PERGAMON

Automatica 37 (2001) 1305–1321

automatica

www.elsevier.com/locate/automatica

Adaptive robust control of MIMO nonlinear systems in semi-strict feedback forms☆

Bin Yao[a,*,1], Masayoshi Tomizuka[b]

[a]School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, USA
[b]Mechanical Engineering Department, University of California at Berkeley, CA 94720, USA

Received 13 March 1997; revised 15 August 2000; received in final form 24 February 2001

Adaptive robust control (ARC) laws are developed for MIMO nonlinear systems transformable to two semi-strict feedback forms. The forms allow coupling and appearance of parametric uncertainties in the input matrix of each layer.

Abstract

Adaptive robust control (ARC) of MIMO nonlinear systems transformable to two semi-strict feedback forms are considered. The forms can have both parametric uncertainties and uncertain nonlinearities such as external disturbances. In addition, the forms allow coupling and appearance of parametric uncertainties in the input matrix of each layer. Furthermore, the usual assumption on the linear parametrization of the state equations is relaxed to the extent that the forms are applicable to the control of some mechanical systems. To deal with the complexity and difficulties caused by the coupling and the appearance of parametric uncertainties in the input matrices, an ARC Lyapunov function—an extension of the adaptive control Lyapunov function (aclf)—is used to formalize the viewpoint and the achievable results of the recently proposed ARC approach. Two backstepping designs via ARC Lyapunov functions are presented. The results are then used to construct specific ARC control laws for MIMO nonlinear systems in the semi-strict-feedback forms. By using trajectory initialization, the resulting ARC law achieves a guaranteed output tracking transient performance and final tracking accuracy in general, while keeping all physical states and control inputs bounded. In addition, the control law achieves asymptotic output tracking in the presence of parametric uncertainties without using a discontinuous or infinite-gain feedback term. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Adaptive control; Robust control; Sliding mode control; Nonlinear systems; Uncertainty

1. Introduction

The following two types of uncertainties are of major concern in the control of uncertain nonlinear systems: parametric uncertainties (e.g., gravitational load for robots) and general uncertainties coming from modeling errors (e.g., ignored nonlinear friction) and external disturbances, which are referred to as uncertain nonlinearities or unknown nonlinear functions in this paper. To account for these uncertainties, two nonlinear control methods have been popular: adaptive control (Krstic, Kanellakopoulos, & Kokotovic, 1995; Marino & Tomei, 1995; Krstic, Kanellakopoulos, & Kokotovic, 1992; Sastry & Isidori, 1989; Marino & Tomei, 1993; Pomet & Praly, 1992) and deterministic robust control (Utkin, 1992; Corless & Leitmann, 1981; Zinober, 1990; Qu, 1993; Slotine, 1985; Barmish & Leitmann, 1982; Chen, 1988). The adaptive control achieves asymptotic tracking for reasonably large classes of nonlinear systems without using discontinuous or infinite-gain feedback terms (Krstic et al., 1995; Marino & Tomei, 1993). However,

☆Parts of the paper were presented at the 1995 IEEE Conference on Decision and Control and at the 1996 IFAC World Congress, USA. This paper was recommended for publication in revised form by Associate Editor Jan Willem Polderman under the direction of Editor Frank L. Lewis.

[1] Supported by the National Science Foundation under the CAREER grant CMS-9734345.

* Corresponding author. Tel.: + 1-765-4947746; fax: + 1-765-4940539.

E-mail addresses: byao@ecn.purdue.edu (B. Yao), tomizuka@euler.me.berkeley.edu (M. Tomizuka).

PII: S0005-1098(01)00082-6

adaptive controllers only deal with the ideal case of constant parametric uncertainties and the adaptation law may lose stability even when a small disturbance appears (Reed & Ioannou, 1989). Every physical system is subject to some form of disturbance. Additional effort has to be made to safely implement such adaptive nonlinear controllers. One may apply remedies similar to those used in robust adaptive control of linear systems (Kreisselmeier & Narendra, 1982; Ioannou, 1986). However, although asymptotic tracking is still preserved in the absence of disturbances, such modifications (Reed & Ioannou, 1989; Polycarpou & Ioannou, 1993) do not guarantee tracking accuracy in the presence of disturbances since the steady state tracking error can only be shown to stay within an unknown region, whose size depends on the disturbances. Furthermore, transient performance is unknown. In contrast, the deterministic robust control—e.g., sliding mode control (Utkin, 1992)—can be used to achieve a guaranteed transient performance and a guaranteed final tracking accuracy in the presence of both parametric uncertainties and uncertain nonlinearities. However, it usually involves switching (Utkin, 1992) or infinite-gain feedback (Qu, 1993) terms, which introduces control chattering. Chattering may be avoided at the expense of degraded tracking performance by using some smoothing techniques (Slotine, 1985; Yao & Tomizuka, 1994).

In Yao and Tomizuka (1994), we presented a systematic way to combine the adaptive control and the sliding mode control (SMC) for the trajectory tracking control of robot manipulators to preserve the advantages of the two methods while overcoming their drawbacks.[2] Comparative experimental results for the motion control of robot manipulators (Yao & Tomizuka, 1994) and the high-speed/high-accuracy trajectory tracking control of machine tools (Yao, Al-Majed, & Tomizuka, 1997) have demonstrated the substantially improved performance of the suggested adaptive robust control (ARC) approach. In the motion control of rigid robots (Yao & Tomizuka, 1994), the design was for multivariable nonlinear differential equations with relative degree of *one*. In Yao and Tomizuka (1997), the methodology was extended to a class of single-input single-output (SISO) nonlinear systems with *arbitrary known* "*relative degrees*" in a semi-strict feedback form (Polycarpou & Ioannou, 1993) by combining the backstepping adaptive control (Krstic et al., 1992; Marino & Tomei, 1995) with the general deterministic robust control.

Recently, other researchers have also approached the control of SISO nonlinear systems in the semi-strict feedback form from various perspectives and excellent results have been obtained. Specifically, in Santosuosso (1995), Santosuosso considered the *regulation* problem of SISO nonlinear systems in a semi-strict feedback form; aside from zero-dynamics, the form in Santosuosso (1995) has the same structure as that studied in Yao and Tomizuka (1997) but the allowable disturbance is restricted to the class of L_2-norm bounded disturbances. In Pan and Basar (1996), Pan and Basar cast the problem of robust adaptive control of SISO nonlinear systems in the framework of nonlinear H^{∞} optimal control, where specific measures of asymptotic tracking, transient behavior, and disturbance attenuation were all incorporated into a single cost functional, and an explicit solution was presented. The presented algorithm suffers from the over-parametrization problem and gives the H_{∞}-gain that relates the L_2 norm of the tracking error to the L_2 norm of the disturbance only; in other words, transient performance in terms of L_{∞} norms of the tracking error and the disturbance is not clear. Freeman, Krstic, and Kokotovic (1996) robustify the well-known backstepping nonlinear adaptive control algorithms developed in Krstic et al. (1995) for bounded uncertainties/disturbances. The L_{∞}/L_2 estimates were also given on the effects of bounded uncertainties/disturbances on the tracking error—a much stronger performance result than those achieved in the conventional robust adaptive control schemes (Polycarpou & Ioannou, 1993; Reed & Ioannou, 1989). It is noted that the same strong performance results are also obtained in a recent paper by Marino and Tomei (1998).

It should be realized that there are some subtle but fundamental differences between the proposed adaptive robust control (ARC) approach (Yao & Tomizuka, 1996, 1997) and the tuning function based robust adaptive control (RAC) approach (Freeman et al., 1996). Firstly, in terms of fundamental view point, the proposed ARC (Yao & Tomizuka, 1996, 1997) puts more emphasis on the underline robust control law design in achieving a guaranteed robust output tracking performance. In fact, the parameter adaptation law in ARC (Yao & Tomizuka, 1996, 1997) can be switched off at any time without affecting system stability and sacrificing the guaranteed output tracking transient performance since the resulting controller becomes a deterministic robust controller. Secondly, in terms of the achievable performance, in the proposed ARC (Yao & Tomizuka, 1996, 1997), the upper bound on the absolute value of the output tracking error over entire time-history is given and is related to certain controller design parameters in a *known* form, which is more transparent than that in RAC (Freeman et al., 1996; Polycarpou & Ioannou, 1993). Finally, in terms of specific approaches used for the controller design and the proof of achievable performance, the proposed ARC uses two Lyapunov functions; one the same as that in the deterministic robust control

[2] In this paper, the terminology of adaptive robust control (ARC) is used to represent such a combined design approach; the use of this terminology is to differentiate the approach from conventional robust adaptive control approach for reasons which will become clear later.

(Utkin, 1992; Corless & Leitmann, 1981; Qu, 1993; Slotine, 1985; Barmish & Leitmann, 1982; Chen, 1988) and the other the same as that in adaptive control (Krstic et al., 1995), while the robust adaptive control (Freeman et al., 1996) uses the same Lyapunov function as in adaptive control (Krstic et al., 1995) only. Because of these subtle differences, the terminology of "*adaptive robust control* (*ARC*)" is used for the proposed combined design method to differentiate the approach from the conventional robust adaptive control approach and to reflect the strong emphasis on the robust control law design for robust performance.

Although a large amount of work has been carried out on the construction of backstepping adaptive or robust controllers for SISO nonlinear systems in semi-strict feedback form (Krstic et al., 1995; Marino & Tomei, 1995; Polycarpou & Ioannou, 1993; Yao & Tomizuka, 1997; Santosuosso, 1995; Pan & Basar, 1996; Freeman et al., 1996), very few results are available for MIMO nonlinear systems. Krstic et al. (1995) considered the backstepping adaptive control of MIMO nonlinear systems in a parametric strict feedback form. However, the form assumes no parametric uncertainties in the input matrix.

This paper concerns the systematic construction of adaptive robust controllers (ARC) for a class of MIMO nonlinear systems transformable to two semi-strict feedback forms. The two forms are the natural extensions of the MIMO parametric strict feedback form studied in Krstic et al. (1995) but with practical applications in mind. In particular, the presented forms have the following additional complexities and design difficulties. Firstly, the proposed semi-strict feedback forms allow coupling and appearance of parametric uncertainties in the input matrices of all intermediate layers—a common problem in applications. As a result, the following new problems have to be solved: (i) At each layer, ARC design techniques have to be developed to determine the ARC control functions for all virtual inputs simultaneously in order to attack the problem of coupling among input channels, which is qualitatively different from that in Krstic et al. (1995). In Krstic et al. (1995), since input matrix assumes no parametric uncertainties, realizable nonlinear state transformations and input transformations can be constructed to decouple the coupling among input channels. The resulting MIMO system thus essentially consists of a bunch of "uncoupled" input–output pairs, and the corresponding SISO backstepping adaptive design can be applied to each pair of input–output independently without caring about the control functions needed for other input–output pairs (Krstic et al., 1995); (ii) Backstepping ARC design techniques have to be developed to deal with the appearance of parametric uncertainties in the input matrices. Our early results on the adaptive robust control of SISO nonlinear systems (Yao & Tomizuka, 1997) cannot handle parametric uncertainties in the input channels. It is well known that the appearance of parametric uncertainties in the input matrix normally complicates the controller design significantly. Even in the absence of uncertain nonlinearities, backstepping adaptive control of the proposed MIMO semi-strict feedback forms has not been dealt with; (iii) Due to the complexity associated with coupled uncertain MIMO nonlinear systems, even the presentation of MIMO semi-strict feedback forms in a meaningful manner becomes difficult. The paper gives two typical representations of MIMO semi-strict feedback forms. Both forms are concise and useful since they admit certain practically significant applications.

The second unique feature of the proposed two forms is that, motivated by the extensive research in the control of rigid robot manipulators (Slotine, 1985; Yao & Tomizuka, 1996, 1994), the usual requirement on the linear parametrization of the state equations is relaxed to the extent that the proposed forms are applicable to the control of some multiple degrees-of-freedom (DOF) mechanical systems. For most mechanical systems, such as robot manipulators, their state equations cannot be linearly parametrized in terms of a set of unknown parameters, which prohibits the direct application of the MIMO backstepping adaptive control results in Krstic et al. (1995). Lastly, the proposed forms allow the presence of certain uncertain nonlinearities. This significantly extends the applicability of the proposed approach since most physical systems have uncertain nonlinearities in one form or another.

To overcome the control design difficulties mentioned above, in this paper, an adaptive robust control (ARC) Lyapunov function, which can be considered as an extension of the adaptive control Lyapunov function in Krstic et al. (1995), is used to formalize and systematize the proposed ARC approach. The formulation simplifies the controller design, especially when some systematic design procedures such as the backstepping design (Krstic et al., 1995) are used to enlarge the applicable class of nonlinear systems. As a result, ARC controllers can be synthesized for much more complicated nonlinear systems. The formulation also makes it possible to relax the assumption on the linear parametrization of the state equations. Two general backstepping designs via ARC Lyapunov functions are presented to construct specific ARC controllers. The results are then used to systematically construct ARC controllers for MIMO nonlinear systems in the proposed two semi-strict feedback forms by employing an additional tool-trajectory initialization. Several applications will be briefly mentioned.

2. Formulation of adaptive robust control

In this section, motivated by the adaptive control Lyapunov function introduced in Krstic et al. (1995)—an adaptive extension of the control Lyapunov function (Artstein, 1983; Sontag, 1983) for systems with parametric uncertainties only, an adaptive robust control (ARC) Lyapunov function is defined to concisely describe the objective and the achievable results of the proposed adaptive robust control method for systems with both parametric uncertainties and uncertain nonlinearities.

