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Brief paper

# A globally stable saturated desired compensation adaptive robust control for linear motor systems with comparative experiments[☆]

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**Abstract**

The recently proposed saturated adaptive robust controller is integrated with desired trajectory compensation to achieve global stability with much improved tracking performance. The algorithm is tested on a linear motor drive system which has limited control effort and is subject to parametric uncertainties, unmodeled nonlinearities, and external disturbances. Global stability is achieved by employing back-stepping design with bounded (virtual) control input in each step. A guaranteed transient performance and final tracking accuracy is achieved by incorporating the well-developed adaptive robust controller with effective parameter identifier. Signal noise that affects the adaptation function is alleviated by replacing the noisy velocity signal with the cleaner position feedback. Furthermore, asymptotic output tracking can be achieved when only parametric uncertainties are present.


*Keywords:* Input saturation; Adaptive control; Robust control; Linear motors; Motion control; Nonlinear systems



## 1. Introduction

Significant research has been devoted to the globally stable controls with input saturation due to the unavoidable actuator saturation for any physical devices (Bernstein & Michel, 1995). Research has been done that focuses on either the ad-hoc technique of anti-windup or system stabilization via account of the saturation nonlinearities at the controller design stage. All the studies in Sussmann and Yang (1991), Lin and Saberi (1995), Teel (1992, 1995) assumed that the systems under investigation are linear and the system parameters are all known, which is not true for most physical systems. Nonlinear factors such as friction and hysteresis affect system behavior significantly and are rather difficult to model precisely. It is not unusual that some of the system parameters are unknown or have variable values. Unpredictable external disturbances also affect the system performance. Therefore, it is of practical importance to take these issues into account when solving the actuator saturation problem. Gong and Yao (2000) combined the saturation functions proposed in Teel (1992) with the adaptive robust control (ARC) strategy proposed in Yao and Tomizuka (1997), Yao (1997) and achieved global stability and good performance for a chain of integrators subject to matched model uncertainties. Recently, a new saturated control structure was introduced in Hong and Yao (2005). This new scheme is based on backstepping design (Krstić, Kanellakopoulos, & Kokotovic, 1995) and ARC strategy Yao (1997), Yao and Tomizuka (1997)~~–the~~ control law is designed to ensure fast error convergence during normal working conditions while globally stabilizing the system for a much larger class of modeling uncertainties than those considered in Gong and Yao (2000).

The saturated ARC proposed in Hong and Yao (2005) has been applied to a linear motor positioning system. As revealed

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in Yao (1998), some implementation problems were encountered. For example, a noisy velocity measurement may restrict the achievable performance due to the state-dependent regressor. In this paper, the desired trajectory replaces the actual state in the regressor for both model compensation and parameter adaptation in order to further improve the tracking performance while preserving global stability. Furthermore, by using the "integration by parts" technique (Xu & Yao, 2001), the resulting parameter estimation algorithm uses only the feedback position signal, which has micrometer resolution and much less noise contamination than the velocity signal used in Hong and Yao (2005). Comparative experimental results show that the tracking error is reduced almost by half.

## 2. Problem formulation

The essential dynamics of the linear motor system detailed in Xu and Yao (2001) are

$$M\ddot{y} = K_f u - B\dot{y} - F_{sc}S_f(\dot{y}) + d(t), \tag{1}$$

where $M$ is the inertia of the payload plus the coil assembly, $y$ is the stage position, $u$ is the control voltage with an input gain of $K_f$, $B$ and $F_{sc}$ represent the two major friction coefficients, viscous and Coulomb, respectively, $S_f(\dot{y})$ is a differentiable with uniformly bounded derivatives, non-decreasing function that approximates the discontinuous sign function $\mathrm{sgn}(\dot{y})$ which is normally used in the modeling of Coulomb friction as in Xu and Yao (2001), $d(t)$ represents the lumped friction modeling error, other unmodeled dynamics (e.g., the force ripples of linear motors), and external disturbances.

The above system is subject to unknown parameters due to payload variation, uncertain friction coefficients, and lumped neglected model dynamics and external disturbances. In this paper, the mass term $M$ is assumed to be known since, compared to other terms, it is unlikely to change once the payload is fixed and easy to estimate accurately either off-line or on-line; it is nevertheless noted that the method presented below can be extended to the case when $M$ is unknown without much theoretical difficult. The low frequency component of the lumped uncertainties is modeled as an unknown constant. Therefore, letting $\mathbf{x} = [x_1, x_2]^{\mathrm{T}} := [y, \dot{y}]^{\mathrm{T}}$ be the state vector, the above governing dynamic equations can be rewritten in a state-space form as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -B_m x_2 - F_{scm}S_f(x_2) + d_{0m} + \Delta + \frac{K_f}{M}u \\ &= \boldsymbol{\varphi}^{\mathrm{T}}(\mathbf{x})\boldsymbol{\theta} + \Delta + \bar{u}, \end{aligned} \tag{2}$$

where $\boldsymbol{\varphi}(\mathbf{x}) = [-x_2, -S_f(x_2), 1]^{\mathrm{T}}$ is the regressor, $\boldsymbol{\theta} = [B_m, F_{scm}, d_{0m}]^{\mathrm{T}} = [B, F_{sc}, d_0]^{\mathrm{T}}/M$ is the vector of unknown parameters to be adapted on-line, in which $d_{0m}$ represents the nominal value of the normalized disturbance $d_m(t) = d(t)/M$, $\Delta = d_m(t) - d_{0m}$ represents the variation or high frequency components of $d_m(t)$, and $\bar{u} = K_f * u/M$ is the normalized control input whose upper and lower limits can be calculated from the physical input saturation level. The following practical assumptions are made:

**Assumption 1.** The extents of the parametric uncertainties are known, i.e.,

$$B_m \in [B_{ml}, B_{mu}], \quad F_{scm} \in [F_{scml}, F_{scmu}],$$

where $B_m$, $F_{scm}$ are positive according to the real system and $B_{ml}$, $B_{mu}$, $F_{scml}$, $F_{scmu}$ are known.

**Assumption 2.** The lumped disturbance $d_m(t)$ is bounded, i.e.,

$$|d_m(t)| \leqslant \delta_{dm},$$

where $\delta_{dm}$ is known.

