Adaptive Robust Control of Robot Manipulators: Theory and Comparative Experiments

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Abstract

Adaptive robust control schemes are developed for the trajectory tracking control of robot manipulators. The schemes utilize a dynamic sliding mode and achieve a guaranteed transient performance and final tracking accuracy in the presence of both parametric uncertainties and uncertain nonlinearities. In the absence of uncertain nonlinearities, the schemes also achieve asymptotic tracking. In addition, three conceptually different adaptive and robust control schemes — a very simple nonlinear PID type robust control, a gain-based nonlinear PID type adaptive control, and a combined parameter and gain based adaptive robust control — are derived for comparison. All algorithms, as well as two benchmark adaptive schemes, are implemented and compared on a two-link direct-drive robot. Comparative experimental results show the importance of using both proper controller structure and parameter adaptation in designing high performance controllers. It is observed that in these experiments, the proposed scheme improves tracking performance without increasing control effort.

I. Introduction

Trajectory tracking control of robot manipulators is of practical significance and is the simplest but most fundamental task in robot control [1, 2]. Practically, parameters of the system such as gravitational load vary from a task to another, and, may not be precisely known in advance. The system may also be subjected to uncertain nonlinearities such as external disturbances and joint friction. On the whole, a good control strategy should take into account both parametric uncertainties and uncertain nonlinearities. During the past decade, numerous robust control algorithms have been proposed, such as adaptive control [3, 4, 5], deterministic robust control (SMC) or sliding mode control (SMC) [6, 7, 8], and, recently, adaptive robust control [9, 10, 11, 12, 13].

The adaptive robust control (ARC) [9, 10, 11, 12, 13] proposed by Yao and Tomizuka effectively combines adaptive control with deterministic robust control (DRC). It uses both means — *proper controller structure and parameter adaptation* — to reduce tracking errors. Departing from the model-based adaptive control, the approach puts emphasis on the selection of controller structure as in DRC to attenuate the effect of model uncertainties as much as possible. Thus, the main practical problem of adaptive control [14]–unknown transient performance and non-robustness to uncertain nonlinearities–can be solved painlessly and a guaranteed transient performance can be obtained. Contrary to DRC, the approach discriminates the difference between parametric uncertainties and uncertain nonlinearities and uses parameter adaptation to reduce the model uncertainties. As a result, an improved performance can be obtained–*asymptotic tracking* is achieved without using discontinuous or infinite-gain feedback [15] in the presence of parametric uncertainties. The approach differs fundamentally from the existing robust adaptive control approaches [14, 16, 17] in that it emphasizes robust performance in addition to robust stability and the selection of controller structure. In return, a much stronger performance robustness–guaranteed transient and final tracking accuracy in the presence of both parametric uncertainties and uncertain nonlinearities–can be achieved; in robust adaptive control schemes [14, 16, 17], steady state tracking error can be shown to stay within an unknown ball whose size depends on the disturbances only and nothing can be said about the transient performance.

There are also some adaptive schemes [18, 19, 20] called performance-based (or direct) adaptive control [21], in which adaptation laws are used to adjust controller gains instead of physical parameters. These gain-based schemes are claimed to be simple, computationally efficient and require very little model information. Robustness to bounded disturbances is also guaranteed. However, they can only guarantee tracking errors within certain bounds even when the system is subject to parametric uncertainties only. Asymptotic stability is lost and the system may exhibit relatively larger final tracking errors as in DRC.

Some comparative experiments were carried out in [22] to test some of the model-based (or parameter-based) adaptive algorithms. However, the tested algorithms belonged to the same class. Facing so many algorithms and so many qualitatively different approaches, one has difficulty in choosing a suitable one for a particular application since each algorithm has its own claim. Thus, it is of practical significance to test qualitatively different approaches on the same machine to understand their fundamental advantages and drawbacks.

This paper serves for two purposes: one is to further improve performance of robot control systems and the other is to test qualitatively different algorithms experimentally to set up a standard with which various controllers can be compared. To achieve the first purpose, the adaptive robust control (ARC) scheme proposed by Yao and Tomizuka [23] is revisited and generalized, in which the regressor is calculated by reference trajectory information only, and thus the resulting adaptation law is less sensitive to noisy velocity signals and has a better robustness in addition to largely reduced on-line computation. The idea of using the desired compensation adaptation law was proposed by Sadegh and Horowitz [4] and was experimentally demonstrated by Whitcomb, et al. [22] that it achieves a superior tracking performance among existing adaptive schemes. Theoretically, the main difference between the proposed approach and the desired compensation adaptive algorithm (DCAL) in [4] is that the proposed approach can guarantee a prescribed precision and transient performance even in the presence of uncertain nonlinearities. To serve for the second purpose, a very simple nonlinear PID scheme proposed by Yao and Tomizuka in [24] is revisited, which can guarantee stability and requires little model information. By adjusting feedback gains on-line, a simple gain-based adaptive control [24] is also suggested to remove the requirements in choosing feedback gains in the nonlinear PID scheme. By combining the design techniques of the gain-based adaptive control and the model-based ARC, a new adaptive robust scheme [24] is also proposed to remove the conditions on the selection of the controller gains. Finally, all schemes, as well as two benchmark adaptive control schemes [3, 4], are implemented and compared. Experimental results are presented to show the advantages and the drawbacks of each method.

