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Brief Paper

# Variable Structure Adaptive Motion and Force Control of Robot Manipulators*

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**Key Words**—Variable structure control; adaptive control; robot manipulators; compliant motion; force control.

**Abstract**—A variable structure adaptive method is developed for robust motion and force tracking control of robot manipulators in the presence of uncertainties in parameters of robot dynamics, contact surface, and external disturbances. The method takes advantages of both variable structure control (VSC) and adaptive control which results in a two-loop controller structure. VSC utilized in the inner-loop drives the system to reach and be maintained on a dynamic sliding mode which is provided by the outer-loop design. Adaptive control is used in the outer-loop to estimate contact surface parameters for ensuring the system with force tracking property. Overall stability of the two-loop system is established and simulation results are presented.

## 1. *Introduction*

MANY APPLICATIONS of robots involve tasks in which the robot end-effectors make compliant contact with environment. Simultaneous control of the motion as well as the contact force is required (Whitney, 1987). So far, several approaches have been proposed such as impedance control (Hogan, 1985; Kazerooni *et al.*, 1986; Lasky and Hsia, 1991; Chan *et al.*, 1991), hybrid position/force control (Raibert and Craig, 1981, Khatib, 1987), and constrained motion control (McClamroch and Wang, 1988; Yoshikawa *et al.*, 1988; Mills and Goldenberg, 1989; Yao *et al.*, 1990; Yao *et al.*, 1992b).

Parameters of the system such as gravitational load and environmental stiffness vary from one task to another, and hence may not be precisely known in advance. The system is also subjected to unmodelled disturbances due to joint friction, surface friction, etc. These effects make it difficult to solve motion and force tracking control problems. There are only a few published papers dealing with all these problems together. Carelli *et al.* (1990) proposed an adaptive force control method to estimate unknown parameters of the robot and the environmental stiffness. The inertia matrix of the robot is assumed to remain constant. Due to the nonlinear and coupled nature of the robot dynamics and the wide working range of the robot, this assumption usually cannot be satisfied. Recently Lozano and Brogliato (1992), proposed an adaptive hybrid position/force control scheme for the redundant robot interacting with stiffness environment without using force derivative feedback. Their approach needs exact knowledge of the stiffness matrix.

Variable structure control (VSC) theory, which possesses advantages of perfect adaptation to various perturbations from modelling and disturbances, receives great attention (Utkin, 1987; Slotine, 1985; Young, 1978; Yao *et al.*, 1992a). The underlying principle behind VSC is to alter system dynamics along some surfaces in the state space so that the states of the system are attracted to these surfaces and maintained on them thereafter. The sliding motion on the surfaces can be guaranteed with desired dynamic behavior regardless of the modelling uncertainties. Slotine and Li (1988) introduced VSC technique with fixed sliding surfaces in adaptive motion control of robots to achieve enhanced robustness.

In this paper, we address the problem of designing robust motion and force tracking controller in the presence of external disturbances and parametric uncertainties in both the robot dynamics and the environment. A new concept named variable structure adaptive (VSA) method is developed which results in a two-loop control system. VSC method is utilized in the inner-loop which will force the system to reach and be maintained on a dynamic sliding mode provided by the outer-loop design. In the outer-loop, adaptive control method (Astrom and Wittenmark, 1989; Narendra and Annaswamy, 1989) estimates environmental stiffness and provides the system with good force tracking property. Overall stability of the two-loop system is established. The suggested approach needs to know the normal direction of the surface or the shape of the surface but without knowing its stiffness and exact location. Chattering problems associated with the VSC control law are not addressed in the paper.

## 2. *Dynamic model and problem formulation*

Dynamic equation of a general rigid link manipulator having $n$ degree of freedom can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + J^T(q)F + \tilde{f}(t) = \tau \qquad (1)$$

where all the terms have the same meaning as that in Yao *et al.* (1992a). The robot is assumed to be nonredundant. Suppose that the environment in undeformation is described by a set of $m$ time-varying hypersurfaces (see Fig. 1)

$$\Phi(x,t) = \Phi_e \quad \Phi(x,t) = [\phi_1(x,t),\ldots,\phi_m(x,t)]^T \quad m \le n \qquad (2)$$

