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High Performance Adaptive Robust Control for Nonlinear System with Unknown Input Backlash

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Abstract—**A high performance adaptive robust control (ARC) algorithm is developed for a class of nonlinear system with unknown input backlash, parametric uncertainties and uncertain nonlinearities disturbances. Due to the inherent dynamic nonlinear nature of backlash, existing robust adaptive control methods mainly focus on using on-line parameter adaptation for an approximate inversion of the unknown backlash. As a result, even in the absence of other uncertain nonlinearities and disturbance, asymptotic output tracking can never be achieved by continuous controller for the non-smooth characteristics of backlash. Experimental results also show that a linear controller alone performed better than a controller including the selected backlash inverter with a correctly estimated or overestimated backlash gap. Unlike many existing control schemes, the backlash inverse is not constructed in this paper. A new linearly parameterized model for backlash is presented and the projection algorithm is used for parameters estimation. The proposed adaptive robust control law consists of three parts: model compensation term, the robust control term and robust feedback term. The ARC ensure that all closed-loop signals are bounded and achieves the tracking within the desired precision. Simulations results illustrate the performance of the ARC.**

I. Introduction

BACKLASH characteristics are common in control systems such as servomechanisms, electronic relay circuits and electromagnetic devices with hysteresis. It is one of the most important nonlinearities that limit the performance of speed and position control in industrial, robotics, automotive, automation and other applications. The control of systems with backlash has been the subject of study since 1940s. Linear controllers have been investigated, including PID controllers, high-order linear controllers, state feedback controllers. For backlash nonlinearity, inverse compensation of backlash are designed widely both in a non-adaptive and an adaptive setting[1]. An adaptive inverse of backlash was construct to cancel the effect of backlash nonlinearity in [2],[3], but the initial condition should be strictly limited. A smooth inverse of backlash was developed to compensation the effect of backlash with back-stepping approach in [4], where the derivation of the control input was used to get the controller, which maybe unavailable. Backlash compensation using neural network[5],[6] or fuzzy logic [7],[8] has been used in feedback control system. For those intelligent compensation, neural networks or fuzzy logic were mainly used for cancellation of the inversion error for their excellent nonlinearity approximation ability. The common feature of the inverse schemes is that they rely on the construction of an inverse backlash to mitigate the effect of the backlash nonlinearity. Experiment by [9] shows that a linear controller alone performed better than a controller including the selected backlash inverter with a correctly estimated or overestimated backlash gap, the reason being that measurement noise induced chattering in the inverter. It was noted that the linear controller alone also traverses the backlash gap rapidly since only the motor moment of inertia (and not the load) is driven inside the backlash gap. As pointed out in the recent comprehensive survey paper with 96 references on controlling mechanical systems with backlash[1] that weak action in the backlash gap (i.e., by simply lowering the closed-loop system bandwidth when the gap is open) might be advantageous.

In this paper, a new linearly parameterized model for backlash is presented and a new approach for adaptive robust control (ARC) of linear or nonlinear systems with backlash is introduced without constructing the backlash inverse. Based only on the intuitive concept and piece-wise description of backlash, the unknown backlash nonlinearity is linearly parameterized globally with bounded model error. The ARC controller is consist of three parts: one is the usual model compensation with the physical parameter estimates. For the second part, a simple proportional feedback part is used to stabilize the nominal system. The last one is a robust feedback term used to attenuate the effect of various model uncertainties. The ARC control law ensures that all closed-loop signals are bounded and achieves the tracking within the desired precision. Computer simulations are carried out to illustrate the effectiveness of the approaches.

This paper is organized as follows: Section II states the problem of this note, where the linear model of backlash is introduced. In section III, the proposed ARC scheme is presented. In section IV, simulation results are presented to

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illustrate the effectiveness of the proposed approach. In section V, conclusions are drawn.