II. Dynamic Model

A dynamic equation of a general rigid link manipulator having n degrees of freedom in free space can be written as [2]

$$M(q,\beta)\ddot{q} \ + \ C(q,\dot{q},\beta)\dot{q} + G(q,\beta) + \tilde{f}(q,\dot{q},t) = u \qquad (1)$$

where $\beta \in R^p$ is the vector of a suitably selected set of the robot parameters and $\tilde{f}(q,\dot{q},t) \in R^n$ is the vector of unknown nonlinear functions such as external disturbances and unmodeled joint friction. The following standard properties [2] for a robot dynamics are used in the controller design: *Property 1* $M(q,\beta)$ is a symmetric positive definite (s.p.d.) matrix, and there exists $k_m > 0$ such that $k_m I_{n\times n} \le M(q,\beta)$. Furthermore, for the robot with all joints revolute or prisma, there exists $k_M > 0$ so that $M(q,\beta) \le k_M I_{n\times n}$. For a general robot, $M(q,\beta) \le k_M I_{n\times n}$ is valid for any finite workspace $\Omega_q = \{q : \|q - q_0\| \le q_{max}\}$ where q_0 and q_{max} are some constants; *Property 2* The matrix $N = \dot{M} - 2C$ is a skew-symmetric matrix; *Property 3* $M(q,\beta), C(q,\dot{q},\beta)$, and $G(q,\beta)$ can be linearly parametrized in terms of β. Therefore, we can write

$$M\ddot{q}_r + \ C\dot{q}_r + G = f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta \qquad (2)$$

where $Y \in R^{n\times l_\beta}$, $\dot{q}_r$ and $\ddot{q}_r$ are any reference vectors.

Assumption 1: β lies in a known bounded set Ω_β and $\|\tilde{f}(q,\dot{q},t)\|$ can be bounded by a known function, i.e.,

$$\begin{array}{l} \beta \in \Omega_\beta \stackrel{\Delta}{=} \{\beta : \ \ \beta_{min} < \beta < \beta_{max} \ \} \\ \|\tilde{f}(q,\dot{q},t)\| \le h_f(q,\dot{q},t) \end{array} \qquad (3)$$

where β_{min}, β_{max}, and $h_f(q,\dot{q},t)$ are known (operation $<$ for vectors is defined elementwise and $\|\bullet\|$ denotes a norm of $\bullet$ which is a vector or a matrix).

We can now formulate the trajectory tracking control of robot manipulators as follows: *Suppose $q_d(t) \in R^n$ is given as the desired joint motion trajectory. Let $e = q(t) - q_d(t) \ \in R^n$ be the motion tracking error. For the robot manipulator described by (1), under the Assumption 1, design a control law u so that the system is stable and q tracks $q_d(t)$ as close as possible.*

III. Adaptive Sliding Mode Control

In this section, the smooth adaptive sliding mode control scheme [9] is modified in the way that the resulting control law is more general and revealing. The results will also be utilized in the following sections' design. The scheme combines SMC with adaptive control to take advantages of the two methods while overcoming their drawbacks. A dynamic sliding mode is employed to eliminate the unpleasant reaching transient and to enhance the dynamic response of the system in sliding mode [1]

Let a dynamic compensator be

$$\begin{array}{ll} \dot{z} = A_z z + B_z e, & z \in R^{n_c}, \ A_z \in R^{n_c\times n_c} \\ y_z = C_z z + D_z e, & y_z \in R^n, \ C_z \in R^{n\times n_c} \end{array} \qquad (4)$$

where (A_z, B_z, C_z, D_z) is controllable and observable. Define a switching-function-like term as

$$\xi = \dot{e} + y_z = \dot{q} - \dot{q}_r, \quad \dot{q}_r \stackrel{\Delta}{=} \dot{q}_d(t) - y_z \qquad (5)$$

Transfer function from ξ to e is

$$e = G_\xi^{-1}(s)\xi, \quad G_\xi(s) = sI_n + G_c(s) \qquad (6)$$

where $G_c(s) = C_z(sI_{n_c} - A_z)^{-1}B_z + D_z$ is the dynamic compensator transfer function. It was shown in [9] that by suitably choosing $G_c(s)$, the resulting dynamic sliding mode $\{\xi = 0\}$, i.e., free response of transfer function $G_\xi^{-1}(s)$, can be arbitrarily shaped to possess any exponentially fast converging rate. In addition, when C_z is of full column rank, the initial value $z(0)$ of the dynamic compensator (4) can be chosen to satisfy

$$C_z z(0) = -\dot{e}(0) - D_z e(0) \qquad (7)$$

then $\xi(0) = 0$. It is shown in [9] that choosing the initial value $z(0)$ in such a way guarantees that the system is maintained in the sliding mode all the time and the reaching transient is eliminated when ideal sliding mode control is applied.