Consider the following MIMO nonlinear system[3]

$$\dot{x} = f(x,\theta) + B(x,\theta)u + D(x)\Delta(x,\theta,u,t),$$
$$y = h(x), \tag{1}$$

where $y \in R^m$ and $u \in R^m$ are the output and input vectors, respectively, $x \in R^n$ is the state vector, $\theta \in R^p$ is the vector of unknown parameters, $h(x), f(x,\theta), B(x,\theta)$, and $D(x)$ are known,[4] and $\Delta \in R^{l_\Delta}$ represents the vector of unknown nonlinear functions such as disturbances and modeling errors. The following reasonable and practical assumptions are made, which are satisfied by most applications (Yao & Tomizuka, 1996, 1994; Yao, Majed, & Tomizuka, 1997):

A2.1. The extent of parametric uncertainties and uncertain nonlinearities is known, i.e.,

$$\theta \in \Omega_\theta \triangleq \{\theta\colon \theta_{\min} < \theta < \theta_{\max}\}$$
$$\Delta \in \Omega_\Delta \triangleq \{\Delta\colon \|\Delta(x,\theta,u,t)\| \leqslant \delta(x)\} \tag{2}$$

where $\theta_{\min}$, $\theta_{\max}$ and $\delta(x)$ are known.

Throughout the paper, the following notations will be used. In general, $\bullet_i$ represents the ith component of the vector $\bullet$ and the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors. Let $\hat{\bullet}$ denote the estimate of $\bullet$ (e.g., $\hat{\theta}$ for θ). For any unknown parameter vector $\bullet$ lying in a known bounded region $\Omega_\bullet = \{\bullet\colon \bullet_{\min} < \bullet < \bullet_{\max},\}$ (e.g., Ω_θ), a smooth projection map $\pi_\bullet$ can be defined for $\hat{\bullet}$ and has the following properties: P1: $\forall \hat{\bullet} \in \Omega_\bullet$, $\pi_\bullet(\hat{\bullet}) = \hat{\bullet}$. P2: $\forall \hat{\bullet}$, $\pi_\bullet(\hat{\bullet}) \in \Omega_{\hat{\bullet}} = \{\mu\colon \bullet_{\min} - \varepsilon_\bullet \leqslant \mu \leqslant \bullet_{\max} + \varepsilon_\bullet\}$ *where* $\varepsilon_\bullet$ *is a known vector of positive numbers which can be arbitrarily small.* P3: $\pi_{\bullet i}(\hat{\bullet}_i)$ *is a nondecreasing function of* $\hat{\bullet}_i$. P4: *The derivatives of the projection are bounded up to a sufficiently high-order, i.e.,* $\forall j \leqslant n$, $\pi_\bullet^{(j)}(\hat{\bullet})$ *is bounded.* See Teel (1993) and Yao and Tomizuka (1997) for further details of the smooth projections. For convenience, define $\hat{\bullet}_\pi$ as $\hat{\bullet}_\pi = \pi_\bullet(\hat{\bullet})$ and the projected estimation error as $\tilde{\bullet}_\pi = \hat{\bullet}_\pi - \bullet$. For any index j, let $\bar{\bullet}_\pi^{(j)}$ denote $\bar{\bullet}_\pi^{(j)} = [\pi(\hat{\bullet})^{\mathrm{T}}, \ldots, \pi^{(j)}(\hat{\bullet})^{\mathrm{T}}]^{\mathrm{T}}$.

Let $y_d(t) \in R^m$ be the desired output trajectories, and the output tracking errors as $e_y = y - y_d(t)$. The objective is to construct a bounded control law such that, under assumption (2), the system state x is bounded and the output tracking possesses certain desirable features. Specifically, the control law to be sought consists of two parts and is given by

$$u(x, \bar{\theta}_\pi^{(l_u)}, t) = u_a(x, \bar{\theta}_\pi^{(l_u)}, t) + u_s(x, \bar{\theta}_\pi^{(l_u)}, t), \tag{3}$$

where l_u is an index, u_a functions as an adaptive control law and u_s a robust control law to be designed within an allowable set Ω_u. The control functions u_a and u_s, and an adaptation law are determined to make a positive semi-definite (p.s.d.) function an ARC Lyapunov function defined as follows.

Definition 1. Let $V(x, \theta, \bar{\theta}_\pi^{(l_V)}, t)$ be a p.s.d. function with continuous partial derivatives (l_V is any index). V is called an adaptive robust control (ARC) Lyapunov function for (1) if it satisfies the following three requirements for some continuous control functions u_a and u_s and an adaptation function τ.

R1: Bounded V leads to bounded state x, and guaranteed transient performance of $V(t)$ is equivalent to the guaranteed transient performance of output tracking error e_y (e.g., $V(t) \to 0$ means $e_y \to 0$ or guaranteed exponential convergence of the upper bound of $V(t)$ leads to guaranteed exponential convergence of the upper bound of $|e_y|$).[5]

R2: There exists a continuous control law u_a such that $\forall u_s \in \Omega_u$:

$$\frac{\partial V}{\partial x}[f + B(u_a + u_s)] + \frac{\partial V}{\partial t} \leqslant$$
$$- W + \tilde{\theta}_\pi^{\mathrm{T}} \tau + \frac{\partial V}{\partial \hat{\theta}} \Gamma \tau \tag{4}$$

or, equivalently,

$$\dot{V}|_{\Delta=0} \leqslant - W + \tilde{\theta}_\pi^{\mathrm{T}} \tau + \frac{\partial V}{\partial \hat{\theta}}(\dot{\hat{\theta}} + \Gamma \tau), \tag{5}$$

where $\tau(x, \bar{\theta}_\pi^{(l_\tau)}, u, t)$ is a known function with $l_\tau = \max\{l_V + 1, l_u\}$, $\dot{V}|_{\Delta=0}$ represents the derivative of V when $\Delta = 0$, and W is any continuously differentiable p.s.d. function and satisfies the condition that asymptotic convergence of W leads to asymptotic output tracking (i.e., $W \to 0 \Rightarrow e_y \to 0$).

[3] The functions h, f, B, and D could depend on the time t explicitly as well, as long as these functions and all their partial derivatives are bounded with respect to time t; a function $f(x,\theta,t)$ is called bounded with respect to time t iff there exists a positive function $f_u(x,\theta)$ such that $\|f(x,\theta,t)\| \leqslant f_u(x,\theta)$, $\forall t$. The details can be found in Yao (1996) and are omitted for simplicity. Note that this assumption is different from the assumption that all system functions are bounded.

[4] A vector or matrix is known if all its elements are known functions with respect to their variables.

[5] For simplicity of notation, $\bullet \to 0$ means that $\bullet \to 0$ as $t \to \infty$.

R3: There exists a $u_s \in \Omega_u$ such that $\forall \theta \in \Omega_\theta$ and $\forall \Delta \leqslant \Omega_\Delta$:

$$\frac{\partial V}{\partial x}[f + B(u_a + u_s) + D\Delta] + \frac{\partial V}{\partial t} - \frac{\partial V}{\partial \hat{\theta}}\Gamma\tau \leqslant - \lambda_V V + c_V \tag{6}$$

or, equivalently,

$$\dot{V} \leqslant - \lambda_V V + c_V + \frac{\partial V}{\partial \hat{\theta}}(\dot{\hat{\theta}} + \Gamma\tau), \tag{7}$$

where $\lambda_V > 0$ and $c_V(t)$ is a bounded positive scalar, i.e., $0 \leqslant c_V \leqslant c_{V\max}$. It is further required that both λ_V and c_V can be adjusted freely by certain controller parameters in a known form without affecting $V(0)$, the initial value of V.

Remark 1. The above definition of ARC Lyapunov functions summarizes the type of control laws and the type of responses that the proposed ARC approach is looking for. Specifically, Requirement R1 converts the stability and output tracking performance of the nonlinear system (1) into the study of the stability and performance of a scalar function V, which is much easier to handle. Requirement R2 guarantees that there exists an adaptive control law to achieve asymptotic output tracking in the presence of parametric uncertainties only as shown in the following theorem. Requirement R3 states that a robust control law can be synthesized to attenuate the effect of both parametric uncertainties and uncertain nonlinearities to achieve a guaranteed output tracking transient performance as well as final tracking accuracy.

Theorem 1. *If there exists an* ARC *Lyapunov function V for* (1), *then, by using the control law* (3) *and the adaptation law*

$$\dot{\hat{\theta}} = - \Gamma\tau(x, \bar{\theta}_\pi^{(l_\tau)}, u(x, \bar{\theta}_\pi^{(l_u)}, t), t), \tag{8}$$

the following results hold:

A: *In general, the control input and the system state are bounded with V bounded above by*

$$\begin{aligned} V(t) &\leqslant \exp(-\lambda_V t)V(0) + \int_0^t \exp(-\lambda_V(t - v))c_V(v)\,\mathrm{d}v \\ &\leqslant \exp(-\lambda_V t)V(0) + \frac{c_{V\max}}{\lambda_V}[1 - \exp(-\lambda_V t)]. \end{aligned} \tag{9}$$

Output tracking is guaranteed to have arbitrary good transient performance and final tracking accuracy in the sense that both λ_V, the exponentially converging rate, and $c_{V\max}/\lambda_V$, the bound on $V(\infty)$, can be freely adjusted via certain controller parameters in a known *form without affecting $V(0)$.*

B: *If, after a finite time, there are no uncertain nonlinearities, i.e., $\Delta = 0$, $\forall t \geqslant t_0$, for some finite t_0, then, in addition to the results in A, asymptotic output tracking is achieved.*

Proof of Theorem 1. See Appendix A.

Remark 2. In the absence of parameter adaptation (i.e., $\Gamma = 0$), the proposed ARC law reduces to a DRC law and Result A of Theorem 1 still holds. Therefore, the adaptation loop can be switched off at any time without affecting the stability and the guaranteed output tracking transient performance. However, such a control law does not discriminate the difference between parametric uncertainties and uncertain nonlinearities as in Utkin (1992), Corless and Leitmann (1981), Zinober (1990), Qu (1993), Barmish and Leitmann (1982) and Chen (1988) and results in a conservative design since Result B of Theorem 1 is lost. As for adaptive control (Krstic et al., 1995, 1992; Sastry & Isidori, 1989; Marino & Tomei, 1993; Pomet & Praly, 1992) the proposed ARC uses certain coordination mechanisms (e.g., smooth projection) and robust feedback control u_s to achieve a guaranteed output tracking transient performance even in the presence of uncertain nonlinearities (A of Theorem 1) while without losing its nominal performance (B of Theorem 1).

Corollary 3. *If there exists an* ARC *Lyapunov function $V(x, \theta, t)$, which is not a function of $\hat{\theta}$, then, by using the control law* (3) *and the modified adaptation law*

$$\dot{\hat{\theta}} = - \Gamma[l_\theta(\hat{\theta}) + \tau(x, \bar{\theta}_\pi^{(l_\tau)}, u(x, \bar{\theta}_\pi^{(l_u)}, t), t)], \tag{10}$$

where $l_\theta(\hat{\theta})$ is any vector of functions satisfying the following conditions:

$$\begin{aligned} &\text{(i)} \quad l_\theta(\hat{\theta}) = 0 \quad \text{if } \hat{\theta} \in \Omega_\theta, \\ &\text{(ii)} \quad \tilde{\theta}_\pi^{\mathrm{T}} l_\theta(\hat{\theta}) \geqslant 0 \quad \text{if } \hat{\theta} \notin \Omega_\theta, \end{aligned} \tag{11}$$

we have the results in Theorem 1.

Proof of Corollary 1. See Appendix B.

Remark 4. The reason for using (10) is that by suitably choosing $l_\theta(\hat{\theta})$, the parameter estimation process can be made more robust and the boundedness of $\hat{\theta}$ can be guaranteed since $l_\theta(\hat{\theta})$ acts like a nonlinear damping term. In fact, when a discontinuous modification law $l_\theta(\hat{\theta})$ is allowed as in the applications studied in Yao and Tomizuka (1996), Yao et al. (1997), the widely used projection method in adaptive systems (Sastry & Bodson, 1989; Goodwin & Mayne, 1989) can be employed since it is shown in Yao and Tomizuka (1996) and Yao (1996) that it satisfies (11). For details, see Yao (1996). Some

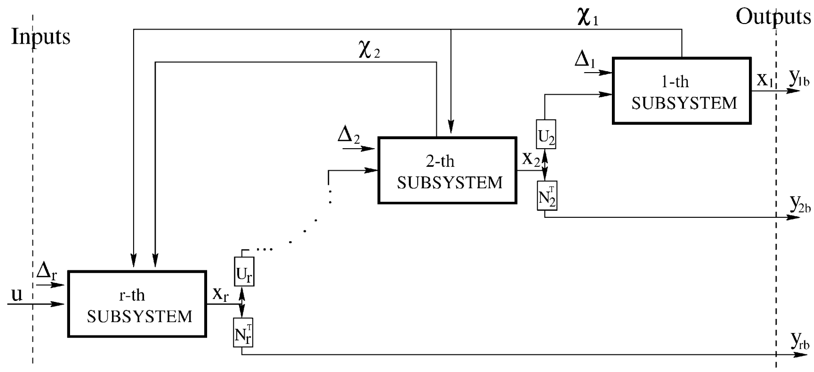

Fig. 1. MIMO semi-strict feedback form I.

continuous modifications are also given in Yao and Tomizuka (1994) by generalizing σ-modification (Reed & Ioannou, 1989).

$M^{-1} = \frac{1}{|M|} \mathrm{adj}(M)$

3. MIMO semi-strict feedback forms

In the previous section, the proposed ARC approach is presented via ARC Lyapunov functions. Thus, the remaining problem is to construct specific ARC Lyapunov functions for a particular application to come out specific control laws and adaptation laws, which is the focus of the rest of the paper. In particular, specific ARC Lyapunov functions will be constructed for MIMO nonlinear systems transformable to the two semi-strict feedback forms introduced below.

