Integrated Direct/Indirect Adaptive Robust Precision Control of Linear Motor Drive Systems with Accurate Parameter Estimates *

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Abstract

The focus of the paper is on the synthesis of nonlinear adaptive robust controllers for precision linear motor drive systems that achieve not only excellent output tracking performance but also accurate parameter estimates for secondary purposes such as machine health monitoring and prognostics. Such an objective is accomplished through the intelligent integration of the output tracking performance oriented direct adaptive robust control (DARC) design with the accurate parameter estimation based indirect adaptive robust control (IARC) design. The newly developed IARC design is first applied to the precision control of linear motor drive systems but with an improved estimation model, in which accurate parameter estimates are obtained through a parameter estimation algorithm that is based on physical dynamics rather than the tracking error dynamics. An integrated direct/indirect adaptive robust controller (DIARC) is then presented that preserves the excellent transient tracking performance of the direct ARC designs as well as the better parameter estimation process of the IARC design. The three controllers, DARC, IARC, and DIARC, are also implemented on a precision linear motor system with a measurement resolution of 1 μm to test the achievable performance of different designs and their limitations. Comparative experimental results demonstrate that the proposed DIARC controller is able to achieve better tracking performance and better parameter estimation than either the DARC or the IARC algorithms.

keyword

Linear Motors, Adaptive Control, Robust Control, Uncertainties, Precision Motion Control

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1 Introduction

Direct drive linear motors show promise for widespread use in high-speed/high-accuracy positioning systems through the elimination of mechanical transmissions and the associated transmission problems such as the structural flexibility and backlash [1, 2, 3]. However, it also loses the advantage of using mechanical transmissions – gear reduction reduces the effect of model uncertainties such as parameter variations(e.g., uncertain payloads) and external disturbance (e.g., cutting forces in machining). Furthermore, certain types of linear motors (e.g., iron core linear motors) are subject to significant force ripple [3]. These uncertain nonlinearities are directly transmitted to the load and thus have significant effect on its motion. Thus, in order for a linear motor system to be able to function and to deliver its high performance potential, a controller which can achieve the required high accuracy in spite of various parametric uncertainties and uncertain nonlinear effects, has to be employed.

A great deal of effort has been devoted to solving some of the difficulties in controlling linear motors [1, 2, 4, 5, 3, 6], ranging from linear optimal robust feedback designs [1, 2], disturbance observer [4, 5], to feedforward compensation based on either an off-line experimentally identified model of first-order approximation of ripple forces [3] or on-line learning for repetitive tasks using neural-networked [6]. Recently, in [7], the idea of adaptive robust control (ARC) [8, 9, 10] is generalized to provide a rigorous theoretic framework for the high performance motion control of linear motors. The controller takes into account the effect of model uncertainties coming from the inertia load, friction, force ripple and electrical parameters, etc. In [11], the proposed ARC algorithm [7] is experimentally tested on an *epoxy* core linear motor. To reduce the effect of velocity measurement noise, a desired compensation ARC algorithm in which the regressor is calculated by reference trajectory information is also presented and implemented. In [12], the proposed desired compensation ARC (DCARC) [11] is extended to the control of an *iron* core linear motor, in which effective ripple force compensation using on-line adaptation is developed. The ARC control of linear motors using output position feedback only is studied in [13]. In all those papers [11, 12, 13], excellent position tracking performance has been obtained. For example, in [14], when tracking a typical industrial point-to-point desired motion trajectory, even with a high acceleration of $a_{max} = 12m/s^2$ and a maximum velocity of $v_{max} = 1\ m/s$, the tracking error during the zero velocity portion of the motion is within the encoder resolution of $1\mu m$ (or $10^{-6}m$) and the maximum tracking error during the entire run is within $\pm 20\mu m$.

The underline parameter adaptation law in ARC controllers in [11, 12, 13] are based on the direct adaptive control designs [15] such as the tuning function based adaptive backstepping [16], in which the adaptive control law and parameter adaptation law are synthesized simultaneously to meet the sole objective of reducing the output tracking error. Such a design normally leads to a controller whose dynamic order is as low as the number of unknown parameters to be adapted while achieving excellent output tracking performance as done in [11, 12, 13]. However, the direct approach also has the drawback that the design of adaptive control law and the parameter estimation law cannot be separated and the choice of the parameter estimation law is limited to the gradient type with certain actual tracking errors as driving signals. It is well known that the gradient type of estimation law may not have as good parameter convergence properties as other types of parameter estimation laws (e.g., the least square method). Furthermore, although the desired trajectory might be persistently exciting and of large signal, for a well designed direct adaptive control law, the actual tracking errors in implementation are normally very small, and thus are more prone to be corrupted by other factors such as the sampling delay and noise that have been neglected when synthesizing the parameter adaptation law. As a result, in implementation, the parameter estimates in the direct adaptive control are normally not accurate enough to be used for secondary purposes such as prognostics and machine component health monitoring, even when the desired trajectory is persistently exciting enough as seen in the experimental results in [11, 12, 13]. To overcome the poor parameter estimates of the direct ARC designs [9, 17, 10], an indirect adaptive robust control design has recently been developed [18], in which the construction of parameter estimation law is totally separated from the design of underline robust control law. As a result, various estimation algorithms having better parameter convergence properties (e.g., the least squares type method) can be used. Furthermore, on-line explicit monitoring of persistent excitation level can be employed in implementation to improve the accuracy of parameter estimates significantly. Because of these algorithm improvement, the resulting parameter estimates are normally accurate enough to be used for secondary purposes such as machine health monitoring and progonostics, which are of significant practical importance for industrial applications.

In this paper, the recently proposed IARC [18] will be first applied to the precision control of the epoxy core linear motor but with an improved estimation model than in [18]. The resulting IARC controller is then experimentally compared with the direct ARC design in [11]. As will be seen from the comparative experimental results shown later, the proposed IARC design has a much better accuracy of parameter estimates than the direct ARC [11], as predicted by the theory in [18]. However, the position tracking performance of the proposed IARC is worse than those in the direct ARC design [11]. To overcome the poor tracking performance problem of IARC, in this paper, an integrated direct/indirect ARC (DIARC) design will be developed. The resulting DIARC controller preserves the excellent tracking performance of direct ARC design [11] while having the same parameter estimation process as in the IARC design [18]. Thorough comparative experiment results will be shown to validate the above theoretical claims.

The paper is organized as follows. The problem formulation is given in Section 2. For illustration and comparison purposes, the direct ARC in [11] is briefly reviewed in Section 3 with the application of the IARC design in [18] given in Section 4. The proposed integrated DIARC is detailed in Section 5. The comparative experimental results are shown in Section 6 with conclusions drawn in Section 7.

