High Performance Adaptive Robust Control Of Nonlinear Systems: A General Framework and New Schemes

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Abstract

A general framework is proposed for the design of a new class of high-performance robust controllers. The framework is based on the recently proposed adaptive robust control (ARC), which effectively combines deterministic robust control with adaptive control. The approach intends to use all available means in achieving high performance; robust filter structures are used to attenuate the effect of model uncertainties as much as possible while learning mechanisms such as parameter adaptation are used to reduce the model uncertainties. Under the proposed general framework, a simple new ARC controller is also constructed for a class of nonlinear systems transformable to a semi-strict feedback form. The new design utilizes the popular discontinuous projection method in solving the conflicts between the deterministic robust control design and the adaptive control design, which is much simpler than the smooth projection or the smooth modifications of adaptation law used in the previously proposed ARC controllers. The controller achieves a guaranteed transient performance and a prescribed final tracking accuracy in the presence of both parametric uncertainties and uncertain nonlinearities while achieving asymptotic stability in the presence of parametric uncertainties without using a discontinuous control law or infinite-gain feedback.

1 INTRODUCTION

Control of uncertain nonlinear dynamics has been one of the mainstream areas of focus in control community during the past twenty years. Two nonlinear control methods have been popular: adaptive control [1, 2, 3, 4] and deterministic robust control (DRC) [5, 6, 7, 8, 9, 10]. The adaptive control achieves asymptotic tracking for reasonably large classes of nonlinear systems without using discontinuous or infinite-gain feedback [1]. Adaptive controllers deal with the ideal case of constant parametric uncertainties only, and uncertain nonlinearities such as unmodeled nonlinear friction force and external disturbances are not considered. The adaptation law may lose stability even when a small disturbance appears [11]. Every physical system is subject to some form of disturbance, and additional effort has to be made to implement such adaptive nonlinear controllers safely. One may apply remedies similar to those used in robust adaptive control of linear systems. However, such modifications [11, 12] do not guarantee tracking accuracy since the steady state tracking error can be shown to stay within an unknown region only, and the size of the region depends on disturbances. Furthermore, transient performance is normally unknown and the actual system response may be too sluggish to be used in the control of high-speed/high-accuracy mechanical systems. In contrast, in general, deterministic robust control–e.g., sliding mode control [5]-can be used to achieve a guaranteed transient performance and final tracking accuracy in the presence of both parametric uncertainties and uncertain nonlinearities. However, it usually involves switching [5] or infinite-gain feedback [8], which introduces chattering. Chattering may be avoided at the expense of degraded tracking performance by using some smoothing techniques [9, 13].

Recently, in [13], Yao and Tomizuka presented a systematic way to combine the adaptive control and the sliding mode control (SMC) for the trajectory tracking control of robot manipulators to preserve the advantages of the two methods while overcoming their drawbacks. Comparative experimental results for the motion control of robot manipulators [14, 15] have demonstrated the improved performance of the suggested methods. In [16, 17, 15], the methodology is generalized to a class of multiple-inputs-multiple-outputs (MIMO) nonlinear systems transformable to semi-strict feedback forms. The forms allow coupling and appearance of parametric uncertainties in the input channels of each layer. Applications include the high performance robust control of robot manipulators in various applications such as the constrained motion and force tracking control [18, 19], coordinated motion and force tracking control of multiple robots grasping a common object [20], and motion and force tracking control of robot manipulators in contact with stiffness surfaces with unknown stiffness [21]. The approach is also applied to the motion control of machine tools [22] by incorporating digital feedforward control [23]. Experimental results show that the maximum tracking errors in high-speed operations can be reduced to the encoder resolution level.

This paper serves for two purposes: one is to formalize and generalize the above ARC approach to establish a general theoretical framework for the design of high-performance robust controllers and the other is to construct simple and practical ARC controllers under the framework. For the first purpose, the paper will focus on the fundamental issues and viewpoints of the proposed ARC, and present a general structure for the ARC controllers. To serve for the second purpose, the paper will present a new ARC controller for a class of SISO nonlinear systems transformable to the semi-strict feedback form [24, 4]. Instead of using the smooth pro-

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jection [24, 16, 17, 15] or the smooth modification of adaptation law such as the generalized σ-modification [13, 4], the proposed ARC controller employs the widely used discontinuous projection method in adaptive systems [25, 26] to solve the conflicts between the robust control design and adaptive control design. As a result, the resulting controller is simpler and the parameter adaptation process is more robust in the presence of uncertain nonlinearities. The discontinuous projection method has been successfully implemented and tested in the motion control of robot manipulators [13, 14] and the motion control of machine tools [22], in which the design techniques for both systems are essentially for nonlinear systems with "relative degree" of *one*. For nonlinear systems with "relative degree" of more than one, the underlining parameter adaptation laws in the previously proposed ARC controllers [24] and the robust adaptive control designs [4, 12] are based on the tuning function based adaptive backstepping design [1], which needs to incorporate the adaptation law in the design of control functions at each step. As a result, either smooth projections [24, 17, 15] or smooth modifications of adaptation law [4, 13] are necessary since the control functions have to be smooth for backstepping design; either method is technical and is hard to be implemented. In contrast, this paper will show that the simple discontinuous projection method can also be used in the ARC design by strengthening the corresponding robust control design.

