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Adaptive Robust Control of Programmable Valves with Manufacturer Supplied Flow Mapping Only

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Abstract— **The energy-saving programmable valves, a unique combination of five independently controlled poppet type cartridge valves, have been shown to significantly reduce energy usage while maintaining excellent control performance. Due to the fact that the flow characteristics of poppet type cartridge valves are very difficult to model, pressure compensated flow mapping via off-line system identification were used in all previous controller designs. However, individually calibrating each valve in use would prohibit their widespread industrial applications. It is desirable to have a controller requiring only the manufacturer supplied valve flow mappings, which may differ from the true individual valve flow characteristics to certain degrees. This paper presents an Adaptive Robust Controller to control the multi-input valve system with the manufacturer supplied flow mapping only. Robust stability is guaranteed under bounded modelling errors, and the effect of modelling errors is effectively attenuated by the use of nonlinear robust feedback structure that achieves a prescribed transient performance and final tracking accuracy. In addition, real-time adaptation is used to reduce the degree of modelling errors for an improved performance. Experimental results are obtained to validate the proposed scheme.**

I. INTRODUCTION

The advent of electro-hydraulic valves and the incorporation of complex digital control have significantly improved the performance of hydraulic systems. A new problem arises as the applications of electro-hydraulic systems become increasingly widespread: is it possible to reduce the energy usage while still having the desired performance? The question was answered by the energy-saving programmable valves [1], a unique combination of five independently controlled poppet-type cartridge valves, as shown in Fig.1. The unique features of the programmable valves come from i) breaking the mechanical linkage between the meter-in and meter-out orifices [2], [3], [4] through valves number 1,2 and 4,5; and ii) enabling the precise control of the regeneration flow through the cross-port valve number 3 for significant energy saving .

The significant energy saving and the excellent control performance of the programmable valves have been shown in [1], [5], [6]. Due to the difficulty to precisely model the flow characteristics of the cartridge valves, a nonlinear pressure compensated flow mapping through off-line system identification was used to for the mathematical model of each valve involved. This method is practical and effective, and the tracking performances obtained in [6] were at least as good as, if not better than, the control performances using the expensive servo valves. However, individually identifying and calibrating each valve in use would prohibit the widespread industrial applications of the programmable valves. To make this programmable valves useful for industry, a controller requiring only the manufacturer supplied flow mapping is desirable.

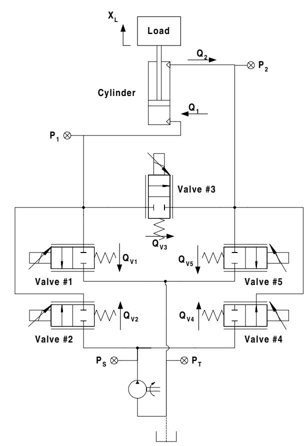

Fig. 1. Programmable valves layout.

The difficulties to control the programmable valves with the manufacturer supplied flow mappings lie in several aspects. First of all is the significant modelling error. The cartridge valves are of fast response and low cost but low accuracy, and are designed originally not for precise control [7]. Therefore the individual valve's flow characteristics may be quite different from the manufacturer supplied universal flow mappings. Such flow modelling errors may have significant effects on the controller design in term of robust performance and stability. Other difficulties in the controller design come from the highly nonlinear hydraulic and mechanical dynamics, large parameter variations [8], significant uncertain nonlinearities such as external disturbances, flow leakages, seal frictions [9], [10], and so on.

The objective of the current work is to design a robust controller with only the manufacturer supplied flow mappings to stabilize the closed-loop system as well as to reduce the effect of various modelling errors to achieve a guaranteed control performance. To explicitly take into account the effect of significant but bounded flow modelling errors,

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the adaptive robust control (ARC) technique proposed by Yao and Tomizuka in [11], [12], [13] is customized and applied to the programmable valves controlled hydraulic system. Robust stability is guaranteed in the presence of bounded flow modelling errors and disturbances, and the system performance is further improved through certain on-line adaptation processes.

II. DYNAMIC MODEL AND PROBLEM FORMULATION

In this paper, the programmable valves are used to control the boom motion of a three degree-of-freedom (DOF) electro-hydraulic robot arm that was built to mimic the industrial backhoe or excavator arms in [10]. The boom motion dynamics with other two joints fixed can be described by [14], [1].

$$\begin{aligned}(J_c+m_L\ell_e{}^2)\ddot{q} = & \tfrac{\partial x}{\partial q}(P_1A_1-P_2A_2)-G_c(q)\\ & -m_Lg\ell_g(q)-D_f\cdot\dot{q}+T(t,q,\dot{q})\end{aligned} \tag{1}$$

where q represents the boom joint angle, J_c is the moment of inertia of the boom without payload, m_L represents the mass of the unknown payload, G_c is the gravitational load of the boom without payload, x represents the boom hydraulic cylinder piston displacement, P_1 and P_2 are the head and rod end pressures of the cylinder respectively, A_1 and A_2 are the head and rod end ram areas of the cylinder respectively, D_f is the damping and viscous friction coefficient and T represents the lumped disturbance torque including external disturbances and terms like the unmodelled friction torque. The specific forms of J_c, G_c, ℓ_g, and ℓ_e are given in [14].

