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Brief paper

Adaptive robust control of linear motors with dynamic friction compensation using modified LuGre model☆

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ABSTRACT

LuGre model has been widely used in dynamic friction modeling and compensation. However, there are some practical difficulties when applying it to systems experiencing large range of motion speeds such as, the linear motor drive system studied in the article. This article first details the digital implementation problems of the LuGre model based dynamic friction compensation. A modified model is then presented to overcome those shortcomings. The proposed model is equivalent to LuGre model at low speed, and the static friction model at high speed, with a continuous transition between them. A discontinuous projection based adaptive robust controller (ARC) is then constructed, which explicitly incorporates the proposed modified dynamic friction model for a better friction compensation. Nonlinear observers are built to estimate the unmeasurable internal state of the dynamic friction model. On-line parameter adaptation is utilized to reduce the effect of various parametric uncertainties, while certain robust control laws are synthesized to effectively handle various modeling uncertainties for a guaranteed robust performance. The proposed controller is also implemented on a linear motor driven industrial gantry system, along with controllers with the traditional static friction compensation and LuGre model compensation. Extensive comparative experimental results have been obtained, revealing the instability when using the traditional LuGre model for dynamic friction compensation at high speed experiments and the improved tracking accuracy when using the proposed modified dynamic friction model. The results validate the effectiveness of the proposed approach in practical applications.

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1. Introduction

Friction modeling and compensation have been studied extensively, but is still full of interesting problems due to their practical significance and the complex behavior of friction. It has been well known that to have high accuracy of motion control at low speed movement, friction cannot be simply modeled as a static nonlinear function of velocity alone, but rather a *dynamic* function of velocity and displacement. Thus, during the past decade, significant efforts have been devoted to solve the difficulties in modeling and compensation of dynamic friction with various types of models proposed (Canudas de Wit, Olsson, Astrom, & Lischinsky, 1995; Dupont, Armstrong, & Hayward, 2000; Lampaert, 2003). Among them, the so called LuGre model by Canudas de Wit et al. (1995) can describe major features of dynamic friction, including presliding displacement, varying break-away force and Stribeck effect. In Olsson (1996), the modification for passivity has been added into the LuGre model. Dupont et al. (2000) proposed a modification for the LuGre model, which can describe the non-drifting effect of dynamic friction. In Swevers, Al-Bender, Ganseman, and Prajogo (2000), a so called Leuven model was proposed, which added the modeling of hysteresis into the LuGre model. But both Dupont et al. (2000) and Swevers et al. (2000) complicated the form of the friction models significantly and make them harder to use for real-time controls.

Due to its relatively simpler form and its ability to simulate major dynamic friction behaviors, LuGre model has been widely used in control with dynamic friction compensation (Canudas de Wit & Lischinsky, 1997; Tan & Kanellakopoulos, 1999; Xu & Yao, 2008). Although many good application results have been reported (Bona, Indri, & Smaldone, 2006), some practical problems are also discovered, especially when applying the LuGre model to systems experiencing large ranges of motion speeds such as, the linear motor

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drive system studied in this article. Namely, the traditional LuGre model could become very stiff when the velocity is large. This leads to some unavoidable implementation problems, since dynamic friction compensation can be only implemented digitally due to its highly nonlinear characteristics. For example, it has been reported in Freidovich, Robertsson, Shiriaev, and Johansson (2006) that the observer dynamics to recover the unmeasurable internal state of the LuGre model could become unstable at high speed motions.

On the other hand, no matter how accurate the mathematical models of dynamic friction are, it is impossible to capture the entire nonlinear behaviors of actual friction to have a perfect friction compensation. So, advanced control techniques have to be used in parallel with appropriate selection of dynamic friction models for effective friction compensation and attenuation. A good control algorithm should have features of both strong disturbance rejection and performance robustness to model uncertainties as well as the ability of on-line learning (e.g., parameter adaptation) in reducing model uncertainties to maximize the achievable control performance. The idea of adaptive robust control (ARC) (Yao & Tomizuka, 1996, 1997) incorporates the merits of deterministic robust control (DRC) and adaptive control (AC) and serves well to meet such a requirement. It is noted that the proposed ARC strategy has been well validated in various applications without having any dynamic friction compensations (Hong & Yao, 2007; Xu & Yao, 2001; Yao, Bu, Reedy, & Chiu, 2000).

In this article, we first revisit the LuGre model and discuss the digital implementation problems when using the model for dynamic friction compensation. Based on the analysis, a modified version of LuGre model is proposed for dynamic friction compensation, in which the estimation of internal states is automatically stopped at high speed movements to by-pass the instability problem of the LuGre model based observer dynamics. A continuous function is designed to make a continuous transition from the LuGre model based low speed dynamic friction compensation to the static friction model based high speed friction compensation. We then utilize the ARC strategy along with the proposed modified LuGre model based dynamic friction compensation to achieve accurate trajectory tracking for both low-speed and high-speed movements. The proposed ARC algorithm, along with ARC algorithms with friction compensations using the LuGre model and the static friction model, respectively, are tested on a linear motor driven industrial gantry system. Comparative experimental results are presented to illustrate the effectiveness of the proposed modified LuGre model based dynamic friction compensation in practical applications and the excellent tracking performance of the proposed ARC algorithm.