[1] Due to space limitation, only the main sketch of all the controller designs will be given. The detailed controller designs and stability proofs can be found in [13]

Let $\hat{\beta}_\pi$ be the smooth projection of $\hat{\beta}$, the estimate of β (The smooth projection is defined in [10, 11]). Then $\hat{\beta}_\pi \in \Omega_{\hat{\beta}} = \{\hat{\beta} : \beta_{min} - \varepsilon_\beta \le \hat{\beta} \le \beta_{max} + \varepsilon_\beta\}$ where ε_β is a known constant which can be arbitrarily small. Let h_β be a bounding function satisfying

$$\|Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{\beta}_\pi\| \le h_\beta(q,\dot{q},\dot{q}_r,\ddot{q}_r), \qquad \forall \hat{\beta}_\pi \in \Omega_{\hat{\beta}} \qquad (8)$$

where $\tilde{\beta}_\pi = \hat{\beta}_\pi - \beta$. For example, choose $h_\beta = \|Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\| \ \beta_M$ where $\beta_M = \|\beta_{max} - \beta_{min} + \varepsilon_\beta\|$. Define

$$h_s(q,\dot{q},\dot{q}_r,\ddot{q}_r,t) = h_f(q,\dot{q},t) + h_\beta(q,\dot{q},\dot{q}_r,\ddot{q}_r) \qquad (9)$$

Definition 1 *For any discontinuous vector like $-h\frac{\bullet}{\|\bullet\|}$ where h is a positive scalar function and $\bullet$ is a vector of functions, its continuous approximation, $\hbar(-h\frac{\bullet}{\|\bullet\|})$, with an approximation error $\varepsilon(t)$ is defined to be a vector of functions that satisfies the following two conditions:*

$$\begin{array}{ll} i. & \bullet^T \ \hbar(-h\frac{\bullet}{\|\bullet\|}) \le 0 \\ ii. & h\|\bullet\| + \bullet^T \hbar(-h\frac{\bullet}{\|\bullet\|}) \le \varepsilon(t) \end{array} \qquad (10)$$

Continuous and smooth examples of the approximation function $\hbar$ are given in [13, 9].

Theorem 1 *Choose a continuous control law as*

$$\begin{array}{lll} u & = & u_a + u_s \\ u_a & = & f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\beta}_\pi - K_\xi \xi \\ u_s & = & \hbar(-h_s\frac{\xi}{\|\xi\|}) \end{array} \qquad (11)$$

where K_ξ is any s.p.d. matrix, $\hbar(-h_s\frac{\xi}{\|\xi\|})$ is any continuous approximation of the ideal SMC control, $-h_s\frac{\xi}{\|\xi\|}$, with an approximation error $\varepsilon(t)$, and $\hat{\beta}$ is updated by

$$\dot{\hat{\beta}} = -\Gamma_\beta[l_\beta(\hat{\beta}) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)^T\xi] \qquad (12)$$

where $l_\beta(\hat{\beta})$ is any vector of modification functions [2] that satisfies the following conditions

$$\begin{array}{lll} i. \ l_\beta(\hat{\beta}) = 0 & if & \hat{\beta} \in \Omega_\beta \\ ii. \ \tilde{\beta}_\pi^T l_\beta(\hat{\beta}) \ge 0 & if & \hat{\beta} \notin \Omega_\beta \end{array} \qquad (13)$$

Then, the following results hold:

A. *In general, all the signals in the system remain bounded and tracking errors, e and $\dot{e}$, exponentially converge to some balls with size proportional to ε. Furthermore, the tracking error ξ is bounded by*

$$\|\xi(t)\|^2 \le \frac{2}{k_m}[exp(-\lambda_V t)V(0) + \int_0^t exp(-\lambda_V(t-\nu))\varepsilon(\nu)d\nu] \qquad (14)$$

where $\lambda_V = \frac{2\lambda_{min}(K_\xi)}{k_M}$, and V is a positive semi-definite (p.s.d.) function given by

$$V = \frac{1}{2}\xi^T M(q,\beta)\xi \qquad (15)$$

In addition, if (7) is satisfied, then $V(0) = 0$ in (14).

B. *If after a finite time, $\tilde{f} = 0$, then, $\xi \longrightarrow 0$, $e \longrightarrow 0$, $\dot{e} \longrightarrow 0$ when $t \longrightarrow \infty$ i.e., the robot follows the desired motion trajectories asymptotically.* △

Remark 1 *Note that, comparing to the adaptive control algorithms [25, 3, 4, 5, 26, 27, 22], the above theorem guarantees transient performance and final tracking accuracy even in the presence of uncertain nonlinearities (Results in A). Comparing to deterministic robust control [7], the method achieves asymptotic tracking or zero final tracking error in the presence of parametric uncertainties (Results in B).* □

Remark 2 *By setting $u_s = 0$ in (11), without using parameter projection and any modification to the adaptation law, and taking off the dynamic compensator (i.e., letting $C_z = 0, A_z = 0, B_z = 0, D_z > 0$ in (4)), the control law (11) reduces to Slotine and Li's well-known adaptive algorithm (SLAC), which is also tested later for comparison.* ◇

[2] Examples of l_β are given in [13, 9, 10]

IV. Desired Compensation Adaptive Robust Control (DCARC)

As in section III, the desired compensation adaptive robust control (DCARC) in [23] will be generalized in this section.