The inertial moment and the gravity force both depend on the unknown element m_L. As a result, the inertial moment and the gravity force are split into two components. The terms J_c and $G_c(q)$ contain only calculable quantities and the terms $m_Lgl_g(q)$ and $m_Ll_e^2$ which contain the unknown quantity m_L.

Neglecting cylinder flow leakages, the hydraulic cylinder equations can be written as [9],

$$\begin{aligned}\tfrac{V_1(x)}{\beta_e}\dot{P}_1 &= -A_1\dot{x}+Q_1 = -A_1\tfrac{\partial x}{\partial q}\dot{q}+Q_1\\ \tfrac{V_2(x)}{\beta_e}\dot{P}_2 &= A_2\dot{x}-Q_2 = A_2\tfrac{\partial x}{\partial q}\dot{q}-Q_2\end{aligned} \tag{2}$$

where $V_1(x) = V_{h1}+A_1x$ and $V_2(x) = V_{h2}-A_2x$ are the total cylinder volumes of the head and rod ends including connecting hose volumes respectively, V_{h1} and V_{h2} are the initial control volumes when $x=0$, β_e is the effective bulk modulus. Q_1 and Q_2 are the supply and return flows respectively.

When the programmable valves in Fig.1 are used, Q_1 and Q_2 are given by,

$$\begin{aligned}Q_1 &= Q_{v2}-Q_{v1}-Q_{v3}\\ Q_2 &= -Q_{v3}-Q_{v4}+Q_{v5}\end{aligned} \tag{3}$$

where Q_{vi} is the orifice flow through the *ith* cartridge valve. Neglecting the cartridge valve dynamics, which is very fast (more than 30Hz) when compared with the system dynamics, the orifice flow can be described as a non-linear function of the pressure drop across the the valve orifice and the control input to the valve:

$$\begin{aligned}Q_{vi} &= f_{vi}(\Delta P_{vi}, u_{vi})\\ &= f_m(\Delta P_{vi}, u_{vi})+[f_{vi}(\Delta P_{vi}, u_{vi})-f_m(\Delta P_{vi}, u_{vi})]\\ &= Q_{vim}+\tilde{Q}_{vi}, \quad i=1,2,...5\end{aligned} \tag{4}$$

where ΔP_{vi}, u_{vi} and f_{vi} represent the pressure drop, the control input and the actual orifice flow function of the *ith* cartridge valve, while $f_m(\Delta P_{vi}, u_{vi})$, simplified as Q_{vim}, represents the manufacturer supplied flow mapping obtained based on the manufacturer supplied flow characteristic curves and interpolation and extrapolation for arbitrary inputs and pressure drops, $\tilde{Q}_{vi}$ represents the flow modelling error, i.e., $\tilde{Q}_{vi} = f_{vi}(\Delta P_{vi}, u_{vi})-f_m(\Delta P_{vi}, u_{vi})$. Therefore, the supply and return flows Q_1 and Q_2 can be rewritten as:

$$\begin{aligned}Q_1 &= Q_{v2M}-Q_{v1M}-Q_{v3M}+(\tilde{Q}_{v2}-\tilde{Q}_{v1}-\tilde{Q}_{v3})\\ &= Q_{1M}+\tilde{Q}_1\\ Q_2 &= -Q_{v3M}-Q_{v4M}+Q_{v5M}+(-\tilde{Q}_{v3}-\tilde{Q}_{v4}+\tilde{Q}_{v5})\\ &= Q_{2M}+\tilde{Q}_2\end{aligned} \tag{5}$$

where Q_{1M} and Q_{2M} represent the nominal flows obtained from the manufacturer provided flow mapping and $\tilde{Q}_1$ and $\tilde{Q}_2$ represent the lumped flow modelling errors.

To explicitly deal with the lumped flow modelling errors $\tilde{Q}_1$ and $\tilde{Q}_2$ and the disturbances T, let us split those uncertain terms into two parts — the constant or slow changing part and the fast changing part, as follows.

$$\begin{aligned}T &= T_n+\Delta\\ \tilde{Q}_i &= \tilde{Q}_{in}+\Delta_{Qi} \quad i=1,\ 2\end{aligned} \tag{6}$$

The nominal parts T_n, $\tilde{Q}_{1n}$ and $\tilde{Q}_{2n}$ as well as the terms containing the unknown quantity m_L would be estimated and compensated via on-line adaptation; while the fast changing parts will be deal with by the robust control law. For simplicity, only the above four terms are adapted, other system coefficients, such as effective bulk modulus β_e and viscous and damping friction coefficient D_f, can be adapted in the same way if needed.

The following notations are employed throughout this paper. θ will be used to denote the unknown parameter while $\hat{\theta}$ denotes the estimate of θ and $\tilde{\theta}$ the estimation error, i.e., $\tilde{\theta} = \hat{\theta}-\theta$. θ_{min} and θ_{max} represent the lower and upper bounds of θ. $\bullet_i$ represents the *ith* component of the vector $\bullet$.

Since the extents of the parametric uncertainties are normally known or can be obtained *a priori*, the following practical assumption is made in the paper:

Assumption 1: Though the flow modelling errors $\tilde{Q}_{vi}$, $i = 1,\ldots,5$ may be quite significant due to the inaccuracy nature of the low cost cartridge valves, they are bounded unless the corresponding valve is broken, which is not considered in this paper. Mathematically, this is equivalent to saying that $\|\tilde{Q}_{vi}(\Delta P_{vi}, u_{vi})\|_\infty = \|f_{vi}(\Delta P_{vi}, u_{vi})-f_m(\Delta P_{vi}, u_{vi})\|_\infty$ is bounded with known bound for all input u_{vi} and pressure drop ΔP_{vi}. Therefore, the lumped flow modelling errors $\tilde{Q}_1$ and $\tilde{Q}_2$ are also bounded with known bounds.