2. Dynamic model of linear motor systems

The linear motor dynamics can be captured well by Lu, Chen, Yao, and Wang (2008)

$$\dot{x}_1 = x_2 \tag{1}$$

$$m\dot{x}_2 = u - f + \bar{\Delta} \tag{2}$$

where $x = [x_1\, x_2]^{\mathrm{T}}$ represents the state vector consisting of the position and velocity, m denotes the inertia of the system normalized with respect to the control input unit of voltages, $u(t)$ is the control input, f represents the normalized friction, and $\bar{\Delta}$ represents the lumped unknown nonlinear functions including the friction modeling errors and the external disturbances. For certain linear motors with permanent magnets, it may be necessary to explicitly consider the effect of cogging forces when the desired trajectory spans a large travel distance. Here, to focus on the main issue of dynamic friction compensation, for simplicity of presentation and without loss of generality, the effect of cogging forces is not explicitly modeled and is lumped into the lumped uncertainties term $\bar{\Delta}$. Using the technique in Lu et al. (2008), the effect of cogging forces can be incorporated easily into the proposed control algorithm as done in some of the experimental results detailed later.

3. Modified LuGre model and problem formulation

With the LuGre model (Canudas de Wit et al., 1995), the friction f in (2) is given by

$$f = \sigma_0 z + \sigma_1 h(v)\dot{z} + \alpha_2 v \tag{3}$$

$$\dot{z} = v - \frac{|v|}{g(v)} z \tag{4}$$

$$g(v) = \alpha_0 + \alpha_1 \mathrm{e}^{-(v/v_s)^2} \tag{5}$$

where z represents the unmeasurable internal friction state, σ_0, $\bar{\sigma}_1(v) = \sigma_1 h(v)$, α_2 are constant or varying friction force parameters that can be physically explained as the stiffness, the damping coefficient of bristles, and viscous friction coefficient. $v = x_2$ is the velocity of linear motor. The function $g(v)$ is positive and it describes the Stribeck effect: $\sigma_0\alpha_0$ and $\sigma_0(\alpha_0 + \alpha_1)$ represent the levels of the Coulomb friction and stiction force, respectively, and v_s is the Stribeck velocity. It is shown in Olsson (1996) that the LuGre model is passive if $\sigma_1 h(v) < \frac{4\sigma_0 g(v)}{|v|}$, where $h(v)$ is an exponentially decay or fractionally decay function with respect to velocity, satisfying $h(v) < h(0) = 1$.

Direct use of the above LuGre model for friction compensation may have some implementation problems. Namely, as the internal friction state z is unmeasurable, it is necessary to construct observers to estimate z for dynamic friction compensation. With LuGre model, the observe dynamics would be of the form of

$$\dot{\hat{z}} = v - \frac{|v|}{g(v)}\hat{z} + \gamma\tau \tag{6}$$

where γ represents the observer gain and τ is the observer error correction function to be selected. Since the observer dynamics (6) are highly nonlinear, the only way to implement the observer is through microprocessors using its discretized version assuming certain sampling rate. With the digital implementation of (6), to avoid instability due to discretization with a finite sampling rate, it is necessary that the equivalent gain $\frac{|v|}{g(v)}$ in (6) is not too large. In Freidovich et al. (2006), it is shown that if the velocity exceeds a critical value which is proportionally related to the sampling rate, digital implementation of the above observer dynamics will become unstable.

On the other hand, the dynamic friction effect is noticeable only when the relative velocity is low. For high speed motions, it is enough to use the following traditional static friction model:

$$f = F_c \operatorname{sgn}(v) + F_v v. \tag{7}$$

It should be noted that at constant speed motion, F_c is related to $\sigma_0|z_{ss}|$ and F_v is related to α_2 in (3)–(5). It is worth noting that Canudas de Wit (1998) also briefly mentioned the possibility of stopping the integration of z and using its steady-state value $\hat{z}_{ss} = \frac{F_c}{\sigma_0}\operatorname{sgn}(v)$ when the speed is above certain critical value. In this case, the friction term is exactly the same as (7). But this rather simplistic modification may result in discontinuous internal state estimation when the speed transits between high and low ranges. In addition, no experimental results have been provided to validate such a modification. With all these facts in mind, in the following, a modified LuGre model will be proposed, which is essentially equivalent to LuGre model (3) at low speeds, and the static friction model (7) at high speeds, with a continuous transition between these two models from low speeds to high speeds. Specifically, the proposed modified model has the form of

$$f = \sigma_0 s(|v|) z + \sigma_1 h(v)\dot{z} + F_c \operatorname{sgn}(v)[1 - s(|v|)] + \alpha_2 v \tag{8}$$

$$\dot{z} = s(|v|)\left(v - \frac{|v|}{g(v)} z\right) \tag{9}$$

$$g(v) = \alpha_0 + \alpha_1 \mathrm{e}^{-(v/v_s)^2} \tag{10}$$

where $s(|v|)$ is a non-increasing continuous function of $|v|$ with the

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following properties:

P1: $s(|v|) = 1$ if $|v| < l_1$ and $s(|v|) = 0$ if $|v| > l_2$, in which $l_2 > l_1 > 0$.

In the above, l_1 and l_2 are the cutoff velocities to be selected based on the particular characteristics of the system studied and the sampling rate of digital implementation. The essence of this modified LuGre model is to make the internal dynamics stop updating when the speed is high enough. This solves the instability problem of the original LuGre model in digital implementation. Different from Canudas de Wit (1998), we do not force z to be its static value at high speeds. Thus, the estimation of z will be continuous. Furthermore, with the proposed model, using similar techniques as in Canudas de Wit et al. (1995) and Olsson (1996), it can be shown that the following desirable properties hold:

Property 1. *With the initial internal state chosen such that* $|z(0)| \leq \alpha_0 + \alpha_1$, *the internal states of the modified model (8)–(10) are always bounded above by the same upper bound, i.e.,* $|z(t)| \leq \alpha_0 + \alpha_1, \forall t \geq 0$.

Property 2. *The mapping from* v *to* f *is dissipative if* $\sigma_1 h(v) < \frac{4\sigma_0 g(v)}{|v|}$.