which are mutually independent for any $t$. $\Phi_e = [\phi_{e1},\ldots,\phi_{em}]^T$ represents the equilibrium position of the environment which is considered to be constant but unknown. In the case of $\Phi_e = \Phi_e(t)$, the analysis is still valid if $\dot{\Phi}_e(t)$ and $\ddot{\Phi}_e(t)$ are known. Suppose that there exists a set of $(n-m)$ scalar functions $\{\psi_1(x,t),\ldots,\psi_{n-m}(x,t)\}$ such that $\{\phi_i(x,t),\ i=1,\ldots,m;\ \psi_j(x,t),\ j=1,\ldots,n-m\}$ are mutually independent for any $t$. The task space is defined as (Yao *et al.*, 1992a)

$$r = [r_f^T, r_p^T]^T \quad r_f = [\phi_1(x,t),\ldots,\phi_m(x,t)]^T \qquad (3)$$
$$r_p = [\psi_1(x,t),\ldots,\psi_{n-m}(x,t)]^T.$$

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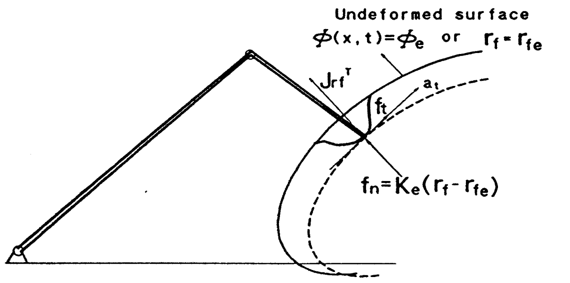


FIG. 1. Robot interaction with a stiff environment.

The same form of (3) is also used by Yoshikawa *et al.* (1988) to study constrained motion of the robot. Differentiate (3)

$$\dot{r} = J_r\dot{x} + v_t = J_q\dot{q} + v_t \tag{4}$$

where

$$J_r = \frac{\partial r(x,t)}{\partial x} \quad J_r = [J_{rf}^T J_{rp}^T]^T \quad J_{rf} \in R^{m\times n} \quad J_{rp} \in R^{(n-m)\times n}$$
$$J_q = \frac{\partial r(x(q),t)}{\partial q} \quad J_q = J_r J \quad J_q, J_r \in R^{n\times n} \quad v_t = \frac{\partial r(x,t)}{\partial t} \in R^n. \tag{5}$$

Using the transformation (3) and (4) in (1), multiplying both sides by $J_q^{-T}$

$$H(r,t)\ddot{r} + C(r,\dot{r},t)\dot{r} + G(r,t) + F_t + \tilde{F}(r,t) = T_r - F_r \tag{6}$$

where

$$\begin{aligned} H(r,t) &= J_q^{-T}(q,t)M(q)J_q^{-1}(q,t)\\ C(r,\dot{r},t) &= J_q^{-T}C(q,\dot{q})J_q^{-1} - J_q^{-T}M(q)J_q^{-1}\dot{J}_qJ_q^{-1}\\ G(r,t) &= J_q^{-T}G(q) \quad \tilde{F}(r,t) = J_q^{-T}\tilde{f}(t) \end{aligned} \tag{7}$$
$$F_t = -H(r,t)\dot{v}_t - C(r,\dot{r},t)v_t \quad F_r = J_r^{-T}F \quad T_r = J_q^{-T}(q,t)\tau.$$

In the definition of the task space (3), directions of curvilinear coordinates $r_f$ are aligned with normal directions of the environment in undeformation (Fig. 1). Therefore, subspace $r_f \in R^m$ in fact represents the constrained subspace in which force tracking control is required and subspace $r_p \in R^{n-m}$ can be considered as the unconstrained subspace in which motion control is needed. Thus definition of the task space has a clear physical meaning.