Assumption 2: The external payload m_L is bounded with known bounds.

Assumption 3: The lumped disturbances T is bounded with known bounds.

Assumption 4: The pump and valves are able to provide sufficient supply pressure and the flows for any given motion trajectory.

III. CONTROLLER DESIGN

Define $P_L = P_1A_1 - P_2A_2$ as the load force to move the cylinder with certain trajectory. For precise motion control purpose, one needs to control P_L precisely. The fact that both P_1 and P_2 can be controlled independently results in tremendous flexibility to control the system. The desired solution here is to control the P_L to track the desired motion trajectory while maintain P_1 and P_2 as low as possible. Therefore, one of the two cylinder chamber is desired to be kept at a low pressure, which is referred as the off-side; while the other chamber's pressure would be critical to the motion and thus is referred as working-side.

The programmable valves controlled system is essentially a multi-input, multi-working-mode and multi-objective system under severe modelling error and parameter variations. The difficulties in the coordinate control of five cartridge valves for precision motion and pressure control are dealt with through a task level and valve level controllers. Given the current system states and desired motion trajectory, the task level controller determines the configurations of programmable valves that would enable significant energy saving while without losing hydraulic circuit controllability for motion tracking. The details about the task level controller can be found in [1], [6].

The valve level controller uses an adaptive robust control technique to control the pressures in both chambers independently with selected working mode to obtain the dual objectives.

A. Adaptation Law

Parameter adaptation would be performed at both off-side and working-side controllers. The adaptation law is defined in (7)

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\tau) \tag{7}$$

where Γ is positive definite diagonal adaptation rate matrix, τ the adaptation function, and the $Proj_{\hat{\theta}}(\bullet)$ function is defined as:

$$Proj_{\hat{\theta}}(\bullet_i) = \begin{cases} 0, & \text{if } \hat{\theta}_i = \theta_{imax} \text{ and } \bullet_i > 0 \\ 0, & \text{if } \hat{\theta}_i = \theta_{imin} \text{ and } \bullet_i < 0 \\ \bullet_i, & \text{otherwise} \end{cases} \tag{8}$$

It can be shown [15], [16], [11] that for any adaptation function τ, the discontinuous projection mapping (8) guarantees:

$$\begin{array}{ll} \text{P1} & \hat{\theta} \in \Omega_\theta \triangleq \left\{\hat{\theta} : \quad \theta_{min} \leq \hat{\theta} \leq \theta_{max}\right\} \\ \text{P2} & \tilde{\theta}^T[\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\tau) - \tau] \leq 0, \quad \forall\tau \end{array} \tag{9}$$

B. Off-side Pressure Regulator

The objective of the off-side pressure regulator is to keep the off-side pressure at a desired low pressure P_0. This subsection designs a pressure regulator for those working modes, for which P_2 is the off-side. The regulator design for P_1 follows the same procedure and is omitted here.

The dynamics of P_2 is described in (2) and (3). In order to use parameter adaptation to reduce parametric uncertainties to improve performance, it is necessary to linearly parameterize the system dynamics in terms of the unknown parameter $\theta_Q \triangleq \beta_e\tilde{Q}_{2n}$. Viewing (2), (5) and (6), and noting P_0 is a constant and its derivative is zero, the pressure error dynamics ($e_{p_2} \triangleq P_2 - P_0$) can be written as follows.

$$\dot{e}_{p_2} = \frac{\beta_e}{V_2}\left(-Q_{2M} + A_2\frac{\partial x}{\partial q}\dot{q} - \theta_Q - \Delta_{Q2}\right) \tag{10}$$

From assumption 1, it is known that θ_Q and the lumped disturbance term Δ_{Q2} are bounded and the bounds can be calculated with *a priori* information. Let us call the upper bounds of the unknown parameters as θ_M and the disturbance as δ_Q.

The goal is to have the cylinder pressure P_2 regulated to a desired constant low pressure P_0, i.e., e_{p_2} converges to zero or as small as possible. The ARC control law Q_{2M} is therefore given as:

$$\begin{array}{l} Q_{2M} = Q_{2Ma} + Q_{2Ms} \\ \quad Q_{2Ma} = A_2\frac{\partial x}{\partial q}\dot{q} - \hat{\theta}_Q \\ \quad Q_{2Ms} = Q_{2Ms1} + Q_{2Ms2}, \quad Q_{2Ms1} = \frac{V_2}{\beta_e}k_{p2}e_{p_2} \end{array} \tag{11}$$

where Q_{2Ma} is the model compensation term and Q_{2Ms} the robust stabilizing term, $k_{p2} > 0$, and Q_{2Ms2} is to dominate the parameter estimation error and unmodelled disturbances, which is chosen to satisfy the following conditions:

$$\begin{array}{ll} \text{i} & -e_{p_2}\frac{\beta_e}{V_2}\left(Q_{2Ms2} - \tilde{\theta}_Q + \Delta_{Q2}\right) \leq \varepsilon_p \\ \text{ii} & -e_{p_2}\frac{\beta_e}{V_2}Q_{2Ms2} \leq 0 \end{array} \tag{12}$$

where $\varepsilon_p > 0$ is a preset positive design parameter related to final tracking accuracy. The robust term Q_{2Ms2} can be chosen as (13) to make (12) satisfied.