Property 3. *When* $|v| > l_2$, *then the proposed model simplifies into the static friction model given by (7), and when* $|v| < l_1$, *the model is the exactly the same as the LuGre model of (3)–(5).*

For any constant speed v, the steady-state friction can be obtained by letting $\dot{z} = 0$ in (9):

$$f_{ss} = \{\sigma_0 s(|v|)g(v) + F_c[1 - s(|v|)]\}\,\mathrm{sgn}(v) + \alpha_2 v. \tag{11}$$

It is thus easy to see that another good property of the proposed modified model is that F_c can be different from $\sigma_0\alpha_0$. Additionally, $\alpha_2 v$ can also be replaced by $\alpha_{21} v s(|v|) + \alpha_{22} v[1 - s(|v|)]$, which makes the viscous term at high speed different from that at the low speed. As such, the descriptions of friction at low and high speeds can be completely separated, but with a continuous transition region in between. This gives one greater flexibility in fitting the friction measurement data over a large range of motion speeds.

With the modified LuGre model for friction, the overall system dynamics to be controlled are given by

$$\dot{z} = s(|x_2|)\left[x_2 - \frac{|x_2|}{g(x_2)} z\right] \tag{12}$$

$$\dot{x}_1 = x_2 \tag{13}$$

$$\begin{aligned} m\dot{x}_2 &= u - \sigma_0 s(|x_2|)z - \sigma_1 h(x_2)s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}z\right) \\ &\quad - F_c \mathrm{sgn}(x_2)[1 - s(|x_2|)] - \alpha_2 x_2 + \bar{\Delta}. \end{aligned} \tag{14}$$

Let $y_d(t)$ be the desired motion trajectory, which is assumed to be known, bounded, with bounded derivatives up to the second order. Under the assumption that the proposed dynamic friction model has known structure (i.e., the shape functions $g(x_2)$, $h(x_2)$ and $s(|x_2|)$ are known) but unknown model parameters of σ_0, σ_1, F_c, and α_2, the objective is to synthesize a bounded control input u such that the actual position x_1 tracks $y_d(t)$ as closely as possible in spite of the assumed model uncertainties.

4. Adaptive robust control (ARC)

To solve the control problem posted in the previous section, a set of unknown parameters are defined as $\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^{\mathrm{T}} = [m, \sigma_0, \sigma_1, F_c, \alpha_2, -\bar{\Delta}_0]^{\mathrm{T}}$ in which $\bar{\Delta}_0$ can be thought as the constant nominal value of the lumped uncertainties $\bar{\Delta}$ in (14). Denote the time-varying portion of $\bar{\Delta}$ as $\tilde{\bar{\Delta}} = \bar{\Delta} - \bar{\Delta}_0$. The Eq. (14) can be re-written as

$$\begin{aligned} \theta_1\dot{x}_2 &= u - \theta_2 s(|x_2|)z - \theta_3 h(x_2)s(|x_2|)(x_2 - \frac{|x_2|}{g(x_2)}z) \\ &\quad - \theta_4 \mathrm{sgn}(x_2)[1 - s(|x_2|)] - \theta_5 x_2 - \theta_6 + \tilde{\bar{\Delta}}. \end{aligned} \tag{15}$$

The following practical assumption is made[1]:

Assumption 1. The extent of parametric uncertainties is known, more precisely, $\theta \in \Omega = \{\theta : \theta_{\min} < \theta < \theta_{\max}\}$, where $\theta_{\min} = [\theta_{1\min}, \ldots, \theta_{6\min}]^{\mathrm{T}}$ and $\theta_{\max} = [\theta_{1\max}, \ldots, \theta_{6\max}]^{\mathrm{T}}$ are known. The uncertain nonlinearity $\tilde{\bar{\Delta}}$ is bounded by a known shape function $\delta(x, t)$ multiplied by an unknown but bounded time-varying disturbance $d(t)$, i.e., $\tilde{\bar{\Delta}} \in \Omega_{\tilde{\bar{\Delta}}} = \{\tilde{\bar{\Delta}} : |\tilde{\bar{\Delta}}(x, z, u, t)| \leq \delta(x, t)d(t)\}$.

Following the ARC design procedure in Xu and Yao (2008), the control law is developed as follows. Let $e(t) = x_1(t) - y_d(t)$ be the position tracking error. Define a tracking-error-index-like variable p as:

$$p = \dot{e} + k_1 e = x_2 - x_{2eq}, \qquad x_{2eq} = \dot{y}_d - k_1 e, \tag{16}$$

where $k_1 > 0$ is a feedback gain. From (15), the derivative of p is:

$$\begin{aligned} \theta_1\dot{p} &= u - \theta_1\dot{x}_{2eq} - \theta_2 s(|x_2|)z - \theta_3 h(x_2)s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}z\right) \\ &\quad - \theta_4 \mathrm{sgn}(x_2)[1 - s(|x_2|)] - \theta_5 x_2 - \theta_6 + \tilde{\bar{\Delta}} \end{aligned} \tag{17}$$

where $\dot{x}_{2eq} = \ddot{y}_d - k_1\dot{e}$ is calculable. In Canudas de Wit and Lischinsky (1997), an adaptive scheme has been proposed for dynamic friction compensation using LuGre model. However, all the parameters that enter the model through nonlinear functions are assumed to be known in that article. This is a relatively strong requirement in practical applications. Our subsequent design does not make this strong assumption. Instead, all the friction parameters α_2, σ_0, σ_1 and F_c can be unknown. With these parameters being unknown, the estimation of the friction internal state z as well as those parameters becomes rather difficult, as we have to somewhat deal with the nonlinear estimation problem caused by the terms like $\sigma_0 z$ and $\sigma_1 h(v)\dot{z}$ in (3) as opposed to the linear estimation problem in Canudas de Wit and Lischinsky (1997), where σ_0 and σ_1 are known. To solve this nonlinear estimation problem, the dual-observer structure concept in Tan and Kanellakopoulos (1999) is utilized to estimate z. In addition, the discontinuous projection mapping is applied to this dual-observer structure to make the estimation process robust to modeling errors as in Xu and Yao (2008):