II. Problem Statement

A. System Model

For simplicity, the following class of second-order nonlinear system is considered:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= w + \theta^T \phi(x,t) + \Delta(x,t) \qquad (1) \\ y &= x_1 \quad w = B(u) \end{aligned}$$

where $x = [x_1, x_2]^T$ is the state vector, w, y are the input and output respectively. $\phi(x,t) = [\phi_1(x,t), \cdots, \phi_p(x,t)]^T$ is the known shape functions, $\theta = [\theta_1, \cdots, \theta_p]^T$ is the vector of unknown constant parameters, and $\Delta(x,t)$ represents the lumped uncertain nonlinearity including external disturbances. The actuator nonlinearity $w = B(u)$ is described as a backlash characteristic.

The control objective is to design a control law for $u(t)$ to ensure that all closed-loop signals are bounded and the tracking error $y(t) - y_r(t)$ is adjustable within the desired precision.

Assumption 1: θ lies in a known bounded set Ω_θ:

$$\theta \in \Omega_\theta \triangleq \{\theta : \theta_{j\min} \le \theta_j \le \theta_{j\max}, j = 1, \cdots, p\}$$

Assumption 2: The uncertain nonlinearity can be bounded by

$$|\Delta(x,t)| \le B$$

where B is a positive constant.

B. Backlash Characteristic

A backlash nonlinearity is shown as Fig.1

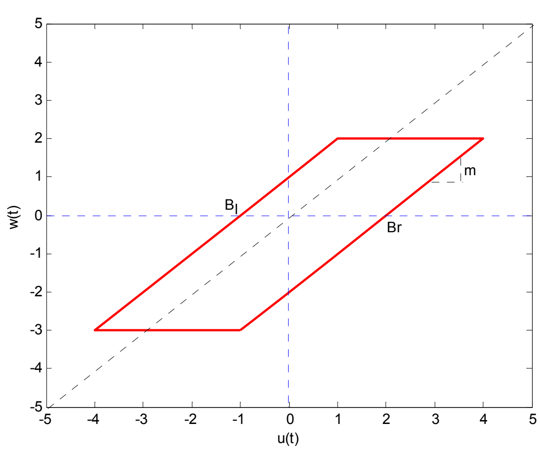

Fig.1 Backlash Model

Traditionally, it can be described by:

$$w(t) = B(u) = \begin{cases} m(u(t) - B_r) & \text{if } \dot{u} > 0 \text{ and } w(t) = m(u(t) - B_r) \\ m(u(t) - B_l) & \text{if } \dot{u} < 0 \text{ and } w(t) = m(u(t) - B_l) \\ w(t\_) & \text{otherwise} \end{cases} \qquad (2)$$

where m is the constant slope of the lines, B_r, B_l are constant parameters. $w(t\_)$ means no change occurs in $w(t)$.

For the development of control law, the following assumptions are made:

Assumption 3: The backlash parameters m, B_r, B_l are unknown, but their signs are known. In general, let $m > 0$, $mB_r > 0, mB_l < 0$.

Assumption 4: The backlash parameters are within known bounded:

$$\begin{aligned} &0 < m_{\min} \le m \le m_{\max} \\ &0 < (mB_r)_{\min} \le mB_r \le (mB_r)_{\max} \\ &(mB_l)_{\min} \le mB_l \le (mB_l)_{\max} < 0 \end{aligned}$$

Assumption 5: The backlash output $w(t)$ is not available for measurement.

From the above, we can rewrite the backlash model (2) as

$$w(t) = B(u) = mu(t) + d_b(u(t)) \qquad (3)$$

Where $d_b(u(t))$ is model error form linearly parameterized of backlash and it can be calculated from (2) and (3) as

$$d_b(u(t)) = \begin{cases} -mB_r & \text{if } \dot{u} > 0 \text{ and } w(t) = m(u(t) - B_r) \\ -mB_l & \text{if } \dot{u} < 0 \text{ and } w(t) = m(u(t) - B_l) \\ w(t\_) - mu(t) & \text{otherwise} \end{cases} \qquad (4)$$