In the normal directions of contact surfaces, the environment is assumed to be an elastic model with unknown constant symmetric positive definite (s.p.d.) stiffness matrix $K_e$ (Carelli *et al.*, 1990):

$$f_n = K_e(r_f - r_{fe}) \quad \text{or} \quad r_f = K_f f_n + r_{fe} \quad f_n \le 0, \tag{8}$$

where $r_{fe} = \Phi_e$ represents unknown equilibrium position, $f_n \in R^m$ are normal contact force components, $K_f = K_e^{-1} \in R^{m\times m}$ is constant s.p.d. compliance matrix. Vector $f_n \le 0$ is true in element, that is, $(f_n)_i \le 0$, $i = 1, \ldots, m$. We denote by $|A|$ the matrix (or vector) whose elements are the absolute values of the components of $A$, that is, $|A|_{ij} = |A_{ij}|$. We shall say that matrices (or vector) $A \le B$ if this relation holds for corresponding components of $A$ and $B$, that is, $A_{ij} \le B_{ij}$. It is assumed that the end-effector is initially in contact with the surfaces, and the control exercised over contact force is such that the force will always hold the end-effector on the surfaces, that is, $f_n \le 0$ will never be violated.

The total interaction force $F$ in (1) or $F_r$ in (6) is supposed to be given by

$$F = \frac{J_{rf}^T}{\|J_{rf}\|}f_n + a_t f_t(\mu, v_{\text{end}}, f_n) \triangleq L(x,\dot{x},\mu,t)f_n$$
$$F_r = L_r(x,\dot{x},\mu,t)f_n \quad L_r \triangleq J_r^{-T}L(x,\dot{x},\mu,t) \quad L, L_r \in R^{n\times m} \tag{9}$$

where $a_t \in R^{n\times m}$ represent the unit tangent directions of the surfaces, $f_t \in R^m$ is the tangent friction force whose magnitude depends on unknown friction coefficient $\mu$ and $f_n$, and direction depends on end-effector velocity $v_{\text{end}}$.

Due to parametric uncertainties of the robot and the environment, we know only the estimated values $\hat{H}(r,t)$, $\hat{C}(r,\dot{r},t)$, $\hat{G}(r,t)$, $\hat{F}_t$, and $\hat{L}_r$ of $H$, $C$, $G$, $F_t$ in (6) and $L_r$ in (9), and $\hat{K}_e$ of $K_e$ in (8) respectively. It is assumed that the modelling errors in (6) are bounded by

$$\begin{gathered} |\Delta H(r,t)| \le \delta_H(r,t) \quad |\Delta C(r,\dot{r},t)| \le \delta_C(r,\dot{r},t)\\ |\Delta G(r,t)| \le \delta_G(r,t) \quad |\Delta F_t| \le \delta_{F_t}\\ |\tilde{F}(r,t)| \le \delta_{\tilde{F}}(r,t) \quad |\Delta L_r| \le \delta_{L_r} \quad |\Delta K_e| \le \delta_{K_e} \end{gathered} \tag{10}$$

where $\Delta\bullet$ reprsents the estimated error of $\bullet$ given by $\bullet - \hat{\bullet}$. The bounds, matrices (or vectors) $\delta_\bullet$ are assumed to be known. These bounds can be directly determined by off-line estimation. They can also be calculated from the modelling errors in the joint space (Yao *et al.*, 1992a).

The following properties can be obtained (Yao *et al.*, 1992a).

*Property* 1. For any finite work space $\Omega = \{q : |q - q_0| \le q_{\max}\}$ in which $J_q$ is nonsingular, $H(r,t)$ is a s.p.d. matrix with $k_r' I_{n\times n} \le H(r,t) \le k_r'' I_{n\times n}$ $\forall q \in \Omega$, $t \in R$.

*Property* 2. The matrix $N(r,\dot{r},t) = \dot{H}(r,t) - 2C(r,\dot{r},t)$ is a skew-symmetric matrix.

3. *VSA motion and force control of robot manipulators*

Suppose $r_{pd}(t) \in R^{n-m}$ is given as the desired motion trajectory in the unconstrained subspace, and $f_{nd}(t) \in R^m$ is the desired force trajectory in the constrained subspace. Let $e_p = r_p(t) - r_{pd}(t)$, $e_f = f_n(t) - f_{nd}(t)$, $e_p \in R^{n-m}$, $e_f \in R^m$ be the tracking errors of motion and interaction force. Consider the robot manipulator described by (6), whose end-effector is in contact with the environment and interaction force is given by (8) and (9). The robot is under the modelling errors (10) which account for parametric uncertainties and external disturbances. The VSA motion and force controller design problem of the robot can be stated as that of designing a VSC control law and a parameter update law so that $e_p \to 0$, $e_f \to 0$ as $t \to \infty$.