$$Q_{2Ms2} = \frac{V_2}{\beta_e}k_{p_{2s}}e_{p_2}, \quad k_{p_{2s}} = \frac{1}{2\varepsilon_p}\left(\theta_M^2 + \delta_Q^2\right) \tag{13}$$

The adaptation function for the off-side ARC is defined as

$$\tau_Q = -\frac{\beta_e}{V_2}\cdot e_{p_2} \tag{14}$$

C. Working-side Motion Controller

The dynamics of the boom motion and pressures at both chambers are described in (1), (2), (5) and (6). Define a set of parameters as $\theta = [\theta_1, \quad \ldots \quad, \theta_4]^T$, $\theta_1 = \frac{1}{1+\frac{l_e^2}{J_c}m_L}$, $\theta_2 = \frac{T_n}{J_c+m_Ll_e^2}$, $\theta_3 = \tilde{Q}_{1n}$, and $\theta_4 = \tilde{Q}_{2n}$. From the assumptions, it is easy to check that all the unknown parameters are bounded with known bounds θ_{max} and θ_{min}.

The system dynamics equations can be rewritten as:

$$\begin{aligned}
\ddot{q} &= \theta_1\left[\frac{1}{J_c}\left(\frac{\partial x}{\partial q}P_L - G_c - D_f\dot{q}\right) + \frac{1}{l_e^2}gl_g\right] + \theta_2 - \frac{1}{l_e^2}gl_g + \tilde{\Delta}\\
\dot{P}_1 &= \frac{\beta_e}{V_1}\left(Q_{1M} - A_1\frac{\partial x}{\partial q}\dot{q} + \theta_3 + \Delta_{Q_1}\right)\\
\dot{P}_2 &= \frac{\beta_e}{V_2}\left(-Q_{2M} + A_2\frac{\partial x}{\partial q}\dot{q} - \theta_4 - \Delta_{Q_2}\right)
\end{aligned} \tag{15}$$

where $\tilde{\Delta} = \frac{\Delta}{J_c + m_L l_e^2}$, Δ_{Q1} and Δ_{Q2} representing the lumped disturbance effects, whose magnitudes are also bounded with known upper bounds denoted as δ, δ_1 and δ_2, respectively. Since the system has both parametric uncertainties θ_1 through θ_4 and uncertain nonlinearity $\tilde{\Delta}$ and Δ_{Qi} , the ARC approach proposed by Yao [12] will be customized to accomplish this system.

To illustrate the adaptive robust motion controller design, this section presents a design procedure for those working modes, whose working side is the head end chamber, i.e. P_1. The controller design for P_2 follows the same procedure and is omitted here.

Step 1 Define a switching-function-like quantity as

$$z_2 = \dot{z}_1 + k_1 z_1 = \dot{q} - \dot{q}_r, \quad \dot{q}_r \triangleq \dot{q}_d - k_1 z_1 \tag{16}$$

where $z_1 = q - q_d(t)$ is the output tracking error with $q_d(t)$ being the reference trajectory. Define a positive definite scalar function $V_2(t) = \frac{1}{2}(z_1^2 + z_2^2)$, differentiate $V_2(t)$ while noting (16) and (15)

$$\begin{aligned}
\dot{V}_2(t) &= z_1\dot{z}_1 + z_2\dot{z}_2\\
&= z_1(z_2 - k_1 z_1) + z_2\{\theta_1\left[\frac{1}{J_c}\left(\frac{\partial x}{\partial q}P_L - G_c - D_f\dot{q}\right) + \frac{1}{l_e^2}gl_g\right]\\
&\quad + \theta_2 - \frac{1}{l_e^2}gl_g - \ddot{q}_r + \tilde{\Delta}\}
\end{aligned} \tag{17}$$

If we treat P_L as the control input to (17), we can synthesize a virtual control law P_{Ld} such that $V_2(t)$ converges to zero or as small as possible.

The resulting control law P_{Ld} consists of two parts given by

$$\begin{aligned}
&P_{Ld}(q,\dot{q},\hat{\theta}_1,\hat{\theta}_2,t) = P_{Lda} + P_{Lds}\\
&P_{Lda} = \frac{\partial q}{\partial x}\left[G_c + D_f\dot{q} + \frac{J_c}{\hat{\theta}_1}\left(-\frac{\hat{\theta}_1}{l_e^2}gl_g - \hat{\theta}_2 + \frac{1}{l_e^2}gl_g + \ddot{q}_{2r} - z_1\right)\right]\\
&P_{Lds} = P_{Lds1} + P_{Lds2}, \quad P_{Lds1} = -\frac{J_c}{\theta_{1min}}\frac{\partial q}{\partial x}k_2 z_2
\end{aligned} \tag{18}$$

in which P_{Lda} functions as an adaptive model compensation, and P_{Lds} is a robust control law with $k_2 > 0$, and P_{Lds2} is chosen to satisfy the following robust performance conditions as in [10]

$$\begin{aligned}
&\text{i} \quad z_2\left[\frac{1}{J_c}\theta_1\frac{\partial x}{\partial q}P_{Lds2} - \tilde{\theta}^T\phi_2 + \tilde{\Delta}\right] \le \varepsilon_2\\
&\text{ii} \quad z_2\frac{\partial x}{\partial q}P_{Lds2} \le 0
\end{aligned} \tag{19}$$

where ε_2 is a positive design parameter, and where ϕ_2 is given as follows:

$$\phi_2 \triangleq \left[\frac{1}{J_c}\left(\frac{\partial x}{\partial q}P_{Lda} - G_c - D_f\dot{q}\right) + \frac{1}{l_e^2}gl_g,\ 1,\ 0,\ 0\right]^T \tag{20}$$