The desired trajectory, position $x_{1d}$, velocity $x_{2d} = \dot{x}_{1d}$ and acceleration $\ddot{x}_{1d}$, are assumed to be known and bounded. Let $\bar{u}_{bd}$ represent the normalized bound of the actuator authority. *The saturation control problem can be stated as: under the above assumptions and the normalized input constraint of* $|\bar{u}(t)| \leqslant \bar{u}_{bd}$, *design a control law that globally stabilizes the system and makes the output tracking error* $z_1 = x_1 - x_{1d}(t)$ *as small as possible.*

## 3. Saturated desired compensation ARC

### *3.1. Controller structure*

From Assumptions 1 and 2, it is obvious that the parameter vector $\boldsymbol{\theta}$ belongs to a set $\Omega_\theta$ as $\boldsymbol{\theta} \in \Omega_\theta = \{\boldsymbol{\theta}_{\min} \leqslant \boldsymbol{\theta} \leqslant \boldsymbol{\theta}_{\max}\}$, where $\boldsymbol{\theta}_{\min} = [B_{ml}, F_{scml}, -\delta_{dm}]^{\mathrm{T}}$, $\boldsymbol{\theta}_{\max} = [B_{mu}, F_{scmu}, \delta_{dm}]^{\mathrm{T}}$, and the operation $\leqslant$ for two vectors is performed in terms of their corresponding elements. Let $\hat{\boldsymbol{\theta}}$ denote the estimate of $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{\theta}}$ be the estimation error $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$. The following projection-type parameter adaptation law (Yao & Tomizuka, 1997) is used

$$\dot{\hat{\boldsymbol{\theta}}} = Proj_{\hat{\theta}}(\Gamma\boldsymbol{\tau}), \quad \hat{\boldsymbol{\theta}}(0) \in \Omega_\theta, \tag{3}$$

$$Proj_{\hat{\theta}}(\bullet_i) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{i\max} \text{ and } \bullet_i > 0, \\ 0 & \text{if } \hat{\theta}_i = \theta_{i\min} \text{ and } \bullet_i < 0, \\ \bullet_i & \text{otherwise,} \end{cases} \tag{4}$$

where $\Gamma$ is a diagonal matrix of adaptation rates and $\boldsymbol{\tau}$ is an adaptation function to be synthesized further on. Such a parameter adaptation law has the following desirable properties (Yao, 1997).

(P1) The parameter estimates are always within the known bound at any time instant, i.e., $\hat{\boldsymbol{\theta}}(t) \in \Omega_\theta$.

(P2) $\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\boldsymbol{\tau}) - \boldsymbol{\tau}) \leqslant 0, \forall\boldsymbol{\tau}$.

The controller design follows the same back-stepping procedure as in Hong and Yao (2005). Define $z_1 = x_1 - x_{1d}$ as the tracking error, $\alpha_1$ as the bounded virtual control law designed for $z_1$ dynamics, which is $\dot{z}_1 = x_2 - \dot{x}_{1d}$. Define $z_2 = x_2 - \alpha_1$, then $z_1$ dynamics become

$$\dot{z}_1 = z_2 + \alpha_1 - \dot{x}_{1d}. \tag{5}$$



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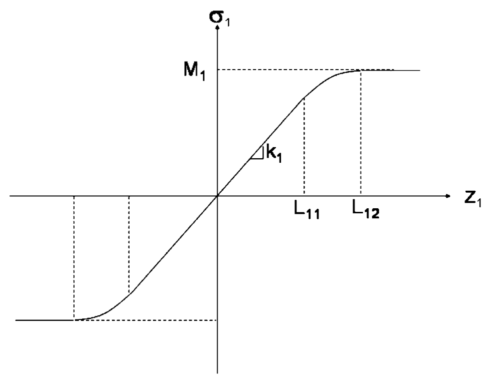


Fig. 1. Saturated robust control term for $z_1$.

The adaptive robust control law for $\alpha_1$ is proposed as

$$\alpha_1 = \alpha_{1a} + \alpha_{1s}, \quad \alpha_{1a} = \dot{\mathbf{x}}_{1d}, \quad \alpha_{1s} = -\sigma_1(z_1), \tag{6}$$

where $\sigma_1(z_1)$ is a saturation function and will be described in detail further on. Substituting (6) into (5) gives,

$$\dot{z}_1 = z_2 - \sigma_1(z_1), \tag{7}$$

where $\sigma_1(z_1)$ is designed to be a smooth (first order differentiable), non-decreasing, saturation function with respect to $z_1$ and have the following four properties:

(i) If $|z_1| < L_{11}$, then $\sigma_1(z_1) = k_1 z_1$,
(ii) $z_1\sigma_1 > 0, \forall z_1 \neq 0$,
(iii) $|\sigma_1(z_1)| \leqslant M_1, \forall z_1 \in \mathbb{R}$,
(iv) $\partial\sigma_1/\partial z_1 \leqslant k_1$ if $|z_1| < L_{12}$, and $\partial\sigma_1/\partial z_1 = 0$ if $|z_1| \geqslant L_{12}$.

Graphically, this function is shown in Fig. 1 and $L_{11}$, $L_{12}$, $k_1$, $M_1$ are the design parameters. From (2) and (6) the dynamics of $z_2$ become

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = \boldsymbol{\varphi}^{\mathrm{T}}(x)\boldsymbol{\theta} + \Delta + \bar{u} - \ddot{x}_{1d} + \frac{\partial\sigma_1}{\partial z_1}(z_2 - \sigma_1). \tag{8}$$

Let $\bar{u} = \bar{u}_a + \bar{u}_s$, where $\bar{u}_a$ and $\bar{u}_s$ represent the model compensation and the robust term, respectively. The essential idea is to use $\bar{u}_a$ to compensate the known model dynamics and $\bar{u}_s$ to deal with the model mismatch plus disturbance. Meanwhile, both $\bar{u}_a$ and $\bar{u}_s$ should be bounded to make sure that the total control effort stays within its limit.

Naturally, it is intuitive to make the model compensation term $\bar{u}_a$ cancel the model dynamics $\boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}}\hat{\boldsymbol{\theta}}$. However, since the regressor depends on the actual state, it is impossible to put a bound on $\bar{u}_a$, which contradicts the essence of saturated control. Furthermore, the adaptation function $\boldsymbol{\tau}$ has to be synthesized as $\boldsymbol{\tau} = \boldsymbol{\varphi} z_2$, where both terms involve the feedback velocity signal. As pointed out in Xu and Yao (2001), such an adaptation structure has potential implementation issues. For example, if the velocity feedback is calculated via the backward difference method from the position signal, a slow adaptation rate has to be used because of the effect of severe noise and this may degrade the achievable tracking performance. To overcome all these problems, it is proposed here to re-formulate the regressor as a function of the desired trajectory ~~(9)~~ only

$$\bar{u}_a = -\boldsymbol{\varphi}_d^{\mathrm{T}}\hat{\boldsymbol{\theta}} + \ddot{x}_{1d} + \frac{\partial\sigma_1}{\partial z_1}\sigma_1, \tag{9}$$

where $\boldsymbol{\varphi}_d = \boldsymbol{\varphi}(\mathbf{x}_d) = [-x_{2d}, -S_f(x_{2d}), 1]^{\mathrm{T}}$.