First, the switching function is chosen as

$$\begin{gathered} s = \dot{r} - z \quad s = [s_f^T, s_p^T]^T \quad z = [z_f^T, z_p^T]^T\\ s_f, z_f \in R^m \quad s_p, z_p \in R^{n-m} \end{gathered} \tag{11}$$

where $z(r, \dot{r}, e_f, e_p, \dot{e}_p)$ are design functions which are chosen to ensure that the robot tracks the desired motion and force trajectories in the resulted dynamic sliding mode. During sliding motion $\{s = 0\}$, which is described by

$$\dot{r}_f = z_f \tag{12}$$
$$\dot{r}_p = z_p. \tag{13}$$

By choosing $z_p = \dot{r}_{pd}(t) - D_p e_p$ where $D_p$ is any s.p.d. matrix, the motion sliding mode (13) is governed by $\dot{e}_p(t) + D_p e_p = 0$ which is asymptotically stable, that is, $e_p \to 0$.

Combining (12) and (8), the force sliding mode equation is

$$K_f \dot{f}_n = z_f \tag{14}$$

in which $K_f$ is unknown. Therefore, adaptive control method is used to estimate the compliance matrix $K_f$ and to guarantee force tracking. Denote the independent unknown parameter set of $K_f$ as $\beta \in R^k$, $k \le \frac{1}{2}m(m+1)$, and let

$$K_f\dot{f}_{nd} = Y_1(\dot{f}_{nd})\beta + Y_2(\dot{f}_{nd}) \quad \hat{K}_f\dot{f}_{nd} = Y_1(\dot{f}_{nd})\hat{\beta} + Y_2(\dot{f}_{nd}), \tag{15}$$

where $Y_2(\dot{f}_{nd}) \in R^m$ are the known terms, $Y_1(\dot{f}_{nd}) \in R^{m\times k}$, and $\hat{\beta}$ is the estimate of $\beta$. The following choices of the force design function and an adaptation law are suggested:

$$z_f = \hat{K}_f\dot{f}_{nd}(t) - D_f e_f = Y_1(\dot{f}_{nd})\hat{\beta} + Y_2(\dot{f}_{nd}) - D_f e_f \tag{16}$$
$$\dot{\hat{\beta}} = -\Gamma Y_1^T(\dot{f}_{nd})e_f \tag{17}$$

where $D_f \in R^{m\times m}$, $\Gamma \in R^{k\times k}$ are any constant s.p.d. matrices.

*Theroem* 1. If the desired force trajectory $f_{nd}(t)$, $\dot{f}_{nd}(t)$ are continuous and bounded, the suggested design function (16) and adaptation law (17) guarantees the force sliding mode equation (14) to track the desired force $f_{nd}(t)$ with transient satisfying

$$\|e_f(t)\| \le \sqrt{\frac{2}{\lambda_{\min}(K_f)} V_1(t_r)} \quad \forall t \ge t_r$$
$$\|\Delta\beta(t)\| \le \sqrt{2\lambda_{\max}(\Gamma) V_1(t_r)} \tag{18}$$
$$V_1(t_r) = \tfrac{1}{2}[e_f^T(t_r) K_f e_f(t_r) + \Delta\beta^T(t_r)\Gamma^{-1}\Delta\beta(t_r)]$$

where $\Delta\beta = \beta - \hat{\beta}$, $t_r$ is the time when the system reaches the sliding mode, and $\lambda(\bullet)$ means eigenvalue of matrix $\bullet$.

*Proof.* For (14), choose a positive definite function $V_1 = \frac{1}{2}[e_f^T K_f e_f + \Delta\beta^T \Gamma^{-1}\Delta\beta]$ and differentiate

$$\dot{V}_1 = e_f^T[K_f \dot{f}_n - K_f \dot{f}_{nd}(t)] + \Delta\beta^T \Gamma^{-1}\Delta\dot{\beta}$$
$$= e_f^T[z_f - K_f \dot{f}_{nd}(t)] + \Delta\beta^T \Gamma^{-1}\Delta\dot{\beta}. \tag{19}$$