The state space realization of (6) is

$$\dot{x}_\xi = A_\xi x_\xi + B_\xi \xi, \qquad y_\xi = C_\xi x_\xi \tag{16}$$

where $x_\xi = [z^T \ , \ e^T]^T \ \in R^{n_c+n}$. Then, there exists an s.p.d. solution P_ξ for any s.p.d. matrix Q_ξ for the following Lyapunov equation [23],

$$A_\xi^T P_\xi + P_\xi A_\xi = -Q_\xi \tag{17}$$

It can be shown [23] that there are known non-negative bounded scalars γ_1, γ_2, γ_3, and γ_4, which depend on the reference trajectory and A_ξ only, such that the following inequality is satisfied

$$\begin{array}{l} \|f_0(q,\dot{q},\dot{q}_r,\ddot{q}_r) + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\beta - f_{0d}(t) - Y_d(t)\beta\| \\ \quad \le \gamma_1\|x_\xi\| + \gamma_2\|\xi\| + \gamma_3\|\xi\|\|x_\xi\| + \gamma_4\|x_\xi\|^2 \end{array} \tag{18}$$

where $f_{0d} = f_0(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d)$ and $Y_d = Y(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d)$. Similar to (8) and (9), there exists a known scalar function $h_\beta(q_d,\dot{q}_d,\ddot{q}_d)$ such that

$$\|Y_d(t)\tilde{\beta}_\pi\| \le h_\beta(q_d,\dot{q}_d,\ddot{q}_d), \quad \forall\hat{\beta}_\pi \in \Omega_{\hat{\beta}} \tag{19}$$

and define $h_s(q,\dot{q},t) = h_f + h_\beta(q_d,\dot{q}_d,\ddot{q}_d)$

Theorem 2 *Choose a continuous control law as*

$$\begin{array}{l} u = u_a + u_s \\ u_a = f_{0d} + Y_d\hat{\beta}_\pi - K_\xi\xi - K_x x_\xi - \gamma_5\|x_\xi\|^2\xi \\ u_s = \hbar(-h_s\frac{\xi}{\|\xi\|}) \end{array} \tag{20}$$

where $K_\xi > 0$ is an s.p.d. matrix, γ_5 is a positive scalar, $K_x = B_\xi^T P_\xi$, in which P_ξ is determined by (17), $\hbar$ is a continuous approximation of $-h_s\frac{\xi}{\|\xi\|}$ with an approximation error ε, and $\hat{\beta}$ is updated by

$$\dot{\hat{\beta}} = -, [l_\beta(\hat{\beta}) + Y_d^T\xi] \tag{21}$$

If controller parameters K_ξ, Q_ξ, and γ_5 are large enough such that

$$\begin{array}{l} \lambda_{min}(K_\xi) \ge \varepsilon_3 + \gamma_2 + \gamma_6 + \gamma_8 \\ \lambda_{min}(Q_\xi) \ge 2(\varepsilon_3 + \gamma_7 + \gamma_{10}) \\ \gamma_5 \ge \gamma_9 + \gamma_{11} \end{array} \tag{22}$$

where ε_3 is any positive scalar, and $\gamma_6, \gamma_7, \gamma_8, \gamma_9, \gamma_{10}$, and γ_{11} are any positive scalars satisfying

$$\gamma_6\gamma_7 = \tfrac{1}{4}\gamma_1^2, \quad \gamma_8\gamma_9 = \tfrac{1}{4}\gamma_3^2, \quad \gamma_{10}\gamma_{11} = \tfrac{1}{4}\gamma_4^2 \tag{23}$$

then, the following results hold:

A. *In general, all signals in the system remain bounded and tracking errors, e and $\dot{e}$, exponentially converge to some balls with size proportional to ε. Furthermore, the p.s.d function V defined by*

$$V = \tfrac{1}{2}\xi^T M(q,\beta)\xi + \tfrac{1}{2}x_\xi^T P_\xi x_\xi \tag{24}$$

is bounded by

$$V \le exp(-\lambda_V t)V(0) + \int_0^t exp(-\lambda_V(t-\nu))\varepsilon(\nu)d\nu \tag{25}$$

where λ_V is a positive scalar which can be arbitrarily large and satisfies

$$\lambda_V \ge \frac{2\varepsilon_3}{max\{k_M \ , \ \lambda_{max}(P_\xi)\}} \tag{26}$$

B. *If after a finite time, $\tilde{f} = 0$, then the same results as in B of Theorem 1 can be obtained.* △

Remark 3 *By setting $u_s = 0$ in (20), without using parameter projection and any modification to the adaptation law, and taking off the dynamic compensator (i.e., letting $C_z = 0, A_z = 0, B_z = 0, D_z > 0$ in (4)), the control law (20) reduces to the well-known desired compensation adaptation law (DCAL) by Sadegh and Horowitz [4], which is also implemented for comparison.* ♢

V. Nonlinear PID Robust Control

In this section, a simple robust control with nonlinear PID feedback structure is designed. The following simple control structure is suggested

$$u = f_c - (K_\xi(t) + \gamma_5\|x_\xi\|^2)\xi - K_x x_\xi \tag{27}$$

where f_c is any constant vector that is used to cancel the low frequency component, $K_\xi > 0$ is an s.p.d. matrix, γ_5 is a positive scalar, and $K_x = B_\xi^T P_\xi$, in which P_ξ is determined by (17).