Given property (P1) of the parameter identifier, the known desired trajectory and properties (iii) and (iv) of $\sigma_1$, it is easy to determine $\bar{u}_{abd}$, the upper bound of $|\bar{u}_a|$. Obviously, for the desired trajectory to be physically trackable, $\bar{u}_{abd}$ has to be less than the bound of the actuator authority, i.e., $|\bar{u}_a| \leqslant \bar{u}_{abd} < \bar{u}_{bd}$. Based on this formulation, the adaptation function can be chosen as $\boldsymbol{\tau} = \boldsymbol{\varphi}_d z_2$, which is now linearly dependent on the noise-contaminated velocity feedback. To further reduce the noise effect in implementation, the cleaner high-resolution position signal will be employed instead of the velocity feedback as detailed later. As a result, higher adaptation rates can be used and smaller steady state tracking errors are expected.

Returning to $z_2$ dynamics (8) and applying the model compensation $\bar{u}_a$ in (9) gives,

$$\dot{z}_2 = \boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}}\boldsymbol{\theta} - \boldsymbol{\varphi}_d^{\mathrm{T}}\hat{\boldsymbol{\theta}} + \Delta + \frac{\partial\sigma_1}{\partial z_1}z_2 + \bar{u}_s. \tag{10}$$

$\boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}}\boldsymbol{\theta} - \boldsymbol{\varphi}_d^{\mathrm{T}}\hat{\boldsymbol{\theta}}$ can be written as $[\boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}} - \boldsymbol{\varphi}_d^{\mathrm{T}}]\boldsymbol{\theta} - \boldsymbol{\varphi}_d^{\mathrm{T}}\tilde{\boldsymbol{\theta}}$. Furthermore, by applying the Mean Value Theorem,

$$(\boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}} - \boldsymbol{\varphi}_d^{\mathrm{T}})\boldsymbol{\theta} = -B_m\dot{z}_1 - F_{scm}g(x_2, t)\dot{z}_1, \tag{11}$$

where $g(x_2, t)$ is bounded and non-negative as $S_f(x_2)$ is a non-decreasing function with uniformly bounded derivatives. Substituting (11) and (7) into (10),

$$\begin{aligned}\dot{z}_2 &= (B_m + F_{scm}g)\sigma_1 + (-\boldsymbol{\varphi}_d^{\mathrm{T}}\tilde{\boldsymbol{\theta}} + \Delta) \\ &\quad + \left(-B_m - F_{scm}g + \frac{\partial\sigma_1}{\partial z_1}\right)z_2 + \bar{u}_s.\end{aligned} \tag{12}$$

In order to actively take into account the actuator saturation problem when the control law is designed, another non-decreasing function $\sigma_2(z_2)$ is used to construct $\bar{u}_s$. Let $\bar{u}_s = -\sigma_2(z_2)$, where $\sigma_2(z_2)$ has the following properties:

(i) $\forall z_2 \in \{z_2 : |z_2| < L_{21}\}$, $\sigma_2(z_2) = k_{21}z_2$.
(ii) $\forall z_2 \in \{z_2 : L_{21} \leqslant |z_2| \leqslant L_{22}\}$, $\partial\sigma_2/\partial z_2 \geqslant k_{21}$, $\sigma_2(L_{22}) = M_2$ and $\sigma_2(-L_{22}) = -M_2$.
(iii) $\forall z_2 \in \{z_2 : |z_2| > L_{22}\}$, $|\sigma_2(z_2)| \geqslant M_2$.

Notice this is the last channel of the system and $\bar{u}_s$ is a part of the real control input, therefore $\sigma_2(z_2)$ only needs to be continuous instead of smooth.Fig. 2 shows an example of $\sigma_2(z_2)$ that has all the required properties. The reason that two linear segments with different gains are used for $\sigma_2(z_2)$ is because low noise effect at steady-state tracking and high disturbance attenuation during transient could both be achieved in this way, as detailed in Remark 1 in the following. The design parameters are $L_{21}$, $k_{21}$, $L_{22}$, $k_{22}$. The complete form of control input is thus as follows:

$$\bar{u} = -\boldsymbol{\varphi}_d^{\mathrm{T}}\hat{\boldsymbol{\theta}} + \ddot{x}_{1d} + \frac{\partial\sigma_1}{\partial z_1}\sigma_1 - \sigma_2(z_2). \tag{13}$$



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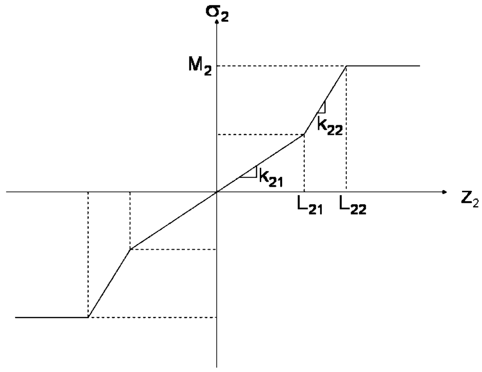


Fig. 2. Saturated robust control term for $z_2$.

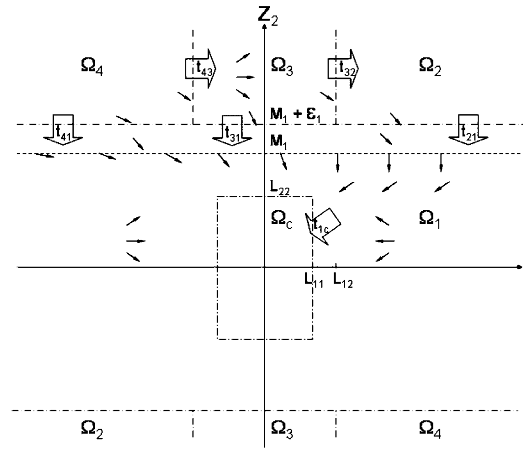


Fig. 3. $z_1$–$z_2$ plane.