2 Dynamic Models and Problem Formulation

Neglecting fast electrical dynamics, the dynamics of a linear motor can be written as[1] [11, 12]:

$$M\ddot{y} \;=\; u - B\dot{y} - A_f S_f(\dot{y}) + d(y,\dot{y},t), \tag{1}$$

where y represents the position of the inertia load, M is mass of the inertia load plus the coil assembly, u represents the control input voltage to the motor, B is the equivalent viscous friction coefficient, $A_f S_f(\dot{y})$ represents the nonlinear Columb friction, in which the amplitude A_f may be unknown but the continuous shape function $S_f(\dot{y})$ is known, and d represents the lumped uncertain nonlinearities including various model approximation errors and external disturbances (e.g. cutting force in machining). The same as in [11], define the unknown parameter set $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]$ as $\theta_1 = M$, $\theta_2 = B$, $\theta_3 = A_f$ and $\theta_4 = d_n$, which represents the effect of unknown mass, viscous damping coefficient, Columb friction magnitude, and the nominal value of lumped uncertain nonlinearities respectively. Letting the two state variables x_1 and x_2 be the position and velocity respectively, the state space

[1]For simplicity, all quantities are normalized with respect to the unit of control input voltage.

representation of (1) can be linearly parameterized in terms of θ as

$$\dot{x}_1 = x_2, \tag{2}$$

$$\theta_1 \dot{x}_2 = u - \theta_2 x_2 - \theta_3 S_f + \theta_4 + \tilde{d}, \tag{3}$$

where $\tilde{d} = d - d_n$.

The following nomenclature is used throughout this paper: $\hat{\bullet}$ is used to denote the estimate of $\bullet$, $\tilde{\bullet}$ is used to denote the parameter estimation error of $\bullet$, e.g., $\tilde{\theta} = \hat{\theta} - \theta$, $\bullet_i$ is the i^{th} component of the vector $\bullet$, $\bullet_{max}$ and $\bullet_{min}$ are the maximum and minimum value of $\bullet(t)$ for all t respectively, and the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors. The following practical assumptions are made:

Assumption 1 *The unknown parameter vector θ is within a known bounded convex set Ω_θ. Without loss of generality, it is assumed that $\forall \theta \in \Omega_\theta,\ \ \theta_{imin} \leq \theta_i \leq \theta_{imax}, i = 1, \ldots, 4$, where θ_{imin} and θ_{imax} are some known constants.*

Assumption 2 *The uncertain nonlinearity $\tilde{d}(y, \dot{y}, t)$ can be bounded by*

$$\tilde{d} \in \Omega_d \stackrel{\Delta}{=} \{\ \tilde{d} : \ \ |\tilde{d}| \leq \delta_d\ \} \tag{4}$$

where $\delta_d(t)$ is a known bounded function.

Let $y_d(t)$ be the reference motion trajectory, which is assumed to be known, bounded with bounded derivatives up to the third order. *The objective is to synthesize a control input u such that the output position tracking error $e = y - y_d(t)$ is as small as possible under the assumptions 1 and 2.*

3 Direct Adaptive Robust Control (DARC) of Linear Motors

To motivate the proposed DIARC, the direct ARC in [11] is briefly reviewed in this section and will be experimentally compared with the proposed DIARC later.

3.1 Projection Type Adaptation Law

Let $\hat{\theta}$ denote the estimate of θ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta} = \hat{\theta} - \theta$). One of the key elements of the ARC design [17, 10] is to use the practical available prior information (1) to construct the projection type adaptation law for a controlled learning process even in the presence of disturbances. As in [8, 10], the widely used projection mapping $Proj_{\hat{\theta}}(\bullet)$ will be used to keep the parameter estimates within the known bounded set $\bar{\Omega}_\theta$, the closure of the set Ω_θ. The standard projection mapping is [19, 20, 8, 16]:

$$Proj_{\hat{\theta}}(\zeta) = \begin{cases} \zeta, & \text{if} \quad \hat{\theta} \in \overset{\circ}{\Omega}_\theta \ \text{ or } \ n_{\hat{\theta}}^T \zeta \leq 0 \\ \left(I - \Gamma \frac{n_{\hat{\theta}} n_{\hat{\theta}}^T}{n_{\hat{\theta}}^T \Gamma n_{\hat{\theta}}} \right) \zeta, & \hat{\theta} \in \partial \Omega_\theta \ \text{ and } \ n_{\hat{\theta}}^T \zeta > 0 \end{cases} \tag{5}$$

where $\zeta \in R^p$, $\Gamma(t) \in R^{p\times p}$, $\mathring{\Omega}_\theta$ and $\partial\Omega_\theta$ denote the interior and the boundary of Ω_θ respectively, and $n_{\hat{\theta}}$ represents the outward unit normal vector at $\hat{\theta} \in \partial\Omega_\theta$. It is proven in [8] that the following lemma holds:

Lemma 1 *Suppose that the parameter estimate $\hat{\theta}$ is updated using the following projection type adaptation law:*

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}\left(\Gamma\tau\right), \quad \hat{\theta}(0) \in \Omega_\theta \tag{6}$$

where τ is any adaptation function and $\Gamma(t) > 0$ is any continuously differentiable positive symmetric adaptation rate matrix. With this adaptation law, the following desirable properties hold:

P1. The parameter estimates are always within the known bounded set $\bar{\Omega}_\theta$, i.e., $\hat{\theta}(t) \in \bar{\Omega}_\theta$, $\forall t$. Thus, from Assumption 1, $\forall t$, $\theta_{imin} \leq \hat{\theta}_i(t) \leq \theta_{imax}, i = 1, \ldots, 4$.

P2.

$$\tilde{\theta}^T\left(\Gamma^{-1}Proj_{\hat{\theta}}\left(\Gamma\tau\right)) - \tau\right) \leq 0, \quad \forall\tau \tag{7}$$