2 GENERAL PHILOSOPHY AND STRUCTURE OF ARC CONTROLLERS

The focus of the proposed ARC is to achieve high performance in practical situations. As such, the problem will be formulated under the general setting that the system is subjected to both parametric uncertainties and uncertain nonlinearities as in DRC. Furthermore, the approach will seek other means to reduce model uncertainties to overcome the conservative design of DRC to improve performance. For example, the previously proposed ARC controllers differentiate between parametric uncertainties and uncertain nonlinearities and use parameter adaptation to reduce the effect of parametric uncertainties. In summary, *the proposed ARC intends to seek all available means to achieve high performance: (i) robust filter structures will be employed to attenuate the effect of model uncertainties as much as possible to guarantee certain transient performance and final tracking accuracy in general, and (ii) mechanisms which will reduce model uncertainties (e.g., parameter adaptation) will be sought and introduced in the design whenever possible to further improve performance.*

The first mean is normally accomplished by robust feedback control design and the second mean is normally done through some learning processes such as parameter adaptation. In general, the two means will interact with each other and the design techniques associated with them may have serious philosophical conflicts. Thus, one of the major difficulties in designing ARC controllers lies in being able to solve the conflicts to integrate the two means. In some situations, some compromises have to be made. *The general design philosophy is that nominal robust performance provided by the first mean should not be lost when introducing learning mechanisms.* In other words, the approach takes the viewpoint that a control law has to be robust first (in the sense of not only stability but also performance). Learning mechanisms are introduced only when their destabilizing effects can be controlled. Such a design philosophy differs from other schemes in the literature that use both the adaptation and control terms normally used in DRC. In all those schemes [27, 28, 29, 30, 31], transient performance was not guaranteed.

The new viewpoint of the utilization of parameter adaptation differs from that in adaptive control field. Adaptive control heavily relies on the on-line parameter estimation to achieve certain stability results. For example, the feedback control law used in the self-tuning regulator [32] is designed based on the on-line parameter estimates. As a result, the performance of the system depends heavily on the quality of the on-line parameter estimates. However, because of the occasional and unavoidable unidentifiability of the system caused by non-persistent excitation, the quality of parameter estimates cannot be guaranteed in general. In return, adaptive controllers may exhibit poor transient performance and are not robust to disturbances. In contrast, the new viewpoint puts emphasis on robust feedback control design. As a result, transient performance and the closed-loop system's robustness can be improved in general.

To achieve the objectives described above, an ARC controller needs the following four components: (i) adjustable model compensation; (ii) robust control law; (iii) learning mechanisms; and (iv) coordination mechanisms. The general structure is illustrated in Fig.1. For simplicity in the discussion, learning mechanisms will be restricted to parameter adaptation in this paper.

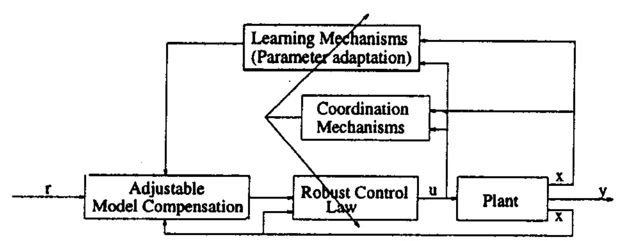

Figure 1: General Structure of ARC controllers

Plant Characterization: For simplicity, the following general MIMO nonlinear plant model will be considered

$$\begin{aligned} \dot{x} &= f(x,\theta,t) + B(x,\theta,t)u + D(x,t)\Delta(x,\theta,u,t) \\ y &= h(x,t) \end{aligned} \quad (1)$$

where $y \in R^m$ and $u \in R^m$ are the output and input vectors respectively, $x \in R^n$ is the state vector, $\theta(t) \in R^p$ is the vector of unknown parameters, $h(x,t), f(x,\theta,t), B(x,\theta,t)$, and $D(x,t)$ are known [1], and $\Delta(x,\theta,u,t) \in R^{l_d}$ represents the vector of unknown nonlinear functions such as disturbances and modeling errors. The following practical assumption is normally made:

Assumption 1 *The unknown parameters lie in a known bounded region Ω_θ and the unknown nonlinear functions Δ are bounded by some known functions $\delta(x,t)$, i.e.,*

$$\begin{aligned} \theta &\in \Omega_\theta \triangleq \{\theta : \ \theta_{min} < \theta < \theta_{max} \} \\ \Delta &\in \Omega_\Delta \triangleq \{\Delta : \ \|\Delta(x,\theta,u,t)\| \le \delta(x,t) \} \end{aligned} \quad (2)$$

[1] A vector or matrix is called known if each of its components is a known function of its variables

where
$\theta_{min} = [\theta_{1min}, \ldots, \theta_{pmin}]^T$, $\theta_{max} = [\theta_{1max}, \ldots, \theta_{pmax}]^T$, and $\delta(x, t)$ are known. ◇

Adjustable Model Compensation: In order to track a reference signal $r(t)$, model compensation is necessary to provide the correct desired control action. However, since the plant model has parametric uncertainties, exact model compensation is not possible. Unlike DRC design, where fixed parameter estimates are used in the model compensation, here, parameter estimates will be adjusted by learning mechanisms to provide an improved model compensation.

Robust Control Law: A robust control law is used to attenuate the effect of model uncertainties in general. The difficulty here is that the usual DRC design technique cannot be directly applied to synthesize the needed robust control law since DRC normally handles fixed model compensation only. Thus robust control techniques, which can account for the effect of adjustable model compensation, have to be sought; this will be illustrated later by specific design examples.

Learning Mechanisms: On-line parameter adaptation is necessary for providing correct model compensation to achieve zero tracking error in the presence of parametric uncertainties. The adaptation law used to update the parameter estimates can be synthesized by usual adaptive control techniques.

Coordination Mechanisms: Coordination mechanisms are used to solve the inherent conflicts between the robust control law design and the adaptive control design. The parameter estimates provided by adaptive control may go unbounded when the plant has uncertain nonlinearities and thus may destabilize the system since the robust control law cannot attenuate an unbounded model uncertainty, which needs infinite control strength. Intelligent utilization of the prior information, such as the bounds of parametric uncertainties, is the key to solving the destabilizing effect of parameter adaptation problem while retaining its nominal learning capability.

The above general concept about ARC controllers has been partially utilized in the previously proposed ARC controllers [13, 24, 16, 17, 15]. In the subsequent sections, the concept will be illustrated by specific examples and will be used to construct new simple and practical ARC controllers.

3 ARC OF A FIRST-ORDER SYSTEM

In this section, robust tracking control of a simple first-order system will be used to illustrate the general philosophy of the proposed ARC and the function of each part. The system is described by

$$\dot{x} = \varphi^T(x,t)\theta + \Delta(x,t) + u \tag{3}$$

where $x, u \in R$, and θ and Δ satisfy Assumption 1. The objective is to let x track its desired trajectory $x_d(t)$.

As illustrated in Fig.1, the control law consists of two parts given by

$$\begin{array}{l} u = u_f + u_s \\ u_f = \dot{x}_d(t) - \varphi^T\hat{\theta}_\pi(t) \\ u_s = u_{s1} + u_{s2}, \qquad u_{s1} = -kz \end{array} \tag{4}$$

where $z = x - x_d$ is the tracking error, $\hat{\theta}_\pi$ is the adjustable parameter needed for achieving correct model compensation, u_f represents the adjustable model compensation, and u_s is the robust control law consisting of two parts: u_{s1} is used to stabilize the nominal system, which is a simple proportional feedback in this case; and u_{s2} represents robust feedback used to attenuate the effect of model uncertainties, which will be synthesized later. Substituting (4) into (3), the error equation is

$$\dot{z} + kz = -\varphi^T\tilde{\theta}_\pi + \Delta(x,t) + u_{s2} \tag{5}$$

where $\tilde{\theta}_\pi = \hat{\theta}_\pi - \theta$ represents the parametric uncertainties. It is thus clear that if we can design a robust feedback u_{s2} such that the following condition is satisfied

$$\text{condition i} \qquad z[-\varphi^T(x,t)\tilde{\theta}_\pi(t) + \Delta(x,t) + u_{s2}] \le \varepsilon \tag{6}$$

where ε is a design parameter, then, the derivative of $V_s = \frac{1}{2}z^2$ is

$$\dot{V}_s = -kz^2 + z[-\varphi^T\tilde{\theta}_\pi + \Delta + u_{s2}] \le -kz^2 + \varepsilon \le -2kV + \varepsilon \tag{7}$$

So

$$|z|^2 \le exp(-2kt)|z(0)|^2 + \tfrac{\varepsilon}{k}[1 - exp(-2kt)] \tag{8}$$

Thus the tracking error exponentially decays to a ball. The exponential converging rate $2k$ and the size of the final tracking error ($|z(\infty)| \le \sqrt{\frac{\varepsilon}{k}}$) can be freely adjusted by the controller parameters ε and k in a known form. In other words, transient performance is guaranteed.