Lemma 1: with (4) and assumption 3, $d_b(u(t))$ is bounded, and satisfies

$$|d_b(u(t))| \le \rho \qquad (5)$$

where ρ is the upper-bound, which can be chosen as

$$\rho = \max\{(mB_r)_{\max}, -(mB_l)_{\min}\}$$

Combing (1) and (3), one obtains

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = mu + \theta^T \phi(x) + d_b(t) + \Delta(x,t) \\ y = x_1 \end{cases} \qquad (6)$$

Define a constant d_c and time-varying $\Delta^*(t)$ such that

$$d_c + \Delta^*(t) = d_b(t) + \Delta(x,t) \qquad (7)$$

Conceptually, (7) divides the backlash modeling error d_b and the original system uncertain nonlinearity $\Delta(x,t)$ into the static component(or low frequency component in reality) d_c and the high frequency component $\Delta^*(t)$. By

Assumption 2 and Assumption 4 and (9), one obtains

$$d_c \in \Omega_{dc} \triangleq \{d_c : |d_c| \le \delta_d\}$$
$$\Delta^*(t) \in \Omega_\Delta \triangleq \{\Delta^* : |\Delta^*| \le \delta_\Delta\} \quad (8)$$

where δ_d and δ_Δ are known.

III. Design of Adaptive Robust Controller

In this section, the adaptive robust control (ARC) strategy [10][11] will be designed. As in [12][13], the first step is to use a projection type adaptation law structure to achieve a controlled learning or adaptation process as detailed in the following.

Let $\hat{\theta}$ denote the estimation of θ, and $\tilde{\theta}$ be the estimation error $\tilde{\theta} = \theta - \hat{\theta}$. The following projection-type parameter adaption law[14] is used:

$$\dot{\hat{\theta}} = \mathrm{Proj}_{\hat{\theta}}(\Gamma\tau),\ \hat{\theta}(0) \in \Omega_\theta$$

$$\mathrm{Proj}_{\hat{\theta}}(\bullet_i) = \begin{cases} 0 & if\ \hat{\theta}_i = \theta_{i\max}\ and\ \bullet_i > 0 \\ 0 & if\ \hat{\theta}_i = \theta_{i\min}\ and\ \bullet_i < 0 \\ \bullet_i & otherwise \end{cases}$$

where Γ is a diagonal matrix of adaption rates and τ is an adaption function to be synthesized further on. Such a parameter adaption law has the following desirable properties. At any time instant, i.e., $\forall t$:

(P1) $\hat{\theta}(t) \in \Omega_\theta$

(P2) $\tilde{\theta}^T(\tau - \Gamma^{-1}\mathrm{Proj}_{\hat{\theta}}(\Gamma\tau)) \le 0$

Define

$$p = \dot{e} + k_1 e = \dot{y} - \dot{y}_d + k_1 e$$
$$= \dot{y} - (\dot{y}_d - k_1 e) \triangleq x_2 - x_{2eq} \quad (9)$$

where : $x_{2eq} \triangleq \dot{y}_d - k_1 e$, $e = y - y_d$ is the output tracking error and $k_1 > 0$ is a positive gain. If p is small or converge to zero, the output tracking error e will be small or converge to zero since $G_p(s) = E(s)/P(s) = 1/(s+k_1)$ is a stable transfer function. So, the rest of the design is to make p as small as possible.

Differentiating (9) and noting (6) (7), one obtains

$$\dot{p} = \dot{x}_2 - \dot{x}_{2eq}$$
$$= mu + \theta^T\phi - \dot{x}_{2eq} + d_c + \Delta^*(t) \quad (10)$$

Thus:

$$\frac{1}{m}\dot{p} = u + (\frac{\theta}{m})^T\phi - \frac{1}{m}\dot{x}_{2eq} + \frac{1}{m}d_c + \frac{1}{m}\Delta^*(t) \quad (11)$$
$$= u + \vartheta^T\varphi + \overline{\Delta}(t)$$

where

$$\vartheta = [\frac{\theta}{m} \quad \frac{1}{m} \quad \frac{d_c}{m}]^T$$
$$\varphi = [\phi \quad -\dot{x}_{2eq} \quad 1]^T$$
$$\overline{\Delta}(t) = \frac{1}{m}\Delta^*(t)$$