$\triangle$

3.2 Desired Compensation ARC Law

With the projection type adaptation law (6), a direct adaptive robust control law is synthesized in [11] for the system (1) that achieves a guaranteed transient and final tracking accuracy. Furthermore, to reduce the effect of measurement noises, desired compensation ARC (DCARC) [21] is used. The resulting DCARC control law has the following form:

$$u = u_a + u_s, \quad u_a = -\varphi_d^T\hat{\theta}, \tag{8}$$

where u_a is the adjustable model compensation needed for achieving perfect tracking, $\varphi_d^T = [-\ddot{y}_d, -\dot{y}_d, -S_f(\dot{y}_d), 1]$ is the regressor that depends on the reference trajectory $y_d(t)$ only and thus is free of measurement noise effect, and u_s is a robust control function having the form of

$$u_s = u_{s1} + u_{s2}, \quad u_{s1} = -k_{s1}p, \tag{9}$$

where p can be thought as a measure of the output tracking performance given by

$$p = \dot{e} + k_1 e = x_2 - x_{2eq}, \quad x_{2eq} \stackrel{\Delta}{=} \dot{y}_d - k_1 e, \;\; k_1 > 0, \tag{10}$$

k_{s1} is a nonlinear gain large enough such that the matrix A_1 defined below is positive definite

$$A_1 = \begin{bmatrix} k_{s1} - k_2 - \theta_1 k_1 + \theta_2 + \theta_3 g & -\frac{1}{2}k_1(\theta_2 + \theta_3 g) \\ -\frac{1}{2}k_1(\theta_2 + \theta_3 g) & \frac{1}{2}\theta_1 k_1^3 \end{bmatrix} \tag{11}$$

in which g is defined by $S_f(x_2) - S_f(\dot{y}_d) = g(x_2, t)\dot{e}$, and u_{s2} is synthesized to satisfy the following robust performance conditions

$$\begin{array}{ll} \text{i} & p\{u_{s2} - \varphi_d^T\tilde{\theta} + \tilde{d}\} \leq \varepsilon \\ \text{ii} & p u_{s2} \leq 0 \end{array} \tag{12}$$

in which ε is a design parameter. With this ARC control law, it is shown in [11] that the tracking error dynamics are

$$\theta_1 \dot{p} \quad = \quad \underbrace{u_{s1} + (\theta_1 k_1 - \theta_2 - \theta_3 g)\dot{e}}+\underbrace{u_{s2} - \varphi_d^T \tilde{\theta} + \tilde{d}} \tag{13}$$

and the following theoretical performance holds:

Theorem 1 *With the projection type adaptation law (6) and an adaptation function of $\tau = \varphi_d p$, the DCARC law (8) guarantees that*

A. *In general, all signals are bounded. Furthermore, the positive definite function V_s defined by*

$$V_s = \frac{1}{2}\theta_1 p^2 + \frac{1}{2}\theta_1 k_1^2 e^2 \tag{14}$$

is bounded above by

$$V_s \leq \exp(-\lambda t) V_s(0) + \frac{\varepsilon}{\lambda}[1 - \exp(-\lambda t)], \tag{15}$$

where $\lambda = \min\{2k_2/\theta_{1max}, k_1\}$.

B. *If after a finite time t_0, there exist parametric uncertainties only (i.e., $\tilde{d} = 0$, $\forall t \geq t_0$), then, in addition to results in A, zero final tracking error is also achieved, i.e, $e \longrightarrow 0$ and $p \longrightarrow 0$ as $t \longrightarrow \infty$.*

4 Indirect Adaptive Robust Control of Linear Motor Drive Systems

Recently, in [18], an indirect adaptive robust control (IARC) design is developed for nonlinear systems transformable to semi-strict feedback forms to overcome the poor parameter estimates problem of the above direct ARC design. The control of linear motor drive systems is also used as a case study. In this section, the application of the IARC design strategy in [18] to the linear motor systems (1) will be thoroughly investigated and a better estimation model than that in [18] will be developed.

4.1 Projection Type Adaptation Law with Rate Limits

In order to achieve a complete separation of estimator design and robust control law design, in addition to the projection-type parameter adaptation law (6), it is also necessary to use the preset adaptation rate limits for a controlled estimation process [18]. For this purpose, for any $\zeta \in R^p$, define a saturation function as:

$$sat_{\dot{\theta}_M}(\zeta) = s_0 \zeta, \quad s_0 = \begin{cases} 1, & \|\zeta\| \leq \dot{\theta}_M \\ \frac{\dot{\theta}_M}{\|\zeta\|}, & \|\zeta\| > \dot{\theta}_M \end{cases} \tag{16}$$

where $\dot{\theta}_M$ is a pre-set rate limit. It can be verified that the following lemma holds [18]:

Lemma 2 *Suppose that the parameter estimate $\hat{\theta}$ is updated using the following projection type adaptation law with a pre-set rate limit $\dot{\theta}_M$:*

$$\dot{\hat{\theta}} = sat_{\dot{\theta}_M}\left(Proj_{\hat{\theta}}\left(\Gamma\tau\right)\right), \quad \hat{\theta}(0) \in \Omega_\theta \tag{17}$$

where τ is any adaptation function and $\Gamma(t) > 0$ is any continuously differentiable positive symmetric adaptation rate matrix. With this adaptation law, the following desirable properties hold:

P1. The parameter estimates are always within the known bounded set $\bar{\Omega}_\theta$, i.e., $\hat{\theta}(t) \in \bar{\Omega}_\theta$, $\forall t$. Thus, from Assumption 1, $\forall t$, $\theta_{imin} \le \hat{\theta}_i(t) \le \theta_{imax}, i = 1, \ldots, 4$.

P2.

$$\tilde{\theta}^T\left(\Gamma^{-1}Proj_{\hat{\theta}}\left(\Gamma\tau\right) - \tau\right) \le 0, \quad \forall\tau \tag{18}$$

P3. The parameter update rate is uniformly bounded by $\|\dot{\hat{\theta}}(t)\| \le \dot{\theta}_M$, $\forall t$ $\triangle$

4.2 Desired Compensation Adaptive Robust Control Law

With the use of the projection type adaptation law with rate limit (17), the parameter estimates and their derivatives are bounded with known bounds, regardless of the estimation function τ to be used. As such, the same design techniques as in the direct ARC design [10, 11] can be used to construct a desired compensation adaptive robust control function that achieves a guaranteed transient and final tracking accuracy, independent of the specific identifier to be used later. The resulting control function is of the desired compensation type having the same form as (8) with the same form of u_s as in (9). The only difference is that the regressor used in the parameter adaptation function of the proposed IARC will be based on the actual system dynamics for better parameter estimates, i.e., $\varphi = [-\ddot{y}, -\dot{y}, -S_f(\dot{y}), 1]^T$, instead of the desired regressor vector φ_d as in DARC. As such, in addition to the condition (9), k_{s1} is also required to be large enough such that

$$A_2 = \begin{bmatrix} \frac{\theta_1}{\hat{\theta}_1}(k_{s1} + \hat{\theta}_2 + \hat{\theta}_3 g) - k_2 - \theta_1 k_1 & -\frac{1}{2}\frac{\theta_1}{\hat{\theta}_1}(\hat{\theta}_2 + \hat{\theta}_3 g)k_1 \\ -\frac{1}{2}\frac{\theta_1}{\hat{\theta}_1}(\hat{\theta}_2 + \hat{\theta}_3 g)k_1 & \frac{1}{2}\theta_1 k_1^3 \end{bmatrix} \ge 0 \tag{19}$$

With this ARC control law, using similar derivations as in [11], the following theorem can be obtained.