$$\begin{aligned} \dot{\hat{z}}_1 &= Proj_{\hat{z}_1}\left\{s(|x_2|)\left[x_2 - \frac{|x_2|}{g(x_2)}\hat{z}_1 - \gamma_1 p\right]\right\} \\ \dot{\hat{z}}_2 &= Proj_{\hat{z}_2}\left\{s(|x_2|)\left[x_2 - \frac{|x_2|}{g(x_2)}\hat{z}_2 + \gamma_2\frac{h(x_2)|x_2|}{g(x_2)}p\right]\right\} \end{aligned} \tag{18}$$

where the projection mapping is defined as

$$Proj_{\hat{\zeta}}(\bullet) = \begin{cases} 0 & \text{if } \hat{\zeta} = \zeta_{\max}, \bullet > 0 \text{ or } \hat{\zeta} = \zeta_{\min}, \bullet < 0 \\ \bullet & \text{otherwise} \end{cases} \tag{19}$$

in which ζ stands for z_1 and z_2, respectively. The observation bounds are set as $z_{1\max} = z_{2\max} = \alpha_0 + \alpha_1$, $z_{1\min} = z_{2\min} = -\alpha_0 - \alpha_1$, which correspond to the physical bounds of the internal state of dynamic friction. In addition, the following projection type on-line adaptation law is used to estimate the unknown parameters

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\tau), \quad \tau = \varphi p, \tag{20}$$

[1] The following notations will be used throughout the article: $\bullet_{\min}$ and $\bullet_{\max}$ for the minimum and maximum value of $\bullet$, respectively. $\hat{\bullet}$ denotes the estimate of $\bullet$ and $\tilde{\bullet} = \hat{\bullet} - \bullet$ the estimation error.

where $\varphi = \left[-\dot{x}_{2eq}, -s(|x_2|)\hat{z_1}, -h(x_2)s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z_2}\right),\right.$ $\left.-\mathrm{sgn}(x_2)[1-s(|x_2|)], -x_2, -1\right]^{\mathrm{T}}$ and $\Gamma > 0$ is a diagonal matrix. Using these discontinuous projection based dual-observer structure and the parameter adaptation law, the unknown friction parameters α_2, σ_0, σ_1, F_c and the unmeasured z can be estimated simultaneously. Furthermore, it can be shown as in Xu and Yao (2008) that the above observers and the parameter adaptation law have the following desirable properties:

$$\theta_{\min} \le \hat{\theta} \le \theta_{\max}, \tag{21}$$

$$z_{\min} \le \hat{z} \le z_{\max}, \tag{22}$$

$$\tilde{\theta}^{\mathrm{T}}[\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\varphi p) - \varphi p] \le 0, \tag{23}$$

$$\begin{aligned}\tilde{z_1}\Bigg\{&Proj_{\hat{z_1}}\left[s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z_1} - \gamma_1 p\right)\right]\\ &- s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z_1} - \gamma_1 p\right)\Bigg\} \le 0,\end{aligned} \tag{24}$$

$$\begin{aligned}\tilde{z_2}\Bigg\{&Proj_{\hat{z_2}}\left[s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z_2} + \gamma_2\frac{h(x_2)|x_2|}{g(x_2)}p\right)\right]\\ &- s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z_2} + \gamma_2\frac{h(x_2)|x_2|}{g(x_2)}p\right)\Bigg\} \le 0.\end{aligned} \tag{25}$$

(21) and (22) imply that the estimates of parameters and states are always bounded with known bounds. As such, certain robust control law can be synthesized to achieve a guaranteed robust performance in general. In addition, the properties by (23) – (25) enable us to use adaptive algorithms to eliminate the effect of parametric uncertainties for a much improved steady-state tracking performance — asymptotic output tracking. Specifically, the following ARC law is proposed:

$$u = u_a + u_s, \quad u_a = -\hat{\theta}^{\mathrm{T}}\varphi, \quad u_s = u_{s1} + u_{s2}, \quad u_{s1} = -k_{s1}p. \tag{26}$$

In (26), u_a is the model compensation term. u_s is a robust control law, in which u_{s1} is used to stabilize the nominal system and u_{s2} is a robust feedback term used to attenuate the effect of various model uncertainties. u_{s2} is required to satisfy the following two robust performance conditions

i. $pu_{s2} \le 0$

$$\begin{aligned}\text{ii. } p&\left[u_{s2} - \tilde{\theta}^{\mathrm{T}}\varphi + \theta_2 s(|x_2|)\tilde{z_1} - \theta_3 h(x_2)s(|x_2|)\frac{|x_2|}{g(x_2)}\tilde{z_2} + \tilde{\tilde{\Delta}}\right]\\ &\le \epsilon_0 + \epsilon_1\|d\|_\infty^2\end{aligned} \tag{27}$$

where ϵ_0 and ϵ_1 are two design parameters. The specific u_{s2} satisfying the above two conditions have been given in Yao and Tomizuka (1997).