Theorem 3 *If controller parameters K_ξ and γ_5 in (27), and Q_ξ in (17) are large enough such that*

$$\begin{array}{l} \lambda_{min}(K_\xi) \ge \varepsilon_3 + \gamma_2 + \gamma_6 + \gamma_8 + \frac{c_0^2}{4\varepsilon(t)} \\ \lambda_{min}(Q_\xi) \ge 2(\varepsilon_3 + \gamma_7 + \gamma_{10}) \\ \gamma_5 \ge \gamma_9 + \gamma_{11} \end{array} \tag{28}$$

where ε_3 is any positive scalar, $\gamma_6, \gamma_7, \gamma_8, \gamma_9, \gamma_{10}$, and γ_{11} are defined in Theorem 2, and

$$c_0 = \|f_c - f_{0d} - Y_d\beta\| + h_f \tag{29}$$

then, the control law (27) guarantees that all signals in the system remain bounded and tracking errors, $e(t)$ and $\dot{e}(t)$, exponentially converge to some balls, the sizes of which are proportional to ε. Furthermore, V defined by (24) is bounded by (25). ♢

Remark 4 *By choosing the dynamic compensator as an integrator, x_ξ consists of e and $\int_0^t e$; thus, control law (27) may be considered as a nonlinear PID feedback control, which is quite easy to implement since it does not require any model information, except some bounds in choosing controller parameters.* ♢

VI. Nonlinear PID Adaptive Control

Feedback gains in the nonlinear PID robust controller are required to satisfy the condition (28), in which the lower bounds are not quite straightforward to calculate. Although analytic formula exist to calculate them, as given in the above development, often the calculated lower bounds are so conservative and so large that they actually may not be used in implementation because of the limited bandwidth of physical systems. Also, the constant feedforward control term f_c may not quite match the low frequency component of the feedforward term because of parametric uncertainties. In this section, a gain-based nonlinear PID adaptive controller is proposed to solve these difficulties. We assume that only bounded disturbances appear — i.e., h_f in (3) is a constant instead of a function of states.

First, choose any $Q_\xi > 2\varepsilon_3 I$ and determine $K_x = B_\xi^T P_\xi$ by (17). There exist γ_7 and γ_{10} satisfying (28), and γ_6 and γ_{11} satisfying (23). Thus there exist constant $\bar{K}_\xi$ and $\bar{\gamma}_5$ such that (28) is satisfied. In the following, we do not need to calculate $\bar{K}_\xi$ and $\bar{\gamma}_5$, but only need to know their existence. The following control law is suggested:

$$u = \hat{f}_c - (\hat{K}_\xi + \hat{\gamma}_5\|x_\xi\|^2)\xi - K_x x_\xi \tag{30}$$

Let β_K be the independent components of K_ξ. For example, if we want a diagonal K_ξ, β_K consists of the n diagonal elements only. $\hat{\beta}_K$ represents its estimate. Then we can write

$$\begin{array}{l} \bar{K}_\xi\xi = Y_K(\xi)\bar{\beta}_K, \qquad \hat{K}_\xi\xi = Y_K(\xi)\hat{\beta}_K \\ \tilde{K}_\xi\xi = (\hat{K}_\xi - \bar{K}_\xi)\xi = Y_K(\xi)\tilde{\beta}_K, \qquad \tilde{\beta}_K = \hat{\beta}_K - \bar{\beta}_K \end{array} \tag{31}$$

where $Y_K(\xi)$ is a matrix of known functions. The gain adaptation law is chosen as

$$\begin{array}{l} \dot{\hat{f}}_c = ,'_f[-,''_f(\hat{f}_c - f_{c0}) - \xi] \\ \dot{\hat{\beta}}_K = ,'_{\beta K}[-,''_{\beta K}(\hat{\beta}_K - \beta_{K0}) + Y_K(\xi)^T\xi] \\ \dot{\hat{\gamma}}_5 = ,'_\gamma[-,''_\gamma(\hat{\gamma}_5 - \gamma_{50}) + \|x_\xi\|^2\|\xi\|^2] \end{array} \tag{32}$$

where $,'_f$, $,''_f$, $,'_{\beta K}$, $,''_{\beta K}$, $,'_\gamma$, and $,''_\gamma$ are any constant s.p.d. matrices or scalars; f_{c0}, β_{K0}, and γ_{50} are the corresponding initial estimates. Choose a p.s.d. function as

$$V_a = V + \frac{1}{2}\tilde{f}_c^T,'^{-1}_f\tilde{f}_c + \frac{1}{2}\tilde{\beta}_K^T,'^{-1}_{\beta K}\tilde{\beta}_K + \frac{1}{2}\tilde{\gamma}_5^T,'^{-1}_\gamma\tilde{\gamma}_5 \tag{33}$$

where $\tilde{f}_c = \hat{f}_c - f_c$, $\tilde{\gamma}_5 = \hat{\gamma}_5 - \bar{\gamma}_5$, and V is defined by (24).

