_{m,2}, \ldots, \upsilon_{0,2}]$$
$$\psi_{(2)} = [\psi_{p,2}, \ldots, \psi_{1,2}] \qquad (27)$$

in which $\bullet_{i,j}$ represents the jth element of $\bullet_i$. Substituting (27) back in (25), we obtain,

$$\dot{z}_1 = b_m^p\upsilon_{m,2} + \xi_{n,2} + \theta^T\bar{\omega} - \dot{y}_r + \bar{\Delta}_1 \qquad (28)$$

where $\omega^T = [\xi_{(2)}, \upsilon_{(2)}, \psi_{(2)}] + e_1^{*T}y$, $\bar{\omega} = \omega - e_{n+1}^*\upsilon_{m,2}$, $\bar{\Delta}_1 = a_{n-1}d_y + \dot{d}_y + \varepsilon_2 + \varepsilon_{u2}$ and e_1^{*T} is the ith standard basis vector in $\mathbb{R}^{n+m+p+1}$. (28) suggests a natural choice for the virtual input is $\upsilon_{m,2}$, which will be used for synthesizing the virtual control law α_1,

$$\alpha_1(y, \eta, \bar{\lambda}_{m+1}, \psi, \hat{\theta}, t) = \alpha_{1a} + \alpha_{1s},$$
$$\alpha_{1a} = -\frac{1}{\hat{b}_m^p}\{\xi_{n,2} + \hat{\theta}^T\bar{\omega} - \dot{y}_r\} \qquad (29)$$

where $\bar{\lambda}_i = [\lambda_1, \ldots, \lambda_i]^T$ obtained from (14). In (29), α_{1a} is the model compensation component of the control law used to achieve an improved model compensation through on-line parameter adaptation given by (21-22). Since, we assume the sign of b_m^p is known, without loss of generality, one can assume $b_m^p > 0$ and it is lower bounded by a non-zero positive constant i.e., $(b_m^p)_{min} = (\theta_{n+1})_{min} > 0$ where $(\theta_{n+1})_{min}$ is independent of the failure pattern. Note that $(\theta_{n+1})_{min}$ is known from assumption A4. Then, the projection mapping (23) guarantees that $\hat{b}_m^p \geq (b_m^p)_{min} > 0$, which implies that the control law (29) is well defined. Let $z_2 = \upsilon_{m,2} - \alpha_1$ denote the input discrepancy. Substituting (29) into (28), we get

$$\dot{z}_1 = b_m^p(z_2 + \alpha_1) - \tilde{\theta}^T\phi_1 + \bar{\Delta}_1 \qquad (30)$$

where $\phi_1 \triangleq \bar{\omega} + e_{n+1}^*\alpha_{1a}$.

Now we present the design of the robust component of the control law α_{1s}, which suppresses the potential destabilizing effect of parameter estimation transients, state-estimation error and as well as other bounded modeling errors.

$$\alpha_{1s} = \alpha_{1s1} + \alpha_{1s2} + \alpha_{1s3}, \quad \alpha_{1s1} = -\frac{1}{(b_m^p)_{min}}k_{1s}z_1 \qquad (31)$$

where k_{1s} is a nonlinear gain, such that,

$$k_{1s} \geq g_1 + \| C_{\phi_1}\Gamma\phi_1 \|^2, \quad g_1 \geq 0 \qquad (32)$$

in which C_{ϕ_1} is a positive definite constant diagonal matrix to be specified later. Substituting (31) into (28) gives

$$\dot{z}_1 = b_m^p z_2 - \frac{b_m^p}{(b_m^p)_{min}}k_{1s}z_1 + b_m^p(\alpha_{1s1} + \alpha_{1s2}) - \tilde{\theta}^T\phi_1 + \bar{\Delta}_1 \qquad (33)$$

Next, we design α_{1s2} and α_{1s3} as follows. Consider the positive semi-definite (p.s.d) function $V_1 = \frac{1}{2}z_1^2$. Its time derivative along the trajectory of the system (33) satisfies,

$$\dot{V}_1 \leq b_m^p z_1 z_2 - k_{1s}z_1^2 + z_1(b_m^p\alpha_{1s2} - \tilde{\theta}^T\phi_1) + z_1(b_m^p\alpha_{1s3} + \bar{\Delta}_1) \qquad (34)$$