3.1. MIMO semi-strict feedback form I

The MIMO form I shown in Fig. 1 is an inter-connection of r subsystems described by

$$\dot{x}_i = f_{i0}(\bar{\chi}_i) + F_i(\bar{\chi}_i)\theta + B_i(\bar{\chi}_i,\theta)\bar{x}_{i+1,m_i} + D_i(\bar{\chi}_i)\Delta_i(\chi,\theta,t),$$
$$\dot{\eta}_i = \Phi_{i0}(\bar{\chi}_i) + \Phi_{i\theta}(\bar{\chi}_i)\theta, \quad 1 \leqslant i \leqslant r-1,$$
$$\dot{x}_r = M^{-1}(\bar{\chi}_{r-1},\beta)\,[f_{r0} + F_r\theta + F_\beta(\chi)\beta + B_r(\chi,\theta,\beta)u$$
$$+ D_r(\chi)\Delta_r],\ \dot{\eta}_r = \Phi_r(\chi,\theta,\beta),$$
$$y = [y_{1b}^{\mathrm{T}},\ldots,y_{rb}^{\mathrm{T}}]^{\mathrm{T}} \in R^m, \tag{12}$$

where $x_i \in R^{m_i}$, $\eta_i \in R^{n_i}$, $\chi_i = [x_i^{\mathrm{T}},\eta_i^{\mathrm{T}}]^{\mathrm{T}}$, $\bar{\chi}_i = [\chi_1^{\mathrm{T}},\ldots,\chi_i^{\mathrm{T}}]^{\mathrm{T}}$, and $\chi = \bar{\chi}_r$. In (12), $\beta \in R^{l_\beta}$ is a vector of some unknown parameters, which satisfies the same assumption as in (2), i.e., $\beta \in \Omega_\beta \triangleq \{\beta\colon \beta_{\min} < \beta < \beta_{\max}\}$ where Ω_β is a known set. Form (12) is obtained as follows. For all i, the ith subsystem is a MIMO nonlinear system with the state vector χ_i, the input vector $v_i \in R^{m_i}$, and the output vector x_i, where it is assumed that $0 = m_0 \leqslant m_1 \leqslant m_2 \leqslant \cdots \leqslant m_r = m$. The $(i+1)$th subsystem is connected to the ith subsystem in the following way: the first m_i outputs of the $(i+1)$th subsystem are connected to the inputs of the ith subsystem (i.e., $v_i = \bar{x}_{i+1,m_i} = U_{i+1}x_{i+1}$), and the remaining outputs of the $(i+1)$th subsystem, $y_{i+1b} = N_{i+1}^{\mathrm{T}}x_{i+1}$, become the $(i+1)$th block of the system outputs, where $U_{i+1} = [I_{m_i}\ 0]$ and $N_{i+1} = [0\ I_{m_{i+1}-m_i}]^{\mathrm{T}}$, $\forall i$. The first $(r-1)$ subsystems, which are described by the first two equations of (12), have the same structure. The rth subsystem, which is described by the third and fourth equations of (12), has the additional complexity that it involves the inversion of an unknown positive definite matrix M. The vectors or matrices, f_{i0}, F_i, B_i, D_i, Φ_{i0}, and $\Phi_{i\theta}$ are known functions of their variables, which include $\bar{\chi}_{i-1}$, the states of all its previous subsystems. Δ_i is the vector of unknown nonlinear functions.

The following assumptions are made for system (12):

A3.1. $\forall i \leqslant r-1$, B_i is nonsingular for any $\theta \in \Omega_\theta$, and B_r is nonsingular for any $\theta \in \Omega_\theta$ and $\beta \in \Omega_\beta$.

A3.2. M is an s.p.d. matrix and there exist positive scalars k_m and k_M such that $k_m I_m \leqslant M \leqslant k_M I_m$.

A3.3. $\forall i \leqslant r-1$, B_i can be linearly parametrized by θ, i.e., $B_i = B_{i0}(\bar{\chi}_i) + B_{i\theta}(\bar{\chi}_i,\theta)$ where $B_{i\theta}(\bar{\chi}_i,\theta)$ is linear w.r.t. θ.[6] Similarly, $B_r = B_{r0}(\chi) + B_{r\theta}(\chi,\theta) + B_{r\beta}(\chi,\beta)$, where $B_{r\theta}$ and $B_{r\beta}$ are linear w.r.t. θ and β respectively. $M = M_0(\bar{\chi}_{r-1}) + M_\beta(\bar{\chi}_{r-1},\beta)$ in which M_β is linear w.r.t. β.

A3.4. The η_i-subsystem is bounded-input bounded-state (BIBS) stable w.r.t. the inputs $(\bar{\chi}_{i-1}, x_i)$.

A3.5. There exist known functions $\delta_i(\bar{\chi}_i)$ such that

$$\|\Delta_i(\chi,\theta,t)\| \leqslant \delta_i(\bar{\chi}_i), \quad i = 1,\ldots,r. \tag{13}$$

Remark 5. Assumption A3.1 is to make sure that the problem is well posed in the sense that it guarantees that each subsystem has "relative degree" one from its input $v_i = \bar{x}_{i+1,m_i}$ to its output x_i. Assumption A3.2 is to capture the physical phenomenon that the inertia matrix M of a multi-DOF mechanical system is always an s.p.d.

[6] For a matrix •, • is said to be linear w.r.t. θ if all its elements are linear functions of θ.

matrix. Assumption A3.4 is to assure that the internal dynamics of each subsystem in (12) is stable. Assumption A3.5 is the semi-strict feedback structural assumption, which is similar to the parametric-strict feedback assumption in Krstic et al. (1995). However, this assumption is a much less restrictive structural assumption than the strict feedback assumption in Krstic et al. (1995); this is due to the fact that only the bounding functions of the uncertain nonlinearities Δ_i are required to be the function of $\bar{\chi}_i$, and Δ_i can contain bounded functions of χ_j, $j > i$ and u. In other words, bounded interactions among the ith subsystem and the subsequent subsystems may be allowed, which violates the strict-feedback property.

Remark 6. It is noted that the MIMO strict feedback form studied in Krstic et al. (1995) is a subset of the proposed semi-strict feedback form (12). By assuming that the input matrices of the first $(r-1)$ subsystem are identity matrices (i.e., $B_i = I_{m_i \times m_i}$, $\forall i \leqslant r-1$), the input matrix of the last subsystem assumes no parametric uncertainties (i.e., B_r is a function of χ only), the inertia matrix M is an identity matrix, no internal dynamics for each subsystem, and no uncertain nonlinearities (i.e., assuming $\Delta_i = 0$, $\forall i$), the proposed semi-strict feedback form (12) reduces to the MIMO strict feedback form in Krstic et al. (1995). It is thus clear that, apart from the control issues caused by the appearance of uncertain nonlinearities, the major difference between the proposed MIMO semi-strict feedback form and the MIMO strict feedback form in Krstic et al. (1995) is that the proposed form allows coupling and appearance of parametric uncertainties in the input matrix of each layer. As a result, our previous SISO ARC design (Yao & Tomizuka, 1997) cannot be straightforwardly generalized to solve the problem, which is qualitatively different from that in Krstic et al., 1995. In Krstic et al. (1995), since input matrix assumes no parametric uncertainties, simple realizable input transformation like $u = B_r(\chi)^{-1}v$ can be used to decouple the system, and the SISO backstepping adaptive design can thus be straightforwardly applied to each input–output pair to solve the problem.

Remark 7. For SISO systems in the parametric-strict feedback form (i.e., assuming $m_i = m = 1$, and $\Delta_i = 0$), the case of B_i being an unknown positive scalar b_i is also studied in Krstic et al. (1995). However, over-parametrization about b_i is used—two parameter estimates are needed for one b_i. In addition, b_i is assumed to be different from the unknown parameter set θ. In contrast, our subsequent designs do not need over-parametrization for unknown parameters in the input matrices, the unknown parameters in the input matrices do not have to be different from θ, and the input gain function B_i does not have to be a constant scalar.

Remark 8. In (12), in the absence of uncertain nonlinearities, in viewing the Assumption A3.3, the state equations of the first $(r-1)$ subsystems can be linearly parametrized by the unknown parameter vector θ. However, the state equations of the last subsystem may not be linearly parametrized by any set of unknown parameters because of the appearance of $M^{-1}(x_I, \beta)$ in the state equations. The reason is that, although M is assumed to be linearly parametrized in terms of β in A3.3, in general, M^{-1} may not be linearly parametrized. Introducing M greatly expands the applicability of the method since, as shown in Yao (1996), most mechanical systems, including robot manipulators, satisfy (12) but not the usual strict feedback forms (Krstic et al., 1995) where linearly parametrizing state equations is required.

Remark 9. In Eq. (12), the output vector y is partitioned into r blocks, and the outputs of the ith block, y_{ib} (empty if $m_i = m_{i-1}$), have a "relative degree"[7] of $r - i + 1$. In this way, we can have relative degrees ranging from 1 to r and solve the problem that different outputs of a MIMO system may have different relative degrees.

3.2. MIMO semi-strict feedback form II

In the above MIMO semi-strict-feedback form, the output y is partitioned into r blocks to generate different "relative degrees". For some applications, it may be more natural to partition the input u into r blocks, which is shown in Fig. 2 and described by

$$\begin{aligned}
\dot{x}_i &= f_{i0}(\bar{\chi}_i) + F_i(\bar{\chi}_i)\theta + B_i(\bar{\chi}_i,\theta)\begin{bmatrix} u_{ib} \\ x_{i+1} \end{bmatrix} \\
&\quad + D_i(\bar{\chi}_i)\Delta_i(\chi,\theta,t), \\
\dot{\eta}_i &= \Phi_{i0}(\bar{\chi}_i) + \Phi_{i1}(\bar{\chi}_i)\theta, \quad 1 \leqslant i \leqslant r-1, \\
\dot{x}_r &= M^{-1}(\bar{\chi}_{r-1},\beta)[f_{r0} + F_r\theta + F_\beta(\chi)\beta \\
&\quad + B_r(\chi,\theta,\beta)u_{rb}D_r + \Delta_r], \\
\dot{\eta}_r &= \Phi_r(\chi,\theta,\beta), y = x_1 \in R^m \quad \text{and} \quad u = [u_{1b}^{\mathrm{T}},\dots,u_{rb}^{\mathrm{T}}]^{\mathrm{T}},
\end{aligned} \tag{14}$$

in which the ith and rth subsystems have the same form as those in Section 3.1 but with $m = m_1 \geqslant m_2 \geqslant \cdots \geqslant m_r \geqslant 0$, u is partitioned into r blocks with $u_{ib} \in R^{m_i - m_{i+1}}$, and the output of the entire system is the output of the first subsystem. The same assumptions as in Section 3.1 are made. Details are given in Yao (1996) and omitted.

[7] The notion of "relative degree" should not be understood in the usual sense (Isidori, 1989) since Δ_i are uncertain and may depend on the control input. The notion should be interpreted in the way introduced in Yao and Tomizuka (1997), which is more natural.

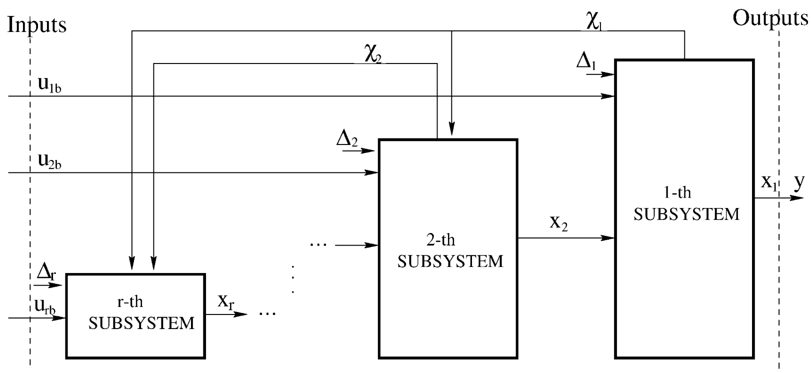

Fig. 2. MIMO semi-strict feedback form II.

Remark 10. Due to the complexity associated with the coupled uncertain MIMO nonlinear systems, even the presentation of the system in a meaningful manner becomes difficult. The proposed two semi-strict feedback forms, (12) and (14), are quite natural and are in fact motivated by various practically significant applications. One practical application of form (12) is the motion and force tracking control in Yao and Tomizuka (1998), and a practical application of the form (14) will be a multi-axes mechanical device (e.g., machine tools) driven by different types of actuators.

4. Backstepping designs via ARC Lyapunov functions

To systematically construct ARC Lyapunov functions for the two semi-strict feedback forms, we use the backstepping design procedure (Krstic et al., 1995). Namely, we assume that an ARC Lyapunov function is known for an initial system and construct a new ARC Lyapunov function for an augmented system by adding another nonlinear system to the back of the initial system. The main design difficulties here are due to the strong coupling and the appearance of parametric uncertainties in the input matrices. Two backstepping designs are presented in this section. The results are self-contained and independent from the semi-strict feedback forms.

4.1. Initial MIMO nonlinear systems

Consider the following initial MIMO system:

$$\dot{x}_I = f_{I0}(x_I) + F_{I\theta}(x_I)\theta + B_I(x_I,\theta)u_I + D_I(x_I)\Delta_I,$$

$$y_I = h_I(x_I), \quad u_I, y_I \in R^{m_I},\ x_I \in R^{n_I},\ F_I \in R^{n_I \times p}, \tag{15}$$

in which Δ_I is assumed to be bounded by a known function δ_I, i.e., $\|\Delta_I\| \leqslant \delta_I(x_I)$. In addition, as in Assumption A3.3, $B_I(x_I,\theta) = B_{I0}(x_I) + B_{I\theta}(x_I,\theta)$ where $B_{I\theta}(x_I,\theta)$ is linear w.r.t. θ. The following notations will be used throughout the paper. For a system matrix $\bullet$, $\hat{\bullet}$ is obtained by substituting the projected parameter estimates for the unknown parameters in $\bullet$ (e.g., $\hat{B}_I = B_I(x_I,\hat{\theta}_\pi)$). $\tilde{\bullet}$ refers to the estimation error of $\bullet$, i.e., $\tilde{\bullet} = \hat{\bullet} - \bullet$. Since $B_{I\theta}$ is linear w.r.t. θ, there exist known matrices $G_{Ir}(x_I,\bullet)$ and $G_{Il}(x_I,\bullet)$, which are linear w.r.t. $\bullet$, such that

$$B_{I\theta}(x_I,\theta)v_r = G_{Ir}(x_I,v_r)\theta, \quad \forall v_r \in R^{m_I},$$

$$v_l^{\mathrm{T}} B_{I\theta}(x_I,\theta) = \theta^{\mathrm{T}} G_{Il}(x_I,v_l), \quad \forall v_l \in R^{n_I}, \tag{16}$$

G_{Ir} and G_{Il} are called the right and the left substitution matrices of $B_{I\theta}$ w.r.t. θ, respectively.