Theorem 2 *Consider the ARC law (8) with the projection type adaptation law with rate limits (17), in which τ could be any adaptation function. Then, the same robust performance results as in A of Theorem 1 can be obtained.*

4.3 Indirect Parameter Estimation Algorithms

In the above subsection, an adaptive robust control law which can admit any estimation function τ has been constructed and a guaranteed transient and final tracking performance is achieved even in the presence of uncertain nonlinearities. Thus, the reminder of the IARC design is to construct suitable estimation functions τ so that an improved final tracking accuracy–asymptotic tracking or zero final tracking error in the presence of parametric uncertainties only–can be obtained with an emphasis on good parameter estimation process as well. As such, in this subsection, it is assumed the system is absence of uncertain nonlinearities, i.e., let$\tilde{d} = 0$ in (3).

Let $H_f(s)$ be the stable transfer function of a filter with a relative degree larger or equal to 2. Then, when$\tilde{d} = 0$, applying the filter to both sides of (3), one obtains

$$\theta_1 \ddot{y}_f = u_f - \theta_2 \dot{y}_f - \theta_3 S_{ff} + \theta_4 1_f \tag{20}$$

where y_f, u_f, S_{ff}, and 1_f represent the filtered output, input, the shape function, and 1 respectively, i.e., $y_f(t) = H_f(p)[y(t)], u_f = H_f(p)[u(t)], S_{ff} = H_f(p)[S_f(\dot{y}(t))]$, and $1_f = H_f(p)[1]$. From (20), a linear regression model can be obtained as

$$u_f = -\varphi_f^T \theta \tag{21}$$

where the regressor is $\varphi_f^T = [-\ddot{y}_f, -\dot{y}_f, -S_{ff}, 1_f]$. Thus, by defining the predicted output and the prediction error as

$$\begin{aligned} \hat{u}_f &= -\varphi_f^T \hat{\theta} \\ \epsilon &= \hat{u}_f - u_f \end{aligned} \tag{22}$$

one obtains the following prediction error model

$$\epsilon = -\varphi_f^T \tilde{\theta} \tag{23}$$

With this static linear regression model, various estimation algorithms can be used to identify unknown parameters, of which the gradient estimation algorithm and the least squares estimation algorithm [22, 16] are given below.

4.3.1 Gradient Estimator

With the gradient type estimation algorithm, the resulting adaptation law is given by (17), in which Γ can be chosen as a constant positive diagonal matrix, i.e., $\Gamma = diag[\gamma_1, ..., \gamma_4]$, and τ is defined as

$$\tau = \frac{1}{1 + \nu \|\varphi_f\|^2} \varphi_f \epsilon, \quad \nu \geq 0 \tag{24}$$

where by allowing $\nu = 0$, one encompasses unnormalized adaptation function.

4.3.2 Least Squares Estimator

When the least squares type estimation algorithm with co-variance re-setting [23] and exponential forgetting [22] is used, the resulting adaptation law is given by (17), in which $\Gamma(t)$ is updated by

$$\dot{\Gamma} = \alpha \Gamma - \frac{1}{1 + \nu \varphi_f^T \Gamma \varphi_f} \Gamma \varphi_f \varphi_f^T \Gamma, \quad \Gamma(0) = \Gamma^T(0) > 0, \;\; \Gamma(t_r^+) = \rho_0 I, \;\; \nu \geq 0 \tag{25}$$

where $\nu = 0$ leads to the unnormalized algorithm, and τ is defined as

$$\tau = \frac{1}{1 + \nu \varphi_f^T \Gamma \varphi_f} \varphi_f \epsilon \tag{26}$$

In (25), α is the forgetting factor, t_r is the covariance resetting time, i.e., the time when $\lambda_{min}(\Gamma(t)) = \rho_1$ where ρ_1 is a pre-set lower limit for $\Gamma(t)$ satisfying $0 < \rho_1 < \rho_0$. In practice, the above least square estimator may lead to estimator windup (i.e., $\lambda_{max}(\Gamma(t)) \longrightarrow \infty$) when the regressor is not persistently exciting. To prevent this estimator windup and take into account the effect of the rate-limited adaptation law (17), (25) is modified to

$$\dot{\Gamma} = \begin{cases} \alpha\Gamma - \frac{1}{1+\nu\varphi_f^T\Gamma\varphi_f}\Gamma\varphi_f\varphi_f^T\Gamma, & \text{if} \quad \lambda_{max}(\Gamma(t)) \leq \rho_M \text{ and } \| Proj_{\hat{\theta}}(\Gamma\tau) \| \leq \dot{\theta}_M \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

where ρ_M is the pre-set upper bound for $\|\Gamma(t)\|$ with $\rho_M > \rho_0$. With these practical modifications, $\rho_1 I \leq \Gamma(t) \leq \rho_M I, \ \forall t$.

To prove that asymptotic output tracking can be achieved, the following lemma which summarizes the properties of the estimators is needed:

Lemma 3 *When the rate-limited projection type adaptation law (17) with either the gradient estimator (24) or the least squares estimator (26) is used, the following results hold:*

$$\tilde{\theta} \in \mathcal{L}_\infty[0,\infty) \tag{28}$$

$$\epsilon \in \mathcal{L}_2[0,\infty) \cap \mathcal{L}_\infty[0,\infty) \tag{29}$$

$$\dot{\hat{\theta}} \in \mathcal{L}_2[0,\infty) \cap \mathcal{L}_\infty[0,\infty) \tag{30}$$

The proof of the lemma is given in Appendix.

Theorem 3 *In the presence of parametric uncertainties only, i.e.,$\tilde{d} = 0$, by using the control law (8) and the adaptation law (17) with either the gradient type estimation function (24) or the least squares type estimation function (26), in addition to the robust performance results stated in Theorem 2, an improved final tracking performance, asymptotic tracking, is also achieved, i.e., $e \to 0$ as $t \to \infty$.* $\triangle$

The proof of the theorem is given in Appendix.

5 Integrated Direct/Indirect Adaptive Robust Control of Linear Motors

As shown in the comparative experimental results in section 6, compared to the fast response of the actual tracking error dynamics (13), the adaptation of the parameters in the IARC design (17) is relatively slower, thus causing a significant transient tracking error due to the parameter estimation error$\tilde{\theta}$. While the direct ARC (DARC) design presented in section 3 does not necessarily produce good parameter estimates, it has been observed in practice that DARC normally has a better tracking performance than IARC. For this reason, it is logical to integrate the DARC design and the IARC design to see if one can construct an ARC controller that would produce better parameter estimates than DARC through the use of estimators similar to those in IARC, while preserving the good transient performance properties of DARC via certain rapid dynamic compensation terms as in DARC. Such an integrated ARC design is presented in this section for (3).