From assumption **A4**, we have

$$\| \tilde{\theta}^T \phi_1 \| \leq \| \theta_M \| \| \phi_1 \| \tag{35}$$

where $\theta_M = \theta_{max} - \theta_{min}$. Thus, $\| \tilde{\theta}^T \phi_1 \|$ is bounded by a known function, which ensures that there exists a robust control function satisfying the following conditions [13]:

$$\begin{aligned} &(a) \quad z_1\{b_m^p \alpha_{1s2} - \tilde{\theta}^T \phi_1\} \leq \varepsilon_{11} \\ &(b) \quad z_1 \alpha_{1s2} \leq 0 \end{aligned} \tag{36}$$

where ε_{11} is a positive design parameter.

Remark 2: Essentially, condition (a) of (36) shows that α_{1s2} is synthesized to attenuate the effect of parametric uncertainties $\tilde{\theta}$ with the level of control accuracy being measured by ε_{11}. Condition (b) is to make sure that α_{1s2} is dissipative in nature so that it does not interfere with the functionality of adaptive control law α_{1a}. One smooth example of α_{1s2} satisfying (36) can be found in the following way. Let h_1 be any function satisfying

$$h_1 \geq \| \theta_M \|^2 \| \phi_1 \|^2 \tag{37}$$

Then, α_{1s2} can be chosen as [9, 10],

$$\alpha_{1s2} = -\frac{h_1}{4(b_m^p)_{min}\varepsilon_{11}} z_1 \tag{38}$$

Similarly, from assumption **A4** and (19-20), we can obtain,

$$|\bar{\Delta}_1| \leq \bar{\delta}_1(t) \triangleq |a_{n-1}|\delta_d(t) + \delta_f(t) + \delta_{\varepsilon 2}(t) + \delta_{\bar{u}}(t) + \varepsilon_2 \tag{39}$$

Note that $\bar{\delta}_1$ is an unknown but bounded function, and the same strategy as in (36) can be used to design a robust control law. However, since the bound of $\bar{\Delta}_1$ is not unknown, it is impossible to prespecify the level of control accuracy. So, a more relaxed requirement compared to the condition (a) of (36) is given by,

$$z_1\{b_m^p \alpha_{1s3} + \bar{\Delta}_1\} \leq \varepsilon_{12} \bar{\delta}_1^2 \tag{40}$$

Remark 3: As for α_{1s2}, a choice of smooth α_{1s3} satisfying (40) is given by [14],

$$\alpha_{1s3} = -\frac{1}{4(b_m^p)_{min}\varepsilon_{12}} z_1 \tag{41}$$

Step 2: From (29), (14), (15) and rearrangement of (25-29), the derivative of α_1 can be written as,

$$\begin{aligned} \dot{\alpha}_1 &= \dot{\alpha}_{1c} + \dot{\alpha}_{1u} \\ \dot{\alpha}_{1c} &= \frac{\partial \alpha_1}{\partial y}(\xi_{n,2} + \hat{\theta}^T \omega) + \frac{\partial \alpha_1}{\partial \eta}\dot{\eta} + \sum_{j=1}^{m+1} \frac{\partial \alpha_1}{\partial \lambda_j}\dot{\lambda}_j \\ &\quad + \sum_{j=1}^{m+1} \frac{\partial \alpha_1}{\partial \psi_{j,2}}\dot{\psi}_{j,2} + \frac{\partial \alpha_1}{\partial t} \\ \dot{\alpha}_{1u} &= \frac{\partial \alpha_1}{\partial y}(-\tilde{\theta}^T \omega + \bar{\Delta}_1) + \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} \end{aligned} \tag{42}$$

Using (13) and (14), $\dot{\alpha}_{1c}$ is calculable and can be used in the design of control functions. However, $\dot{\alpha}_{1u}$ is not calculable due to various uncertainties and hence, will be dealt with via robust feedback in this step. From (13) and (42), the derivative of the $z_2 = \upsilon_{m,2} - \alpha_1$ is

$$\dot{z}_2 = \upsilon_{m,3} - k_2 \upsilon_{m,1} - \dot{\alpha}_{1c} - \dot{\alpha}_{1u} \tag{43}$$