Theorem 1. *If the ARC law (26) is applied, then*
A. In general, all signals are bounded. The output tracking has a guaranteed transient and steady-state performance with the tracking error index $V_s = \frac{1}{2}mp^2$ bounded above by

$$V_s \le \exp(-\lambda_V t)V_s(0) + \frac{\epsilon_0 + \epsilon_1\|d\|_\infty^2}{\lambda_V}[1 - \exp(-\lambda t)], \tag{28}$$

where $\lambda_V = 2k_{s1}/\theta_{1\max}$.
B. If after a finite time t_0, there exist parametric uncertainties only (i.e., $\tilde{\tilde{\Delta}} = 0$, $\forall t \ge t_0$), then, in addition to results in A, zero final tracking error is also achieved, i.e., $e \longrightarrow 0$ and $p \longrightarrow 0$ as $t \longrightarrow \infty$.

5. Experimental results

5.1. System setup and identification

In the Precision Mechatronics Lab at Zhejiang University, a two-axes commercial Anorad Gantry by Rockwell Automation has been setup. The gantry has two built-in linear encoders providing each axis a position measurement resolution of 0.5 μm. To study dynamic friction and its compensation in low speed motions, a Renishaw RLE10-SX-XC laser position measurement system with a laser encoder compensation kit RCU10-11ABZ is used as well, which provides a direct measurement of load position with a resolution of 20 nm. The entire system is controlled through a dSPACE DS1103 controller board with a sampling frequency $f_s = 5$ kHz for the following experiments (see Lu et al., 2008 for further details).

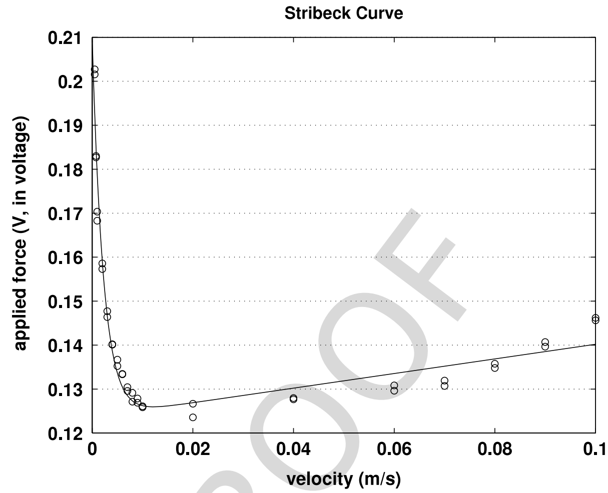

Fig. 1. Stribeck curve of friction.

The experiments have been conducted on the upper X-axis. When the power amplifier for the axis is turned on, the load carriage has a vibration amplitude around 150 nm at zero input control voltage, revealing some imperfections on the electrical sub-system, which may be caused by relatively low switching frequency of three-phase PWM wave. Off-line parameter identification is then carried out at high speed first, in which the proposed dynamic friction model simplifies into (7). It is found that the nominal value of m is 0.12 volt/m/s^2, and the value of F_c is 0.15 volt. In Canudas de Wit et al. (1995), a systematic way of estimating Stribeck function and σ_0, σ_1, α_2 are proposed. Since our model is a modified version of LuGre model, we follow the same procedures in our estimation process. First, we use the feedback control algorithm to set the speed constant at different values, so as to get Stribeck curve shown in Fig. 1. From Stribeck curve plot, we get $\sigma_0 g(x_2) = 0.1236 + 0.0861\mathrm{e}^{-|x_2/0.0022|}$ and $\alpha_2 = 0.166$. It is observed that the Stribeck effect is evident during motion with speeds less than 0.08 m/s, and beyond that the traditional static friction model describes the static friction curve well. Thus, we set l_1 to be 0.08 m/s and l_2 to be 0.1 m/s. Further study shows that using a sampling rate of 5 kHz, the observer dynamics is marginally stable at 0.11 m/s when the original LuGre model is used to construct observers like (6). So setting l_2 to be a little less than this critical velocity is reasonable. The part of $s(|x_2|)$ in $[l_1\ l_2]$ and $[-l_2\ -l_1]$ is simply chosen as a line section. Higher order choice is also possible. The function of $h(x_2)$ is chosen to be $h(x_2) = \frac{0.00013}{0.00013+|x_2|}$. It can be easily verified that this selection, together with the range of parameter variations for σ_0 and σ_1 to be given in the next subsection, satisfies the passivity condition $\sigma_1 h(v) < \frac{4\sigma_0 g(v)}{|v|}$. To obtain σ_0 and σ_1, we operate the system around zero velocity, give it a step input and measure the output response. With these experiments, we get $\sigma_0 = 7000$ and $\sigma_1 = 1176$.

5.2. Comparative experimental results

Two algorithms – C1: ARC with the proposed modified LuGre model based dynamic friction compensation; and C2: ARC with

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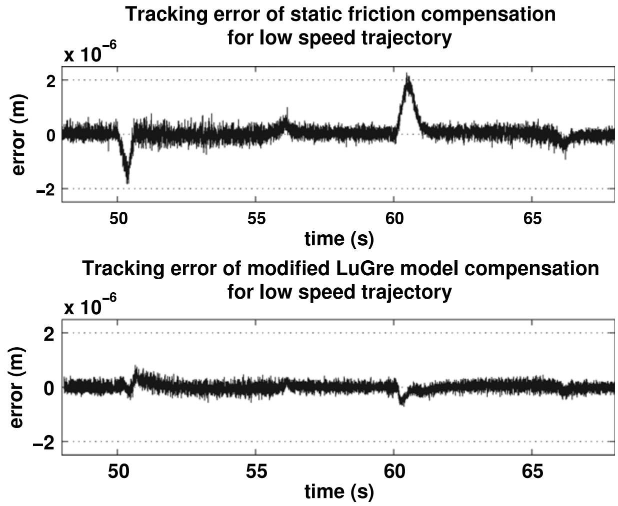

Fig. 2. Tracking errors in low speed movements.