Since $\beta$ is constant, $\Delta\dot{\beta} = -\dot{\hat{\beta}}$. Substitute (16) and (17) into the above, and noticing (15)

$$\dot{V}_1 = e_f^T[-D_f e_f - Y_1(\dot{f}_{nd})\Delta\beta] - \Delta\beta^T\Gamma^{-1}\dot{\hat{\beta}}$$
$$= -e_f^T D_f e_f - \Delta\beta^T[Y_1^T e_f + \Gamma^{-1}\dot{\hat{\beta}}]$$
$$= -e_f^T D_f e_f \le 0. \tag{20}$$

Therefore, $e_f \in L_\infty^m \cup L_2^m$ and $\Delta\beta \in L_\infty^k$. Thus, $z_f$, $\dot{f}_n \in L_\infty^m$, which imply $e_f$ is uniform continuous. From (20), $e_f \to 0$ and

$$\tfrac{1}{2}[\lambda_{\min}(K_f)\|e_f(t)\|^2 + \lambda_{\min}(\Gamma^{-1})\|\Delta\beta(t)\|^2] \le V_1(t) \le V_1(t_r)$$

which leads to (18). ■

The above choice of $z = [z_f^T, z_p^T]^T$ guarantees the sliding mode track the desired motion $r_{pd}(t)$ and the desired force $f_{nd}(t)$. What remains is to determine control torque such that the system reaches the sliding mode in finite time and stays on it thereafter. From (16) and (8),

$$\dot{z}_f = Y_1(\dot{f}_{nd})\dot{\hat{\beta}} + \dot{Y}_1 - (\dot{f}_{nd})\hat{\beta} + \dot{Y}_2 - D_f(K_e \dot{r}_f - \dot{f}_{nd}(t)).$$

Denote

$$\dot{z} = \dot{z}_1 + \dot{z}_2$$
$$\dot{z}_1 = \begin{bmatrix} Y_1(\dot{f}_{nd})\dot{\hat{\beta}} + \dot{Y}_1(\dot{f}_{nd})\hat{\beta} + \dot{Y}_2 - D_f(\hat{K}_e \dot{r}_f - \dot{f}_{nd}(t)) \\ \ddot{r}_{pd}(t) - D_p \dot{e}_p \end{bmatrix} \tag{21}$$
$$\dot{z}_2 = \begin{bmatrix} -D_f \Delta K_e \dot{r}_f \\ 0 \end{bmatrix},$$

where $\hat{K}_e$ can be chosen constant or updated by $\hat{K}_e = \hat{K}_f^{-1}$ in which $\hat{K}_f$ is calculated from (17) if $\hat{K}_f$ is nonsingular. In either case, we assume its estimation error satisfies assumption (10). It is seen that $z$, $\dot{z}_1$ can be calculated from (16) and (17) by using only measurements of position, velocity and force, and $\dot{z}_2$ is bounded by

$$|\dot{z}_2| \le \delta_{\dot{z}} \quad \delta_{\dot{z}} = \begin{bmatrix} |D_f|\,\delta_{K_e}|\dot{r}_f| \\ 0 \end{bmatrix}. \tag{22}$$

*Theorem* 2. For the robot manipulator described by equation (6) with the modelling errors (10), the system follows the desired motion trajectory $r_{pd}(t)$ while exerting the desired force trajectory $f_{nd}(t)$ if the following control torque is applied:

$$T_r = \hat{H}(r,t)\dot{z}_1 + \hat{C}(r,t)z + \hat{G}(r,t) + \hat{F}_t + \hat{L}_r f_n$$
$$- T_d - K_s s - \varepsilon sgn(s) \tag{23}$$

where $\varepsilon > 0$ and

$$sgn(s) = [sgn(s_1), \ldots, sgn(s_n)]^T$$
$$T_d = [(T_d)_1, \ldots, (T_d)_n]^T \tag{24}$$
$$(T_d)_i = (\delta_T)_i sgn(s_i) \quad i = 1, \ldots, n.$$

The bound $\delta_T = [(\delta_T)_1, \ldots, (\delta_T)_n]^T$ satisfies

$$\delta_T \ge \delta_H(r,t)|\dot{z}_1| + (|\hat{H}| + \delta_H)\delta_{\dot{z}} + \delta_C|z| + \delta_G$$
$$+ \delta_{F_t} + \delta_{\tilde{F}} + \delta_{L_r}|f_n|. \tag{25}$$