Now, consider the augmented p.s.d function $V_2 = V_1 + \frac{1}{2}z_2^2$. From (34) and (43), the derivative of V_2 is given by

$$\dot{V}_2 \leq \dot{V}_1|_{\alpha_1} + z_2\{b_m^p z_1 + \upsilon_{m,3} - k_2\upsilon_{m,1} - \dot{\alpha}_{1c} - \dot{\alpha}_{1u}\} \tag{44}$$

where $\dot{V}_1|_{\alpha_1} = -k_{1s}z_1^2 + z_1(b_m^p \alpha_{1s2} - \tilde{\theta}_1\phi_1) + z_1(b_m^p \alpha_{1s3} + \bar{\Delta}_1)$. As in (29), the ARC control function α_2 for the virtual control input $\upsilon_{m,3}$ in(43) consists of

$$\begin{aligned} &\alpha_2(y, \eta, \bar{\lambda}_{m+2}, \psi, \hat{\theta}, t) = \alpha_{2a} + \alpha_{2s} \\ &\alpha_{2a} = -\hat{b}_m^p z_1 + k_2 \upsilon_{m,1} + \dot{\alpha}_{1c} \\ &\alpha_{2s} = \alpha_{2s1} + \alpha_{2s2} + \alpha_{2s3} \quad \alpha_{2s1} = -k_{2s} z_2 \\ &k_{2s} \geq g_2 + \| \frac{\partial \alpha_1}{\partial \hat{\theta}} C_{\theta 2} \| + \| C_{\phi 2} \Gamma \phi_2 \|^2 \end{aligned} \tag{45}$$

where $g_2 > 0$ is a constant and $C_{\theta 2}$ and $C_{\phi 2}$ are positive definite constant diagonal matrices, α_{2s2} and α_{2s3} are robust control functions to be chosen later. Substituting (45) and (42) in (44), and using similar techniques as in (30), we have

$$\begin{aligned} \dot{V}_2 &\leq \dot{V}_1|_{\alpha_1} + z_2 z_3 - k_{2s} z_2^2 + z_2(\alpha_{2s2} - \tilde{\theta}^T \phi_2) \\ &\quad + z_2(\alpha_{2s3} + \bar{\Delta}_2) - z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} \end{aligned} \tag{46}$$

where $z_3 = \upsilon_{m,3} - \alpha_2$ represents the input discrepancy and

$$\phi_2 = e_{n+1}^* z_1 - \frac{\partial \alpha_1}{\partial y}\omega \ , \ \bar{\Delta}_2 = -\frac{\partial \alpha_1}{\partial y}\bar{\Delta}_1 \tag{47}$$

From (39), it follows that $\bar{\Delta}_2 \leq |\partial \alpha_1 / \partial y| \bar{\delta}_1$. Similar to (36) and (40), the robust control functions α_{2s2} and α_{2s3} are chosen to satisfy

$$\begin{aligned} &(a) \ z_2(\alpha_{2s2} - \tilde{\theta}^T \phi_2) \leq \varepsilon_{21} \\ &(b) \ z_2(\alpha_{2s3} + \bar{\Delta}_2) \leq \varepsilon_{22} \bar{\delta}_1^2 \\ &(c) \ z_2 \alpha_{2s2} \leq 0 \ , \ z_2 \alpha_{2s3} \leq 0 \end{aligned} \tag{48}$$

where ε_{21} and ε_{22} are positive design parameters. As in step 1, α_{2s2} and α_{2s3} can be chosen as,

$$\alpha_{2s2} = -\frac{h_2}{4\varepsilon_{21}} z_2 \ , \ \alpha_{2s3} = -\frac{1}{4\varepsilon_{21}} \left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 \tag{49}$$

where h_2 is any smooth function satisfying $h_2 \geq \| \theta_M \|^2 \| \phi_2 \|^2$. From (34) and h_2 defined above, the derivative of V_2 satisfies

$$\begin{aligned} \dot{V}_2 &\leq z_2 z_3 - \sum_{j=1}^{2} k_{js} z_j^2 + z_1(b_m^p \alpha_{1s2} - \tilde{\theta}_1 \phi_1) + z_1(b_m^p \alpha_{1s3} + \bar{\Delta}_1) \\ &\quad + z_2(\alpha_{2s2} - \tilde{\theta}^T \phi_2) + z_2(\alpha_{2s3} + \bar{\Delta}_2) - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} z_2 \end{aligned} \tag{50}$$