**Remark 1.** $\sigma_2(z_2)$ has two regions with different gains whereas $\sigma_1(z_1)$ only has one linear gain. This is because model mismatch and uncertainties only appear in $z_2$ dynamics. The region with moderate gain $k_{21}$ represents the normal operation of the system and is selected to achieve high performance like short transient periods and small tracking errors while remaining insensitive to noise effects. When $|z_2|$ is between $L_{21}$ and $L_{22}$, for example under some unexpected disturbance or large model mismatch, more aggressive gain $k_{22}$ is employed to improve the disturbance rejection performance. This high gain $k_{22}$ could also be designed as a certain nonlinear function for further improvement. When an emergency happens, such as an overpowering random strike on the positioning stage that drags system states far away from the normal operation region, i.e.$|z_2| \gg L_{22}$, $\sigma_2$ can be a constant as the example Fig. 2 so that the overall control effort is always guaranteed to stay within the physical limit of the actuator. Or, for better transient performance, no upper bound is imposed on $\sigma_2$ and when large error occurs, just let the actuator get saturated since no integral action is introduced to this part of the control law.

### 3.2. Global stability and constraints on design parameters

Combining (7) and (12), the error dynamics can be rewritten as follows:

$$\begin{aligned}\dot{z}_1 &= z_2 - \sigma_1(z_1),\\ \dot{z}_2 &= \underbrace{(B_m + F_{scm}g)\sigma_1}+(-\boldsymbol{\varphi}_d^{\mathrm{T}}\tilde{\boldsymbol{\theta}} + \Delta)\\ &\quad + \left(\underbrace{-B_m - F_{scm}g} + \frac{\partial \sigma_1}{\partial z_1}\right) z_2 - \sigma_2(z_2). \qquad (14)\end{aligned}$$

It can be seen that the underbraced terms in (14) are new compared to the error dynamics in Hong and Yao (2005). The global stability of such a system is proved as follows. The essential idea is to divide the plane into four regions and analyze the error dynamics in each region. The conclusion is that no matter where the initial state starts, the trajectory will converge to a preset region $\Omega_c = \{z_1, z_2 : |z_1| \leqslant L_{11}, |z_2| \leqslant L_{22}\}$ in finite time with the upper bound of the convergence time estimated accordingly.

As to the additional terms, $(B_m + F_{scm}g)\sigma_1$ can be lumped with the model mismatch $-\boldsymbol{\varphi}_d^{\mathrm{T}}\tilde{\boldsymbol{\varphi}} + \Delta$ and the total effect can be found bounded by a constant $h$, i.e., $|(B_m + F_{scm}g)\sigma_1 - \boldsymbol{\varphi}_d^{\mathrm{T}}\tilde{\boldsymbol{\theta}} + \Delta| \leqslant h$, since all the signals involved are bounded. Furthermore, $(-B_m - F_{scm}g)z_2$ actually acts as a damping term, which helps to preserve stability even though its effect is trivial compared to the high gain robust term.

To prove the global stability of the controlled system, the following constraints are required for design parameters: (a) $k_{21} > k_1$, (b) $k_1 L_{11} > L_{22}$, (c) $h < M_2 - k_1 M_1$, and (d) $M_2 \leqslant \bar{u}_{bd} - \bar{u}_{abd}$.

**Theorem 1.** *With the proposed controller* (6), (13) *satisfying conditions* (a)–(d), *all signals are bounded. Furthermore, the error state* $[z_1, z_2]^{\mathrm{T}}$ *reaches the preset region* $\Omega_c$ *in a finite time and stay within thereafter. At steady state, the final tracking error is bounded above by* $|z_1(\infty)| \leqslant \frac{h}{k_1(k_{21}-k_1)}$.

**Proof.** With conditions (b) and (c), there exist positive $\varepsilon_1$,$\varepsilon_2$ and $\varepsilon_3$, such that $h + k_1(M_1 + \varepsilon_1) + \varepsilon_2 < M_2$ and $L_{22} + \varepsilon_3 < k_1 L_{11}$. Noting $M_1 > k_1 L_{11} > L_{22}$, as shown in Fig. 3, the entire $z_1$–$z_2$ plane is divided into four regions $\Omega_1$–$\Omega_4$ defined as follows:

$$\Omega_c = \{z : |z_1| \leqslant L_{11}, |z_2| \leqslant L_{22}\},$$

$$\Omega_1 = \{z : |z_2| \leqslant M_1 + \varepsilon_1\},$$

$$\Omega_2 = \{z : z_2(z_1 - sign(z_2)L_{12}) > 0, |z_2| > M_1 + \varepsilon_1\},$$

$$\Omega_3 = \{z : |z_1| \leqslant L_{12}, |z_2| > M_1 + \varepsilon_1\},$$

$$\Omega_4 = \{z : z_2(z_1 + sign(z_2)L_{12}) < 0, |z_2| > M_1 + \varepsilon_1\}.$$

Notice that $\Omega_c \subset \Omega_1$.

**Claim 1.** *Any trajectory starting from* $\Omega_1$ *will enter* $\Omega_c$ ~~i~~in a finite time $t_{1c}$ *and stay within thereafter.*

**Proof.** Consider the trajectory with the state satisfying $L_{22} \leqslant |z_2(t)| \leqslant M_1 + \varepsilon_1$ first. Then, noting the properties of $\sigma_1(z_1)$ and $\sigma_2(z_2)$, the following inequality can be established according to the error dynamics (14):

$$\begin{aligned} z_2\dot{z}_2 &\leqslant |z_2|(h - (B_m + F_{scm}g)|z_2| + k_1|z_2| - |\sigma_2(z_2)|)\\ &\leqslant |z_2|(h + k_1(M_1 + \varepsilon_1) - M_2) \leqslant -\varepsilon_2|z_2|. \qquad (15)\end{aligned}$$



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Eq. (15) indicates that any trajectory starting with an initial state of $L_{22} \leqslant |z_2(0)| \leqslant M_1 + \varepsilon_1$ will reach the region $\Omega_5 = \{z : |z_2(t)| \leqslant L_{22}\}$ in a finite time $t_{1c,2}$ and stay within $\Omega_5$ thereafter. Furthermore, the upper bound of the reaching time $t_{1c,2}$ is

$$t_{1c,2} \leqslant \max\left\{0, \frac{|z_2(0)| - L_{22}}{\varepsilon_2}\right\}. \tag{16}$$

Within the region $\Omega_5$, i.e., $|z_2(t)| \leqslant L_{22}$, if $|z_1(t)| > L_{11}$, from (14) and properties (i) and (ii) of the non-decreasing function $\sigma_1(z_1)$,

$$z_1\dot{z}_1 \leqslant |z_1|L_{22} - |\sigma_1(z_1)| \leqslant |z_1|(L_{22} - k_1 L_{11}) \leqslant -\varepsilon_3|z_1|. \tag{17}$$

Thus, any trajectory starting within $\Omega_5$ with $|z_1(0)| > L_{11}$ will reach the region $\Omega_c$ in a finite time $t_{1c,1}$ and stay within $\Omega_c$ thereafter. Furthermore, the upper bound of the reaching time $t_{1c,1}$ can be obtained from (17) as,

$$t_{1c,1} \leqslant \max\left\{0, \frac{|z_1(t_{1c,2})| - L_{11}}{\varepsilon_3}\right\}. \tag{18}$$

Combine (16) and (18), the upper bound of the reaching time for the trajectory starting within $\Omega_1$ to $\Omega_c$ is obtained

$$\begin{aligned} t_{1c} &= t_{1c,2} + t_{1c,1} \\ &\leqslant \max\left\{0, \frac{|z_2(0)| - L_{22}}{\varepsilon_2}\right\} \\ &\quad + \max\left\{0, \frac{|z_1(t_{1c,2})| - L_{11}}{\varepsilon_3}\right\}. \end{aligned} \tag{19}$$

Via similar analysis as above, the following Claim 2–4 can be proved. The details are skipped due to the page limit.