Since our intention is to use backstepping design procedure (Krstic et al., 1995), we assume that there exists an ARC Lyapunov function $V_I(x_I,\overline{\theta}_\pi^{(l_I)},t)$ for system (15) with an associated ARC control law and an adaptation function given by $u_I = \alpha_I = \alpha_{Ia}(x_I,\overline{\theta}_\pi^{(k_I)},t) + \alpha_{Is}(x_I,\overline{\theta}_\pi^{(k_I)},t)$ and $\tau_I(x_I,\overline{\theta}_\pi^{(k_I)},\alpha_I,t)$, respectively. By definition, V_I satisfies Requirements R1–R3, in which Requirements R2 (4) and R3 (6) are rewritten as

$$\text{B1:}\quad \frac{\partial V_I}{\partial x_I}[f_{I0} + F_{I\theta}\theta + B_I\alpha_I] + \frac{\partial V_I}{\partial t} \leqslant -W_I + \tilde{\theta}_\pi^{\mathrm{T}}\tau_I + \frac{\partial V_I}{\partial \hat{\theta}}\Gamma\tau_I,$$

$$\text{B2:}\quad \frac{\partial V_I}{\partial x_I}[f_{I0} + F_{I\theta}\theta + B_I\alpha_I + D_I\Delta_I] + \frac{\partial V_I}{\partial t} \leqslant -\lambda_{V_I}V_I + c_{V_I} + \frac{\partial V_I}{\partial \hat{\theta}}\Gamma\tau_I, \tag{17}$$

where λ_{V_I} and c_{V_I} can be freely adjusted by some controller parameters in a known form. For convenience, denote

$$DV_I = \frac{\partial V_I}{\partial x_I}[f_{I0} + F_{I\theta}\theta + B_I\alpha_I + D_I\Delta_I] + \frac{\partial V_I}{\partial t} - \frac{\partial V_I}{\partial \hat{\theta}}\Gamma\tau_I. \tag{18}$$

4.2. Augmented MIMO nonlinear systems I

Now consider the following augmented system with the state vector $x = [x_I^T, x_e^T, \eta^T]^T \in R^n$ and $n = n_I + m + n_\eta$

$$\dot{x}_I = f_{I0} + F_{I\theta}\theta + B_I \bar{x}_{em_I} + D_I \Delta_I,$$
$$\dot{x}_e = f_{e0}(x) + F_{e\theta}(x)\theta + B_e(x,\theta)u + D_e\Delta_e, \quad x_e \in R^m,$$
$$\dot{\eta} = \Phi_{\eta 0}(x) + \Phi_{\eta\theta}(x)\theta, \quad \eta \in R^{n_\eta},$$
$$y = [\, y_I^T, (N_e^T x_e)^T]^T, \tag{19}$$

where $\bar{x}_{em_I} = U_e x_e$ with $U_e = [I_{m_I}\ 0] \in R^{m_I \times m}$, $u \in R^m$, $y \in R^m$, and $N_e = [0\ I_{m-m_I}]^T \in R^{m \times (m-m_I)}$. Note that (19) can be rewritten in the standard form (1) where $f = [(f_{I0} + F_{I\theta}\theta + B_I \bar{x}_{em_I})^T, (f_{e0} + F_{e\theta}\theta)^T, (\Phi_{\eta 0} + \Phi_{\eta\theta}\theta)^T]^T$, $B = [0, B_e^T, 0]^T$, and $\Delta = [\Delta_I^T, \Delta_e^T]^T$, respectively.

As in Assumptions A3.1–5, the following assumptions are made for the added system:

A4.1. $B_e(x,\theta)$ is nonsingular for any $\theta \in \Omega_\theta$ and can be linearly parametrized by θ, i.e., $B_e = B_{e0} + B_{e\theta}(x,\theta)$ where $B_{e\theta}$ is linear w.r.t. θ.
A4.2. The η-subsystem is bounded-input bounded-state (BIBS) stable w.r.t. the input (x_I, x_e).
A4.3. Uncertain nonlinearities Δ_e are bounded by $\|\Delta_e\| \leqslant \delta_e(x)$ where $\delta_e(x)$ is known.

For a guaranteed output tracking transient performance, the following compatibility assumption is also made:

A4.4. The initial output of the added system is compatible with the required initial ARC input of the original system, i.e., $\bar{x}_{em_I}(0) = \alpha_I(0)$.

4.3. Backstepping design I

In this subsection, using backstepping design, an ARC Lyapunov function is constructed for the augmented system (19) based on the ARC Lyapunov function V_I for the initial system (15). Since α_I is an ARC control law for the initial system (15) and the inputs of (15) are $\bar{x}_{em_I}$, the key point is to design an ARC control law for the second equation of system (19) such that $\bar{x}_{em_I}$ tracks α_I and other outputs track their desired trajectories with a guaranteed transient performance. To this end, define the output tracking error $z \in R^m$ and V as

$$z(x, \bar{\theta}_\pi^{(k_I)}, t) = x_e - \bar{\alpha}(x_I, \bar{\theta}_\pi^{(k_I)}, t),$$
$$\bar{\alpha} \triangleq [\alpha_I^T, (N_e^T y_d(t))^T]^T, \tag{20}$$
$$V(x, \bar{\theta}_\pi^{(l_V)}, t) = V_I(x_I, \bar{\theta}_\pi^{(l_I)}, t) + \tfrac{1}{2} z^T E z,$$

where E is any s.p.d. matrix and $l_V = \max\{l_I, k_I\}$. The following three lemmas show that V given by (20) satisfies the three requirements that are needed for V to be an ARC Lyapunov function for (19). The proofs are given in Appendix C.

Lemma 11. *V in (20) satisfies Requirement R1 for system (19).*

Lemma 12. *Define L, α_{ea}, and ϕ as*

$$L(x, \bar{\theta}_\pi^{(l_\alpha)}, t) = \hat{B}_e + B_{e\theta}\left(x, \Gamma\left(\frac{\partial V_I}{\partial \hat{\theta}}\right)^T\right) - U_e^T E_B,$$

$$\alpha_{ea}(x, \bar{\theta}_\pi^{(l_\alpha)}, t) = \alpha_{e0} - \phi_0 \hat{\theta}_\pi - \phi_0 \Gamma\left(\frac{\partial V_I}{\partial \hat{\theta}}\right)^T - U_e^T \frac{\partial \alpha_I}{\partial \hat{\theta}} \Gamma(\tau_I - \phi_0^T E z),$$

$$\phi(x, \bar{\theta}_\pi^{(l_V)}, u, t) = \phi_0 + G_{er}(x,u), \tag{21}$$

where $l_\alpha = \max\{l_V, k_I + 1\}$,

$$E_B = \begin{bmatrix} z^T E B_{e\theta}\left(x, \Gamma\left(\frac{\partial \alpha_{I1}}{\partial \hat{\theta}}\right)^T\right) \\ \vdots \\ z^T E B_{e\theta}\left(x, \Gamma\left(\frac{\partial \alpha_{Im_I}}{\partial \hat{\theta}}\right)^T\right) \end{bmatrix},$$

$$\alpha_{e0} = -E^{-1}\left(\frac{\partial V_I}{\partial x_I} B_{I0} U_e\right)^T + U_e^T \frac{\partial \alpha_I}{\partial x_I}[f_{I0} + B_{I0}\bar{x}_{em_I}] - f_{e0} + \frac{\partial \bar{\alpha}}{\partial t},$$

$$\phi_0 = E^{-1} U_e^T G_{Il}^T\left(x_I, \left(\frac{\partial V_I}{\partial x_I}\right)^T\right) - U_e^T \frac{\partial \alpha_I}{\partial x_I}[F_{I\theta} + G_{Ir}(x_I, \bar{x}_{em_I})] + F_{e\theta} \tag{22}$$

and G_{er} is the right substitution matrix of $B_{e\theta}$. If L is nonsingular, by letting $\Omega_u = \{u_s\colon z^T E L u_s \leqslant 0\}$ and choosing u_a as

$$u_a(x, \bar{\theta}_\pi^{(l_\alpha)}, t) = L^{-1}[-E^{-1}Qz + \alpha_{ea}], \tag{23}$$

where Q is any s.p.d. matrix, Requirement R2 (5) is satisfied by V for the system (19) with an adaptation function given by

$$\tau(x, \bar{\theta}_\pi^{(l_V)}, u, t) = \tau_I - \phi^T E z. \tag{24}$$

Lemma 13. *If $u_s \in \Omega_u$ is chosen such that*

$$z^T[E(L - \tilde{B}_{e\theta})u_s + \alpha_{es}] \leqslant \varepsilon_e(t), \tag{25}$$

where ε_e is a design parameter and

$$\alpha_{es} = E\left[-\phi(x, \bar{\theta}_\pi^{(l_V)}, u_a, t)\tilde{\theta}_\pi - U_e^T \frac{\partial \alpha_I}{\partial x_I} D_I \Delta_I + D_e \Delta_e\right], \tag{26}$$

then, Requirement R3 (6) is satisfied by V for system (19) with $\lambda_V = \min\{\lambda_{V_I}, 2\lambda_{\min}(Q)/\lambda_{\max}(E)\}$ and $c_V = c_{V_I} + \varepsilon_e$.

Remark 14. One solution to (25) can be found in the following way. Let h be a function satisfying

$$h(x,\bar{\theta}_\pi^{(l_x)},t) \geqslant \sup_{\theta\in\Omega_\theta,\,\Delta\in\Omega_\Delta} \|\alpha_{es}(x,\bar{\theta}_\pi^{(l_x)},u_a,t)\|. \tag{27}$$

For example, let

$$h \geqslant \theta_M\|E\phi(x,\bar{\theta}_\pi^{(l_V)},u_a,t)\| + \left\|EU_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial x_I}D_I\right\|\delta_I + \|ED_e\|\delta_e, \tag{28}$$

where $\theta_M = \|\theta_{\max} - \theta_{\min} + \varepsilon_\theta\|$. Let ρ_u be a positive scalar satisfying

$$\rho_u(x,\bar{\theta}_\pi^{(l_x)},t) \geqslant \sup_{\theta\in\Omega_\theta} \|EB_{e\theta}(x,\tilde{\theta}_\pi,t)L^{-1}E^{-1}\|, \tag{29}$$

which may not be difficult to calculate since $B_{e\theta}$ is linear w.r.t. $\tilde{\theta}_\pi$. We assume that $\rho_u < 1$.[8] Then, by choosing

$$u_s(x,\bar{\theta}_\pi^{(l_x)},t) = -\frac{1}{4(1-\rho_u)\varepsilon_e}h^2L^{-1}E^{-1}z \tag{30}$$

from (28) and (29), we have

$$\begin{aligned}\text{LS of (25)} &\leqslant \frac{1}{4(1-\rho_u)\varepsilon_e}h^2(-\|z\|^2 + \|z\|\,\|EB_{e\theta}\\ &\quad\times(x,\tilde{\theta}_\pi,t)L^{-1}E^{-1}\|\,\|z\|) + \|z\|\,\|\alpha_{es}\|\\ &\leqslant -\left(\frac{1}{2\sqrt{\varepsilon_e}}h\|z\| - \sqrt{\varepsilon_e}\right)^2 + \varepsilon_e \leqslant \varepsilon_e.\end{aligned} \tag{31}$$

Thus, (25) is satisfied.

Lemmas 11–13 lead to the following theorem:

Theorem 2. *If L is nonsingular and* (25) *is satisfied, V defined by* (20) *is an* ARC *Lyapunov function for the augmented system* (19) *with control functions u_a given by* (23) *and u_s determined from* (25), *and an adaptation function τ given by* (24).

Remark 15. Since $\Omega_{\hat\theta}$ can be chosen arbitrarily close to Ω_θ and B_e is nonsingular for any $\theta\in\Omega_\theta$, $\hat{B}_e$ is nonsingular. Noting that the last two terms of L in (21) are linear w.r.t. Γ, nonsingularity of L can thus be guaranteed by using a small adaptation rate Γ.

4.4. Augmented MIMO nonlinear systems II

In the previous subsection, we constructed an ARC Lyapunov function for the augmented system (19). The state equations of the added system have the same form as those in the first $(r-1)$ subsystem in (12), which are required to be linearly parametrized by the unknown parameter vector θ when $\Delta_e = 0$. In this subsection, the added system will take the form of the rth subsystem in (12) so that the results will be applicable for most mechanical systems as mentioned in Remark 8. The main difficulty here is that the state equations of the added system may not be linearly parametrized by any group set of unknown parameters. To solve this problem, the idea of using an energy-like Lyapunov function, which is originally developed in the control of robot manipulators (Slotine, 1985), is adopted as follows.