Step i ($3 \leq i < \rho$): Mathematical induction will be used to prove the general result for all the intermediate steps. At each step i, the ARC control function α_i will be constructed for virtual control input $\upsilon_{m,i+1}$. For any $j \in [3, i-1]$, let $z_j = \upsilon_{m,j} - \alpha_{j-1}$ and recursively design

$$\phi_j = -\frac{\partial \alpha_{j-1}}{\partial y}\omega \ , \ \bar{\Delta}_j = -\frac{\partial \alpha_{j-1}}{\partial y}\bar{\Delta}_1 \tag{51}$$

Lemma 1: At step i, choose the desired ARC control function α_i as

$$\begin{aligned}
&\alpha_i(y, \eta, \bar{\lambda}_{m+i}, \psi, \hat{\theta}, t) = \alpha_{ia} + \alpha_{is}\\
&\alpha_{ia} = -z_i + k_i \upsilon_{m,i} + \dot{\alpha}_{(i-1)c}\\
&\alpha_{is} = \alpha_{is1} + \alpha_{is2} + \alpha_{is3} \quad \alpha_{is1} = -k_{is} z_i\\
&k_{is} \geq g_i + \| \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} C_{\theta i} \| + \| C_{\phi i} \Gamma \phi_i \|^2
\end{aligned} \tag{52}$$

where $g_i > 0$ is a constant, and $C_{\theta i}$ and $C_{\phi i}$ are positive definite constant diagonal matrices, α_{is2} and α_{is3} are robust control functions satisfying,

$$\begin{aligned}
&(a) \ z_i(\alpha_{is2} - \tilde{\theta}^T \phi_i) \leq \varepsilon_{i1}\\
&(b) \ z_i(\alpha_{is3} + \bar{\Delta}_i) \leq \varepsilon_{i2} \bar{\delta}_i^2\\
&(c) \ z_i \alpha_{is2} \leq 0 \ , \ z_i \alpha_{is3} \leq 0
\end{aligned} \tag{53}$$

and

$$\begin{aligned}
\dot{\alpha}_{(i-1)c} = \ &\frac{\partial \alpha_{i-1}}{\partial y}(\xi_{n,2} + \hat{\theta}^T \omega) + \frac{\partial \alpha_{i-1}}{\partial \eta}\dot{\eta} + \sum_{j=1}^{m+i+1} \frac{\partial \alpha_{i-1}}{\partial \lambda_j}\dot{\lambda}_j\\
&+ \sum_{j=1}^{p} \frac{\partial \alpha_{i-1}}{\partial \psi_j}\dot{\psi}_j + \frac{\partial \alpha_{i-1}}{\partial t}
\end{aligned} \tag{54}$$

Then, the ith error subsystem is

$$\begin{aligned}
\dot{z}_i = \ &z_{i+1} - z_{i-1} - k_{is} z_i + (\alpha_{is2} - \tilde{\theta}^T \phi_i)\\
&+ (\alpha_{is3 + \bar{\Delta}_i}) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}
\end{aligned} \tag{55}$$

and the derivative of the augmented p.s.d function $V_i = V_{i-1} + 1/2 z_i^2$ satisfies,

$$\begin{aligned}
\dot{V}_i \ \leq \ &z_i z_{i+1} - \sum_{j=1}^{i} k_{js} z_j^2 + z_1(b_m^p \alpha_{1s2} - \tilde{\theta}^T \phi_1) + \sum_{j=2}^{i} z_j(b_m^p \alpha_{js2} - \tilde{\theta}^T \phi_j)\\
&+ z_1(b_m^p \alpha_{1s3} + \bar{\Delta}_1) + \sum_{j=2}^{i} z_j(\alpha_{js3} + \bar{\Delta}_j) - \sum_{j=2}^{i} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} z_j
\end{aligned} \tag{56}$$

The lemma can be easily verified by recursively writing the various expressions and substituting the expressions obtained in step 1 and 2.