**Claim 2.** *Any trajectory starting from $\Omega_2$ will enter $\Omega_1$ in a finite time $t_{21} \leqslant (|z_2(0)| - (M_1 + \varepsilon_1))/M_2 - h$.*

**Claim 3.** *Any trajectory starting from $\Omega_3$ will enter either $\Omega_1$ in a finite time $t_{31}$, or $\Omega_2$ in a finite time $t_{32}$, with $t_{31} \leqslant t_{32} \leqslant |L_{12}\,\mathrm{sign}(z_2) - z_1(0)|/\varepsilon_1$.*

**Claim 4.** *Any trajectory starting from $\Omega_4$ will enter either $\Omega_1$ in a finite time $t_{41} \leqslant (|z_2(0)| - (M_1 + \varepsilon_1))/M_2 - h$, or $\Omega_3$ in a finite time $t_{43} \leqslant (|z_1(0)| - L_{12})/2M_1 + \varepsilon_1$.*

In all, with Claims 1–4, no matter where the trajectory starts, it will enter $\Omega_c$ in a finite time and stay within thereafter. As shown in the upper half plane of Fig. 3, little black arrows represent the phase portrait and big hollow arrows indicate the state travelling from one region to another with the reaching time marked on. The global stability is thus proved.

Once the trajectory enters $\Omega_c = \{z : |z_1| \leqslant L_{11}, |z_2| \leqslant L_{22}\}$, the error dynamics become

$$\begin{aligned} \dot{z}_1 &= z_2 - k_1 z_1, \\ \dot{z}_2 &= (-\varphi_b^{\mathrm{T}}\tilde{\theta} + \Delta) + (-B_m + k_1)z_2 - \sigma_2(z_2). \end{aligned} \tag{20}$$

Define a positive semi-definite function $V_2 = z_2^2/2$ and let $k_s = k_{21} - k_1$. From the second equation of (20), the derivative of $V_2$ is given by

$$\begin{aligned} \dot{V}_2 &= z_2\dot{z}_2 \leqslant |z_2|(h + k_1|z_2| - k_{21}|z_2|) \\ &\leqslant -\frac{k_s}{2}\left(|z_2| - \frac{h}{k_s}\right)^2 + \frac{h^2}{2k_s} - \frac{k_s}{2}z_2^2 \\ &\leqslant -k_s V_2 + \frac{h^2}{2k_s} \end{aligned} \tag{21}$$

which leads to the following inequality:

$$V_2(t) \leqslant \exp(-k_s t)V_2(0) + \frac{h^2}{2k_s^2}[1 - \exp(-k_s t)]. \tag{22}$$

From (22), the steady state of $z_2$ is bounded by $|z_2(\infty)| \leqslant h/k_s$. Then, according to the first equation of (20), the steady state tracking error $z_1$ is bounded by $|z_1(\infty)| \leqslant h/k_1 k_s = h/k_1(k_{21} - k_1)$.

**Remark 2.** Constraint (a) requires the gain in the second channel to be greater than $k_1$ in order to overpower the term $\partial\sigma_1/\partial z_1$ in $z_2$ dynamics (14). Constraint (b) guarantees that once $z_2$ is bounded within a pre-set range, $z_1$ is ensured to decrease and be bounded accordingly. Constraint (c) implies that, for the design problem to be meaningful, the level of lumped modeling error and disturbance should be within the limit of the control authority available for robust feedback. Constraint (d) implies that a trade-off has to be made between the amount of model uncertainties to which the system can be made robust, and the aggressiveness of the trajectory that it can follow. Overall, these constraints are easy to meet practically and are favorably posed to attain global stability as well as good local performance. To be more specific, theoretically there is no absolute restriction on how large the feedback gains $(k_1, k_{21}, k_{22})$ can be. Ideally, this means the steady-state tracking error can be made arbitrarily small as seen from Theorem 1. In terms of robustness, the controlled system can tolerate a large class of modeling error and external disturbances, even those with the magnitude close to $M_2$. Whereas in Gong and Yao (2000), significant conservativeness exists on this matter.

### 3.3. Asymptotic tracking

If the considered system is subject to only parametric uncertainties, better performance such as zero final tracking error can be achieved. To prove the asymptotic tracking, a strengthened constraint denoted as (a*) is posed along with constraints (b)–(d):

(a*) $\quad k_{21} - k_1 > \frac{1}{2}(B_m + F_{scm}g + 1)^2$.

**Theorem 2.** *With the proposed controller* (6), (13) *satisfying conditions* (a*), (b)–(d) *and the adaptation law* (3), (4), *asymptotic output tracking is achieved if the system is only subject to parametric uncertainty, i.e.,* $\Delta = 0, \forall t$.