One of the multiple possible solutions of the robust feedback term P_{Lds2} is to choose

$$P_{Lds2} = -\frac{J_c}{\theta_{1min}}\frac{\partial q}{\partial x}k_{2s}z_2,\ k_{2s} = \frac{1}{2\varepsilon_2}(\|\phi_2\|^2\|\theta_M\|^2 + \delta^2) \tag{21}$$

With the virtual control law (18) and (21), the derivative of $V_2(t)$ would be:

$$\begin{aligned}
\dot{V}_2(t) = &-k_1 z_1^2 - \frac{\theta_1}{\theta_{1min}}k_2 z_2^2 + \frac{\theta_1}{Jc}\frac{\partial x}{\partial q}z_2 z_3\\
&+ z_2\left\{-\frac{\theta_1}{\theta_{1min}}k_{2s}z_2 - \tilde{\theta}^T\phi_2 + \Delta\right\}
\end{aligned} \tag{22}$$

where $z_3 = P_L - P_{Ld}$ denotes the discrepancy between the virtual control law P_{Ld} and the true system state P_L.

Step 2 Differentiate z_3 while noting (15),

$$\begin{aligned}
\dot{z}_3 &= \dot{P}_L - \dot{P}_{Ld}\\
&= \beta_e\{-\left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2}\right)\frac{\partial x}{\partial q}\dot{q} + \frac{A_1}{V_1}(Q_{1M} + \theta_3 + \Delta_{Q1})\\
&\quad + \frac{A_2}{V_2}(Q_{2M} + \theta_4 + \Delta_{Q2})\} - \dot{P}_{Ldc} - \dot{P}_{Ldu}
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\dot{P}_{Ldc} &= \frac{\partial P_{Ld}}{\partial q}\dot{q} + \frac{\partial P_{Ld}}{\partial \dot{q}}\hat{\ddot{q}}_2 + \frac{\partial P_{Ld}}{\partial t}\\
\dot{P}_{Ldu} &= \frac{\partial P_{Ld}}{\partial \dot{q}}\{-\tilde{\theta}_1[\frac{1}{J_c}(\frac{\partial x}{\partial q}P_L - G_c - D_f\dot{q}) - \frac{1}{l_e^2}gl_g]\\
&\quad - \tilde{\theta}_2 + \tilde{\Delta}\} + \frac{\partial P_{Ld}}{\partial \hat{\theta}}\dot{\hat{\theta}}
\end{aligned} \tag{24}$$

in which $\hat{\ddot{q}}_2$ represent the calculable part of $\ddot{q}$ given by

$$\hat{\ddot{q}}_2 = \hat{\theta}_1[\frac{1}{J_c}(\frac{\partial x}{\partial q}P_L - G_c - D_f\dot{q}) + \frac{1}{l_e^2}gl_g] + \hat{\theta}_2 - \frac{1}{l_e^2}gl_g \tag{25}$$

In (24), $\dot{P}_{Ldc}$ is calculable and can be used in the construction of control functions, but $\dot{P}_{Ldu}$ cannot due to various uncertainties.

Define a positive definite scalar function $V_3(t) = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2) = V_2(t) + \frac{1}{2}z_3^2$, differentiate $V_3(t)$ while noting (23),

$$\begin{aligned}
\dot{V}_3(t) &= \dot{V}_2(t) + z_3\dot{z}_3\\
&= -k_1 z_1^2 - \frac{\theta_1}{\theta_{1min}}k_2 z_2^2 + z_2\left\{\frac{\theta_1}{\theta_{1min}}k_{2s}z_2 - \tilde{\theta}^T\phi_2 + \tilde{\Delta}\right\} + \frac{\theta_1}{Jc}\frac{\partial x}{\partial q}z_2 z_3\\
&\quad + z_3\{\beta_e[-\left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2}\right)\frac{\partial x}{\partial q}\dot{q} + \frac{A_1}{V_1}(Q_{1M} + \theta_3 + \Delta_{Q1})\\
&\quad + \frac{A_2}{V_2}(Q_{2M} + \theta_4 + \Delta_{Q2})] - \dot{P}_{Ldc} - \dot{P}_{Ldu}\}
\end{aligned} \tag{26}$$

In viewing (26), Q_{1M} can be thought as the control input for (26) and step 2 is to synthesize a control function Q_{1Md} for Q_{1M} such that $V_3(t)$ converges to zero or as small as possible, i.e., equivalent to say that P_L tracks the desired control function P_{Ld} synthesized in Step 1 with a guaranteed transient performance.