In the above development, u_{s2} is synthesized to dominate the model uncertainties coming from both the parametric uncertainties and uncertain nonlinearities to guarantee transient performance as seen from (6). For any given u_{s2}, the actual tracking error z will be proportional to the extent of actual model uncertainties as seen from (5). So in the following, we will use parameter adaptation to reduce the model uncertainties to further improve the tracking performance.

Conventionally, the adaptation law synthesized by adaptive control [1] is given by

$$\dot{\hat{\theta}} = \Gamma\varphi z \tag{9}$$

where Γ is any p.d. matrix. If we let $\hat{\theta}_\pi = \hat{\theta}$ in (4), such an adaptation law will eliminate the effect of parametric uncertainties (i.e., $\varphi^T\tilde{\theta}_\pi \longrightarrow 0$ in (5) as $t \longrightarrow \infty$) in the presence of parametric uncertainties only and asymptotic tracking can be achieved [15]. However, such an adaptation law can go unbounded even in the presence of a small disturbance [11]. As a result, it cannot be directly used in the robust control law (4) since no finite robust control term u_{s2} can be found to attenuate an unbounded model uncertainty in (6). To solve this conflict, several coordination mechanisms can be used. The first one is to use the bounded smooth projection of the estimated parameter in the robust control law only as done in [24, 17, 15], i.e., let $\hat{\theta}_\pi = \pi(\hat{\theta})$ where π is a bounded smooth projection map defined in [24, 33]. Such a modification ensures that $\hat{\theta}_\pi$ stays in a known bounded range all the time no matter if the estimated parameter will go unbounded or not. As a result, a finite robust control law can be determined from (4) and (6) to guarantee the transient performance and final tracking accuracy in general. Furthermore, in the presence of parametric uncertainties only, the smooth projection will not interfere with the normal identification process of the adaptation law and the nominal performance of adaptive control–asymptotic tracking–is preserved. However, this method suffers from the drawback that the internal parameter $\hat{\theta}$ may still go unbounded although it does not affect the stability of the actual system.

The second coordination mechanism is to modify the adaptation law to make it robust to uncertain nonlinearities so that we can let $\hat{\theta}_\pi = \hat{\theta}$ in (4). This can be done by either the variation of σ-modification [13], which is a smooth modification, or the discontinuous projection method [13]. The variations of σ-modification method used in [13, 4] suffers from the problem that the bound on the estimated parameter $\hat{\theta}$ can not be known in advance. Thus, the robust control law cannot be determined from (6) for a predetermined ε. As a result, transient performance cannot be pre-specified. In contrast, the discontinuous projection method used in [13] guarantees that $\hat{\theta}$ stays in a known bounded region all the time [25, 26]. Thus, it does the same job as the smooth projection but does not have the problem associated with the smooth projection mentioned before. The resulting adaptation law for $\hat{\theta}_\pi = \hat{\theta}$ is

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\varphi z) \tag{10}$$

where the projection mapping $Proj_{\hat{\theta}}(\bullet)$ is defined by [25, 26] (for simplicity, assume that Γ is a diagonal matrix in the following)

$$Proj_{\hat{\theta}}(\bullet) = \begin{cases} 0 & if \quad \begin{cases} \hat{\theta}_i = \hat{\theta}_{imax} & and \quad \bullet > 0 \\ \hat{\theta}_i = \hat{\theta}_{imin} & and \quad \bullet < 0 \end{cases} \\ \bullet & \quad otherwise \end{cases} \tag{11}$$

It can be shown [25, 26, 13] that the projection mapping has the following nice properties

$$\begin{array}{ll} \text{P1} & \hat{\theta} \in \hat{\Omega}_\theta = \{\hat{\theta} : \theta_{min} \le \hat{\theta} \le \theta_{max}\} \\ \text{P2} & \tilde{\theta}^T(\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\bullet) - \bullet) \le 0, \quad \forall\bullet \end{array} \tag{12}$$

The last step in ARC design is to make sure that the robust control term will not interfere with the nominal identification process of parameter adaptation. This can be easily satisfied by putting a trivial passive-like constraint on the selection of u_{s2} as follows:

$$\text{condition ii} \qquad zu_{s2} \le 0 \tag{13}$$

which can be satisfied easily as shown in the later's design.