**Proof.** Following Theorem 1, the trajectory will eventually enter $\Omega_c = \{z : |z_1| \leqslant L_{11}, |z_2| \leqslant L_{22}\}$, then the asymptotic tracking under condition $\Delta = 0$ is proved as follows. Define a



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positive semi-definite function $V_a = \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} + \frac{1}{2}k_1 z_1^2$, with property (P2) of the parameter estimation law, its derivative $\dot{V}_a$ becomes

$$\begin{aligned}\dot{V}_a &= z_2\dot{z}_2 + \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\tilde{\theta}} + k_1 z_1 \dot{z}_1 \\ &= z_2((B_m + F_{scm}g)k_1 z_1 - (B_m + F_{scm}g - k_1)z_2 - \sigma_2 \\ &\quad - \varphi_d^{\mathrm{T}}\tilde{\theta}) + \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\varphi_d z_2) + k_1 z_1(z_2 - k_1 z_1) \\ &\leqslant -z_2^2(B_m + F_{scm}g + k_{21} - k_1) \\ &\quad + z_1 z_2(k_1(B_m + F_{scm}g + 1)) - z_1^2 k_1^2. \end{aligned} \tag{23}$$

Define matrix A as follows:

$$A = \begin{pmatrix} B_m + F_{scm}g + k_{21} - k_1 - k_a & -\frac{1}{2}k_1(B_m + F_{scm}g + 1) \\ -\frac{1}{2}k_1(B_m + F_{scm}g + 1) & \frac{1}{2}k_1^2 \end{pmatrix},$$

where $k_a$ is a positive constant. As long as $k_{21}$ is large enough to satisfy the following inequality:

$$k_{21} \geqslant \frac{1}{2}(B_m + F_{scm}g + 1)^2 + k_1 + k_a - B_m - F_{scm}g \tag{24}$$

which is guaranteed by condition (a*), matrix A is positive semi-definite. Therefore (23) can be rewritten as,

$$\dot{V}_a \leqslant -k_a z_2^2 - \frac{1}{2}k_1^2 z_1^2. \tag{25}$$

As a result, both $z_1$ and $z_2$ approach to the origin asymptotically. □

## 4. Comparative experiments

### 4.1. System setup

The linear motor system under study is described in Xu and Yao (2001). The saturated desired compensation ARC algorithm is designed and tested on the $Y$-axis of the stage. The mass of the stage and coil assembly is 3.34 kg and the input gain $K_f$ is 27.79. The corresponding bound $\bar{u}_{bd}$ is 83.32. The sampling frequency is 2.5 kHz and the resolution of the position sensor is 1 µm.

### 4.2. Implementation issues and design parameters

As mentioned in the controller structure section, there are two main issues which need further consideration in implementation. The motor system is normally equipped with a high-resolution encoder which provides very clean position feedback in contrast to a noisy velocity signal. To further alleviate the noise effect, the parameter estimates can be updated as in Xu and Yao (2001), with position measurements only.

Another modification is made to function $\sigma_2$. The example $\sigma_2$ as shown in Fig. 2 has a constant control effort as $|z_2| > L_{22}$, which introduces a certain degree of conservativeness. In other words, the actual amount of control effort for the model compensation $\bar{u}_a$ during most of the running period is much smaller than the estimated upper bound. Therefore the actuator has not put in all available power to attenuate the disturbance although the robust control term reaches its maximum value. To improve the system's ability to cope with large disturbances, the magnitude restriction on function $\sigma_2$ is removed so that $\sigma_2$ keeps increasing monotonically and the physical actuator becomes saturated naturally. Such a $\sigma_2$ still satisfies properties (i)–(iii) and thus the overall system's global stability is guaranteed.

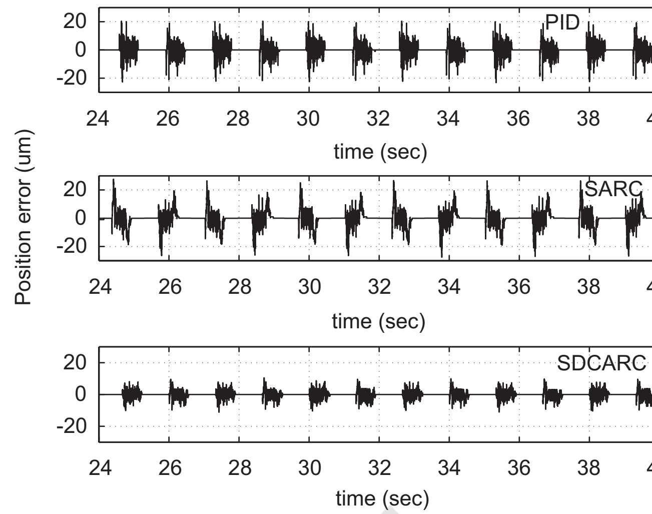


Fig. 4. Tracking error W/O disturbance.

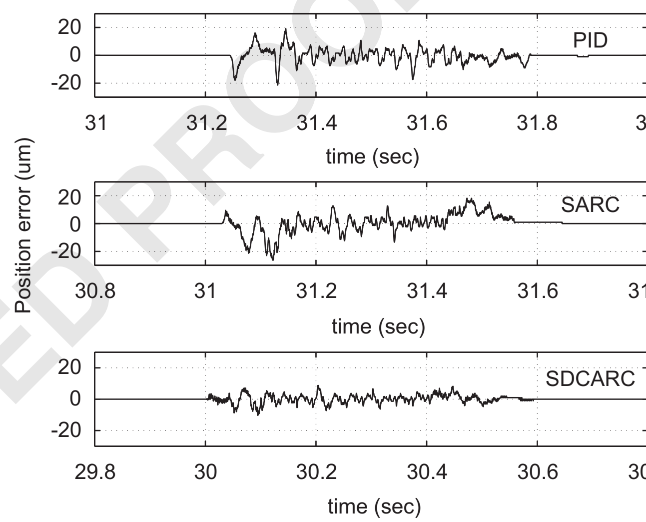


Fig. 5. Tracking error W/O disturbance (zoomed in portion).

Design parameters are determined from conditions (a)–(d) described before. Furthermore, with a clean position signal employed in the parameter adaptation law, higher robust gain and adaptation rates can be utilized for this scheme.

The control parameters are selected as follows, $L_{11} = 25$ µm, $L_{12} = 30$ µm, $k_1 = 750$, $M_1 = 0.0206$, $L_{21} = 0.0094$ m, $k_{21} = k_1 + 600$, $k_{22} = k_{21} + 200$, $M_2 = 0.99(\bar{u}_{bd} - \bar{u}_{abd})$, $L_{22} = (M_2 - k_{21}L_{21})/k_{22} + L_{21}$. It can be verified that these parameters satisfy the design constraints (a)–(d).

### 4.3. Comparative experiment results

The desired trajectory is a point-to-point movement, with a distance of 0.4 m, a maximum velocity of 1 m/s and a maximum acceleration of 12 m/s$^2$. In implementation, the system is at rest and there is no control input during roughly the first 20 s.