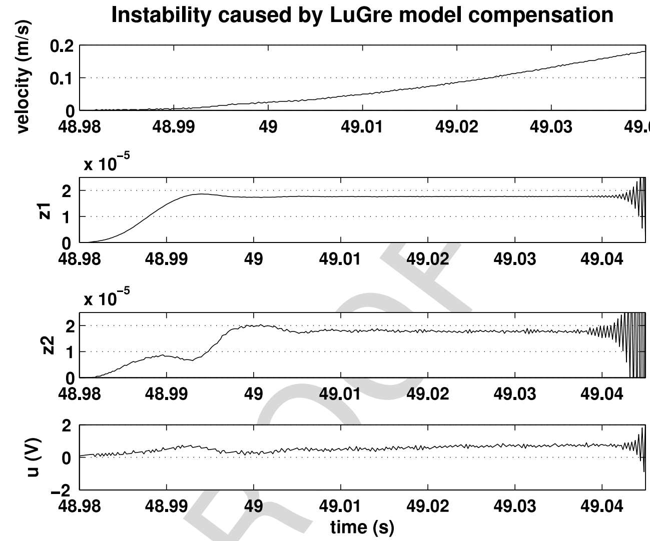

Fig. 3. Instability caused by using LuGre model based compensation in high speed motions.

static friction compensation as done in Yao, Hu, and Wang (2007) — are first implemented and compared for the following two different classes of trajectories.

Low speed motions: The desired trajectory represents a point-to-point movement, with a maximum velocity of only 0.0002 m/s, a maximum acceleration of 0.0002 m/s^2 and a traveling distance of 0.001 m. Due to the small travel distance of this motion, the cogging force effect is negligible and not explicitly accounted for in the ARC controllers. For dynamic friction compensation with the proposed modified LuGre model, the bounds of the parameter variations in the experiments are chosen as $\theta_{\min} \triangleq [0.1, 4000, 500, 0.1, 0, -0.5]^T$ and $\theta_{\max} = [0.2, 10000, 1500, 0.3, 0.5, 0.5]^T$. Theoretically, we should use the form of $u_{s2} = -k_{s2}(x)p$ with $k_{s2}(x)$ being a nonlinear proportional feedback gain as given in Yao and Tomizuka (1997) to satisfy the robust performance requirement (27) globally. In implementation, a large enough constant feedback gain k_{s2} is used instead to simplify the resulting control law. With such a simplification, though the robust performance condition (27) may not be guaranteed globally, the condition can still be satisfied in a large enough working range which is normally acceptable to practical applications as done in Yao et al. (2000). With this simplification, we choose $u_s = -k_s p$, $k_s = 60$ in the experiments where k_s represents the combined gain of u_{s1} and u_{s2}. Other controller parameters and adaptation rates used in C1 are: $k_1 = 250$, $\Gamma = \mathrm{diag}\{1, 2.5 \times 10^{10}, 2.5 \times 10^8, 100, 10, 1000\}$, $\gamma_1 = \gamma_2 = 0.2$, with $\hat{\theta}(0) = [0.12, 7000, 1176, 0.15, 0.166, 0]^T$ and $\hat{z}_1 = \hat{z}_2 = 0$. The bounds and controller parameters used in C2 are the same as in C1 except without using the parameters related to the internal states of dynamic friction model.

The tracking errors of the two ARC controllers are shown in Fig. 2. As seen from the plots, during transient periods when the velocity changes directions during the back-and-forth point-to-point motions, the tracking error peaks shown in the upper figure for C2 almost disappear after using the proposed modified LuGre model based dynamic friction compensation — C1 achieves a maximum tracking error of about 700 nm, while C2 has a maximum tracking error around 2 μm.

High speed motions: The desired trajectory has a maximum velocity of 0.3 m/s, a maximum acceleration of 5 m/s^2, and a traveling distance of 0.4 m. Due to the large travel distance of this motion, the cogging force effect should be considered as the Anorad gantry is powered by iron-core linear motors. The cogging force compensation terms of $\sum_{i=1}^{n}[\hat{A}_{ris}\sin(\frac{2\pi i}{P}x_1)+\hat{A}_{ric}\cos(\frac{2\pi i}{P}x_1)]$ are added into the proposed algorithms as done in Yao et al. (2007). For C1, bounds of the parameter variations are chosen as: $\theta_{\min} = [0.1, 4000, 500, 0.1, 0, -0.5]^T$ and $\theta_{\max} = [0.2, 10\,000, 1500, 0.3, 0.5, 0.5]^T$. Other controller and adaptation parameters used are: $k_1 = 250$, $k_s = 60$, $\Gamma = \mathrm{diag}\{1, 2.5 \times 10^{10}, 10\,000, 100, 10, 2000\}$, and $\gamma_1 = \gamma_2 \triangleq 0.2$ with $\hat{\theta}(0) = [0.12, 7000, 1176, 0.15, 0.166, 0]^T$ and $\hat{z}_1 = \hat{z}_2 = 0$. C2 uses the same bounds and parameters except without usng the parameters related to the internal states of dynamic friction model.

To verify the implementation problems of the original LuGre model based observer designs in high speed motions, we also implemented the proposed ARC with LuGre model based dynamic friction compensation, i.e., assuming $s(|v|) = 1$ for all velocity in the proposed ARC controller. As shown in Fig. 3, the estimation of internal states quickly becomes unstable due to the digital implementation.