$sgn(.)$ is the sign function and $K_s$ is any s.p.d. matrix. Furthermore, the time $t_r$ when the system reaches the sliding mode is

$$t_r \le \frac{2}{c_3}\ln\left(1 + \frac{c_3}{c_4}\sqrt{V_0}\right) \tag{26}$$

where

$$c_3 = \frac{2\lambda_{\min}(K_s)}{k_r''}, \quad c_4 = \varepsilon\sqrt{\frac{2}{k_r''}}, \quad V_0 = \tfrac{1}{2}s^T(0)H(r(0),0)s(0), \tag{27}$$

and the reaching transient response is shaped by

$$\|s\| \le \sqrt{\frac{2}{k_r'}}\left[\left(\sqrt{V_0} + \frac{c_4}{c_3}\right)e^{(-c_3/2)t} - \frac{c_4}{c_3}\right]. \tag{28}$$

*Proof.* For (6), choose $V = \frac{1}{2}s^T H(r,t)s$. From property 1,

$$\tfrac{1}{2}k_r'\|s\|^2 \le V \le \tfrac{1}{2}k_r''\|s\|^2. \tag{29}$$

Differentiating $V$ with respect to time yields

$$\dot{V} = s^T H\dot{s} + \tfrac{1}{2}s^T\dot{H}s$$
$$= s^T(H(r,t)\ddot{r} - H(r,t)\dot{z}) + s^T C(r,\dot{r},t)s$$
$$= s^T[T_r - H(r,t)\dot{z} - C(r,\dot{r},t)z - G(r,t) - F_t - \tilde{F}(r,t) - L_r f_n] \tag{30}$$

where Property 2 has been used to eliminate the term $\frac{1}{2}s^T\dot{H}s$. Substituting control torque (23) into it and noticing (29), we have

$$\dot{V} = -s^T K_s s - \varepsilon s^T sgn(s) - s^T T_d$$
$$- s^T[\Delta H(r,t)\dot{z}_1 + H(r,t)\dot{z}_2 + \Delta C(r,\dot{r},t)z$$
$$+ \Delta G(r,t) + \Delta F_t + \tilde{F} + \Delta L_r f_n]$$
$$\le -s^T K_s s - \varepsilon s^T sgn(s) - s^T T_d + |s|^T\delta_T$$
$$\le -\lambda_{\min}(K_s)\|s\|^2 - \varepsilon\sum_{i=1}^{n}|s_i|$$
$$\le -\lambda_{\min}(K_s)\|s\|^2 - \varepsilon\|s\|$$
$$\le -c_3 V - c_4\sqrt{V}. \tag{31}$$

Thus

$$\sqrt{V} \le \left(\sqrt{V_0} + \frac{c_4}{c_3}\right)e^{(-c_3/2)t} - \frac{c_4}{c_3} \tag{32}$$

and this means in finite time $V = 0$, that is, $s = 0$. Moreover, from (29), the reaching transient response is shaped by (28). The upper limit of the reaching time $t_r$ is solved by setting the right-hand side of (32) equal to zero which is given by (26). ■

*Remark* 1. From Theorem 2, $s$ reaches to zero exponentially with a rate determined by the controller parameter $K_s$ and the reaching time $t_r$ is inversely dependent on $\lambda_{\min}(K_s)$ and $\varepsilon$. Therefore, by suitable choices of $K_s$ and $\varepsilon$, the reaching transition can be guaranteed with prescribed quality. □

*Remark* 2. Control law (23) is discontinuous across sliding surface which may lead to control chattering. Smooth implementation of VSC law can be used, for example, the concept of boundary layer (Slotine, 1985), in which $sgn(s_i)$ is replaced by the saturation function $sat(s_i/\Delta_i)$ where $\Delta_i$ is the boundary layer thickness. Such a modification guarantees $s$ within a small ball around the sliding mode $s = 0$, that is, $|s| \le \Delta$. In such a case, the force sliding mode equation (14) should be changed to $K_f \dot{f}_n = z_f + s_f$. Since $s_f$ can be considered as a small disturbance, system performance will not be unduly affected. Mathematically, a small disturbance may cause the adaptive control law (16) with (17) to be unstable due to parameter estimation drifting when force trajectory is not persistently exciting. This problem can be dealt with by applying standard techniques such as $\sigma$-modification (Narendra and Annaswamy, 1989) to the adaptation law (17). The drawback is that the tracking error can only be guaranteed to converge to a ball whose size depends on the unknown parameters. □