Let us consider the following augmented system:

$$\begin{aligned}\dot{x}_I &= f_{I0}(x_I) + F_{I\theta}(x_I)\theta + B_I(x_I,\theta)\bar{x}_{em_I} + D_I(x_I)\Delta_I,\\ \dot{x}_e &= M^{-1}(x_I,\beta)[f_{e0} + F_{e\theta}\theta + F_\beta\beta + B_e(x,\theta,\beta)u + D_e\Delta_e],\\ \dot{\eta} &= \Phi_\eta(x,\theta,\beta,t), \quad \eta\in R^{n_\eta},\\ y &= [y_I^{\mathrm{T}},(N_e^{\mathrm{T}}x_e)^{\mathrm{T}}]^{\mathrm{T}}.\end{aligned} \tag{32}$$

In viewing Assumptions A3.1–5, Δ_e satisfies Assumption A4.3, the η-subsystem satisfies Assumption A4.2, and

A4.5. B_e is nonsingular and $B_e(x,\theta,\beta) = B_{e0}(x) + B_{e\theta}(x,\theta) + B_{e\beta}(x,\beta)$ where $B_{e\theta}(x,\theta)$ and $B_{e\beta}(x,\beta)$ are linear w.r.t. θ and β, respectively.
A4.6. M is an s.p.d. matrix and there exist positive scalars k_m and k_M such that $k_mI_m \leqslant M \leqslant k_MI_m$.
A4.7. $M(x_I,\beta) = M_0(x_I) + M_\beta(x_I,\beta)$ in which M_β is linear w.r.t. β.

We assume that the compatibility assumption A4.4 is satisfied and proceed to construct an ARC Lyapunov function for (32) as in Section 4.3. Define z as in (20) and V as

$$V(x,\bar{\theta}_\pi^{(l_V)},\beta,t) = V_I(x_I,\bar{\theta}_\pi^{(l_I)},t) + \tfrac{1}{2}z^{\mathrm{T}}M(x_I,\beta)z. \tag{33}$$

Similar to (16), let $G_{Mr}(x,\bullet)$ and $G_{Ml}(x,\bullet)$ denote the right and left substitution matrices of the matrix $M_\beta(x,\beta)$ in terms of β. Let $G_{\theta r}(x,\bullet)$ and $G_{\theta l}(x,\bullet)$ denote the right and left substitution matrices of the matrix $B_{e\theta}(x,\theta)$ (in terms of θ) respectively, and $G_{\beta r}(x,\bullet)$ and $G_{\beta l}(x,\bullet)$ for $B_{e\beta}(x,\beta)$ (in terms of β).

From (32), each component of $\dot{M}$ has the following form:

$$\begin{aligned}\dot{M}_{ij} &= \frac{\partial M_{ij}}{\partial x_I}[f_{I0} + F_{I\theta}\theta + B_{I0}\bar{x}_{em_I} + G_{Ir}(x_I,\bar{x}_{em_I})\theta]\\ &\quad + \frac{\partial M_{ij}}{\partial x_I}D_I\Delta_I + \frac{\partial M_{ij}}{\partial t}.\end{aligned} \tag{34}$$

Since $M(x_I,\beta)$ can be linearly parametrized in terms of β, so do $\partial M_{ij}/\partial x_I$ and $\partial M_{ij}/\partial t$. Therefore, when $\Delta_I = 0$, from (34), $\dot{M}(x_I,\beta)$ can be linearly parametrized in terms of the augmented parameter set $\theta_e = [\theta^{\mathrm{T}},\beta^{\mathrm{T}},\vartheta^{\mathrm{T}}]^{\mathrm{T}}$, where $\vartheta = [\beta_1\theta^{\mathrm{T}},\beta_2\theta^{\mathrm{T}},\ldots,\beta_{l_\beta}\theta^{\mathrm{T}}]^{\mathrm{T}}\in R^{l_\beta p}$. Thus, there

[8] In the absence of input channel parametric uncertainties (i.e., $B_{e\theta}=0$), $\rho_u = 0$. Therefore, as long as the input channel uncertainties are not so large, the assumption that $\rho_u < 1$ can be satisfied.

exist known vector $d_M(x_I,\bullet)$ and matrix $D_M(x_I,\bullet) = [D_{M\theta}, D_{M\beta}, D_{M\vartheta}]$ such that, $\forall v \in R^m$,

$$\tfrac{1}{2}v^{\mathrm{T}}\dot{M}(x_I,\beta)v = v^{\mathrm{T}}[d_M(x_I,v) + D_M(x_I,v)\theta_e + \tilde{\Delta}_M], \quad (35)$$

where $\tilde{\Delta}_M$ linearly depends on Δ_I and can be bounded by a known function $\delta_M(x,v)$, i.e., $\|\tilde{\Delta}_M\| \leqslant \delta_M$. Since M_β is linear w.r.t. β, there exists a known matrix $D_{p\vartheta}(x)$ such that

$$M_\beta(x_I,\beta)U_e^{\mathrm{T}}[F_{I\theta}\theta + B_{I\theta}(x_I,\theta)\bar{x}_{em_I}] = D_{p\vartheta}(x)\vartheta. \quad (36)$$

Since $\theta \in \Omega_\theta$ and $\beta \in \Omega_\beta$, we have $\vartheta \in \Omega_\vartheta$, where Ω_ϑ is a known bounded set having the form $\Omega_\vartheta = \{\vartheta: \vartheta_{\min} < \vartheta < \vartheta_{\max}\}$. Thus, we can define $\hat{\vartheta}_\pi = \pi_\vartheta(\hat{\vartheta})$, the projection of $\hat{\vartheta}$, in the same way as $\hat{\theta}_\pi$ for $\hat{\theta}$. Let θ_e represent the augmented unknown parameter vector (i.e., $\theta_e = [\theta^{\mathrm{T}}, \beta^{\mathrm{T}}, \vartheta^{\mathrm{T}}]^{\mathrm{T}}$), and $\hat{\theta}_{e\pi}$ be the projection of its estimate. Corresponding to Lemmas 11–13, we have the following three lemmas. The proofs of the three lemmas are given in Appendix D.

Lemma 16. *V in (33) satisfies Requirement R1 for system (32).*

Lemma 17. *Define*

$$Z_B(x,\bar{\theta}_\pi^{(l_z)},t) = \begin{bmatrix} z^{\mathrm{T}}B_{e\theta}\left(x,\Gamma\left(\frac{\partial\alpha_{I1}}{\partial\hat{\theta}}\right)^{\mathrm{T}}\right) \\ \vdots \\ z^{\mathrm{T}}B_{e\theta}\left(x,\Gamma\left(\frac{\partial\alpha_{Im_I}}{\partial\hat{\theta}}\right)^{\mathrm{T}}\right) \end{bmatrix},$$

$$\alpha_{e0} = -\left(\frac{\partial V_I}{\partial x_I}B_{I0}U_e\right)^{\mathrm{T}} - f_{e0} + M_0\frac{\partial\bar{\alpha}}{\partial t} + M_0U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial x_I}[f_{I0} + B_{I0}\bar{x}_{em_I}] - d_M(x_I,z),$$

$$\phi_{\theta0} = U_e^{\mathrm{T}}G_{Il}^{\mathrm{T}}\left(x_I,\left(\frac{\partial V_I}{\partial x_I}\right)^{\mathrm{T}}\right) + F_{e\theta} + D_{M\theta}(x_I,z) - M_0U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial x_I}[F_{I\theta} + G_{Ir}(x_I,\bar{x}_{em_I})],$$

$$\phi_{\beta0} = F_\beta - G_{Mr}\left(x_I,\left(U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial x_I}[f_{I0} + B_{I0}\bar{x}_{em_I}] + \frac{\partial\bar{\alpha}}{\partial t}\right)\right) + D_{M\beta}(x_I,z),$$

$$\phi_{\vartheta0}(x,\bar{\theta}_\pi^{(k_I)},t) = D_{M\vartheta}(x_I,z) - D_{p\vartheta}(x). \quad (37)$$

Let L, α_{ea}, ϕ_θ, and ϕ_β be

$$L(x,\bar{\theta}_\pi^{(l_z)},\hat{\beta}_\pi,t) = \hat{B}_e + B_{e\theta}\left(x,\Gamma\left(\frac{\partial V_I}{\partial\hat{\theta}}\right)^{\mathrm{T}}\right) - \hat{M}U_e^{\mathrm{T}}Z_B,$$

$$\alpha_{ea} = \alpha_{e0} - \hat{M}U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial\hat{\theta}}\Gamma[\tau_I(x,\bar{\theta}_\pi^{(k_I)},\alpha_I,t) - \phi_{\theta0}^{\mathrm{T}}z] - \phi_{\theta0}\Gamma\left(\frac{\partial V_I}{\partial\hat{\theta}}\right)^{\mathrm{T}} - \phi_{e0}\hat{\theta}_{e\pi},$$

$$\phi_\theta(x,\bar{\theta}_\pi^{(l_V)},u,t) = \phi_{\theta0} + G_{\theta r}(x,u),$$

$$\phi_\beta(x,\bar{\theta}_\pi^{(l_V)},u,t) = \phi_{\beta0} + G_{Mr}\left(x_I,\left(U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial\hat{\theta}}\Gamma\tau_\theta(x,\bar{\theta}_\pi^{(l_V)},u,t)\right)\right) + G_{\beta r}(x,u), \quad (38)$$

where $\phi_{e0} = [\phi_{\theta0}^{\mathrm{T}}, \phi_{\beta0}^{\mathrm{T}}, \phi_{\vartheta0}^{\mathrm{T}}]^{\mathrm{T}}$. Then, if L is nonsingular, by letting $\Omega_u = \{u_s: z^{\mathrm{T}}Lu_s \leqslant 0\}$ and choosing u_a as

$$u_a(x,\bar{\theta}_\pi^{(l_z)},\hat{\beta}_\pi,\hat{\vartheta}_\pi,t) = L^{-1}[-Qz + \alpha_{ea}(x,\bar{\theta}_\pi^{(l_z)},\hat{\beta}_\pi,\hat{\vartheta}_\pi,t)], \quad (39)$$

V of (33) satisfies (4) of Requirement R2 for system (32) in terms of θ_e with an adaptation function $\tau_e(x,\bar{\theta}_\pi^{(l_V)},u,t) = [\tau_\theta^{\mathrm{T}}, \tau_\beta^{\mathrm{T}}, \tau_\vartheta^{\mathrm{T}}]^{\mathrm{T}}$ given by

$$\tau_\theta(x,\bar{\theta}_\pi^{(l_V)},u,t) = \tau_I(x,\bar{\theta}_\pi^{(k_I)},\alpha_I,t) - \phi_\theta^{\mathrm{T}}(x,\bar{\theta}_\pi^{(l_V)},u,t)z,$$

$$\tau_\beta(x,\bar{\theta}_\pi^{(l_V)},u,t) = -\phi_\beta^{\mathrm{T}}(x,\bar{\theta}_\pi^{(l_V)},u,t)z,$$

$$\tau_\vartheta(x,\bar{\theta}_\pi^{(k_I)},t) = -\phi_{\vartheta0}^{\mathrm{T}}z. \quad (40)$$

Lemma 18. *If $u_s \in \Omega_u$ is chosen such that*

$$z^{\mathrm{T}}[(L - \tilde{L}_u)u_s + \alpha_{es}] \leqslant \varepsilon_e(t), \quad (41)$$

where

$$\tilde{L}_u = B_{e\theta}(x,\tilde{\theta}_\pi) + B_{e\beta}(x,\tilde{\beta}_\pi) - M_\beta(x,\tilde{\beta}_\pi)U_e^{\mathrm{T}}Z_B,$$

$$\alpha_{es} = -\phi_e(x,\bar{\theta}_\pi^{(l_V)},u_a,t)\tilde{\theta}_{e\pi} + \tilde{\Delta}, \quad (42)$$

then, (6) of Requirement R3 is satisfied by V for (32) with $\lambda_V = \min\{\lambda_{V_I}, 2\lambda_{\min}(Q)/k_M\}$ and $c_V = c_{V_I} + \varepsilon_e$.

Remark 19. One solution to (41) can be found in the same way as in Remark 14 except that h and ρ_u are required to satisfy

$$h \geqslant \theta_{eM}\|\phi_e(x,\bar{\theta}_\pi^{(l_V)},u_a,t)\| + k_M\left\|U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial x_I}D_I\right\|\delta_I + \|D_e\|\delta_e + \delta_M(x,z),$$

$$\rho_u(x,\bar{\theta}_\pi^{(l_z)},t) \geqslant \sup_{\theta \in \Omega_\theta}\|\tilde{L}_uL^{-1}(x,\bar{\theta}_\pi^{(l_z)},t) \quad (43)$$

where $\theta_{eM} = \|\theta_{e\max} - \theta_{e\min} + \varepsilon_{\theta e}\|$.

Lemmas 16–18 lead to the following theorem:

Theorem 3. *If L of (38) is nonsingular and (41) is satisfied, V defined by (33) is an ARC Lyapunov function for the* *augmented system* (32) *with control functions* u_a *and* u_s *from* (39) *and* (41), *respectively*, *and an adaptation function* τ_e *given by* (40) *in terms of the augmented parameter vector* θ_e.

Remark 20. Although the state equations of system (38) cannot be linearly parametrized, by introducing M as a weighting matrix in forming V (33) as in the control of robot manipulators (Slotine, 1985), $\dot{V}$ can be linearly reparametrized, which makes the utilization of adaptive control possible.

5. ARC of systems in MIMO semi-strict feedback forms

In this section, the two backstepping designs presented in Section 4 will be applied recursively to solve the ARC of nonlinear systems transformable to the two semi-strict feedback forms in Section 3.