4 SIMPLE ARC CONTROLLERS FOR SISO NONLINEAR SYSTEMS

As seen from the above section, the discontinuous projection method is simple and yet solves the conflict between the robust control and adaptive control. In this section, it will be utilized to construct a simple ARC controller for SISO nonlinear systems transformable to the following semi-strict feedback form [24, 12]

$$\begin{array}{l} \dot{x}_i = x_{i+1} + \theta^T\varphi_i(x_1,\ldots,x_i,t) + \Delta_i(x,t), \quad i \le n-1 \\ \dot{x}_n = \sigma(x)u + \theta^T\varphi_n(x,t) + \Delta_n(x,t) \\ y = x_1 \end{array} \tag{14}$$

where $x = [x_1,\ldots,x_n]^T$. $\varphi_i(x_1,\ldots,x_i,t) \in R^p$, $i = 1,\ldots,n$, are the known shape functions, which are assumed to be sufficiently smooth. θ satisfies (2) and the unknown nonlinearities $\Delta_i(x,t)$ are assumed to be bounded by

$$|\Delta_i(x,t)| \le \delta_i(\bar{x}_i,t), \qquad i = 1,\ldots,n \tag{15}$$

where $\bar{x}_i = [x_1,\ldots,x_i]^T$, and $\delta_i(\bar{x}_i,t)$ are known. The objective is to design a bounded control law for the input u such that the system is stable and the output y tracks the desired output trajectory $y_d(t)$ as close as possible.

In [24], an ARC controller was presented for (15) by using the smooth projection to solve the conflicts. The underlining adaptation law was based on the standard backstepping tuning function design in [1]. The design needs to incorporate the adaptation functions (or tuning functions) in the construction of the smooth control functions at each step, which prohibits the use of any discontinuous modification. In the following, the robust control law will be strengthened so that the adaptation functions will not be needed in the design. As a result, the discontinuous projection method can be used to modify the adaptation law. The design proceeds in the following steps.

4.1 Step 1

Let $\tilde{\Delta}_1(x,t) = \Delta_1(x,t)$. The first equation of (14) can be rewritten as

$$\dot{x}_1 \quad = \quad x_2 + \theta^T\varphi_1(x_1,t) + \tilde{\Delta}_1(x,t) \tag{16}$$

Noticing that $|\tilde{\Delta}_1(x,t)| \le \tilde{\delta}(x_1,t) \triangleq \delta(x_1,t)$, the ARC design in section 3 can be applied to (16) to synthesize a virtual control law α_1 for x_2 so that x_1 tracks its desired trajectory $x_{1d}(t)$ with the desired properties mentioned in section 3. If x_2 were the actual control input, then the adaptation law would be given by (10) and the control law would be given by (4). Since it is not the case, we postpone the specification of the adaptation law and use the first tuning function [1]

$$\tau_1(x_1,t) = \phi_1(x_1,t)z_1, \qquad \phi_1(x_1,t) \triangleq \varphi_1(x_1,t) \tag{17}$$

to denote the essential part of the adaptation law (10), where $z_1 = x_1 - x_{1d}$ is the tracking error. *However, in contrast with [24, 1], the adaptation function τ_1 will NOT be used in the control law design at each step to allow for the use of discontinuous projection later. The control law has thus to be strengthened to compensate for the loss of information as follows.* The new control function is

$$\begin{array}{l} \alpha_1(x_1,\hat{\theta},t) = \alpha_{1f} + \alpha_{1s} \\ \alpha_{1f} = \dot{x}_{1d}(t) - \hat{\theta}^T(t)\varphi_1(x_1,t) \\ \alpha_{1s} = \alpha_{1s1} + \alpha_{1s2}, \qquad \alpha_{1s1} = -(k_1 + g_1\|\Gamma\phi_1\|^2)z_1 \end{array} \tag{18}$$

where $k_1 > 0$, $g_1 \ge 0$, and $\alpha_{1s2}(x_1,\hat{\theta},t)$ is any function satisfying the similar conditions as (6) and (13), i.e.,

$$\begin{array}{ll} \text{i} & z_1[-\tilde{\theta}^T\varphi_1 + \tilde{\Delta}_1 + \alpha_{1s2}(x_1,\hat{\theta},t)] \le \varepsilon_1, \quad \forall\hat{\theta} \in \hat{\Omega}_\theta \\ \text{ii} & z_1\alpha_{1s2} \le 0 \end{array} \tag{19}$$

in which ε_1 is a design parameter. Define $z_2 = x_2 - \alpha_1(x_1,\hat{\theta},t)$. Substituting (18) into (16), the first error subsystem S_1 becomes

$$\dot{z}_1 + (k_1 + g_1\|\Gamma\phi_1\|^2)z_1 = z_2 - \tilde{\theta}^T\phi_1 + \tilde{\Delta}_1 + \alpha_{1s2} \tag{20}$$