The proposed DIARC control law has the following form:

$$u = u_a + u_s, \;\; u_a = u_{a1} + u_{a2}, \;\; u_{a1} = -\varphi_d^T \hat{\theta}, \tag{31}$$

where u_{a1} is the same adjustable model compensation as in the IARC law (8) with the same parameter estimation algorithm (17) for $\hat{\theta}$, u_{a2} is a fast dynamic compensation term to be synthesized later, and u_s is a robust control function having the same form as (9) with the same u_{s1} as in IARC but different robust performance conditions for u_{s2} as specified in the following. Substituting (31) into (3), corresponding to (13), the new tracking error dynamics are

$$\theta_1 \dot{p} = \underbrace{u_{s1} + (\theta_1 k_1 - \theta_2 - \theta_3 g)\dot{e}} + u_{a2} + u_{s2} \underbrace{-\varphi_d^T \tilde{\theta} + \tilde{d}} \tag{32}$$

Define a constant d_0 and time varying function $\tilde{d}^*(t)$ such that

$$d_0 + \tilde{d}^*(t) = -\varphi_d^T \tilde{\theta} + \tilde{d} \tag{33}$$

Conceptually, (33) lumps the disturbance and the model uncertainties due to parameter estimation error together and divides it into the low frequency component d_0 and the higher frequency components, $\tilde{d}^*(t)$, so that the low frequency component d_0 can be compensated for through the fast adaptation of direct ARC design [10, 17] as follows. Substituting (33) into (32),

$$\theta_1 \dot{p} = \underbrace{u_{s1} + (\theta_1 k_1 - \theta_2 - \theta_3 g)\dot{e}} + u_{a2} + u_{s2} + d_0 + \tilde{d}^*(t) \tag{34}$$

Choose the fast compensation term u_{a2} as

$$u_{a2} = -\hat{d}_0 \tag{35}$$

where $\hat{d}_0$ represents the estimate of d_0 updated by

$$\dot{\hat{d}}_0 = Proj_{\hat{d}_0}(\gamma_d \frac{1}{\hat{\theta}_1} p), \qquad |\hat{d}_0(0)| \leq \hat{d}_{max} \tag{36}$$

in which $\hat{d}_{max}$ is a pres-set bound for $\hat{d}_0(t)$. As in DARC in section 3, the projection mapping in (36) guarantees that $|\hat{d}_0(t)| \leq \hat{d}_{max}, \forall t$. Substituting (35) into (34),

$$\theta_1 \dot{p} = \underbrace{u_{s1} + (\theta_1 k_1 - \theta_2 - \theta_3 g)\dot{e}} + u_{s2} - \tilde{d}_0 + \tilde{d}^*(t) \tag{37}$$

Similar to (12), the robust function u_{s2} is now chosen to satisfy the following robust performance conditions:

$$\begin{array}{ll} \text{i} & p\{u_{s2} - \tilde{d}_0 + \tilde{d}^*(t)\} \leq \varepsilon \\ \text{ii} & p u_{s2} \leq 0 \end{array} \tag{38}$$

Theorem 4 *With the same parameter estimation algorithm for $\hat{\theta}$ as in IARC in section 4 and $\hat{d}_0$ updated by (36), the DIARC law (31) achieves the same theoretical performance results as DARC in Theorem 1.* △

The proof of the theorem is given in Appendix.

Remark 1 *One smooth example of u_{s2} satisfying (38) is*

$$u_{s2} = -k_{s2}p, \quad k_{s2} = \frac{1}{4\varepsilon}h^2 \tag{39}$$

where h is any smooth function satisfying

$$h \geq \hat{d}_{max} + \|\theta_M\|\|\varphi_d\| + \delta_d, \tag{40}$$

in which $\theta_M = \theta_{max} - \theta_{min}$. Other smooth or continuous examples of u_{s2} can be worked out in the same way as in [8, 9, 10]. $\diamondsuit$

6 Comparative Experiments

6.1 Experiment Setup

All the control algorithms are implemented on the Y-axis of a precision X-Y stage driven by an Anorad LEM-S-3-S linear motor (epoxy core). The system has a measurement resolution of 1 μm with the detailed experimental set-up given in [11]. Standard least-square identification is performed to obtain the parameters of the Y-axis. The nominal values of M is 0.027 $(V/m/s^2)$. To test the learning capability of the proposed ARC algorithms, a 20lb load is mounted on the motor in experiments and the identified values of the parameters are

$$\theta_1 = 0.1\ (V/m/s^2), \quad \theta_2 = 0.273\ (V/m/s), \quad \theta_3 = 0.09\ (V). \tag{41}$$

The bounds of the parameter variations are chosen as:

$$\begin{aligned} \theta_{min} &= [0.02,\ 0.22,\ 0.02,\ -1]^T \\ \theta_{max} &= [0.12,\ 0.35,\ 0.2,\ 1]^T \end{aligned}$$

The initial parameter estimates of $\theta_0 = [0.05\ 0.24\ 0.05\ 0.0]^T$ are used for all experiments.

6.2 Performance Index

As in [24, 25], the following performance indexes will be used to measure the quality of each control algorithm:

- $L_2[e] = \sqrt{\frac{1}{T_f}\int_0^{T_f} |e|^2 dt}$, the scalar valued L_2 norm, is used as an objective numerical measure of *average tracking performance* for an entire error curve $e(t)$, where T_f represents the total running time;

- $e_M = \max_t\{|e(t)|\}$, the maximal absolute value of the tracking error, is used as an index of measure of *transient performance*;

- $e_F = \max_{T_f-2\le t\le T_f}\{|e(t)|\}$, the maximal absolute value of the tracking error during the last 2 seconds, is used as an index of measure of *final tracking accuracy*;

- $L_2[u] = \sqrt{\frac{1}{T_f}\int_0^{T_f}|u|^2dt}$, the average control input, is used to evaluate the amount of *control effort*;

- $c_u = \frac{L_2[\Delta u]}{L_2[u]}$, the normalized control variations, is used to measure the *degree of control chattering*, where

$$L_2[\Delta u] = \sqrt{\frac{1}{N}\sum_{j=1}^{N}|u(j\Delta T) - u((j-1)\Delta T)|^2} \ .$$

  is the average of control input increments.

6.3 Comparative Experimental Results

Experiments were performed with the Y-axis. The control system is implemented using a dSPACE DS1103 controller board. The controller executes programs at a sampling frequency $f_s = 10kHz$, which results in a velocity measurement resolution of $0.01\ m/sec$.