Tracking errors of the ARCs with the proposed dynamic friction compensation (C1) and the traditional static friction compensation (C2) are plotted in Fig. 4 with the magnified plot over a single back-and-forth movement shown in Fig. 5. The control inputs are shown in Fig. 6. It can be seen from these plots that the peaks in the output tracking error plots in Fig. 5 occur at the beginning and at the end of the travel, where the system velocity is near zero and the dynamic friction effect is more severe. As such, it can be seen from Fig. 9 that the maximum tracking errors have been Q1 reduced from around 13 μm to 7 μm with the proposed modified LuGre model based dynamic friction compensation. These results well demonstrate the effectiveness of the proposed model and the good trajectory tracking ability of our algorithm at high speeds. The estimates of the internal friction state z shown in Fig. 7 reveal a well-behaved observer. All these results validate the effectiveness of the proposed dynamic friction model based compensation.

6. Conclusions

In this article, practical digital implementation problems with existing LuGre model and their variations for dynamic friction compensation are discussed and experimentally verified. A modified version of LuGre was then proposed to solve those implementation problems. An ARC algorithm with dynamic friction compensation using the proposed model was also developed with rigorous closed-loop stability and performance robustness proofs. The proposed ARC algorithm was also implemented on a linear motor driven industrial gantry system and experimentally compared with the previously presented ARC algorithms with static friction

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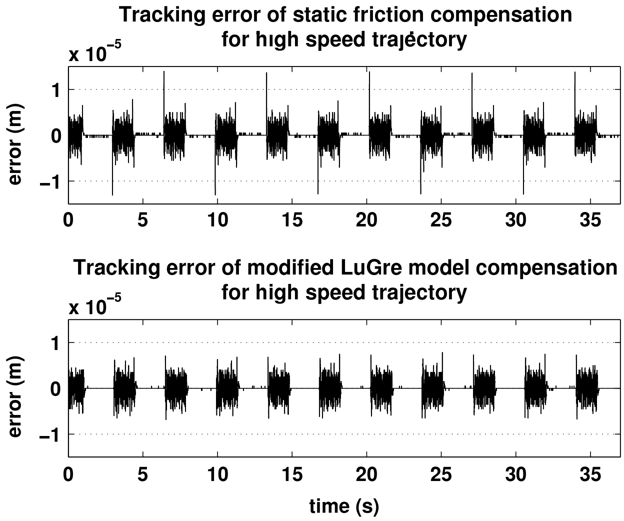

Fig. 4. Tracking errors in high speed movements.

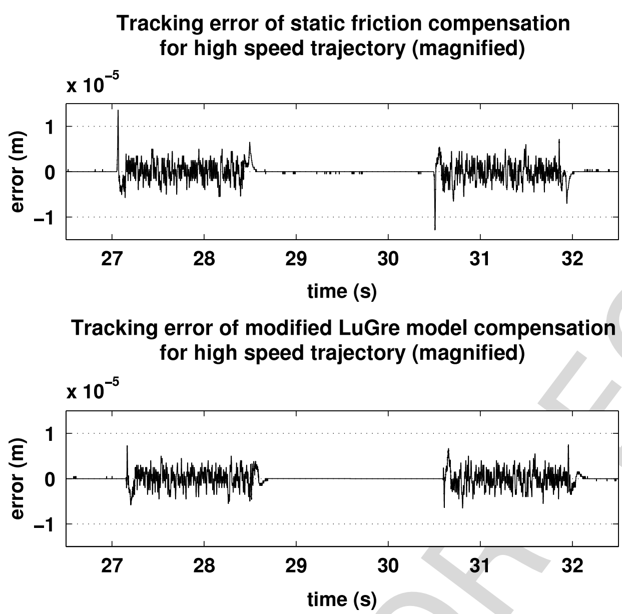

Fig. 5. Magnified tracking errors in high speed movements.

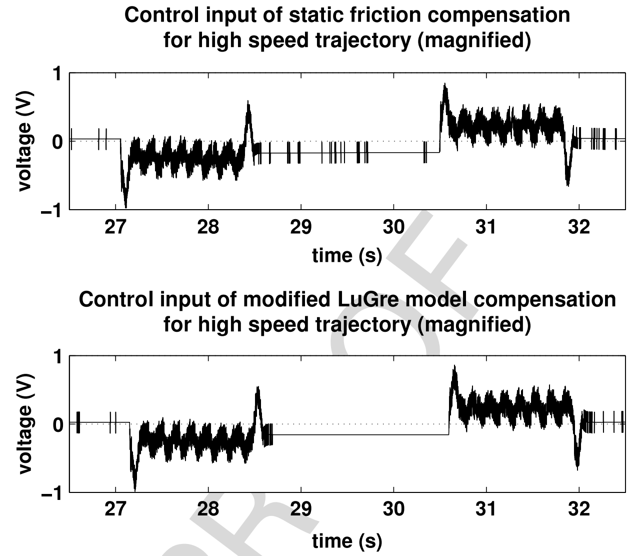

Fig. 6. Control inputs in high speed movements.

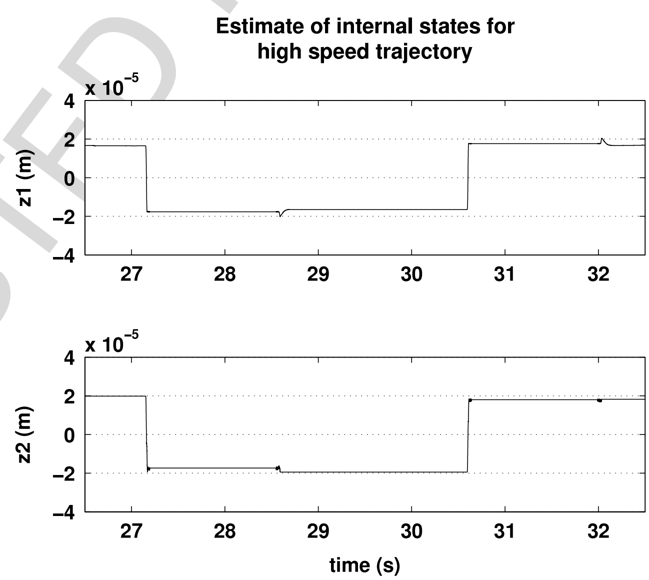

Fig. 7. Estimate of z in high speed movements.

compensation. Comparative experimental results have revealed the substantially improved tracking performance of the proposed ARC algorithm at both low and high speed motions, while without the instability problem of the LuGre model based dynamic friction compensation at high speeds.