5.1. ARC for MIMO semi-strict feedback form I

To recursively apply the two backstepping designs presented in Section 4 to solve the ARC problem for system (12), it is necessary to make sure all assumptions in Section 4 are met. As shown below, all assumptions are straightforwardly met except the compatibility Assumption A4.4 on the connections. To overcome this problem, an additional tool, trajectory initialization Krstic et al., 1995, will be used. Namely, instead of tracking the desired outputs $y_d(t) = [y_{d1b}^T, \ldots, y_{drb}^T]^T$ directly, the controller will be designed to track the filtered outputs $y_t(t) = [y_{t1b}^T, \ldots, y_{trb}^T]^T$, in which the ith block of outputs, $y_{tib} \in R^{m_i - m_{i-1}}$, are generated by the $(r-i+1)$th order stable system having the form of $(y_{tib}^{(r-i+1)} - y_{dib}^{(r-i+1)}) + \cdots + \beta_{i(r-i+1)}(y_{tib} - y_{dib}) = 0$. By doing so, the initial conditions $y_{tib}(0), \ldots, y_{tib}^{(r-i)}(0)$ are free to choose and will be specified in the following to make sure that the compatibility assumptions are met. To accomplish this task, it is necessary to know the explicit dependence of the control law on the filtered desired trajectory y_t and its derivatives. Therefore, in the following, these known time functions will appear as independent variables in the expression of the control law as opposed to appearing implicitly in the control law as in Section 4.

The following notations are used. Similar to (16), $G_{ir}(\bar{\chi}_i, \bullet)$ and $G_{il}(\bar{\chi}_i, \bullet)$ denote the right and the left substitution matrices of the matrix $B_{i\theta}(\bar{\chi}_i, \theta)$, $G_{\theta r}(\chi, \bullet)$ and $G_{\theta l}(\chi, \bullet)$ for the matrix $B_{r\theta}(\chi, \theta)$, $G_{\beta r}(\chi, \bullet)$ and $G_{\beta l}(\chi, \bullet)$ for the matrix $B_{r\beta}(\chi, \beta)$, and $G_{Mr}(\bar{\chi}_{r-1}, \bullet)$ and $G_{Ml}(\bar{\chi}_{r-1}, \bullet)$ for the matrix $M_\beta(\bar{\chi}_{r-1}, \beta)$. $\forall k$, define $\bar{y}_{tib}^{(k)}$ as $\bar{y}_{tib}^{(k)} = [y_{tib}^T, \ldots, (y_{tib}^{(k)})^T]^T$. Recursively define y_{ju} by $y_{1u} = y_{t1b}, \ldots, y_{ju} = [y_{j-1u}^{(1)T}, y_{tjb}^T]^T = [y_{t1b}^{(j-1)T}, \ldots, y_{tjb}^T]^T \in R^{m_j}$, and L_{ju} by $L_{1u} = L_1, \ldots, L_{ju} = [U_j^T L_{j-1u} N_j] L_j$. The design proceeds in the following steps:

5.1.1. Step 1

The first system is defined as the first subsystem of (12), the first two equations of (12) for $i = 1$, whose inputs are $\bar{x}_{2,m_1}$ and outputs are $\bar{y}_1 = y_{1b} = x_1$. Noting Assumptions A3.1–5, the first system can be considered as a special case of the augmented MIMO nonlinear system I (9) with $m_I = 0$. $x_1, \eta_1, \bar{x}_{2,m_1}, f_{10}, F_1, B_1, D_1$, and Δ_1 in (12) correspond to $x_e, \eta, u, f_{e0}, F_e, B_e, D_e$, and Δ_e in (19), respectively. Therefore, the backstepping design results in Section 4.3 can be used to construct an ARC Lyapunov function V_1 for the first system. V_1 is given by $V_1(x_1, y_{t1b}) = \frac{1}{2} z_1^T E_1 z_1$, where $z_1(x_1, y_{t1b}) = x_1 - \alpha_0(y_{t1b})$ and $\alpha_0(y_{t1b}) = y_{t1b}(t)$. An ARC control function can be obtained through Eqs. (23) and (25). The detailed expressions of the resulting control function $\alpha_1(\chi_1, \hat{\theta}_\pi, \bar{y}_{t1b}^{(1)}, t)$ and the adaptation function $\tau_1(\chi_1, \hat{\theta}_\pi, \bar{y}_{t1b}^{(1)}, t)$ are given in Yao (1996) and omitted.

5.1.2. Step i

$\forall i \leqslant r-1$, the control function $\alpha_i(\bar{\chi}_i, \bar{\theta}_\pi^{(i)}, \bar{y}_{t1b}^{(i)}, \ldots, \bar{y}_{tib}^{(1)}, t)$ and adaptation function $\tau_i(\bar{\chi}_i, \bar{\theta}_\pi^{(i)}, \bar{y}_{t1b}^{(i)}, \ldots, \bar{y}_{tib}^{(1)}, t)$ are calculated based on the control functions α_j and adaptation functions τ_j of all previous steps as

$$
\begin{aligned}
\alpha_i &= \alpha_{ia} + \alpha_{is},\\
\alpha_{ia} &= L_i^{-1}\Bigg\{ -E_i^{-1} Q_i z_i - E_i^{-1} U_i^T B_{i-10}^T E_{i-1} z_{i-1}\\
&\quad + U_i^T \sum_{j=1}^{i-1} \left[\frac{\partial \alpha_{i-1}}{\partial x_j}(f_{j0} + B_{j0}\bar{x}_{j+1,m_j}) + \frac{\partial \alpha_{i-1}}{\partial \eta_j}\Phi_{j0} \right]\\
&\quad + U_i^T \left[\sum_{j=1}^{i-1} \sum_{k=0}^{i-j} \frac{\partial \alpha_{i-1}}{\partial y_{tjb}^{(k)}} y_{tjb}^{(k+1)} + \frac{\partial \alpha_{i-1}}{\partial t} \right]\\
&\quad + N_i y_{tib}^{(1)} - f_{i0}\\
&\quad - \phi_{i0}\left[\hat{\theta}_\pi - \Gamma \sum_{j=1}^{i-2} \left(\frac{\partial \alpha_j}{\partial \hat{\theta}} \right)^T U_{j+1} E_{j+1} z_{j+1} \right]\\
&\quad - U_i^T \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma (\tau_{i-1} - \phi_{i0}^T E_i z_i) \Bigg\},\\
\alpha_{is} &= -\frac{1}{4(1-\rho_{ui})\varepsilon_{ei}} h_i^2 L_i^{-1} E_i^{-1} z_i,\\
\tau_i &= \tau_{i-1} - (\phi_{i0} + G_{ir}(\bar{\chi}_i, \alpha_i))^T E_i z_i,
\end{aligned} \tag{44}
$$

where

$$
\begin{aligned}
z_i &= x_i - \bar{\alpha}_{i-1}, \quad \bar{\alpha}_{i-1} = [\alpha_{i-1}^T, y_{tib}^T]^T,\\
\phi_{i0} &= E_i^{-1} U_i^T G_{i-1l}^T(\bar{\chi}_{i-1}, E_{i-1} z_{i-1})\\
&\quad - U_i^T \sum_{j=1}^{i-1} \Bigg\{ \frac{\partial \alpha_{i-1}}{\partial x_j}[F_j + G_{jr}(\bar{\chi}_j, \bar{x}_{j+1,m_j})]\\
&\quad + \frac{\partial \alpha_{i-1}}{\partial \eta_j}\Phi_{j\theta} \Bigg\} + F_i,
\end{aligned}
$$

$$Z_{Bi} = \begin{bmatrix} z_i^{\mathrm{T}} E_i B_{i\theta}\left(\bar{\chi}_i, \Gamma\left(\dfrac{\partial \alpha_{i-1,1}}{\partial \hat{\theta}}\right)^{\mathrm{T}}\right) \\ \vdots \\ z_i^{\mathrm{T}} E_i B_{i\theta}\left(\bar{\chi}_i, \Gamma\left(\dfrac{\partial \alpha_{i-1,m_{i-1}}}{\partial \hat{\theta}}\right)^{\mathrm{T}}\right) \end{bmatrix},$$

$$L_i = \hat{B}_i + B_{i\theta}\left(\bar{\chi}_i, -\Gamma \sum_{j=1}^{i-2} \left(\frac{\partial \alpha_j}{\partial \hat{\theta}}\right)^{\mathrm{T}} U_{j+1} E_{j+1} z_{j+1}\right) - U_i^{\mathrm{T}} Z_{Bi},$$

$$h_i \geqslant \theta_M \|E_i[\phi_{i0} + G_{ir}(\bar{\chi}_i, \alpha_{ia})\| + \sum_{j=1}^{i-1} \|E_i U_i^{\mathrm{T}} \frac{\partial \alpha_{i-1}}{\partial x_j} D_j\|\delta_j + \|E_i D_i\|\delta_i,$$

$$\rho_{ui} \geqslant \sup_{\theta \in \Omega_\theta} \|E_i B_{i\theta}(\bar{\chi}_i, \tilde{\theta}_\pi) L_i^{-1} E_i^{-1}\|, \quad (45)$$

in which, for simplicity, the robust control term α_{is} is chosen according to (30) in Remark 14. Using mathematical induction, the following lemmas are proved in Yao (1996):

Lemma 21. *α_i given by* (44) *has the following structure*:

$$\begin{aligned} \alpha_i = {} & L_{iu}^{-1}(\bar{\chi}_i, \bar{\theta}_\pi^{(i)}, \bar{y}_{t1b}^{(i-1)}, \ldots, \bar{y}_{tib}^{(0)}, t) y_{iu}^{(1)} \\ & + \alpha_{iy}(\bar{\chi}_i, \bar{\theta}_\pi^{(i)}, \bar{y}_{t1b}^{(i-1)}, \ldots, \bar{y}_{tib}^{(0)}, y_{iu}^{(1)}, t) \\ & - \alpha_{ip}(\bar{\chi}_i, \bar{\theta}_\pi^{(i)}, \bar{y}_{t1b}^{(i-1)}, \ldots, \bar{y}_{tib}^{(0)}, t) \end{aligned} \quad (46)$$

with the property that every term in α_{iy} contains a z_k as a factor for some $k \leqslant i$.

Lemma 22. *At step i, by choosing the initial values of the filtered reference trajectories $y_{iu}(0) = [y_{i-1u}^{(1)\mathrm{T}}(0), y_{tib}^{\mathrm{T}}(0)]^{\mathrm{T}}$ as $y_{tib}(0) = N_i^{\mathrm{T}} x_i(0) = [x_{i,m_{i-1}+1}(0), \ldots, x_{i,m_i}(0)]^{\mathrm{T}}$ and $y_{i-1u}^{(1)}(0) = L_{i-1u}(0)[\bar{x}_{i,m_{i-1}}(0) + \alpha_{i-1p}(0)]$, we have, $z_i(0) = 0$.*

Define the ith system to be the interconnection of the first i subsystem, i.e., the $\bar{\chi}_i$-system of (12) with state vector $\bar{\chi}_i$, the input vector $\bar{x}_{i+1,m_i} \in R^{m_i}$ and the output vector $\bar{y}_i = [\bar{y}_{i-1}^{\mathrm{T}}, y_{ib}^{\mathrm{T}}]^{\mathrm{T}} = [y_{1b}^{\mathrm{T}}, \ldots, y_{ib}^{\mathrm{T}}]^{\mathrm{T}} \in R^{m_i}$. It is easy to verify that the $(i-1)$th system has the form of the x_I-subsystem in Section 4.1 with $\bar{\chi}_{i-1}$ corresponding to x_I. Thus, the ith system can be considered as the system obtained by augmenting the $(i-1)$th system by the ith subsystem in the same way as in Section 4.2. x_i, η_i, $\bar{x}_{i+1,m_i}$, f_{i0}, F_i, B_i, D_i, and Δ_i in (12) correspond to x_e, η, u, f_{e0}, F_e, B_e, D_e, and Δ_e in (19) of Section 4.2, respectively. Furthermore, Lemma 22 shows that the trajectory initialization guarantees that the compatibility condition $\bar{x}_{i,m_{i-1}}(0) = \alpha_{i-1}(0)$ is met for the ith system. It is thus straightforward to check that all the assumptions in Sections 4.1–4.3 are satisfied by the ith system. This shows that the backstepping design results in Section 4.3 can be applied, which leads to an ARC Lyapunov function V_i given by $V_i = V_{i-1} + \frac{1}{2} z_i^{\mathrm{T}} E_i z_i = \sum_{j=1}^{i} \frac{1}{2} z_j^{\mathrm{T}} E_j z_j$, $E_i > 0$. After some lengthy substitutions and calculations, the control function and adaptation function are obtained in the form of (44).

5.1.3. Step r

By augmenting the $(r-1)$th system by the rth subsystem in the same way as in the augmented MIMO system II in Section 4.4, we obtain the rth system, which is the same as the entire system (12). Similar to Step i, if $y_{r-1u}^{(1)}(0)$ is chosen as in Lemma 22, the compatibility condition for rth system will be satisfied and it is not difficult to check that all assumptions in Section 4.4 are satisfied. Thus, the backstepping design results in Section 4.4 can be applied to obtain an ARC Lyapunov function for system (12), which has the following form:

$$V_r = \sum_{j=1}^{r-1} \frac{1}{2} z_j^{\mathrm{T}} E_j z_j + \frac{1}{2} z_r^{\mathrm{T}} M^{-1}(\bar{\chi}_{r-1}, \beta) z_r, \quad (47)$$

where $z_r = x_r - \bar{\alpha}_{r-1}$, $\bar{\alpha}_{r-1} = [\alpha_{r-1}^{\mathrm{T}}, y_{trb}^{\mathrm{T}}]^{\mathrm{T}}$. The detailed expressions of the resulting control law α_r and the adaptation function τ_e are quite long and omitted; they are given in Yao (1996).