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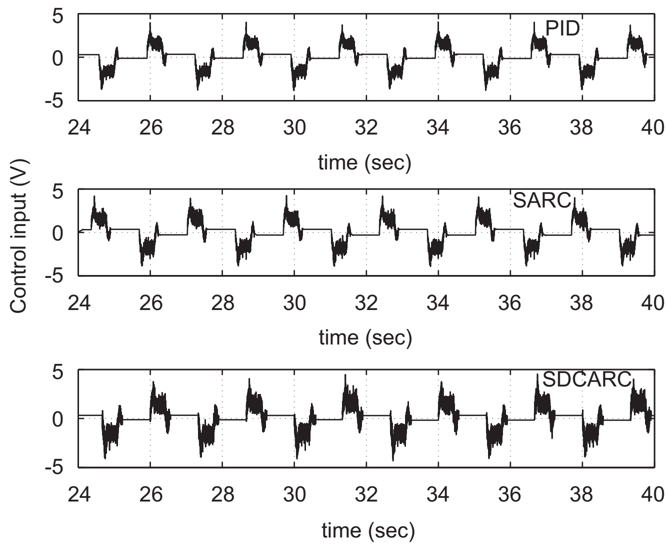

Fig. 6. Control input W/O disturbance.

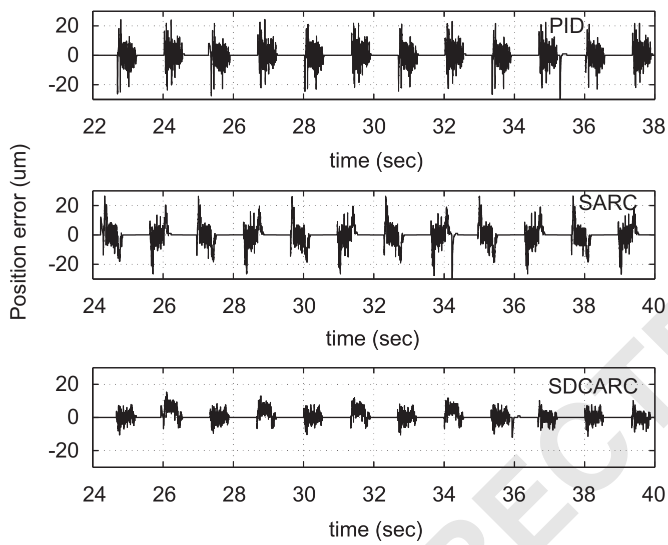

Fig. 7. Tracking error W/1 V disturbance.

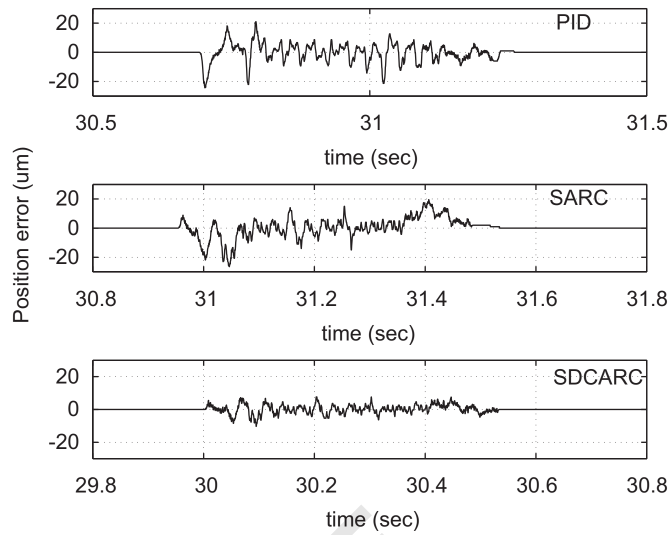

Fig. 8. Tracking error W/1 V disturbance (zoomed in portion).

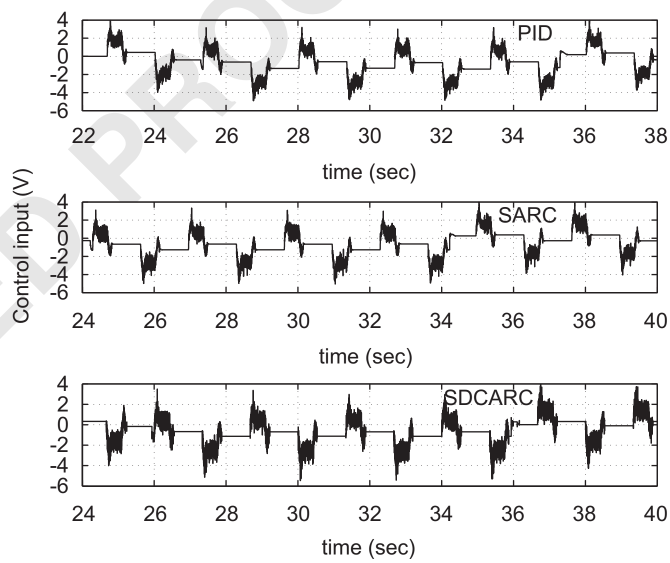

Fig. 9. Control input W/1 V disturbance.



The comparative experiments are conducted with three controllers under two different conditions, with no disturbances and with 1 V input disturbance, respectively. The controllers are: (i) the conventional PID with similar closed-loop bandwidth Xu and Yao, 2001, (ii) the saturated ARC (SARC) described in Hong and Yao (2005), and (iii) the saturated DCARC (SDCARC) proposed in this paper. The tracking errors with three controllers under different conditions are compared in Figs. 4 and 7. Figs. 5 and 8 are the magnified views over one moving period. The control inputs are shown in Figs. 6 and 9. Under both conditions, it is seen that the proposed SDCARC achieves the best position tracking performance. In order to gain more insight in a quantitative manner, the performance indexes used in Xu and Yao (2001) are listed in Table 1 to make the comparison more apparent. These results verify the high-performance nature of the proposed SDCARC, and the performance robustness to disturbances and modeling uncertainties under normal working conditions.

Table 1

| | W/o disturbance | | | W/disturbance | | |
|---|---|---|---|---|---|---|
| Controller | PID | SARC | SDCARC | PID | SARC | SDCARC |
| $e_M$ (μm) | 23.1 | 27.6 | 11.1 | 33.0 | 31.0 | 15.2 |
| $e_F$ (μm) | 22.5 | 26.8 | 10.4 | 23.0 | 26.7 | 11.4 |
| $L_2[e]$(μm) | 3.89 | 5.06 | 1.82 | 4.57 | 5.18 | 2.45 |
| $L_2[u](V)$ | 1.02 | 1.09 | 1.02 | 1.41 | 1.34 | 1.26 |
| $C_u$ | 0.191 | 0.146 | 0.352 | 0.138 | 0.118 | 0.285 |

To illustrate the global stability of the proposed SDCARC, as well as how effectively the controlled system deals with the practical scenario of experiencing an accident, such as a strong but short disturbance, experiments are conducted as follows.