Choose $V_{s1} = \frac{1}{2}z_1^2$. From (20), its time derivative is

$$\dot{V}_{s1} = z_1z_2 - (k_1 + g_1\|\Gamma\phi_1\|^2)z_1^2 + z_1[\tilde{\Delta}_1 + \alpha_{1s2}] - \tilde{\theta}^T\tau_1 \tag{21}$$

4.2 Step i

Mathematical induction will be used to explain the remaining intermediate design steps. At step i, the ARC design used in the above step will be employed to construct a control function α_i for x_{i+1} so that x_i will track its desired ARC control law α_{i-1} synthesized at step $i-1$ (for simplicity, denote $\alpha_0(t) = x_{1d}(t)$) with a desired transient performance. Let $z_j = x_j - \alpha_{j-1}$ and recursively define the following functions for step j from those in the previous steps

$$\begin{array}{rcl} \phi_j(\bar{x}_j,\hat{\theta},t) & = & \varphi_j(\bar{x}_j,t) - \sum_{l=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_l}\varphi_l \\ \tilde{\Delta}_j(x,t) & = & \Delta_j(x,t) - \sum_{l=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_l}\Delta_l(x,t) \\ \tau_j(\bar{x}_j,\hat{\theta},t) & = & \tau_{j-1} + w_jz_j\phi_j \end{array} \tag{22}$$

Lemma 1 *At step $j \le i$, choose the desired control function $\alpha_j(\bar{x}_j, \hat{\theta}, t)$ as*

$$\begin{array}{l} \alpha_j = \alpha_{jf}(\bar{x}_j, \hat{\theta}, t) + \alpha_{js}(\bar{x}_j, \hat{\theta}, t), \qquad \alpha_{js} = \alpha_{js1} + \alpha_{js2} \\ \alpha_{jf} = -\hat{\theta}^T \phi_j + \sum_{l=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_l} x_{l+1} + \frac{\partial \alpha_{j-1}}{\partial t} \\ \alpha_{js1} = -\frac{w_{j-1}}{w_j} z_{j-1} - (k_j + d_j \|\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\|^2 + g_j \|\Gamma \phi_j\|^2) z_j \end{array} \tag{23}$$

where $k_j > 0,\ d_j, g_j \ge 0,\ w_j > 0$ is a weighting factor, and $\alpha_{js2}(\bar{x}_j, \hat{\theta}, t)$ is any function satisfying

$$\begin{array}{ll} \text{i} & z_j[-\tilde{\theta}^T \phi_j(\bar{x}_2, \hat{\theta}, t) + \bar{\Delta}_j(x,t) + \alpha_{js2}] \le \varepsilon_j, \quad \forall \hat{\theta} \in \hat{\Omega}_\theta \\ \text{ii} & z_j \alpha_{js2} \le 0 \end{array} \tag{24}$$

Then, the j-th error subsystem is

$$\begin{array}{rl} \dot{z}_j = & z_{j+1} - \frac{w_{j-1}}{w_j} z_{j-1} - (k_j + d_j \|\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\|^2 + g_j \|\Gamma \phi_j\|^2) z_j \\ & + [-\tilde{\theta}^T \phi_j + \bar{\Delta}_j + \alpha_{js2}] - \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \end{array} \tag{25}$$

and the derivative of the augmented p.d. function

$$V_{sj} = V_{s(j-1)} + \tfrac{1}{2} w_j z_j^2 \tag{26}$$

is given by

$$\begin{array}{rl} \dot{V}_{sj} = & w_j z_j z_{j+1} + \sum_{l=1}^{j} w_l \{-(k_l + d_l \|\frac{\partial \alpha_{l-1}}{\partial \hat{\theta}}\|^2 + g_l \|\Gamma \phi_l\|^2) z_l^2 \\ & + z_l[\bar{\Delta}_l + \alpha_{ls2}] - z_l \frac{\partial \alpha_{l-1}}{\partial \hat{\theta}} \dot{\hat{\theta}}\} - \tilde{\theta}^T \tau_j \end{array} \tag{27}$$