As in [11], a nonlinear robust feedback gain of the following form is used for u_s. Choose a nonlinear feedback gain k_s' large enough such that

$$k_s' = \max\{k_{p1} + \frac{1}{4\varepsilon'}h^2,\ \ k_{p2} + c(|p| - p_0)^2\} \ge k_{s1} + \frac{1}{4\varepsilon'}h^2, \tag{42}$$

where h is defined in (40), c and p_0 are two empirical parameters. Then, the control function $u_s = -k_s'p$ satisfies (9) and (38). In the experiments, the following parameters are used: $k_{p1} = 50$, $k_{p2} = 50$, $\varepsilon' = 2$, $p_0 = 0.01$, $c = 2\times10^6$ whenever $|p| > p_0$ and $c = 0$ whenever $|p| \le p_0$. The following three control algorithms are compared:

DARC: the Direct Adaptive Robust Control (DARC) law reviewed in section 3. As in [11], $S_f(x_2)$ is chosen as $\frac{2}{\pi}\arctan(1000x_2)$ and $k_1 = 500$. The adaptation rates are set as $\Gamma = \textbf{diag}\{25, 100, 5, 1000\}$.

IARC: the Indirect Adaptive Robust Control with the least square type estimation algorithm (26) and (27) in section 4. The initial adaptation rates are set as $\Gamma = \textbf{diag}\{50, 20, 5, 100\}$.

DIARC: the Direct/Indirect Adaptive Robust Control (DIARC) with the least square type estimation algorithm (26) and (27) in section 5. The initial adaptation rates are same as those used in IARC with $\gamma_d = 10^4$.

For the implementation of the last two algorithms, a stable transfer function $H_f(s)$ with a relative degree equal to 2, with a break frequency of 50Hz and damping of 0.7 is used. To test the tracking performance of the proposed algorithms, a typical high-speed/high acceleration motion trajectory for the pick-and-place operations in industry is used in all experiments. The desired trajectory has a movement of $0.4m$ with a maximum speed of $1m/s$ and an acceleration of $12m/sec^2$ as in [11]. The experimental results in terms of the quantitative indexes are given in Table 1 and in terms of time history are given in Figs.1-8.

Table 1

	without load			with load		
controller	DARC	IARC	DIARC	DARC	IARC	DIARC
e_M (μm)	10.4	13.0	10.7	18.4	14.9	10.7
e_F (μm)	10.4	12.7	9.2	10.8	12.7	9.3
$L_2[e]$ (μm)	1.84	3.32	1.66	1.64	3.36	1.76
$L_2[u]$ (V)	0.28	0.29	0.28	0.45	0.46	0.46
$L_2[\Delta u]$ (V)	0.10	0.11	0.11	0.10	0.10	0.10
c_u	0.34	0.38	0.39	0.21	0.23	0.22

As seen from these results, the tracking errors of all the controllers are very small, which are within $20\mu m$ over the entire run. As seen for no load and load cases, the parameter esitmates of IARC and DIARC algorithms are better than that of DARC, especially the inertial load and the friction estimates. However, the tracking performances of DARC and DIARC controllers are better than that of IARC. Overall, DIARC achieves the best tracking performance while having more robust parameter estimation process and accurate parameter estimates than DARC.

7 Conclusions

In this paper, nonlinear adaptive robust controllers that achieve not only excellent output tracking performance but also accurate parameter estimates are synthesized for precision linear motor drive systems. Such an objective is accomplished through the intelligent integration of the output tracking performance oriented direct adaptive robust control (DARC) design with the recently developed accurate parameter estimation based indirect adaptive robust control (IARC) design. All three controllers, DARC, IARC, and DIARC, are implemented on a precision linear motor system to test the achievable performance of different designs and their practical limitations. Comparative experimental results demonstrate that the proposed DIARC controller is able to achieve better tracking performance than either DARC and or IARC while having accurate parameter estimates as in IARC for secondary purposes such as prognostics and machine health monitoring.

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8 Appendix

Proof of Lemma 3:
The boundedness of $\hat{\theta}$ and $\dot{\hat{\theta}}$ is guaranteed by P1 and P3 of Lemma 2 respectively. From Assumption 1, $\tilde{\theta}$ is bounded, i.e., $\tilde{\theta} \in \mathcal{L}_\infty[0,\infty)$. From Theorem 2, y, $\dot{y}$, and u are bounded, which leads to a bounded φ and φ_f. Thus, it is a well-known fact [22, 20] that, for the linear regression model (23), either the gradient type estimation algorithm (24) or the least square type estimation algorithm (26) will guarantee the L_2 boundedness of the prediction error signal ϵ and $\dot{\hat{\theta}}(t)$. $\diamondsuit$

Proof of Theorem 3:
Noting that φ_f is the filtered output of the regressor φ through a stable filter $H_f(s)$, using the standard swapping lemma in adaptive control [22, 20], the fact that $\varphi_f^T\tilde{\theta} = -\varepsilon \in \mathcal{L}_2[0,\infty)$ and $\dot{\hat{\theta}} \in \mathcal{L}_2[0,\infty)$ from Lemma 3 leads to $\varphi^T\tilde{\theta} \in \mathcal{L}_2[0,\infty)$.

From (13), when $\tilde{d} = 0$, by direct substitutions, it is easy to show that the tracking error dynamics can be rewritten as

$$\hat{\theta}_1\dot{p} \quad = \quad \underbrace{u_{s1} + (\hat{\theta}_1k_1 - \hat{\theta}_2 - \hat{\theta}_3 g)\dot{e}} + u_{s2} - \varphi^T\tilde{\theta} \tag{43}$$

Noting $\dot{e} = p - k_1 e$ and $\theta_1 = M$, the time derivative of V_s given by (14) is

$$\begin{aligned} \dot{V}_s &= \frac{\theta_1}{\hat{\theta}_1}p\{u_{s1} + (\hat{\theta}_1k_1 - \hat{\theta}_2 - \hat{\theta}_3 g)\dot{e} + u_{s2} - \varphi^T\tilde{\theta}\} + \theta_1 k_1^2 e\dot{e} & (44)\\ &= -[p,e]A_2\begin{bmatrix} p \\ e \end{bmatrix} - k_2p^2 - \frac{1}{2}\theta_1k_1^3e^2 + \frac{\theta_1}{\hat{\theta}_1}p\{u_{s2} - \varphi^T\tilde{\theta}\} & (45) \end{aligned}$$

Thus, with ii of (12) and (19),

$$\dot{V}_s \le -k_2p^2 - \frac{1}{2}\theta_1k_1^3e^2 - \frac{\theta_1}{\hat{\theta}_1}p\varphi^T\tilde{\theta} \tag{46}$$

It is thus obvious that $\varphi^T\tilde{\theta} \in \mathcal{L}_2[0,\infty)$ leads to $p \in \mathcal{L}_2[0,\infty)$ and $e \in \mathcal{L}_2[0,\infty)$. By Barbalat's Lemma, asymptotic tracking can be proved, which leads to Theorem 3. $\diamondsuit$

Proof of Theorem 4:
Noting (34) and i of (38), results in A of Theorem 1 can be obtained in the same way as in the DARC design in section 3. The following is to prove asymptotic output tracking when $\tilde{\delta} = 0$.