Step ρ*:* In this final step, the actual control law u^* will be synthesized such that $\upsilon_{m,\rho}$ tracks the desired ARC control function $\alpha_{\rho-1}$. The derivative of z_ρ can be obtained as

$$\begin{aligned}
\dot{z}_\rho = \ &\upsilon_{m,\rho+1} + u^* - k_\rho \upsilon_{m,1} - \dot{\alpha}_{(\rho-1)c}\\
&- \frac{\partial \alpha_{\rho-1}}{\partial y}(-\tilde{\theta}^T \omega + \bar{\Delta}_1) - \frac{\partial \alpha_{\rho-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}
\end{aligned} \tag{57}$$

If $\upsilon_{m,\rho+1} + u^*$ were the virtual input, (57) would have the same form as the intermediate step i. Therefore, the general form, (51-57) applies to step ρ. Since u is the actual control input, it can be chosen as,

$$u^* = \alpha_\rho - \upsilon_{m,\rho+1} \tag{58}$$

where α_ρ is given by (52). Then, $z_{\rho+1} = u^* + \upsilon_{m,\rho+1} - \alpha_\rho = 0$.

Theorem 1: Let the parameter estimates be updated using adaptation law (21) in which τ is chosen as

$$\tau = \sum_{j=1}^{\rho} \phi_j z_j \tag{59}$$

If diagonal controller gain matrices $C_{\theta j}$ and $C_{\phi k}$ are chosen such that $c_{\phi kr}^2 \geq \frac{\rho}{4}\sum_{j=1}^{\rho} 1/c_{\theta jr}^2$, where $c_{\theta jr}$ and $c_{\phi kr}$ are the rth diagonal element of $C_{\theta j}$ and $C_{\phi k}$ respectively. Then, the control law (58) guarantees that,

1. In general the control input and all internal signals are bounded. Furthermore, V_ρ is bounded above by,

$$V_\rho(t) \leq exp(-\lambda_\rho t) V_\rho(0) + \frac{\bar{\varepsilon}_{\rho 1} + \bar{\varepsilon}_{\rho 2} \| \bar{\delta}_1 \|_{\inf}^2}{\lambda_\rho}[1 - exp(-\lambda_\rho t)] \tag{60}$$

   where $\lambda_\rho = 2\min\{g_1, \ldots, g_\rho\}$, $\bar{\varepsilon}_{\rho 1} = \sum_{j=1}^{\rho} \varepsilon_{j1}$, $\bar{\varepsilon}_{\rho 2} = \sum_{j=1}^{\rho} \varepsilon_{j2}$ and $\| \bar{\delta}_1 \|_{\inf}^2$ stands for the infinity norm of $\bar{\delta}_1$.
2. If after a finite time t_0, $\tilde{\Delta} = 0$ and $d_y = 0$ (i.e., in the presence of parametric uncertainties and modeled disturbance only) then, in addition to results in (1), asymptotic output tracking control is also achieved.

Proof of the theorem has been omitted due to space restrictions, but can be obtained from the authors upon request and is similar to one presented in [10].

Remark 4: In context of actuator fault compensation, (1) guarantees that the jump in parameter values due to failed actuator does not interfere with the desired transient performance. Furthermore, the accuracy can be improved by choosing suitable values of ε_{j1} and ε_{j2}. Also, the effect of failed actuators is taken into account through the $\bar{\delta}_j$ term, which upper bounds $\bar{\Delta}_j$ containing the effect of actuator failure through the ε_u term defined in (19).

Remark 5: Note that the component of ε_u corresponding to the failed actuator i.e., $\int_0^t e^{A_0(t-\tau)} B_f \bar{u}(\tau) d\tau$ is upper-bounded by a known constant due to the assumptions that the zero dynamics is stable and $\bar{u}_j$ belongs to a known bound. But, other types of actuator fault scenarios can be easily addressed using the same technique.

Remark 6: It may appear that we have neglected the $\rho + 1$ to n states in the present analysis. But, due to the assumption of stable zero dynamics and bounded disturbances as well as actuator faults, it can be easily proved using standard adaptive control arguments that all internal signals remain bounded and do not interfere with the tracking performance.

SIMULATIONS

For simulation purposes, we will use the linearized model of Boeing 747, as given in [15]. It should be noted that the same example was used in [1], and thus, provides a platform to compare the MRAC and backstepping based ARC algorithms for actuator fault accommodation. The details of the model and various assumptions can be found in [15].