Proof: It is easy to check that step 1 satisfies the Lemma. So let us assume that the Lemma is valid for step j, $\forall j \le i-1$, and show that it is also true for step i to complete the induction process. From (15) and (22)

$$|\bar{\Delta}_i(x,t)| \le \bar{\delta}_i(\bar{x}_i, \hat{\theta}, t) \triangleq \delta_i + \sum_{j=1}^{i-1} |\frac{\partial \alpha_{i-1}}{\partial x_j}| \delta_j \tag{28}$$

This insures that there exists an $\alpha_{is2}(\bar{x}_i, \hat{\theta}, t)$ satisfying (24). The control law (23) can then be formed. Viewing the assumption of (14) and (23) for step $j < i$,

$$\begin{array}{rl} \dot{z}_i = & x_{i+1} + \theta^T \varphi_i + \Delta_i - \left\{ \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} [x_{j+1} + \theta^T \varphi_j + \Delta_j] \right. \\ & \left. + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} + \frac{\partial \alpha_{i-1}}{\partial t} \right\} \end{array} \tag{29}$$

Substituting $x_{i+1} = z_{i+1} + \alpha_i$ and (23) into (29), it is straightforward to verify that (25) and (27) are satisfied for i. This completes the induction process. □

Remark 1 *One example of a smooth α_{js2} satisfying (24) can be found in the following way. Let $h_j(\bar{x}_j, \hat{\theta}, t)$ be any smooth function satisfying*

$$h_j \ge \|\theta_{max} - \theta_{min}\| \|\phi_j(\bar{x}_j, \hat{\theta}, t)\| + \bar{\delta}_j(\bar{x}_j, \hat{\theta}, t) \tag{30}$$

Then, using the same technique as in [24], it can be shown that

$$\alpha_{js2} = h_j \tanh\left(\frac{0.2785 h_j z_j}{\varepsilon_j}\right) \tag{31}$$

satisfies (24). Other smooth or continuous examples of α_{js2} can be found in [17, 15]. ◇

4.3 Step n

This is the final design step. By letting $x_{n+1} = \sigma(x) u$, the actual control input u can be chosen as

$$u = \frac{1}{\sigma(x)} \alpha_n(\bar{x}_n, \hat{\theta}, t) \tag{32}$$

where α_n is given by (23).

Lemma 2 *With the following adaptation law*

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma \tau_n) \tag{33}$$

if controller parameters d_j and g_j are chosen such that $g_j \ge \frac{n w_j}{4} \sum_{l=1}^{n} \frac{w_l}{d_l}$, the control law (32) guarantees that

$$V_{sn}(t) \quad \le exp(-2k_v t) V_{sn}(0) + \frac{\varepsilon_v}{2k_v}[1 - exp(-2k_v t)] \tag{34}$$

where $k_v \triangleq min\{k_1, \ldots, k_n\}$, $\varepsilon_v \triangleq \sum_{j=1}^{n} w_j \varepsilon_j$. ◇

Proof: From (32), $z_{n+1} = 0$. From (22), $\tau_n = \sum_{j=1}^{n} w_j z_j \phi_j$. Thus, from (27),

$$\begin{array}{rl} \dot{V}_{sn} = & \sum_{j=1}^{n} w_j \{-(k_j + d_j \|\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\|^2 + g_j \|\Gamma \phi_j\|^2) z_j^2 \\ & + z_j[-\tilde{\theta}^T \phi_j + \bar{\Delta}_j + \alpha_{js2}] - z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}}\} \end{array} \tag{35}$$

Noting (33),

$$\begin{array}{rl} \|\dot{\hat{\theta}}\|^2 = & \|Proj_{\hat{\theta}}(\Gamma \tau_n)\|^2 \le \|\Gamma \tau_n\|^2 \\ & \le \left(\sum_{j=1}^{n} \|\Gamma w_j \phi_j z_j\|\right)^2 = n \sum_{j=1}^{n} \|\Gamma \phi_j\|^2 w_j^2 z_j^2 \end{array} \tag{36}$$

Thus, if $g_j \ge \frac{n w_j}{4} \sum_{l=1}^{n} \frac{w_l}{d_l}$,

$$\begin{array}{l} |\sum_{j=1}^{n} w_j z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}}| \le \sum_{j=1}^{n} (w_j d_j \|\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\|^2 z_j^2 + \frac{w_j}{4 d_j} \|\dot{\hat{\theta}}\|^2) \\ \le \sum_{j=1}^{n} w_j d_j \|\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}}\|^2 z_j^2 + \sum_{j=1}^{n} w_j g_j \|\Gamma \phi_j\|^2 z_j^2 \end{array} \tag{37}$$