Similar to (18), the control function Q_{1Md} consists of two parts given by

$$\begin{aligned}
&Q_{1Md}(q,\dot{q},P_1,P_2,\hat{\theta},t) = Q_{1Mda} + Q_{1Mds}\\
&Q_{1Mda} = -\hat{\theta}_3 + \frac{V_1}{A_1}\{(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2})\frac{\partial x}{\partial q}\dot{q} - \frac{A_2}{V_2}(Q_{2M} + \theta_4)\\
&\qquad + \frac{1}{\beta_e}(\dot{P}_{Ldc} - \frac{\hat{\theta}_1}{J_c}\frac{\partial x}{\partial q}z_2\}\\
&Q_{1Mds} = Q_{1Mds1} + Q_{1Mds2}, \quad Q_{1Mds1} = -\frac{V_1}{A_1\beta_e}k_3 z_3
\end{aligned} \tag{27}$$

where $k_3 > 0$ and like (19), Q_{1Mds2} is a robust control function chosen to satisfy the following two robust performance conditions:

$$\begin{aligned}
&\text{i} \quad z_3\left[\frac{A_1\beta_e}{V_1}Q_{1Mds2} - \tilde{\theta}^T\phi_3 - \frac{\partial P_{Ld}}{\partial \dot{q}}\tilde{\Delta} + \frac{A_1}{V_1}\Delta_{Q_1} + \frac{A_2}{V_2}\Delta_{Q_2}\right] \le \varepsilon_3\\
&\text{ii} \quad z_3\frac{A_1\beta_e}{V_1}Q_{1Mds2} \le 0
\end{aligned} \tag{28}$$

where ε_3 is a positive design parameter and ϕ_3 is defined as:

$$\phi_3 \triangleq \begin{bmatrix} \frac{1}{J_c}\frac{\partial x_L}{\partial q}z_2 - \frac{\partial P_{Ld}}{\partial \dot{q}}[\frac{1}{J_c}(\frac{\partial x_L}{\partial q}P_L - G_c) + \frac{1}{l_e^2}gl_g] \\ -\frac{\partial P_{Ld}}{\partial \dot{q}} \\ \frac{A_1}{V_1}\beta_e \\ \frac{A_2}{V_2}\beta_e \end{bmatrix} \quad (29)$$

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One of the possible solutions of the robust feedback term Q_{1Mds2} is to choose

$$Q_{1Mds2} = -\frac{V_1}{A_1\beta_e}k_{3s}z_3,\ k_{3s} = \frac{1}{2\varepsilon_3}(\|\phi_3\|^2\|\theta_M\|^2 + B^2) \quad (30)$$

where B is the upper bound for the lumped effects of all disturbances and uncertainties.

Similar to (14), the adaptation function for working-side would be

$$\tau = \phi_2 z_2 + \phi_3 z_3 \quad (31)$$

Once the desired control functions Q_{1Md} and Q_{2Md} are synthesized as given in (27) and (11), the desired flow command for each of the five valves can be calculated according to the selected working mode. Based on Assumption 4 that the pump and valves are able to provide sufficient supply pressure and flow, there exists a control input for each valve to obtain the desired flow rate Q_{vi}. The control input can be obtained through the inverse flow mapping. The details can be worked out easily and are omitted.

IV. THEORETICAL PERFORMANCE

Theorem 1: With the adaptive robust control law (11) and the adaptation law (7) and (14), the following results hold:

A. In general, the off-side pressure regulation is stable with prescribed transient performance and accuracy:

$$e_{p_2}^2(t) \leq e_{p_2}^2(0)\cdot exp(-2k_{p_2}t) + \frac{\varepsilon_p}{k_{p_2}}\cdot[1 - exp(-2k_{p_2}t)] \quad (32)$$

B. If after a finite time t_0, $\Delta_{Qi} = 0$ $i = 1,\ 2$, i.e., the model uncertainties are due to parametric uncertainties only, in addition to the results in A, asymptotic tracking ($e_{p_2} \longrightarrow 0$ as $t \longrightarrow \infty$) is obtained for any positive gain k_{p_2} and ε_p.

Theorem 2: With the robust control law (27) and the adaptation law (7) and (31), the following results hold:

A. In general, the over all closed loop system is stable with prescribed transient performance and final tracking accuracy:

$$V_3(t) \leq V_3(0)\cdot exp(-2\lambda t) + \frac{\varepsilon}{2\lambda}\cdot[1 - exp(-2\lambda t)] \quad (33)$$

where $\lambda = \min\{k_1,\ k_2,\ k_3\}$ and $\varepsilon = \varepsilon_2 + \varepsilon_3$.

B. If after a finite time t_0, $\Delta = 0$ and $\Delta_{Qi} = 0$ $i = 1,\ 2$, i.e., the model uncertainties are due to parametric uncertainties only, in addition to the results in A, asymptotic tracking ($z_1 \longrightarrow 0$ as $t \longrightarrow \infty$) is obtained for any positive gain k_i, $k = 1,2,3$ and ε_i, $i = 2,3$.

Remark 1: Results in B of the two theorems imply that the parametric uncertainties, including the large flow mapping errors, may be reduced through parameter adaptation and improved performance can be obtained.

Remark 2: The theoretically proven global stability and performance are based on the assumption that the pump and valves can provide sufficient flow for any trajectory. In reality, the electro-hydraulic system, like any system, has its own limits and working range, such as maximal supply pressure, maximal flow rate and so on. Demanding the system to follow a trajectory, which is beyond its capability, is not practical. A well designed reference trajectory is important to keep the system running in its proper working range.