Remark 23. With the trajectory initialization in Lemma 2, $V_r(0) = 0$. Thus, from Theorem 1,

$$V_r(t) \leqslant \int_0^t \exp(-\lambda_{V_r}(t-v)) c_{V_r}(v)\,\mathrm{d}v \leqslant \frac{c_{V_r\max}}{\lambda_{V_r}} [1 - \exp(-\lambda_{V_r} t)], \quad (48)$$

where $\lambda_{V_r} = \min\{\min_{i \leqslant r-1} \{2\lambda_{\min}(Q_i)/\lambda_{\max}(E_i)\}, 2\lambda_{\min}(Q_i)/k_M\}$ and $c_{V_r} = \sum_{i=1}^{r} \varepsilon_{ei}$. Thus, by increasing Q_i and/or decreasing ε_{ei}, $V_r(t)$ can be made arbitrarily small, which means that the output tracking error of the filtered trajectory $e_t = y - y_t$ can be made arbitrarily small. At the same time, the trajectory planning error, $e_d(t) = y_t(t) - y_d(t)$, converges to zero exponentially and can be guaranteed to possess any good transient behavior when one suitably chooses the Hurwitz polynomials $G_{id}(s) = s^{(r-i+1)} + \cdots + \beta_{i(r-i+1)}$. Same as in Krstic et al. (1992), Yao and Tomizuka (1997), it can be proved (Yao, 1996) that the trajectory initialization process is independent of the selection of controller gains Q_i and ε_{ei}. Therefore, the upper bound of the actual output tracking error $e = e_t + e_d$ is guaranteed to have arbitrarily fast exponentially converging rate and arbitrarily small final value by the trajectory initialization and the selection of the controller gains Q_i and ε_{ei}. □

5.2. MIMO semi-strict feedback form II

Similar techniques as in the above sections can be used to solve the ARC of nonlinear systems transformable to the MIMO semi-strict form II (14). Details are omitted.

6. Conclusion

In this paper, the adaptive robust control (ARC) of MIMO nonlinear systems transformable to semi-strict feedback forms have been solved. Two forms were presented; one partitions the outputs to create various "relative degrees"—a unique feature for MIMO systems—and the other partitions the inputs. Both forms are practical application motivated and have the following unique features. Firstly, the forms allow coupling among states of intermediate layers and the appearance of parametric uncertainties in the input matrices of all layers. Secondly, the usual assumption of linearly parametrizing state equations is relaxed to the extent that the forms are applicable to the control of some multi-DOF mechanical systems. Lastly, the forms admit a class of uncertain nonlinearities satisfying the semi-strict feedback assumption in addition to parametric uncertainties. The unique features of the proposed ARC approach lie in two aspects. Firstly, it uses robust feedback control to achieve a guaranteed output tracking transient performance and final tracking accuracy in general; this overcomes the poor transient performance and non-robustness to uncertain nonlinearities of adaptive control. Secondly, it uses parameter adaptation to reduce model uncertainties to improve performance as compared to the conservative design of robust control.

Acknowledgements

The authors would like to thank the anonymous reviewers for their valuable comments in improving the quality of the paper.

Appendix A

Proof of Theorem 1. Noting (8) and (7) in R3, we have

$$\dot{V} \leqslant -\lambda_V V + c_V(t), \tag{A.1}$$

which leads to (9). Requirement R1 guarantees that x is bounded. Thus, the control input is bounded since $\hat{\theta}$ appears in u through bounded functions like $\bar{\theta}_\pi^{(l_u)}$ only (Yao & Tomizuka, 1997). This proves A of the Theorem.

To prove B of the Theorem, define

$$V_\theta(\tilde{\theta},\theta) = \sum_{i=1}^{p} \frac{1}{\gamma_i}\int_0^{\tilde{\theta}_i} (\pi_{\theta i}(v_i + \theta_i) - \theta_i)\,\mathrm{d}v_i, \quad \gamma_i > 0. \tag{A.2}$$

Then, V_θ is positive definite (p.d.) with respect to (w.r.t.) $\tilde{\theta}$ for each $\theta \in \Omega_\theta$ (Teel, 1993). Furthermore (Yao & Tomizuka, 1997; Teel, 1993), $\partial/\partial\tilde{\theta} V_\theta(\tilde{\theta},\theta) = \tilde{\theta}_\pi^{\mathrm{T}}\Gamma^{-1}$ where $\Gamma = \mathrm{diag}\{\gamma_1,\ldots,\gamma_p\}$.

Now consider the situation in B that $\Delta = 0$, $\forall t \geqslant t_0$. Since x is bounded as shown in A, from (8), $\forall t$, $\hat{\theta}(t) \in L_\infty^p$. Thus, $\hat{\theta}(t_0)$ is bounded. Choose a p.s.d. function as $V_a = V + V_\theta$ Then, $V_a(t_0)$ is bounded. Noting (5) and (8), the derivative of V_a along Eq. (1) is

$$\begin{aligned}\dot{V}_a &= \dot{V}|_{\Delta=0} + \frac{\partial}{\partial\tilde{\theta}} V_\theta(\tilde{\theta},\theta)\dot{\hat{\theta}} \\ &= \dot{V}|_{\Delta=0} + \tilde{\theta}_\pi^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\theta}} \\ &\leqslant -W, \quad \forall t \geqslant t_0.\end{aligned} \tag{A.3}$$

Therefore, $W \in L_1$ and $V_a \in L_\infty$. Since $u \in L_\infty^m$, $\dot{x} \in L_\infty^n$ and $\dot{\hat{\theta}} \in L_\infty^p$. These imply that W is uniformly continuous. By Barbalat's lemma (Slotine & Weiping Li, 1991), $W \to 0$, which leads to asymptotic output tracking due to Requirement R2.B of the Theorem is thus proved. □

Appendix B

Proof of Corollary 1. Since V is not a function of $\hat{\theta}$, $\partial V/\partial\hat{\theta} = 0$. Thus, (A.1) and (9) are not affected, and the results in A of Theorem 1 remain valid. In the case that $\Delta = 0$, $\forall t \geqslant t_0$, noting Requirement R2 and (11), following the same proof as in (A.3), we have

$$\dot{V}_a \leqslant -W - \tilde{\theta}_\pi^{\mathrm{T}} l_\theta(\hat{\theta}) \leqslant -W. \tag{B.1}$$

Thus, the results in B of Theorem 1 remain valid. □

Appendix C

Proof of Lemma 11. If $V \in L_\infty$, then, $V_I \in L_\infty$ and $z \in L_\infty$. Since $V_I \in L_\infty$ results in $x_I \in L_\infty^{n_I}$ and $\alpha_I \in L_\infty^{m_I}$, $x_e = z + \bar{\alpha} \in L_\infty^m$. From Assumption A4.2, $\eta \in L_\infty^{n_\eta}$. Thus, x is bounded. Furthermore, since $0 \leqslant V_I \leqslant V$ and $0 \leqslant z^{\mathrm{T}}Ez \leqslant 2V$, Requirement R1 is satisfied by V for (19). □

Proof of Lemma 12. From (20),

$$\begin{aligned}\frac{\partial V}{\partial x} &= \left[\frac{\partial V_I}{\partial x_I}, 0, 0\right] + z^{\mathrm{T}}E\left[-U_e^{\mathrm{T}}\frac{\partial \alpha_I}{\partial x_I}, I_m, 0\right], \\ \frac{\partial V}{\partial t} &= \frac{\partial V_I}{\partial t} - z^{\mathrm{T}}E\frac{\partial\bar{\alpha}}{\partial t}, \\ \frac{\partial V}{\partial\hat{\theta}} &= \frac{\partial V_I}{\partial\hat{\theta}} - z^{\mathrm{T}}EU_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial\hat{\theta}}.\end{aligned} \tag{C.1}$$

Noting (16) and (18), from (C.1), we have

$$\begin{aligned}&\frac{\partial V}{\partial x}[f + Bu + D\Delta] + \frac{\partial V}{\partial t} \\ &\quad = DV_I + \frac{\partial V_I}{\partial\hat{\theta}}\Gamma\tau_I \\ &\qquad + z^{\mathrm{T}}E\left\{E^{-1}\left(\frac{\partial V_I}{\partial x_I}(B_{I0} + B_{I\theta}(x,\theta))U_e\right)^{\mathrm{T}}\right.\end{aligned}$$

$$- U_e^{\mathrm{T}}\frac{\partial \alpha_I}{\partial x_I}[f_{I0} + F_{I\theta}\theta + (B_{I0} + B_{I\theta}(x_I,\theta))\bar{x}_{em_I} + D_I\Delta_I]$$

$$\left. + f_{e0} + F_{e\theta}\theta + \hat{B}_e u - \tilde{B}_e u + D_e\Delta_e - \frac{\partial\bar{\alpha}}{\partial t}\right\}$$

$$= DV_I + \frac{\partial V_I}{\partial\hat{\theta}}\Gamma\tau_I + z^{\mathrm{T}}E\{-\alpha_{e0} + \phi_0\theta + \hat{B}_e u - G_{er}(x,u)\tilde{\theta}_\pi + \tilde{\Delta}\}, \quad (\mathrm{C.2})$$

where α_{e0} and ϕ_0 are defined in (22), and

$$\tilde{\Delta} = -U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial x_I}D_I\Delta_I + D_e\Delta_e. \quad (\mathrm{C.3})$$

Noticing (C.1), (24), and (21), one can rearrange (C.4) as

$$\frac{\partial V}{\partial x}[f + Bu + D\Delta] + \frac{\partial V}{\partial t}$$

$$= DV_I + \frac{\partial V}{\partial\hat{\theta}}\Gamma\tau - \tilde{\theta}_\pi^{\mathrm{T}}\phi^{\mathrm{T}}Ez + z^{\mathrm{T}} E\left\{\phi_0\Gamma\left(\frac{\partial V_I}{\partial\hat{\theta}}\right)^{\mathrm{T}} + B_{e\theta}\left(x,\Gamma\left(\frac{\partial V_I}{\partial\hat{\theta}}\right)^{\mathrm{T}}\right)u + U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial\hat{\theta}}\Gamma(\tau_I - \phi_0^{\mathrm{T}}Ez) - U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial\hat{\theta}}\Gamma G_{er}^{\mathrm{T}}(x,u)Ez - \alpha_{e0} + \phi_0\hat{\theta}_\pi + \hat{B}_e u + \tilde{\Delta}\right\}$$

$$= DV_I + \frac{\partial V}{\partial\hat{\theta}}\Gamma\tau - \tilde{\theta}_\pi^{\mathrm{T}}\phi^{\mathrm{T}}Ez + z^{\mathrm{T}}E\{-\alpha_{ea} + Lu + \tilde{\Delta}\}, \quad (\mathrm{C.4})$$

in which

$$\frac{\partial\alpha_I}{\partial\hat{\theta}}\Gamma G_{er}^{\mathrm{T}}Ez = \begin{bmatrix} \frac{\partial\alpha_{I1}}{\partial\hat{\theta}}\Gamma G_{er}^{\mathrm{T}}Ez \\ \vdots \\ \frac{\partial\alpha_{Im_I}}{\partial\hat{\theta}}\Gamma G_{er}^{\mathrm{T}}Ez \end{bmatrix} = \begin{bmatrix} u^{\mathrm{T}}B_{e\theta}^{\mathrm{T}}\left(x,\Gamma\left(\frac{\partial\alpha_{I1}}{\partial\hat{\theta}}\right)^{\mathrm{T}}\right)Ez \\ \vdots \\ u^{\mathrm{T}}B_{e\theta}^{\mathrm{T}}\left(x,\Gamma\left(\frac{\partial\alpha_{Im_I}}{\partial\hat{\theta}}\right)^{\mathrm{T}}\right)Ez \end{bmatrix} = E_B u \quad (\mathrm{C.5})$$

is used. When $\Delta = 0$, from (C.3), $\tilde{\Delta} = 0$. Thus, by letting $\Delta = 0$ in (C.4), from B1 of (17) and (23), we have that $\forall u_s \in \Omega_u$

Left side (LS) of (4)

$$\leqslant -W_I + \tilde{\theta}_\pi^{\mathrm{T}}\tau_I + \frac{\partial V}{\partial\hat{\theta}}\Gamma\tau - \tilde{\theta}_\pi^{\mathrm{T}}\phi^{\mathrm{T}}Ez + z^{\mathrm{T}}E\{-E^{-1}Qz + Lu_s\}$$

$$\leqslant -W + \tilde{\theta}_\pi^{\mathrm{T}}\tau + \frac{\partial V}{\partial\hat{\theta}}\Gamma\tau, \quad (\mathrm{C.6})$$

where $W = W_I + z^{\mathrm{T}}Qz$. Noting that $W \to 0$ leads to $W_I \to 0$, which in turn results in $e_y \to 0$, Requirement R2 (5) is thus satisfied. □

Proof of Lemma 13. From (C.4), (23), B2 of (17), and (25),

$$\text{LS of (6)} \leqslant -\lambda_{V_I}V_I + c_{V_I} - \tilde{\theta}_\pi^{\mathrm{T}}\phi^{\mathrm{T}}Ez + z^{\mathrm{T}}E\{-E^{-1}Qz + Lu_s + \tilde{\Delta}\}$$

$$\leqslant -\lambda_V V + c_{V_I} + z^{\mathrm{T}}E\{-\phi_0\tilde{\theta}_\pi - \tilde{B}_{e\theta}u + Lu_s + \tilde{\Delta}\}$$

$$= -\lambda_V V + c_{V_I} + z^{\mathrm{T}}\{E[L - B_{e\theta}(x,\tilde{\theta}_\pi)]u_s + \alpha_{es}\}$$

$$\leqslant -\lambda_V V + c_V, \quad (\mathrm{C.7})$$

which completes the proof. □

Appendix D

Proof of Lemma 16. In viewing Assumption A4.6, Lemma 6 can be proved in the same way as in Lemma 11. □

Proof of Lemma 17**.** Noting (18) and (32), the derivative of V is

$$\dot{V} = \frac{\partial V_I}{\partial x_I}\dot{x}_I + \frac{\partial V_I}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial V_I}{\partial t} + z^{\mathrm{T}}\left(M\dot{x}_e - M\dot{\bar{\alpha}} + \frac{1}{2}\dot{M}z\right)$$

$$= DV_I + \frac{\partial V_I}{\partial\hat{\theta}}(\dot{\hat{\theta}} + \Gamma\tau_I) + z^{\mathrm{T}}\left(\frac{\partial V_I}{\partial x_I}B_I U_e\right)^{\mathrm{T}} + z^{\mathrm{T}}\left\{f_{e0} + F_{e\theta}\theta + F_\beta\beta + B_e u + D_e\Delta_e - MU_e^{\mathrm{T}}\left[\frac{\partial\alpha_I}{\partial x_I}(f_{I0} + F_{I\theta}\theta + B_I\bar{x}_{em_I} + D_I\Delta_I) + \frac{\partial\alpha_I}{\partial\hat{\theta}}\dot{\hat{\theta}}\right] - M\frac{\partial\bar{\alpha}}{\partial t} + \frac{1}{2}\dot{M}z\right\}. \quad (\mathrm{D.1})$$