Noting P1 of (12), (37), and the condition i of (24), (35) becomes

$$\dot{V}_{sn} \quad \le \quad \sum_{j=1}^{n} w_j(-k_j z_j^2 + \varepsilon_j) \le -2k_v V_{sn} + \varepsilon_v \tag{38}$$

which leads to (34). □

4.4 Guaranteed Transient Performance

The same as in [24], the idea of trajectory initialization in [1] will be used to render $z(0) = 0$ independent of the choice of $k = [k_1, \ldots, k_n]^T$ and $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_n]^T$ to achieve a guaranteed transient performance. The detail is omitted and can be worked out in the same way as in [24]. Basically, $x_{1d}(t)$ will be created by an n-th order stable system

$$x_{1d}^{(n)} + \beta_1 x_{1d}^{(n-1)} + \ldots + \beta_n x_{1d} = y_d^{(n)} + \ldots + \beta_n y_d \tag{39}$$

where $y_d(t)$ is the desired output trajectory. Then, by placing the initial conditions $x_{1d}^{(i)}(0)$, $i = 0, \ldots, n-1$ at the best estimate of the initials $y^{(i)}(0)$, which is obtained by substituting $\hat{\theta}(0)$ for θ and neglecting all uncertainties in the calculation of $y^{(i)}$, we can guarantee that $z(0) = 0$.

Theorem 1 *Given the desired trajectory $x_{1d}(t)$ generated by (39) with the initial conditions chosen as in [24], the following results hold if the control law (32) with the adaptation law (33) is applied:*

A. *In general, all signals are bounded. Furthermore,*

$$V_{sn}(t) \quad \le \frac{\varepsilon_v}{2k_v}[1 - exp(-2k_v t)] \tag{40}$$

and the output tracking error is guaranteed to have any prescribed transient performance by increasing k and/or decreasing ε.

B *If after a finite time t_0, $\Delta_i = 0, \forall i$, i.e., in the presence of parametric uncertainties only, in addition to results in A, asymptotic output tracking is also obtained for any gains k and ε.* △

Proof : From the trajectory initialization, $V_{sn}(0) = 0$. From Lemma 2, (40) is true and thus z is bounded. Since $\hat{\theta}$ is always bounded, all control functions α_j and the control input u are bounded, which leads to the boundedness of the state x. Furthermore, from (40), $z_1(t)$ is within a ball whose size can be made arbitrarily small by increasing k and/or decreasing ε in a known form. From (39) and the nature of the trajectory initialization, the trajectory planning error, $e_d(t) = x_{1d}(t) - y_d(t)$, can be guaranteed to possess any good transient behavior by suitably choosing the Hurwitz polynomial $G_d(s) = s^n + \beta_1 s^{n-1} + \ldots + \beta_n$ without being affected by k and ε. Therefore, any good transient performance of the output tracking error $e = y - y_d = z_1(t) + e_d(t)$ can be guaranteed by selecting the controller parameters k and ε in a known form, which proves A of Theorem 1.

Now consider the situation that $\Delta_i(x,t) = 0,\ t \geq t_0,\ \forall i$. From (22), $\tilde{\Delta}_i(x,t) = 0,\ \forall i$. Choose a p.d. function V_{an} as

$$V_{an} = V_{sn} + \tfrac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{41}$$

Noticing (35), (37), (33), condition ii of (24), and P2 of (12,

$$\begin{aligned} \dot{V}_{an} = \dot{V}_{sn} + \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} = \textstyle\sum_{j=1}^{n} w_j \{ -(k_j + d_j \| \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \|^2 \\ + g_j \|\Gamma \phi_j\|^2) z_j^2 + z_j \alpha_{js2} - z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \} + \tilde{\theta}^T \Gamma^{-1} (\dot{\hat{\theta}} - \Gamma \tau_n) \\ \leq -\textstyle\sum_{j=1}^{n} w_j k_j z_j^2 + \tilde{\theta}^T (\Gamma^{-1} Proj_{\hat{\theta}}(\Gamma \tau_n) - \tau_n) \leq -\textstyle\sum_{j=1}^{n} w_j k_j z_j^2 \end{aligned} \tag{42}$$

Therefore, $z = [z_1, \ldots, z_n]^T \in L_2^n$. It is also easy to check that $\dot{z}$ is bounded. So, $z \longrightarrow 0$ as $t \longrightarrow \infty$ by the Barbalat's lemma, which leads to B of Theorem 1. □

5 Conclusions

A general structure of the proposed adaptive robust control has been presented for the design of high-performance robust controllers. Under the proposed general framework, a simple new ARC controller, which is based on the widely used discontinuous projection method, was also constructed for a class of nonlinear systems transformable to the semi-strict feedback form. Simulation results will be presented to illustrate the proposed method.

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