Theorem 4 *If the control law (30) with the gain adaptation law (32) is applied, then*

A . *In general, all signals in the system remain bounded.*

B . *In addition, if the initial estimates β_{K0} and γ_{50} are large enough such that the condition (28) is satisfied for $f_c = f_{c0}$, then, tracking errors exponentially converge to some balls whose sizes are proportional to controller parameter ε.* △

Remark 5 *The above adaptive controller does not require any model information and has a simple nonlinear PID feedback structure. Thus, it can be easily implemented and costs little computation time, however, bounded disturbances are assumed in the development, and asymptotic stability is not guaranteed even in the presence of parameter uncertainties only. Also, when the initial estimates do not satisfy the condition (28), the error bound is not guaranteed to be reduced by suitably choosing controller gains and theoretical performance may thus not be guaranteed.* ◇

VII. Desired Compensation ARC with Adjustable Gains

The DCARC scheme in section IV requires that feedback gains satisfy condition (22), which has the same drawback as the nonlinear PID robust control (NPID) scheme, as pointed out in the above section. In this section, by incorporating the gain-based adaptive control synthesis technique into the design of the DCARC scheme, a new adaptive robust controller is proposed to overcome this difficulty.

As in the above section, since $\bar{K}_\xi$ and $\bar{\gamma}_5$ are unknown, instead of using constant feedback gains K_ξ and γ_5 in (20), they will be adjusted on-line as in the above gain-based adaptive control. The resulting control law is given by

$$\begin{array}{l} u = u_a + \hbar(-h_s \frac{\xi}{\|\xi\|}) \\ u_a = f_{0d} + Y_d \hat{\beta}_\pi - \hat{K}_\xi \xi - K_x x_\xi - \hat{\gamma}_5 \|x_\xi\|^2 \xi \end{array} \quad (34)$$

in which the parameter adaptation law for β is the same as in DCARC, and the gain adaptation laws are suggested as

$$\begin{array}{l} \dot{\hat{\beta}}_K = \Gamma'_{\beta K}[-\Gamma''_{\beta K}(\hat{\beta}_K - \beta_{K0}) + Y_K(\xi)^T \xi] \\ \dot{\hat{\gamma}}_5 = \Gamma'_\gamma[-\Gamma''_\gamma(\hat{\gamma}_5 - \gamma_{50}) + \|x_\xi\|^2 \|\xi\|^2] \end{array} \quad (35)$$

Choose a positive definite (p.d.) function as

$$V_p = V + \frac{1}{2}\tilde{\beta}_K^T \Gamma'^{-1}_{\beta K} \tilde{\beta}_K + \frac{1}{2}\tilde{\gamma}_5^T \Gamma'^{-1}_\gamma \tilde{\gamma}_5 \quad (36)$$

where V is defined by (24).

Theorem 5 *If the control law (34) with adaptation laws (21) and (35) is applied,*

A. *In general, all signals in the system remain bounded and the bounds of the tracking error can be found in [13].*

B. *In addition, if the initial estimates β_{K0} and γ_{50} are large enough such that the condition (22) is satisfied, then*

a). *Tracking errors exponentially converge to some balls whose sizes are proportional to the controller parameter ε.*

b). *If after a finite time, $\tilde{f} = 0$ (no uncertain nonlinearities), asymptotic tracking is also achieved.* △

VIII. Experimental Results

All schemes presented before are implemented and compared. In addition, Slotine and Li's adaptive algorithm (SLAC) [3] and Sadegh and Horowitz's DCAL [4] are also implemented for comparison.

Experiments are conducted on the planar UCB/NSK two axis SCARA direct drive manipulator system. The details of the experimental setup can be found in [9, 13]. The friction term $F_f(q, \dot{q})$ is lumped into $\tilde{f}(q, \dot{q}, t)$ and is bounded by (3), where $h_f = 9$. In the experiment, only payload mass m_p is unknown with the maximum payload, $m_{pmax} = 10kg$. Thus, letting $\beta = m_p$ and $\Omega_\beta = (-0.00001,\ m_{pmax} + 0.00001)$, (2) can be formed. Since all the controllers are supposed to deal with model uncertainties, the initial estimate of the payload is set to $9kg$, with an actual value in experiments being around $0.7kg$. All experiments are conducted with a sampling time $\Delta T = 1ms$.

A. *Performance Indexes*

Since we are interested in tracking performance, sinusoidal trajectories with a smoothed initial starting phase are adopted for each joint. In this experiment, the desired joint trajectories are $q_d = [1.5(1.181 - 0.3343exp(-5t) - cos(\pi t - 0.561))\ ,\ 1.3045 - 0.538exp(-5t) - cos(\frac{4}{3}\pi t - 0.697))]^T$ (rad), which are reasonably fast. Zero initial tracking errors are used and each experiment is run for ten seconds, i.e, $T_f = 10s$.

Commonly used performance measures, such as the rising time, damping and steady state error, are not adequate for nonlinear systems like robots. In [22], the scalar valued L^2 norm given by $L^2[e(t)] = (\frac{1}{T_f}\int_0^{T_f} \|e(t)\|^2 dt)^{1/2}$ is used as an objective numerical measure of tracking performance for an entire error curve $e(t)$. However, it is an average measure, and large errors during the initial transient stage cannot be predicted. Thus, the sum of the maximal absolute value of tracking error of each joint, $e_M = e_{1M} + e_{2M}$, is used as an index of measure of transient performance, in which $e_{iM} = \max_{t\in[0,T_f]}\{|e_i(t)|\}$. The maximal absolute value and the average tracking error of each joint during the last three seconds are defined by $e_{iF} = \max_{t\in[T_f-3,T_f]}\{|e_i(t)|\}$ and $L[e_{if}] = \frac{1}{3}\int_{T_f-3}^{T_f} |e_i| dt$ respectively. Then, $e_F = e_{1F} + e_{2F}$ and $L[e_f] = L[e_{1f}] + L[e_{2f}]$ are used as indexes to measure the steady state tracking error. The average control input of each joint, $L[u_i] = \frac{1}{T_f}\int_0^{T_f} |u_i| dt$, is used to evaluate the amount of control effort. The average of control input increments of each joint is defined by $L[\Delta u_i] = \frac{1}{10000}\sum_{k=1}^{10000} |u_i(k\Delta T) - u_i((k-1)\Delta T)|$. The sum of the normalized control variations of each joint, $c_u = \sum_{i=1}^{2} \frac{L[\Delta u_i]}{L[u_i]}$, is used to measure the degree of control chattering.