Plant Model: The linearized model for the lateral motion of Boeing 747 can be represented as,

$$\begin{aligned} \dot{x}(t) &= Ax(t)+Bu(t), \quad y(t)=x_2(t)=y_r(t) \\ x(t) &= [\beta,\ y_r,\ p,\ \phi]^T, \quad B=[b_1,\ b_2,\ b_3] \end{aligned} \tag{61}$$

where, β is the side-slip angle, y_r is the yaw-rate, p is the roll rate, ϕ is the roll angle, y is the output which needs to follow the reference trajectory $r(t)$ and u is the control input vector consisting of three control signals representing three rudder servos $\delta_{r1}, \delta_{r2}, \delta_{r3}$. Note that the B matrix has been augmented by b_2 and b_3 for studying actuator failure compensation properties of the proposed algorithm. From the data provided in [15] for Boeing 747 in horizontal flight at 40,000 ft and nominal forward speed 774 ft/s, the perturbation dynamics matrices are

$$A = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.598 & -0.115 & -0.0318 & 0 \\ -3.05 & 0.388 & -0.465 & 0 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix} \tag{62}$$

$$b_1 = \begin{bmatrix} 0.00729 \\ -0.475 \\ 0.153 \\ 0 \end{bmatrix} \quad b_2 = \begin{bmatrix} 0.01 \\ -0.5 \\ 0.2 \\ 0 \end{bmatrix} \quad b_3 = \begin{bmatrix} 0.005 \\ -0.3 \\ 0.1 \\ 0 \end{bmatrix} \tag{63}$$

It can be easily verified that this plant satisfies the assumptions A1-A4. Also, for the second set of simulations, we added a small disturbance $d_y(t) = 0.001 sin(4t)$ to the output y and another disturbance was added to the intermediate channel with $d = [1, 1.50, 0.5625, 0.0625]^T$ and $\Delta(y,t) = 0.04 sin(2.1t)$.

Simulation results: Simulations are done using $r(t) = 0.02 sin(0.2t)$ as the reference signals for MRAC and ARC based fault compensation techniques. In these simulations, all disturbances are assumed to be zero i.e., $\Delta(y,t) = d_y(t) = 0$. The dashed line indicates results obtained using MRAC, solid line indicates ARC and dash-dot represents the reference signal. Two faults are introduced during the simulation: $u_2(t)$ fails at $t = 50$ seconds such that $u_2(t) = u_2(50)$ for $t \geq 50$ and $u_3(t)$ fails at 100 seconds such that $u_3(t) = -0.06$ for $t \geq 100$.

In the first set of simulations, both the systems perform well initially, and have similar control input profiles. With the failure of first actuator, the jump in the parameter value is relatively less, and it does not significantly affect the tracking performance. But, with the second actuator failure, which causes a bigger jump in the parameter value, the tracking error stays close to zero for ARC based scheme, but deviates significantly for the MRAC based scheme. This can be explained as follows. The design of robust component of the ARC control law has already incorporated such jumps in parameter values, and hence, is better suited to handle the parametric uncertainties introduced due to actuator failures. This is also the reason that we need not prove separately the stability of the overall system when there is a jump from J_p to J_{p+1} failure pattern.

In the next set of simulations, small disturbances are added to the system model. Fig. 2 shows result with the same set of faults as described earlier for $r(t) = 0.02 sin(0.2t)$. These simulations demonstrate the strength of ARC based schemes in attenuating the effect of disturbance, modeling error, as well as, large parametric uncertainties. In fact there is an order of magnitude difference in the tracking error for MRAC and ARC based scheme. But, this comes at the cost of higher control effort. In fact, there is always a trade-off between good tracking performance and control effort. But, in ARC based approach, this trade-off is more transparent as the robust control law and tracking error, both can be adjusted by tuning ε_{j1} and ε_{j2}.

CONCLUSION

In this paper, an adaptive robust output feedback based ARC scheme is presented for unknown actuator fault accommodation. The proposed scheme is applicable to any linear uncertain system. Adaptation and robust feedback are used simultaneously to maintain tracking performance in face of large parametric uncertainties introduced due to failing actuators, exogenous disturbances and other modeling uncertainties.