Appendix

Proof of Theorem 1. From (17), (26), and ii of (27), the derivative of a non-negative function $V_s = \frac{1}{2}mp^2$ is

$$\begin{aligned} \dot{V}_s &= -k_{s1}p^2 + p\Big[u_{s2} - \tilde{\theta}^{\mathrm{T}}\varphi + \theta_2 s(|x_2|)\tilde{z}_1 \\ &\quad - \theta_3 s(|x_2|)h(x_2)\frac{|x_2|}{g(x_2)}\tilde{z}_2 + \tilde{\tilde{\Delta}}\Big] \\ &\le -\lambda_V V_s + \epsilon_0 + \epsilon_1\|d\|_\infty^2. \end{aligned} \tag{29}$$

By comparison lemma, (28) is true. Thus p is bounded, so do e and $\dot{e}$ because e is related to p through a stable transfer function. Since the desired trajectory is assumed to be bounded and have bounded derivatives up to second order, $x_1 = e + y_d$ and $x_2 = \dot{e} + \dot{y}_d$ are also bounded. By the projection law, $\hat{z}_1, \hat{z}_2, \hat{\theta}$ are bounded, and the control input u is thus bounded. This completes the proof of part (A). For part (B), when $\tilde{\tilde{\Delta}} = 0$, the derivative of a non-negative function defined by

$$V_a = \frac{1}{2}mp^2 + \frac{1}{2\gamma_1}\theta_2\tilde{z}_1{}^2 + \frac{1}{2\gamma_2}\theta_3\tilde{z}_2{}^2 + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} \tag{30}$$

is

$$\begin{aligned} \dot{V}_a &= p\left[-k_{s1}p - \tilde{\theta}^{\mathrm{T}}\varphi + u_{s2} + \theta_2 s(|x_2|)\tilde{z}_1 - \theta_3 h(x_2)s(|x_2|)\frac{|x_2|}{g(x_2)}\tilde{z}_2\right] \\ &\quad + \frac{1}{\gamma_1}\theta_2\tilde{z}_1\left\{Proj_{\hat{z}_1}\left[s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z}_1 - \gamma_1 p\right)\right]\right. \\ &\quad \left. - s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}z\right)\right\} \\ &\quad + \frac{1}{\gamma_2}\theta_3\tilde{z}_2\left\{Proj_{\hat{z}_2}\left[s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z}_2 + \gamma_2\frac{h(x_2)|x_2|}{g(x_2)}p\right)\right]\right. \end{aligned}$$

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$$- s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)} z\right)\Bigg\} + \tilde{\theta}^{\mathrm{T}}\Gamma^{-1} Proj_{\hat{\theta}}(\Gamma\varphi p)$$

$$= -k_{s1}p^2 + u_{s2}p + \frac{1}{\gamma_1}\theta_2\tilde{z}_1 \Bigg\{ Proj_{\hat{z}_1}\left[s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z}_1 - \gamma_1 p\right)\right]$$

$$- s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z}_1 - \gamma_1 p\right)\Bigg\}$$

$$+ \frac{1}{\gamma_2}\theta_3\tilde{z}_2 \Bigg\{ Proj_{\hat{z}_2}\left[s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z}_2 + \gamma_2\frac{h(x_2)|x_2|}{g(x_2)}p\right)\right]$$

$$- s(|x_2|)\left(x_2 - \frac{|x_2|}{g(x_2)}\hat{z}_2 + \gamma_2\frac{h(x_2)|x_2|}{g(x_2)}p\right)\Bigg\}$$

$$- s(|x_2|)\frac{|x_2|}{(g)}\left[\frac{1}{\gamma_1}\theta_2\tilde{z}_1{}^2 + \frac{1}{\gamma_2}\theta_3\tilde{z}_2{}^2\right] + \tilde{\theta}^{\mathrm{T}}[\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\varphi p) - \varphi p].$$

Q2 Using (23)–(25), we have

$$\dot{V}_a \le -k_{s1}p^2 + u_{s2}p - s(|x_2|)\frac{|x_2|}{g(x_2)}\left[\frac{1}{\gamma_1}\theta_2\tilde{z}_1{}^2 + \frac{1}{\gamma_2}\theta_3\tilde{z}_2{}^2\right].$$

Since $g(x_2) > 0$, $s(x_2) \ge 0$ and $u_{s2}p \le 0$, we have

$$\dot{V}_a \le -k_{s1}p^2 \tag{31}$$

Thus, $p \in L_2 \bigcap L_\infty$. It is clear that $\dot{p} \in L_\infty$ based on (17). So, by applying Barbalat's lemma, $p \longrightarrow 0$ as $t \longrightarrow \infty$, so $e \longrightarrow 0$, which proves part (B).

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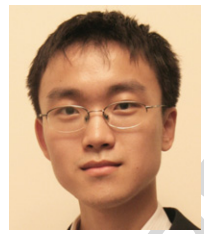

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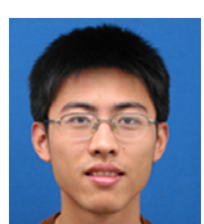

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