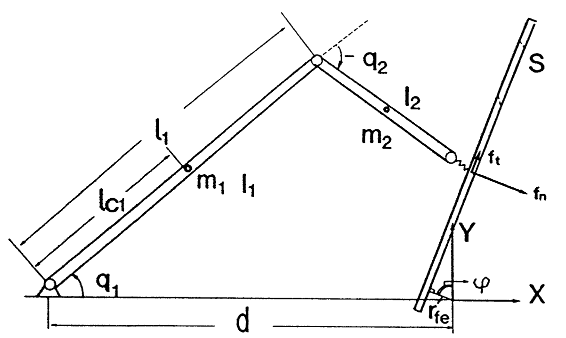


FIG. 2. Configuration of the robot.

## 4. *Simulation*

The robot used is the same as that in Yao *et al.* (1992a) where estimated value, bounds, etc., can be found. The robot is assumed in contact with a surface $S$ which rotates around the $Z$-axis as shown in Fig. 2. The surface $S$ has unknown stiffness $k_e$ with unknown friction coefficient $\mu = 0.2$. The time-varying surface $S$ in undeformation is described by

$$-x\sin(\varphi(t)) + y\cos(\varphi(t)) = r_{fe}$$
$$\varphi(t) = \tfrac{3}{8}\pi - \tfrac{1}{8}\pi\cos(\tfrac{1}{2}\pi t) \tag{33}$$

where the distance between $S$ and $Z$-axis, $r_{fe}$, is an unknown constant. The task space is defined as

$$r = [r_f, r_p]^T \quad r_f = -x\sin(\varphi) + y\cos(\varphi),$$
$$r_p = x\cos(\varphi) + y\sin(\varphi). \tag{34}$$

The interaction force on the surface in the task space is given by (9)

$$F_r = \begin{bmatrix} f_n \\ f_t \end{bmatrix} = L_r f_n \quad L_r = \begin{bmatrix} 1 \\ \mu sgn(f_n) sgn(\dot{r}_p) \end{bmatrix} \tag{35}$$
$$f_n = k_e(r_f - r_{fe}), \quad f_t = \mu\,|f_n|\,sgn(\dot{r}_p).$$

The estimated value of $L_r$ and the bounds $\delta_{L_r}$ in (10) are $\hat{L}_r = [1\ 0]^T$, $\delta_{L_r} = [0\ 0.2]^T$. In this case, $\beta$ of (15) represents $k_f = k_e^{-1}$ with initial estimated value $\hat{\beta}(0) = \hat{k}_e^{-1}$, and $Y_1(f_{nd}) = f_{nd}$, $Y_2(f_{nd}) = 0$. $k_e$ is supposed to be equal to $500\,\mathrm{N\,m^{-1}}$ while real value is $5000\,\mathrm{N\,m^{-1}}$ (silicon rubber) and $\delta_{k_e} = 5000$. Actual $r_{fe}$ is $0.0008$ m and initial $r_f(0)$ is $-0.0002$ m. Sampling time is supposed to be $0.0025$ s. The desired motion and force trajectories are $r_{pd} = 0.14(1 - \cos(0.5\pi t))$ and $f_{nd} = -10 + 5\cos(\pi t)$. Adaptive law is given by (17) and the control torque is calculated by (23), in which $K_s = \mathrm{diag}\{100, 100\}$, $\varepsilon = 1$, $D_f = 0.02$, $D_p = 20$, $\Gamma = 0.01$. In this case, the initial tracking errors of the system are $e_p(0) = 0$, $e_f(0) = 0$, $s(0) = [0, 0]^T$, $\Delta\beta(0) = 0.0018$.

There are several ways to implement VSA law (23). We have chosen to implement it in two ways.

*Case* 1: Smooth $sgn(s)$ by $sat(s/\Delta)$ where $\Delta = [0.01, 0.015]^T$ in the VSA law (23).

*Case* 2: VSA law (23) is directly applied.