B. *Controller Gains*

The choice of feedback gains is crucial to achieve a good tracking performance for all controllers. The detailed discussion of the gain tuning processes for each controller is given in [13]. In general, the larger the feedback gains (especially, the gain K_ξ), the smaller the tracking errors. However, if the gains are too big, the robot will be subject to severe control chattering due to the measurement noise and the neglected high-frequency dynamics and a large noisy sound can be heard. After the gains exceed certain limits, the structural resonance is excited because of severe control chattering and the system goes unstable. Thus, in order to achieve a fair comparison, we tried to tune gains of each controller such that the tracking errors of each controller are minimized while maintaining the same degree of control chattering for all controllers.

C. *Comparative Experimental Results*

As in [22], we first test the reliability of the results by running the same controller several times. It is found that the standard deviation of the error from different runs is negligible.

The experimental results are shown in the following table (unit is rad for tracking errors and Nm for control input torques). In the table, ASMC stands for the adaptive sliding mode control presented in section III; DCARC for the desired compensation ARC in section IV; DCRC(I) and DCRC(NI) for the desired compensation robust control obtained by switching off parameter adaptation in DCARC and DCAL respectively; NPID for nonlinear PID robust control in section V. NPID(I) (or NPID(NI)) for the case with (or without) the dynamic compensator (**??**) respectively; PIDAC for the nonlinear PID adaptive control in section VI; and ARCAG for the controller in section VII.

Table 1: Experimental Results

Controller	e_M	e_F	$L[e_f]$	$L_2[e]$	$L[u_1]$	$L[u_2]$	c_u
ASMC	0.03	0.02	0.006	0.006	32	6	0.54
SLAC	0.05	0.03	0.016	0.013	33	6	0.55
DCARC	0.02	0.01	0.004	0.004	31	6	0.41
DCAL	0.04	0.02	0.009	0.008	30	6	0.43
DCRC(I)	0.03	0.02	0.008	0.008	30	6	0.42
DCRC(NI)	0.07	0.05	0.018	0.021	30	6	0.40
NPID(I)	0.02	0.02	0.007	0.006	31	6	0.41
NPID(NI)	0.04	0.04	0.015	0.015	30	6	0.40
PIDAC	0.07	0.02	0.006	0.007	30	6	0.44
ARCAG	0.04	0.01	0.004	0.005	30	6	0.42

Based on the above experimental data, the following can be concluded:

a . **Parameter Adaptation Improves Tracking Accuracy**
If we compare the parameter-based adaptive controllers with their robust counterparts, i.e., DCARC versus DCRC(I), DCAL versus DCRC(NI), then we can see that, in terms of both final tracking accuracy (Fig. 2) and average tracking errors, parameter adaptation reduces the tracking errors around a factor of 2. The parameter-based adaptive controllers also have better transient performances (Fig. 1). The improvement comes from the fact that the estimated payloads approach their true values (not shown). This result verifies the advantage of introducing *parameter adaptation*. All controllers use almost the same amount of control effort and have the same degree of control chattering, as seen from the table, and thus the comparison is fair.

b . **Dynamic Compensator Improves Tracking Accuracy**
Comparing the controllers having dynamic compensators with their counterparts not employing dynamic compensators, i.e., DCRC(I) versus DCRC(NI) and NPID(I) versus NPID(NI), we can see that introducing dynamic compensators reduces the tracking errors by more than a factor of 2 in terms of all the performance indexes, as shown in Fig. 1 and Fig. 2. The comparison is fair, as shown by the control effort and the degree of control chattering in Table 1. This result supports the importance of employing *proper controller structure*.

c . **Desired Compensation Improves Tracking Accuracy**
Comparing the controllers having desired compensation with their counterparts using actual state in model compensation design, i.e., DCARC versus ASMC and DCAL versus SLAC, we can see that, in terms of all performance indexes (Fig. 1 and Fig. 2), the controllers with desired compensation have better tracking performances. They also have less degrees of control chattering, as shown in Table 1.