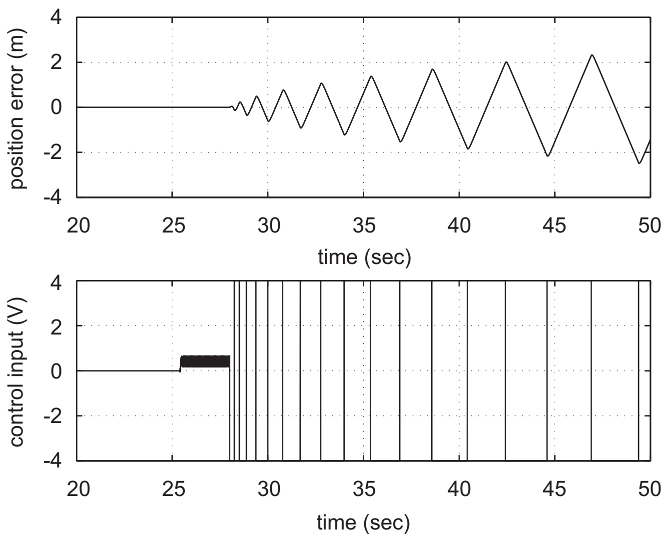


Fig. 10. PID W/6 V disturbance.

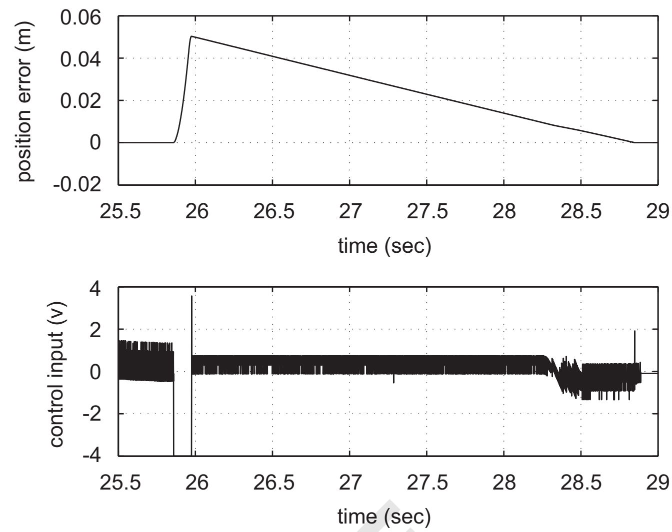


Fig. 12. SDCARC W/6 V disturbance (in the presence of the disturbance).

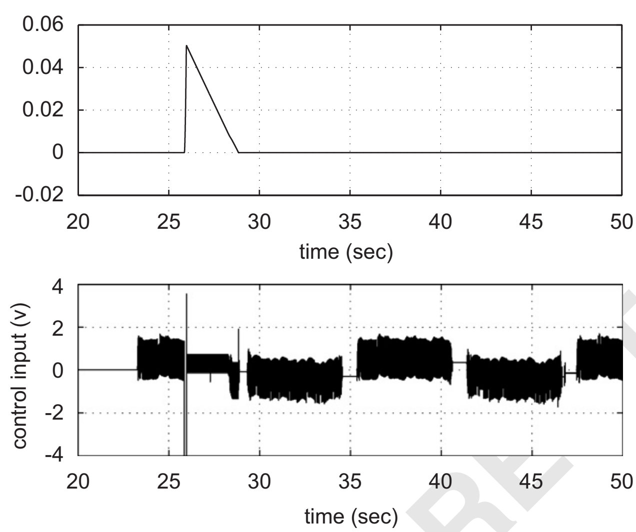


Fig. 11. SDCARC W/6 V disturbance (the whole process).

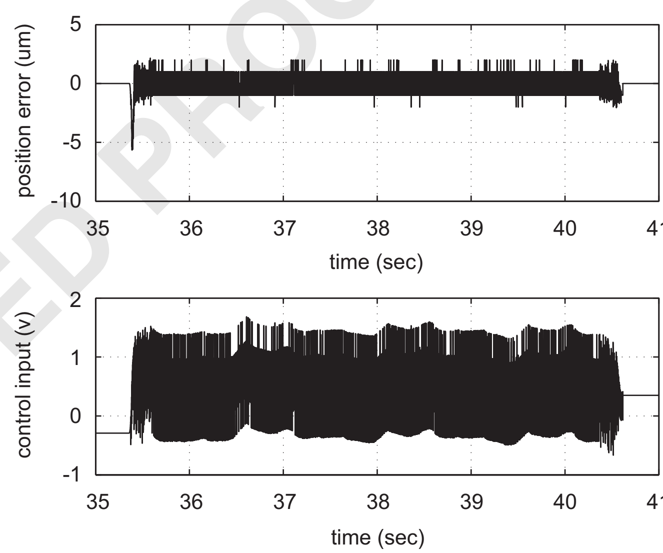


Fig. 13. SDCARC W/6 V disturbance (zoomed in portion at steady-state).



The actual input limit of the hardware is 10 V, therefore the physical limit of the control input is purposely set at 4 V, so that a step input, with the amplitude of 6V and a duration of 0.1 s, can be injected as a disturbance and realized by the hardware. The desired trajectory is also changed to be less aggressive due to the reduced control input authority, with a distance of 0.1 m, a maximum velocity of 0.02 m/s and a maximum acceleration of 0.1 m/s$^2$.

When applying the PID controller with the striking disturbance, in simulation the system goes unstable as shown in Fig. 10 due to "integration wind-up". With the other two controllers it is stable and shows similar performance. Experimental results for the proposed SDCARC are shown in Figs. 11–13. Fig. 11 provides an overall view of the tracking error and control input during total operation time. Fig. 12 emphasizes the period when the strong input disturbance is added. As seen from the plots, when the strong input disturbance is inserted around 25.8584 s, the control input is not sufficient to overpower the disturbance, hence it saturates at the 4 V limit and large tracking error accumulates to around 50 mm. However, after the strong input disturbance is removed 0.1 s later, the tracking error reduces to the encoder resolution level of 1 μm after reaching steady state, Fig. 13. These results verify the guaranteed global stability of the proposed SDCARC.

## 5. Conclusion

A saturated desired compensation adaptive robust control algorithm has been proposed. It is designed to improve tracking performance as well as to preserve global stability for the linear motor system which is subject to limited control authority, model uncertainties, and external disturbances. Such a goal is

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accomplished by utilizing a pre-calculated desired trajectory instead of actual state feedback in the regressor and replacing the use of the velocity signal by the position signal in the adaptation law. This reduces the noise effect and increases the closed-loop bandwidth. Experiments are presented, which compare the improved algorithm with the previous work in Hong and Yao (2005). The results confirm the superiority of the proposed scheme: guaranteed global stability, and excellent tracking performance under normal working conditions.

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