V. EXPERIMENTAL RESULTS

The proposed two level control system is implemented on an electro-hydraulic arm. Experiments are done with a fast point-to-point trajectory for the system with and without 25Kg external payload. The desired trajectory, shown in Fig. 2, has maximal angular acceleration and velocity as $2rad/sec^2$ and $1rad/sec$, which are both close to the physical limits of the system. For comparison, experiments are also done when the parameter adaptation are shut off, which turns to be the deterministic robust control (DRC). The tracking errors of the system with and without payload are shown in Fig. 3.

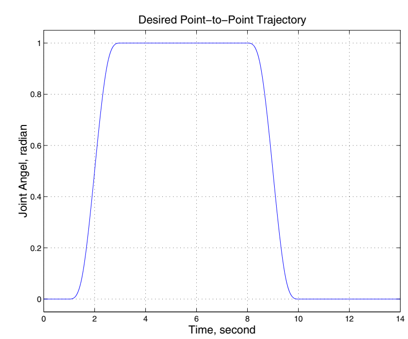

Fig. 2. Desired point-to-point trajectory

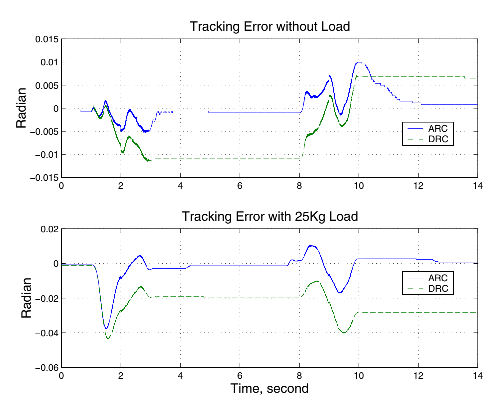

Fig. 3. Experimental tracking performances

It is shown in the experimental results, when parameter adaptation is shut off, large steady state tracking error is resulted due to the severe modelling error; on the other hand, the steady state tracking error is within the preset tolerance when adaptation turned on. The comparison clearly shows the performance improvement obtained via on-line adaptation of slowly changing disturbances and flow mapping error.

VI. CONCLUSIONS AND FUTURE WORKS

In this paper, an adaptive robust controller is designed to control the programmable valves with the manufacture provided flow mapping instead of individually calibrated flow mapping. Though there exists a severe modelling error due to the inaccurate manufacture's flow mapping, robust stability is guaranteed theoretically. Nevertheless, through explicitly dealing with model uncertainties , i.e., on-line estimate the slowing changing lumped disturbances and flow mapping errors, system performance is greatly improved.

Current research still lumps the flow mapping errors of the five cartridge valves together into meter-in and meter-out flow errors $\tilde{Q}_1$ and $\tilde{Q}_2$. Future work includes on-line estimation of the nonlinear pressure compensated flow mappings for each cartridge valve.

REFERENCES

[1] S. Liu and B. Yao, "Energy-saving control of single-rod hydraulic cylinders with programmable valves and improved working mode selection," *SAE Tracnsactions - Journal of Commercial Vehicle*, pp. 51–61, 2002, SAE 2002-01-1343.

[2] A. Jansson and J.-O. Palmberg, "Seperate control of meter-in and meter-outorifices in mobile hydraulic systems," *SAE Transactions*, vol. 99, no. 2, pp. 337–383, 1990.

[3] J. Aardema and D. Koehler, "System and method for controlling an independent metering valve," *United States Patent, 5,947,140*, 1999.

[4] R. Book and C. Goering, "Programmable electrohydraulic valve," *SAE Transactions*, vol. 108, no. 2, pp. 346–352, 1999.

[5] B. Yao and S. Liu, "Energy-saving control of hydraulic systems with novel programmable valves," in *the 4th World Congress on Intelligent Control and Automation (WCICA)*, Shanghai, China, 2002.

[6] S. Liu and B. Yao, "Coordinate control of energy-saving programmable valves," in *ASME International Mechanical Engineering Congree and Exposition*, Wahsington, D.C., 2003, IMECE 2003-42668.

[7] J. J. Pippenger, *Hydraulic Cartridge Valve Technology*. Amalgan Publishing Company, 1990.

[8] J. Watton, *Fluid Power Systems*. Prentice Hall, 1989.

[9] H. E. Merritt, *Hydraulic Control Systems*. John Wiley & Sons, 1967.

[10] B. Yao, F. Bu, J. T. Reedy, and G. T. Chiu, "Adaptive robust control of singal-rod hydraulic actuators: Theory and experiments," *The IEEE/ASME Trans. on Mechatronics*, vol. 5, no. 1, pp. 79–91, 2000.

[11] B. Yao and M. Tomizuka, "Smooth robust adaptive sliding mode control of robot manipulators with guaranteed transient performance," *Trans. of ASME, Journal of Dynamic Systems, Measurement and Control*, vol. 118, no. 4, pp. 764–775, 1996.

[12] B. Yao, "High performance adaptive robust control of nonlinear systems: a general framework and new schemes," in *Proc. of IEEE Conference on Decision and Control*, 1997, pp. 2489–2494.