Firstly, noting that the same system dynamics based identifier is used to obtain $\hat{\theta}$, Lemma 3 remains valid, which leads to $\varphi^T\tilde{\theta} \in \mathcal{L}_2[0,\infty)$ as in the proof of Theorem 3.

Secondly, by direct substitutions, as in (43), it is easy to show that the tracking error dynamics (32) can be rewritten as

$$\hat{\theta}_1\dot{p} \quad = \quad \underbrace{u_{s1} + (\hat{\theta}_1 k_1 - \hat{\theta}_2 - \hat{\theta}_3 g)\dot{e}} - \hat{d}_0 + u_{s2} - \varphi^T\tilde{\theta} \tag{47}$$

Consider a positive function given by

$$V_a = V_s + \frac{\theta_1}{2\gamma_d}\hat{d}_0^2 \tag{48}$$

where V_s is given by (14). Similar to (44), it is easy to verify that the time derivative of V_a is

$$\dot{V}_a \quad = \quad -[p,e]A_1\left[\begin{array}{c} p \\ e \end{array}\right] - k_2p^2 - \frac{1}{2}\theta_1 k_1^3 e^2 + \frac{\theta_1}{\hat{\theta}_1}p\{u_{s2} - \varphi^T\tilde{\theta}\} + \theta_1\hat{d}_0[\frac{1}{\gamma_d}\dot{\hat{d}}_0 - \frac{1}{\hat{\theta}_1}p] \tag{49}$$

Thus, with the parameter adaptation law (36) for $\hat{d}_0$, noting P2 of Lemma 1 with $d_0 = 0$, ii of (12) and (19),

$$\dot{V}_a \leq -k_2p^2 - \frac{1}{2}\theta_1 k_1^3 e^2 - \frac{\theta_1}{\hat{\theta}_1}p\varphi^T\tilde{\theta} \tag{50}$$

It is thus obvious that $\varphi^T\tilde{\theta} \in \mathcal{L}_2[0,\infty)$ leads to $p \in \mathcal{L}_2[0,\infty)$ and $e \in \mathcal{L}_2[0,\infty)$. By Barbalat's Lemma, asymptotic output tracking can be proved, which completes the proof. $\diamondsuit$