Comparative simulation studies are done using a linearized model for lateral motion of Boeing 747 and they confirm the superior performance of the proposed fault accommodation scheme, as compared to that of conventional adaptive schemes. In summary, some of the salient features of the fault accommodation scheme presented in this paper are,

1. capability to handle large parametric uncertainties due to unknown actuator failures with guaranteed transient performance
2. better disturbance rejection properties
3. guaranteed robust performance when adaptation is switched off
4. calculable upper bound for tracking error based on controller parameters and ability to achieve prespecified final tracking accuracy

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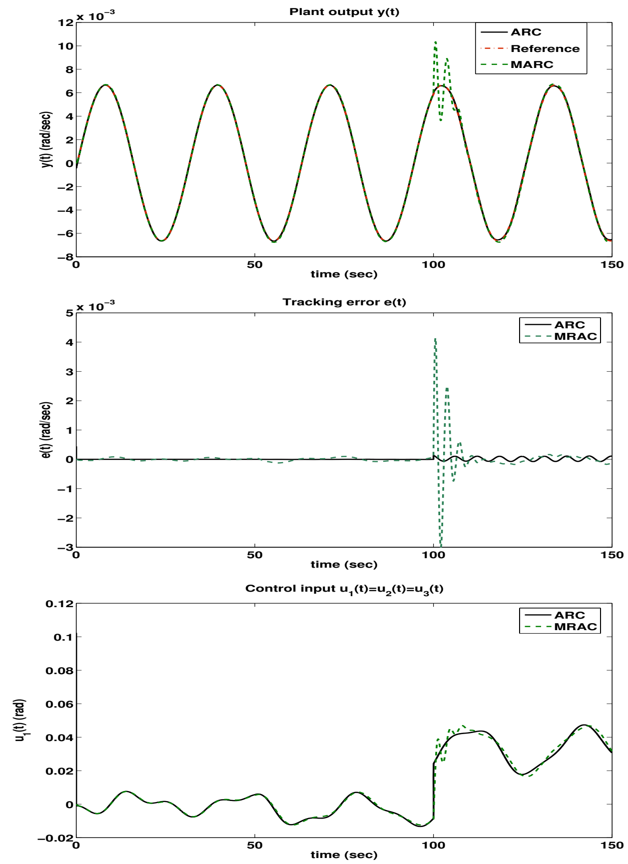

Figure 1. System response, tracking error and control effort without disturbance.

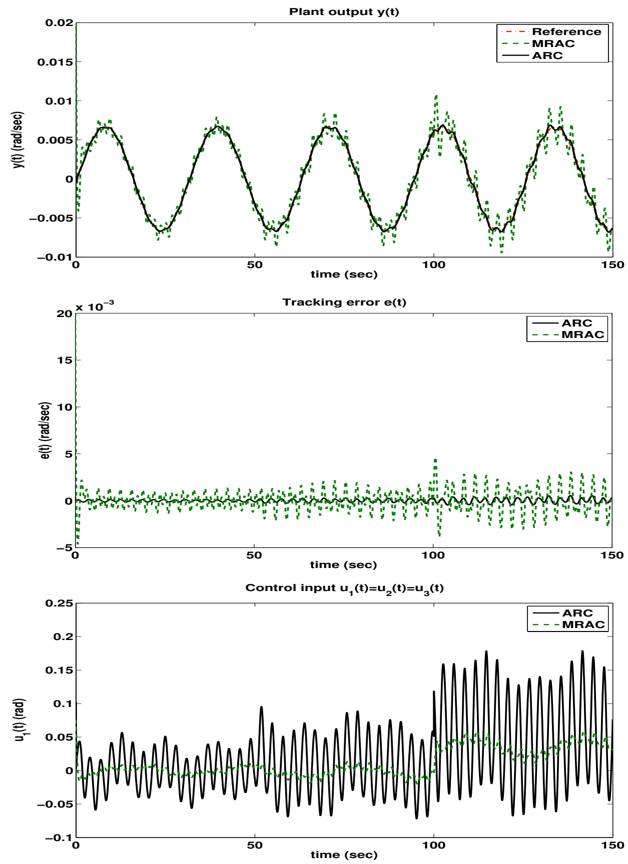

Figure 2. System response, tracking error and control effort with disturbance.

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