[13] B. Yao and M. Tomizuka, "Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form," *Automatica*, vol. 33, no. 5, pp. 893–900, 1997.

[14] F. Bu and B. Yao, "Nonlinear adaptive robust control of hydraulic actuators regulated by proportional directional control valves with deadband and nonlinear flow gain coefficients," in *Proc. of American Control Conference*, Chicago, 2000, pp. 4129–4133.

[15] G. Goodwin and D. Mayne, "A parameter estimation perspective of continuous time model reference adaptive control," *Automatica*, vol. 23, no. 1, pp. 57–70, 1989.

[16] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*. Prentice Hall, Inc, 1989.

VII. APPENDIX

Define a positive definite scalar function $V_p(t) = \frac{1}{2}e_{p2}^2$. Diffentiate V_p while noting (11) and (12), one can obtain:

$$\begin{aligned}\dot{V}_p(t) &= e_{p_2}\cdot\dot{e}_{p_2}\\ &= e_{p_2}\cdot\frac{\beta_e}{V_2}\left(-Q_{2M}+A_2\frac{\partial x}{\partial q}\dot{q}-\theta_Q-\Delta_{Q2}\right)\\ &= -k_{p_2}e_{p_2}^2-e_{p_2}\frac{\beta_e}{V_2}\left\{Q_{2Ms2}-\tilde{\theta}_Q-\Delta_{Q2}\right\}\\ &\le -k_{p_2}e_{p_2}^2+\varepsilon_p\\ &= -2k_{p2}V_p(t)+\varepsilon_p\end{aligned} \tag{34}$$

Therefore,

$$V_p(t)\le V_p(0)\cdot exp(-2k_{p2}t)+\frac{\varepsilon_p}{2k_{p2}}\cdot[1-exp(-2k_{p2}t)] \tag{35}$$

which is equivalent to (32).

To prove part B, define a positive definite scalar function $V_{p\theta}(t) = V_p + \frac{1}{2}\gamma^{-1}\tilde{\theta}_Q^2$. Noting $\Delta_{Q2} = 0$ and $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$, from (14), (P2) of (9) and (i) of (12), one can obtain:

$$\begin{aligned}\dot{V}_{p\theta}(t) &= \dot{V}_p+\tilde{\theta}_Q\gamma^{-1}\dot{\hat{\theta}}_Q\\ &= -k_{p_2}e_{p_2}^2-e_{p_2}\frac{\beta_e}{V_2}Q_{2Ms2}+e_{p_2}\frac{\beta_e}{V_2}\tilde{\theta}_Q+\tilde{\theta}_Q\gamma^{-1}Proj_{\hat{\theta}}(\gamma\tau_Q)\\ &= -k_{p_2}e_{p_2}^2-e_{p_2}\frac{\beta_e}{V_2}Q_{2Ms2}+\tilde{\theta}^T[\gamma^{-1}Proj_{\hat{\theta}}(\gamma\tau_Q)-\tau_Q]\\ &\le -k_{p_2}e_{p_2}^2\end{aligned} \tag{36}$$

Therefore, $e_{p_2}\in L_2^2$. It is also easy to check that $\dot{e}_{p_2}$ is bounded. So, $e_{p_2}\to 0$ as $t\to\infty$ by the Barbalat's lemma.

Proof of Theorem 2:

Substitute the adaptive control law (27) into (26), one can get:

$$\begin{aligned}\dot{V}_3(t) &= -k_1z_1^2-\frac{\theta_1}{\theta_{1min}}k_2z_2^2-k_3z_3^2\\ &\quad +z_2\left\{-\frac{\theta_1}{\theta_{1min}}k_{2s}z_2-\tilde{\theta}^T\phi_2+\Delta\right\}\\ &\quad +z_3\left\{-k_{3s}z_3-\tilde{\theta}^T\phi_3+A_1\Delta_{Q1}+A_2\Delta_{Q2}-\frac{\partial P_{Ld}}{\partial\dot{q}}\tilde{\Delta}\right\}\\ &\le -k_1z_1^2-k_2z_2^2-k_3z_3^2+\varepsilon_2+\varepsilon_3\\ &\le -2\lambda\frac{1}{2}(z_1^2+z_2^2+z_3^2)+\varepsilon\\ &= -2\lambda V_3(t)+\varepsilon\end{aligned} \tag{37}$$

which proves (33).

To prove part B, define a positive definite scalar function $V(t) = V_3+\frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}$. Noting all disturbance terms are zero and $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$, from (14), (P2) of (9), one can obtain:

$$\begin{aligned}\dot{V}(t) &= \dot{V}_3+\tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}}\\ &\le -k_1z_1^2-k_2z_2^2-k_3z_3^2-\frac{\theta_1}{\theta_{1min}}k_{2s}z_2^2-k_{3s}z_3^2\\ &\quad -\tilde{\theta}^T(\phi_2z_2+\phi_3z_3)+\tilde{\theta}^T\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\tau)\\ &\le -\textstyle\sum_{j=1}^3 k_jz_j+\tilde{\theta}^T[\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\tau)-\tau]\\ &\le -\textstyle\sum_{j=1}^3 k_jz_j\end{aligned} \tag{38}$$

Therefore, $z\in L_2^2$. It is also easy to check that $\dot{z}_1$ is bounded. So, $z_1\to 0$ as $t\to\infty$ by the Barbalat's lemma.