Noting (20) and (33),

$$\frac{\partial V}{\partial\hat{\theta}} = \frac{\partial V_I}{\partial\hat{\theta}} - z^{\mathrm{T}}M(x_I,\beta)U_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial\hat{\theta}}. \quad (\mathrm{D.2})$$

Using similar techniques as in (C.2) and substituting (35) and (36) into (D.1), we have

$$\dot{V} = DV_I + \frac{\partial V}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial V_I}{\partial\hat{\theta}}\Gamma\tau_I + z^{\mathrm{T}}\{-\alpha_{e0} + \phi_{\theta0}\theta + \phi_{\beta0}\beta + \phi_{\vartheta0}\vartheta + \hat{B}_e u - G_{\theta r}(x,u)\tilde{\theta}_\pi - G_{\beta r}(x,u)\tilde{\beta}_\pi + \tilde{\Delta}\}, \quad (\mathrm{D.3})$$

where α_{e0}, $\phi_{\theta0}$, $\phi_{\beta0}$, and $\phi_{\vartheta0}$ are defined in (37), and

$$\tilde{\Delta} = D_e\Delta_e - MU_e^{\mathrm{T}}\frac{\partial\alpha_I}{\partial x_I}D_I\Delta_I + \tilde{\Delta}_M. \quad (\mathrm{D.4})$$

It can be checked out that

$$\frac{\partial V_I}{\partial \hat{\theta}} \Gamma \tau_I = \frac{\partial V}{\partial \hat{\theta}} \Gamma \tau_\theta + z^{\mathrm{T}} \left[M_0 U_e^{\mathrm{T}} \frac{\partial \alpha_I}{\partial \hat{\theta}} \Gamma \tau_\theta + G_{Mr}\left(x_I, U_e^{\mathrm{T}} \frac{\partial \alpha_I}{\partial \hat{\theta}} \Gamma \tau_\theta \right)\beta + \phi_\theta \Gamma \left(\frac{\partial V_I}{\partial \hat{\theta}} \right)^{\mathrm{T}} \right]. \tag{D.5}$$

Substituting (D.5) into (D.3) and rearranging terms as in (C.4), we have

$$\dot{V} = DV_I + \frac{\partial V}{\partial \hat{\theta}}(\dot{\hat{\theta}} + \Gamma \tau_\theta) - \tau_I^{\mathrm{T}} \tilde{\theta}_\pi + \tau_e^{\mathrm{T}} \tilde{\theta}_{e\pi} + z^{\mathrm{T}}\{ -\alpha_{ea} + Lu + \tilde{\Delta} \}, \tag{D.6}$$

in which transformations similar to the one in (C.5) have been used. When $\Delta = 0$, from (D.4), $\tilde{\Delta} = 0$. Since V does not depend on $\hat{\beta}$ and $\hat{\vartheta}$, $\partial V / \partial \hat{\theta}_e = [\partial V / \partial \hat{\theta}, 0, 0]$. Then, noting B1 of (17), (39), and (D.6), we have that $\forall u_s \in \Omega_u$

$$\begin{aligned} \text{LS of (4)} &\leqslant -W + \tilde{\theta}_{e\pi}^{\mathrm{T}} \tau_e + \frac{\partial V}{\partial \hat{\theta}} \Gamma \tau_\theta \\ &= -W + \tilde{\theta}_{e\pi}^{\mathrm{T}} \tau_e + \frac{\partial V}{\partial \hat{\theta}_e} \Gamma_e \tau_e, \end{aligned} \tag{D.7}$$

where $W = W_I + z^{\mathrm{T}} Q z$ and $\Gamma_e = \mathrm{diag}\{\Gamma, \Gamma_\beta, \Gamma_\vartheta\}$, in which Γ_β and Γ_ϑ are any s.p.d. matrices. It is thus clear that Requirement R2 (4) is satisfied by V in terms of the augmented unknown parameter vector θ_e. □

Proof of Lemma 18. From (D.6), B2 of (17), and (41), it can be checked out that

$$\begin{aligned} \text{LS of (6)} &\leqslant -\lambda_{V_I} V_I + c_{V_I} - \tilde{\theta}_\pi^{\mathrm{T}} \phi_\theta^{\mathrm{T}} z + \tilde{\beta}_\pi^{\mathrm{T}} \tau_\beta + \tilde{\vartheta}_\pi^{\mathrm{T}} \tau_\vartheta \\ &\quad + z^{\mathrm{T}}\{ -Qz + Lu_s + \tilde{\Delta} \} \\ &= -\lambda_V V + c_{V_I} + z^{\mathrm{T}}[(L - \tilde{L}_u) u_s + \alpha_{es}] \\ &\leqslant -\lambda_V V + c_V, \end{aligned} \tag{D.8}$$

which completes the proof. □

References

Artstein, A. (1983). Stabilization with relaxed controls. *Nonlinear Analysis*, *TMA-7*, 1163–1173.

Barmish, B. R., & Leitmann, G. (1982). On ultimate boundedness control of uncertain systems in the absence of matching assumptions. *IEEE Transactions on Automatic Control*, *27*, 153–158.

Chen, Y. H. (1988). Design of robust controllers for uncertain dynamical systems. *IEEE Transactions on Automatic Control*, *33*, 487–491.

Corless, M. J., & Leitmann, G. (1981). Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems. *IEEE Transactions on Automatic Control*, *26*(10), 1139–1144.

Freeman, R. A, Krstic, M., & Kokotovic, P. V. (1996). Robustness of adaptive nonlinear control to bounded uncertainties. *IFAC World Congress*, Vol. F (pp. 329–334).

Goodwin, G. C., & Mayne, D. Q. (1989). A parameter estimation perspective of continuous time model reference adaptive control. *Automatica*, *23*(1), 57–70.

Ioannou, P. A. (1986). Robust adaptive controller with zero residual tracking errors. *IEEE Transactions on Automatic Control*, *31*(8), 773–776.

Isidori, A. (1989). *Nonlinear control systems: An introduction*. Berlin: Springer.

Kreisselmeier, G., & Narendra, K. S. (1982). Stable model reference adaptive control in the presence of bounded disturbances. *IEEE Transactions on Automatic Control*, *27*(6), 1169–1175.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. V. (1992). Adaptive nonlinear control without overparametrization. *Systems and Control Letters*, *19*, 177–185.

Krstic, M., Kanellakopoulos, I., & Kokotovic, PV. (1995). *Nonlinear and adaptive control design*. New York: Wiley.

Marino, R., & Tomei, P. (1993). Global adaptive output-feedback control of nonlinear systems, part I: Linear parameterization; part II: Nonlinear parameterization. *IEEE Transactions on Automatic Control*, *38*(1), 17–49.

Marino, R., & Tomei, P. (1995). *Nonlinear control design: Geometric, adaptive, and robust*. London; New York: Prentice-Hall.

Marino, R., & Tomei, P. (1998). Robust adaptive state-feedback tracking for nonlinear systems. *IEEE Transactions on Automatic Control*, *43*(1), 84–89.

Pan, Z., & Basar, T. (1996). Adaptive controller design for tracking and disturbance attenuation in parametric-strict-feedback nonlinear systems. *IFAC world congress*, Vol. F (pp. 323–328).

Polycarpou, M. M., & Ioannou, P. A. (1993). A robust adaptive nonlinear control design. *Proceedings of American control conference* (pp. 1365–1369).

Pomet, J. B., & Praly, L. (1992). Adaptive nonlinear regulation: Estimation from the Lyapunov equation. *IEEE Transactions on Automatic Control*, *37*, 729–740.

Qu, Z. (1993). Robust control of nonlinear uncertain systems under generalized matching conditions. *Automatica*, *29*(4), 985–998.

Reed, J. S., & Ioannou, P. A. (1989). Instability analysis and robust adaptive control of robotic manipulators. *IEEE Transactions on Robotics and Automation* 5(3).

Santosuosso, G. L. (1995). Nonlinear h_∞ almost disturbance decoupling and adaptive stabilization without overparamterization. *Proceedings of IEEE conference on decision and control* (pp. 2970–2975).

Sastry, S., & Bodson, M. (1989). *Adaptive control: Stability, convergence and robustness*. Englewood Cliffs, NJ 07632, USA: Prentice-Hall, Inc.

Sastry, S., & Isidori, A. (1989). Adaptive control of linearizable systems. *IEEE Transactions on Automatic Control*, *34*, 1123–1131.

Slotine, J. J. E. (1985). The robust control of robot manipulators. *International Journal of Robotics Research*, *4*, 49–63.

Slotine, J. J. E., & Weiping Li (1991). *Applied nonlinear control*. Englewood Cliffs, NJ: Prentice-Hall.

Sontag, E. D. (1983). A Lyapunov-like characterizations of asymptotic controllability. *SIAM Journal of Control and Optimization*, *21*, 462–471.

Teel, A. R. (1993). Adaptive tracking with robust stability. *Proceedings of 32nd conference on decision and Control* (pp. 570–575).

Utkin, V. I. (1992). *Sliding modes in control optimization*. Berlin: Springer.

Yao, B. (1996). *Adaptive robust control of nonlinear systems with application to control of mechanical systems*. Ph.D. thesis, Mechanical Engineering Department, University of California at Berkeley, Berkeley, USA, January.

Yao, B., Al-Majed, M., & Tomizuka, M. (1997). High performance robust motion control of machine tools: An adaptive robust control approach and comparative experiments. *IEEE/ASME Transactions on Mechatronics*, *2*(2), 63–76.

Yao, B., & Tomizuka, M., (1994). Comparative experiments of robust and adaptive control with new robust adaptive controllers for robot manipulators. *Proceedings of IEEE Conference on Decision and Control* (pp. 1290–1295).

Yao, B., & Tomizuka, M. (1996). Smooth robust adaptive sliding mode control of robot manipulators with guaranteed transient performance. *Proceedings of American control conference* (pp. 1176–1180). 1994. The full paper appeared in *ASME Journal of Dynamic Systems, Measurement and Control*, 118(4), 764–775.

Yao, B., & Tomizuka, M. (1997). Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form. *Automatica*, 33(5), 893–900. Part of the paper appeared in Proceedings of 1995 American control conference (pp. 2500–2505).

Yao, B., & Tomizuka, M. (1998). Adaptive robust motion and force tracking control of robot manipulators in contact with compliant surfaces with unknown stiffness. *ASME Journal of Dynamic Systems, Measurement, and Control*, *120*(2), 232–240.

Zinober, A. S. I. (1990). *Deterministic control of uncertain control system*. London, UK: Peter Peregrinus Ltd.

Bin Yao received the Ph.D. degree in Mechanical Engineering from the University of California at Berkeley in February 1996. Since 1996, he has been an Assistant Professor in the school of Mechanical Engineering at Purdue University. Dr. Yao's research focuses on the development of a general framework for the design of high performance adaptive robust control algorithms. Integration of the approach with actual sensor properties, physical properties, and control-oriented modeling has been carried out for different applications such as the control of electro-hydraulic systems, linear motors, machine tools, and robot manipulators. He experimentally verified the approach on the precision control of various electro-mechanical/hydraulic systems. Dr. Yao is the recipient of the 1997 Caterpillar Engineering Young Faculty Development Fund for his work on the electro-hydraulic control, and the 1998 National Science Foundation CAREER award for his work on the engineering synthesis of high performance adaptive robust controllers for mechanical systems and manufacturing processes. His research interests include design and coordinated control of intelligent high performance electro-mechanical/electro-hydraulic systems, optimal adaptive and robust control, nonlinear observer design and neural networks for virtual sensing, modeling, fault detection, diagnostics, and adaptive fault-tolerant control, and data fusion.

Masayoshi Tomizuka was born in Tokyo, Japan in 1946. He received his B.S. and M.S. degrees in Mechanical Engineering from Keio University, Tokyo, Japan and his Ph.D. degree in Mechanical Engineering from the Massachusetts Institute of Technology in February 1974. In 1974, he joined the faculty of the Department of Mechanical Engineering at the University of California at Berkeley, where he currently holds the Cheryl and John Neerhout, Jr., Distinguished Professorship Chair. At UC Berkeley, he teaches courses in dynamic systems and controls. His current research interests are optimal and adaptive control, digital control, signal processing, motion control, and control problems related to robotics, machining, manufacturing, information storage devices and vehicles. He has served as a consultant to various organizations, including Lawrence Berkeley Laboratory, General Electric, General Motors and United Technologies.

He served as Technical Editor of the ASME Journal of Dynamic Systems, Measurement and Control, J-DSMC (1988–1993), Editor-in-Chief of the IEEE/ASME Transactions on Mechatronics (1997–1999) and an Associate Editor of the Journal of the International Federation of Automatic Control, Automatica. He currently serves as an Associate Editor of European Journal of Control. He was General Chairman of the 1995 American Control Conference, and served as President of the American Automatic Control Council (1998–1999). He is a Fellow of the ASME, the Institute of Electric and Electronics Engineers (IEEE) and the Society of Manufacturing Engineers. He is the recipient of the J-DSMC Best Paper Award (1995), the DSCD Outstanding Investigator Award (1996) and the Charles Russ Richards Memorial Award (ASME, 1997). The Charles Russ Richards Memorial Award, established in 1944, is given to an engineering graduate who demonstrates outstanding achievement in mechanical engineering 20 years or more following graduation.