The following ARC is proposed to design u as follows:

$$u = u_a + u_s$$
$$u_s = u_{s1} + u_{s2}$$
$$u_{s1} = -k_{s1}p \quad (12)$$
$$u_a = -\hat{\vartheta}^T\varphi$$
$$\dot{\hat{\vartheta}} = \mathrm{Proj}(\Gamma_\vartheta\varphi p) \quad (13)$$

In (12), u_a represents the usual model compensation with the physical parameter estimates $\hat{\vartheta}$,which is updated by using an on-line adaptation projection algorithm (13). u_s represents the robust control term in which u_{s1} is a simple proportional feedback to stabilize the nominal system and u_{s2} is a robust feedback term used to attenuate the effect of various model uncertainties for guaranteed robust control performance in general.

Noting Assumption 1, Assumption 4, and (P1), there exists a u_{s2} such that the following two conditions are satisfied:

(i)

$$p \cdot u_{s2} \le 0 \quad (14)$$

(ii)

$$p(u_{s2} + \tilde{\vartheta}^T\varphi + \overline{\Delta}(t)) \le \varepsilon \quad (15)$$

where ε and ε_d are design parameters which can be arbitrarily small. Essential,(15) shows that u_{s2} is synthesized to dominate the model uncertainties coming from both parametric uncertainties and uncertain nonlinearities. And (14) is to make sure that u_{s2} is dissipative in nature so that it doesn't interfere with the functionality of the adaptive control part u_a.

Remark 1: One example of u_{s2} satisfying (14) and (15) can be chosen as:

$$u_{s2} = -\frac{1}{4\varepsilon}[\|\tilde{\vartheta}_{\max}\|\|\varphi\| + \overline{\Delta}_{\max}]^2 p \quad (16)$$

where

$$\tilde{\vartheta}_{\max} = \vartheta_{\max} - \vartheta_{\min},\quad \overline{\Delta}_{\max} = \frac{\delta_\Delta}{m_{\min}}$$

$$\vartheta_{\max} = [\frac{\theta_{\max}}{m_{\min}} \quad \frac{1}{m_{\min}} \quad \frac{\delta_d}{m_{\min}}]^T$$

$$\vartheta_{\min} = [\frac{\theta_{\min}}{m_{\max}} \quad \frac{1}{m_{\max}} \quad \frac{-\delta_d}{m_{\max}}]^T$$

It can be proved easily to show that the above choice of u_{s2} does satisfy (14) and (15).

Proof: The condition (i) is easily to satisfied by the control law (16). For condition (ii), we can get

$$\begin{aligned}
&\varepsilon - pu_{s2} \\
&= [\frac{1}{2\sqrt{\varepsilon}}(\|\tilde{\vartheta}_{\max}\|\|\varphi\| + \overline{\Delta}_{\max})]^2 p^2 + (\sqrt{\varepsilon})^2 \\
&\geq 2 \cdot \frac{1}{2\sqrt{\varepsilon}}(\|\tilde{\vartheta}_{\max}\|\|\varphi\| + \overline{\Delta}_{\max})|p| \cdot \sqrt{\varepsilon} \\
&= |p|\|\tilde{\vartheta}_{\max}\|\|\varphi\| + |p|\overline{\Delta}_{\max} \\
&\geq p(\tilde{\vartheta}^T\varphi + \overline{\Delta}(t))
\end{aligned}$$

Thus:

$$p(u_{s2} + \tilde{\vartheta}^T\varphi + \overline{\Delta}(t)) \leq \varepsilon$$

So the condition (ii) is also satisfied. □

Theorem1: Consider the system (1) consisting of the adaptive robust controller given by (12) and the projection type parameters adaptation law (13), all signals in the resulting closed loop system are bounded, and the output tracking is guaranteed to have a prescribed transient performance and final tracking accuracy in the sense that the tracking error index p is bounded by:

$$p^2(t) \leq e^{-2mk_{s1}t}p^2(0) + \frac{\varepsilon}{k_{s1}}[1 - e^{-2mk_{s1}t}] \tag{17}$$