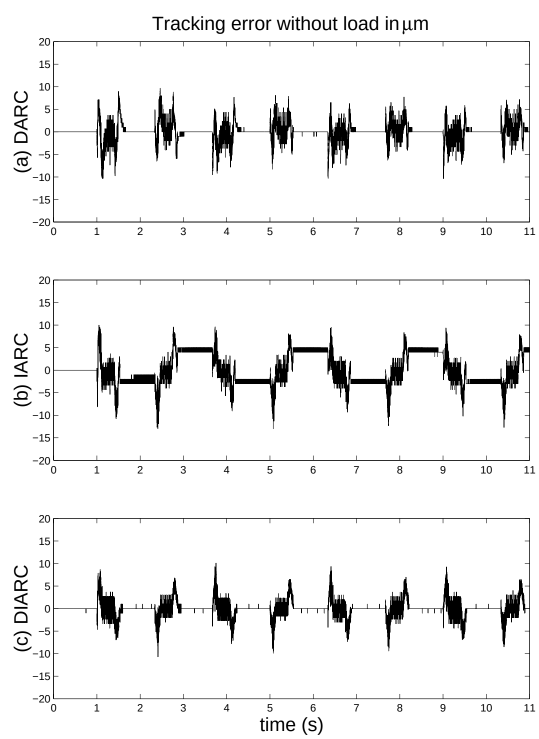

Figure 1: Tracking error for (a)DARC, (b)IARC, (c)DIARC with no load

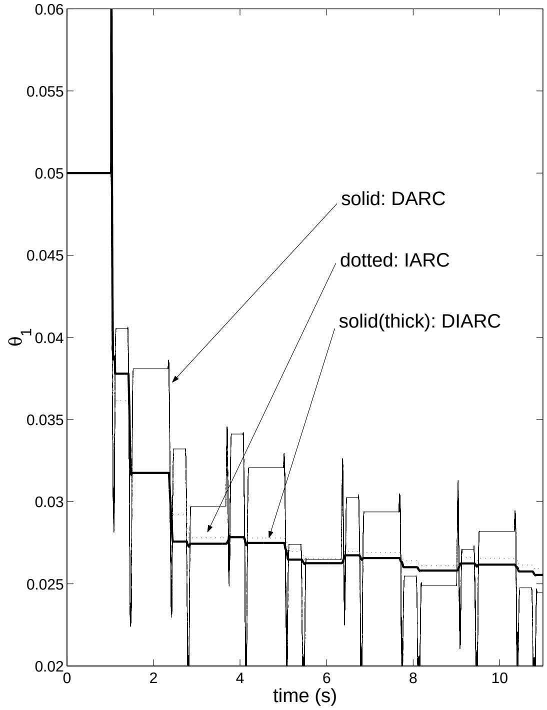

Figure 2: $\hat{\theta}_1$ for (a)DARC, (b)IARC, (c)DIARC with no load

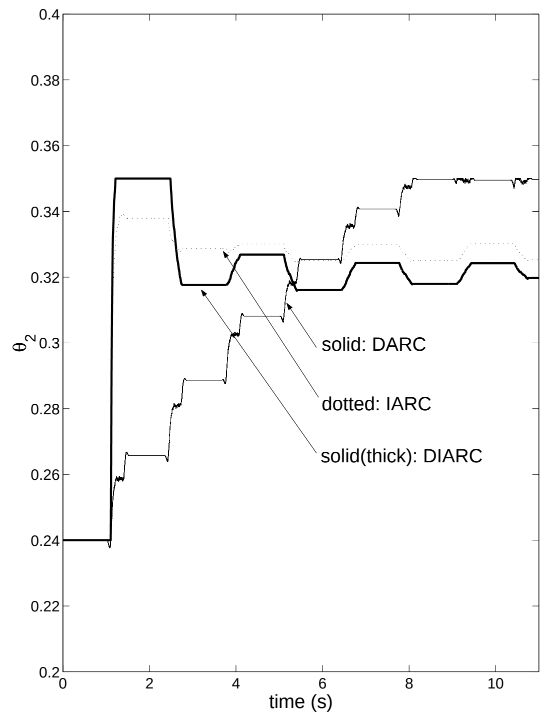

Figure 3: $\hat{\theta}_2$ for (a)DARC, (b)IARC, (c)DIARC with no load

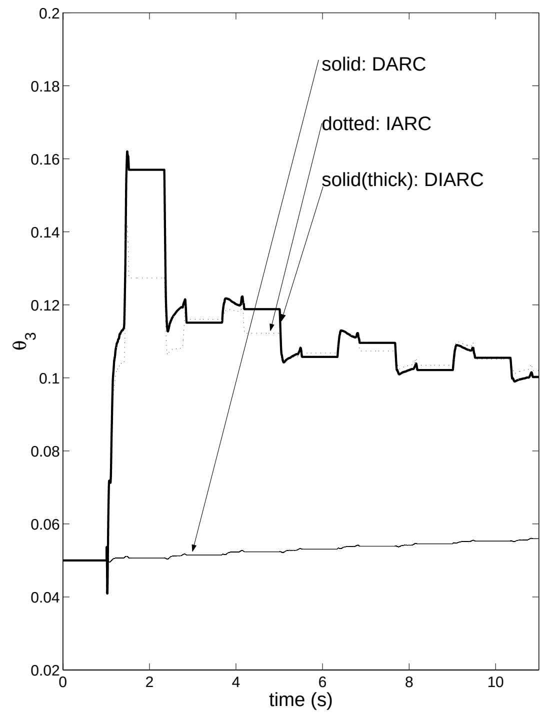

Figure 4: $\hat{\theta}_3$ for (a)DARC, (b)IARC, (c)DIARC with no load

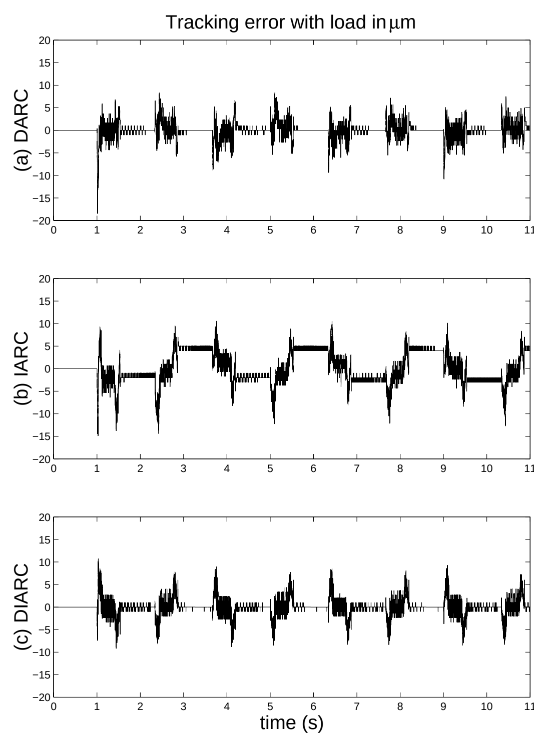

Figure 5: Tracking error for (a)DARC, (b)IARC, (c)DIARC with load

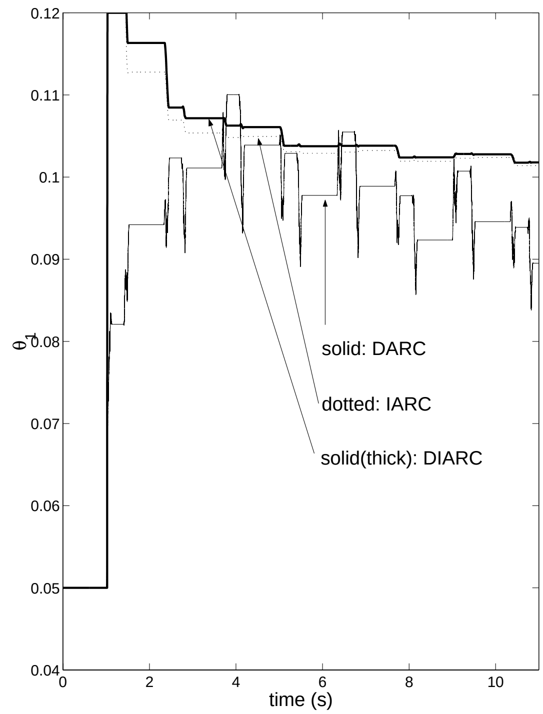

Figure 6: $\hat{\theta}_1$ for (a)DARC, (b)IARC, (c)DIARC with load

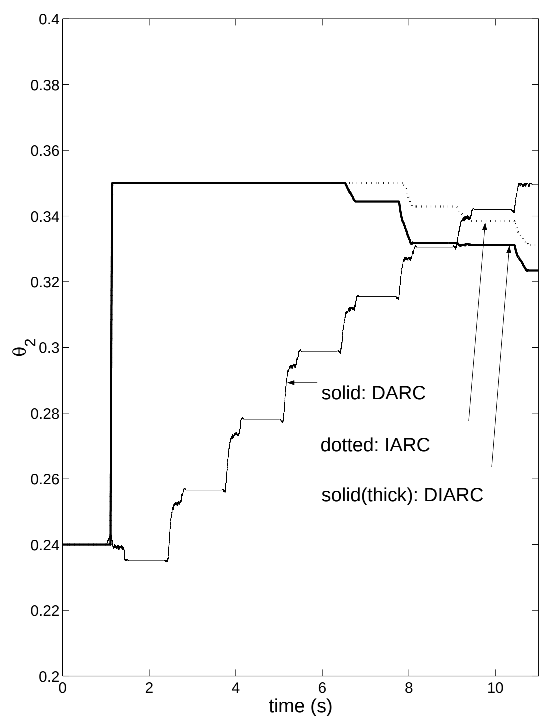

Figure 7: $\hat{\theta}_2$ for (a)DARC, (b)IARC, (c)DIARC with load

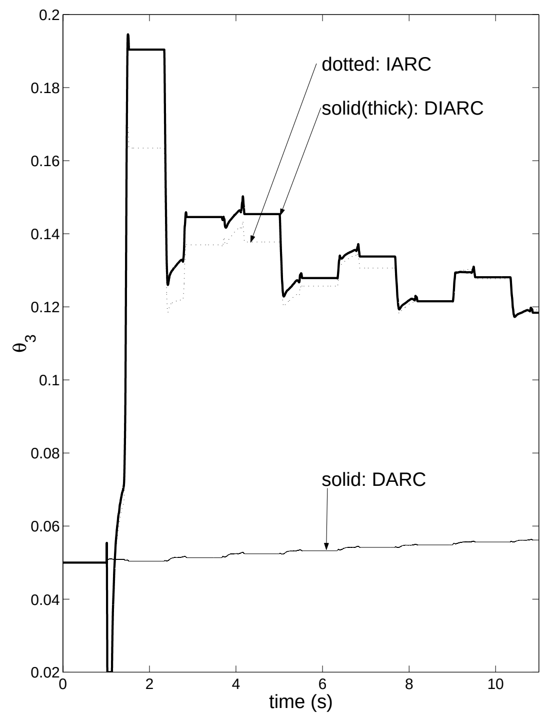

Figure 8: $\hat{\theta}_3$ for (a)DARC, (b)IARC, (c)DIARC with load