Figures 3 and 4 show the motion and force tracking errors, which verify the robust motion and force tracking control of the suggested VSA controller. In both cases, $f_n < 0$, the robot does not lose contact with the environment. $\hat{k}_f$ are shown in Fig. 5 which approximate its true value, but does not converge to it due to the short execution time. Joint torques of the robot are presented in Fig. 6. We see that although both cases give reasonable motion and force tracking accuracy, case 2 results in chattering problems due to unmodelled dynamics such as sampling time, while case 1 gives a smooth control torque and has a better force tracking performance.

For comparing, the simulation is also done for a non-adaptive VSC controller, that is, using $\hat{k}_f(t) = 0.002$ instead of the adaptive law (17) in forming (11). Motion and force tracking errors are shown as case 3 in Figs 3 and 4, which exhibit larger force tracking error.

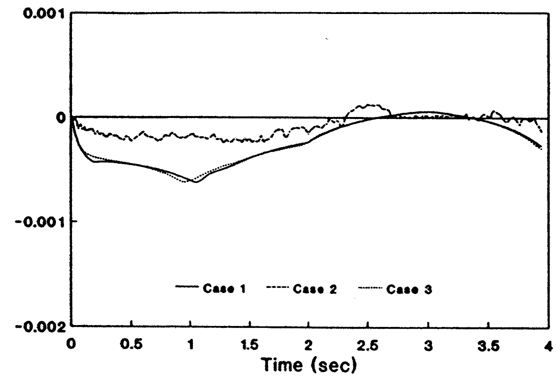


FIG. 3. Tracking error of position $r_p$.

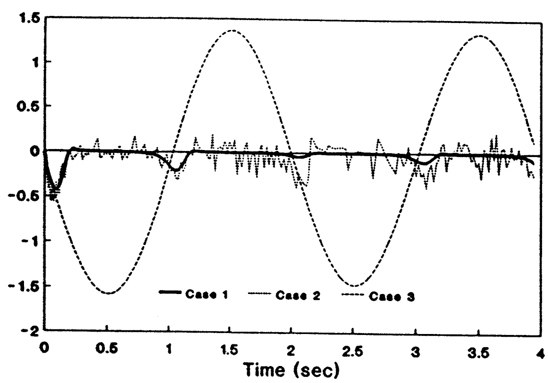


FIG. 4. Tracking error of force $f_n$.

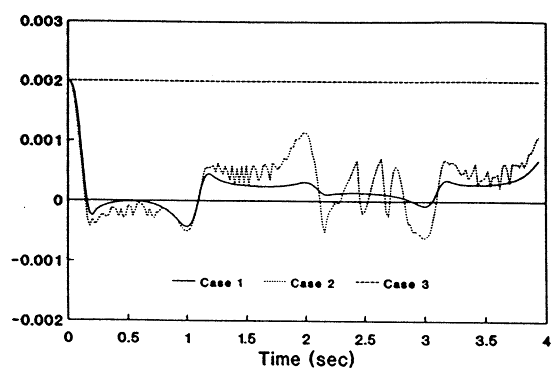


FIG. 5. Estimated value of $k_f$.

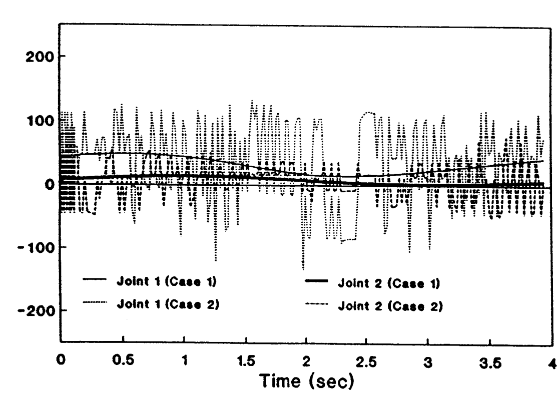


FIG. 6. Joint torque of the robot.

### 5. *Conclusion*

A VSA control scheme is developed which effectively combines VSC method and adaptive control method. Dynamic sliding mode is designed using adaptive control method to estimate unknown parameters of the environment and to ensure the system with force tracking property. Persistency of excitation is not needed. VSC method is then utilized to force the robot system to reach and be maintained on the designed sliding mode with prescribed reaching transient. Overall stability of the system is established. Only measurements of position, velocity and force are needed for implementation. Simulation results illustrates the performance of the proposed method.

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