Proof:

Consider the following Lyapunov function:

$$V = \frac{1}{2m}p^2$$

then the derivation of V

$$\begin{aligned}
\dot{V} &= \frac{1}{m}p\dot{p} \\
&= p(u + \vartheta^T\varphi + \overline{\Delta}(t)) \\
&= -k_{s1}p^2 + p(u_{s2} + \tilde{\vartheta}^T\varphi + \overline{\Delta}(x,t)) \\
&\leq -k_{s1}p^2 + \varepsilon
\end{aligned}$$

Thus:

$$\dot{V} \leq -2mk_{s1}V + \varepsilon \tag{18}$$

$$V(t) \leq \exp(-2mk_{s1}t)V(0) + \frac{\varepsilon}{2mk_{s1}}[1 - \exp(-2mk_{s1}t)] \tag{19}$$

Then (17) is true. The theorem shows that the output tracking precision is guaranteed by setting ε and k_{s1} respectively.

Theorem2: Consider the system (1) consisting of the following parameters and the adaptive robust controller given (12)(13), In the absence of uncertain nonlinearities(i.e., assuming $\overline{\Delta}(t) = 0$), asymptotic position tracking is also achieved.

Proof: Consider the following Lyapunov function:

$$V = \frac{1}{2m}p^2 + \frac{1}{2}\tilde{\vartheta}^T\Gamma_\vartheta^{-1}\tilde{\vartheta}$$

Noting (8) and (1), then the derivation of V

$$\begin{aligned}
\dot{V} &= \frac{1}{m}p\dot{p} + \tilde{\vartheta}^T\Gamma_\vartheta^{-1}\dot{\tilde{\vartheta}} \\
&= p(u + \vartheta^T\varphi) - \tilde{\vartheta}^T\Gamma_\vartheta^{-1}\dot{\hat{\vartheta}} \\
&= -k_{s1}p^2 + p(u_{s2} + \tilde{\vartheta}^T\varphi) - \tilde{\vartheta}^T\Gamma_\vartheta^{-1}\operatorname{Pr}oj(\Gamma_\vartheta\varphi p) \\
&\leq -k_{s1}p^2 + pu_{s2} + \tilde{\vartheta}^T[p\varphi - \Gamma_\vartheta^{-1}\operatorname{Pr}oj(\Gamma_\vartheta\varphi p)] \\
&\leq -k_{s1}p^2
\end{aligned}$$

By the Barbalat's lemma, it follows that $\lim_{t\to\infty} p(t) = 0$, which implies that $\lim_{t\to\infty} e(t) = 0$.

Theorem 2 indicates that the asymptotic position tracking can be achieved if the system works only on the single slope side of the backlash (i.e., $d_b(t)$ is constant as $d_b(t) = mb_r$ or $d_b(t) = mb_l$).

IV. Simulation

In this section, the proposed ARC algorithm is applied to the following nonlinear systems:

$$\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = w(t) + a_1\dfrac{1 - e^{-x_1}}{1 + e^{-x_1}} + a_2x_2\sin(\dot{x}_1) + a_3x_1\sin(3t) \\
\quad + \Delta(x,t) \\
y = x_1
\end{cases}$$

where $w(t)$ is the output of a backlash described by:

$$w(t) = \begin{cases}
1.2[u(t) - 0.5] & \dot{u}(t) > 0 \ and \ w(t) = 1.2[u(t) - 0.5] \\
1.2[u(t) - (-0.8)] & \dot{u}(t) < 0 \ and \ w(t) = 1.2[u(t) + 0.8] \\
w(t_) & otherwise
\end{cases}$$