d . **Gain-based Adaptive Controllers via Fixed-gain Robust Controllers**
If we compare the gain-based adaptive controllers with their robust counterparts, i.e., PIDAC versus NPID(I) and ARCAG versus DCARC, we can see that gain-based adaptive controllers can have a large stability margin for the choice of feedback gains since they can use small initial gain estimates. Because of the small initial estimates, they have larger initial tracking errors or poorer transient response, as seen from Fig. 1. The estimated feedback gains (e.g., $\hat{K}_\xi$ shown in Fig. 3) increase quickly to some values that are slightly larger than the fixed feedback gains used in their robust counterparts (e.g., when $t = 10s$, $\hat{K}_\xi(t) = diag\{180,\ 12.6\}$ for PIDAC but $K_\xi = diag\{160, 12\}$ for NPID(I)). This is the reason that they achieve a slightly better final tracking accuracy. We should keep in mind, however, that this advantage comes from the slightly increased degree of control chattering, as shown in Table 1. Therefore, in practice, gain-based adaptive controllers do not offer much advantage in improving tracking performance. They may be used in the initial gain-tuning process to obtain the lower bound of the stabilizing feedback gains instead of using a troublesome and conservative theoretical formula like (28). However, caution should be taken. Large dampings (e.g., $,''_{\beta K}$ and $,''_{\gamma}$ in (32)) should be used; otherwise, the resulting final estimates may be too big that they may exceed the practical limits and destabilize the system because of their gain adaptation nature.

Since the proposed DCARC possesses all the desirable good qualities — parameter adaptation, dynamic compensator, and desired compensation — it is natural that it achieves the best tracking performance, as seen from Table 1 (or Fig. 1 and 2), by using the same amount of control effort and control chattering. These facts show again the importance of using the both means, parameter adaptation and proper controller structure, in designing high performance controllers, which is the main theme of the proposed ARC. Using either one of them alone is not enough — in fact, in these experiments, probably because the effect of link dynamics is not so severe and the disturbances and measurement noise are not so small, the proposed simple NPID robust controller outperforms DCAL, the adaptive controller that achieves the best tracking performance among existing adaptive controllers tested.

The tracking errors of DCARC are plotted in Fig. 4. Those spikes of the tracking errors after the initial transient occur at the time when the joint velocities change their directions. Thus, they are mainly caused by the discontinuous Columb friction

IX. Conclusions

In this paper, the proposed adaptive robust control is applied to the trajectory tracking control of robot manipulators. Two schemes are developed: ASMC is based on the conventional adaptation structure and DCARC is based on the desired compensation adaptation structure. A dynamic sliding mode is used to enhance the system response. In addition, several conceptually different robust and adaptive controllers are also constructed for comparison — a simple nonlinear PID type robust control, and a simple gain-based adaptive control, which requires almost no model information, and a combined parameter and gain-based adaptive robust control. All algorithms, as well as two existing adaptive control algorithms, SLAC and DCAL, are implemented on a two-link SCARA type robot manipulator to study their advantages and disadvantages. Comparative experimental results show the importance of using both *proper controller structure* and *parameter adaptation* in designing high performance controllers, which is the main feature in the newly developed adaptive robust control [9, 12, 13]. It is observed that in these experiments, the proposed DCARC improves tracking performance without increasing control effort. Thus, the work in this paper serves for the two purposes: improving tracking performance of robot control systems and setting up a standard with which various control algorithms could be compared.

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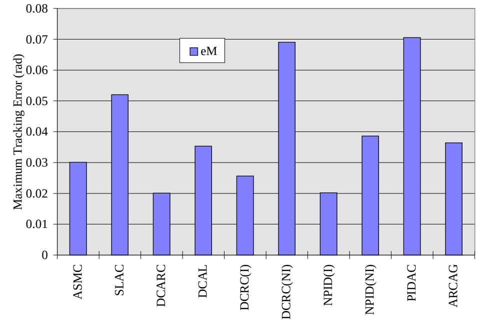

Figure 1: Transient Performance

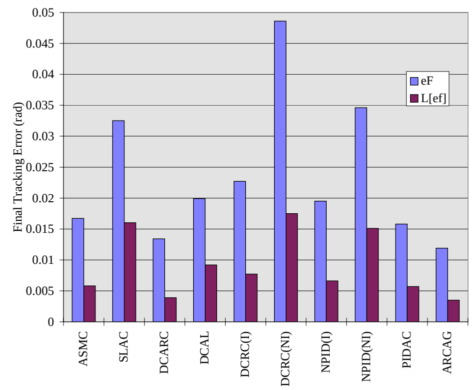

Figure 2: Final Tracking Accuracy

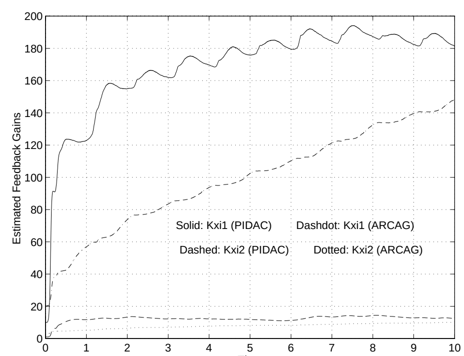

Figure 3: Estimated Feedback Gains $\hat{K}_\xi$

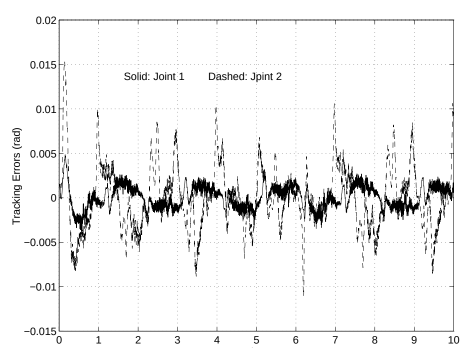

Figure 4: Joint Tracking Errors