The actual values of physical parameters are set as $a_1 = 1$, $a_2 = -2$, $a_3 = -0.5$ and the bounds of them are chosen to be $a_1 \in [0.5,1.5]$, $a_2 \in [-3,-1]$, $a_3 \in [-1.5,-0.1]$. The backlash parameters of $m = 1.2$, $mB_r = 0.6$ and $mB_l = -0.96$ are assumed not known but within the known range of $m \in [0.5,1.8]$, $mB_r \in [0.1,1.2]$, and $mB_l \in [-1.5,-0.5]$ respectively. The disturbance $\Delta(x,t) = 0.05\sin 10t$. The control objective is to let the system follow the desired trajectory:

$$y_d(t) = 0.5[\sin(t) + \sin(2t) + \sin(3t)]$$

The initial values are chosen as $x(0) = [0.1,1]^T$, $u(0) = 0$, $\hat{m}(0) = 1$, $\Gamma_g = 10I_5$, $k_{s1} = 8$, $\varepsilon = 0.01$.

Assuming a sampling period T=0.001*sec*, the simulation results are shown in figures 2-4. Fig.2 shows the position tracking performance of ARC. Fig.3 shows the corresponding tracking error and Fig.4 shows the input control signal of backlash $u(t)$.

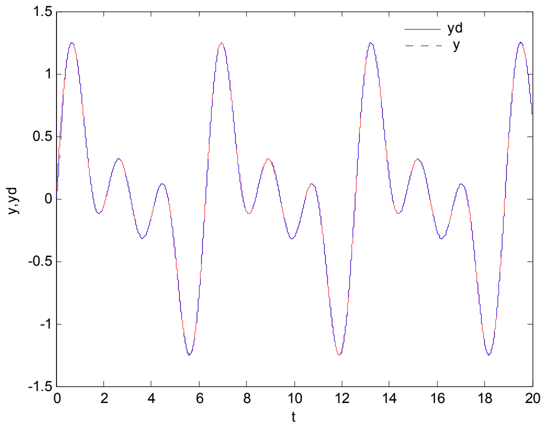

Fig.2 Tracking performance of ARC

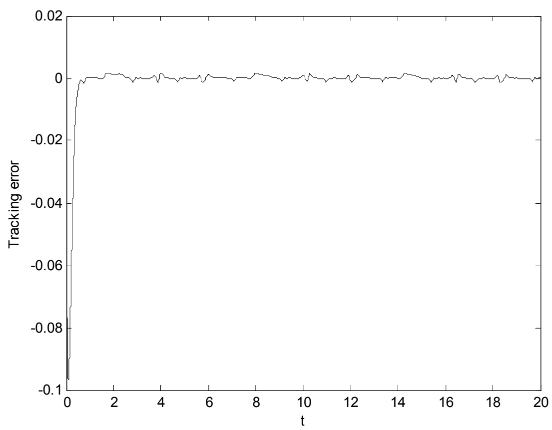

Fig.3 Output tracking error of ARC

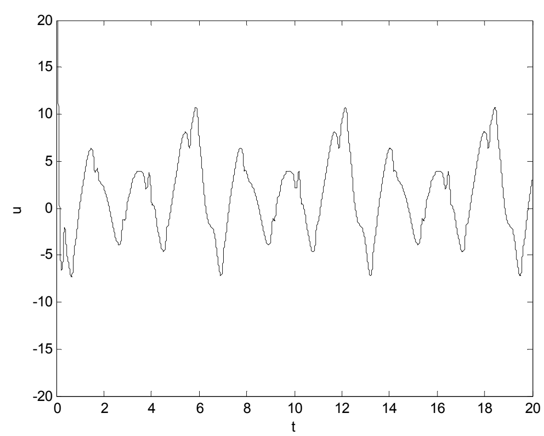

Fig.4 Control signal *u(t)* as the input of backlash of ARC

V. Conclusion

In practical control systems, backlash with unknown parameters in physical components may severely limit the performance of control. In this paper, an adaptive robust control is proposed for a class of continuous-time uncertain nonlinear dynamic systems preceded by an unknown input backlash. By using a global linear model of backlash, the ARC controller is developed without constructing a backlash inverse. The new control law ensures all closed-loop signals are bounded and achieves the tracking precision within the desired precision.

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