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# Output feedback adaptive robust precision motion control of linear motors☆

Li Xu, Bin Yao*

*School of Mechanical Engineering, Purdue University, 1288 Mechanical Engineering Building, West Lafayette, IN 47907-1288, USA*

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*An output feedback adaptive robust controller is constructed for precision motion control of linear motor drive systems. The control law theoretically achieves a guaranteed high tracking accuracy for high-acceleration/high-speed movements, as verified through experiments also.*

**Abstract**

This paper studies high performance robust motion control of linear motors that have a negligible electrical dynamics. A discontinuous projection based adaptive robust controller (ARC) is constructed. Since only output signal is available for measurement, an observer is first designed to provide exponentially convergent estimates of the unmeasurable states. This observer has an extended filter structure so that on-line parameter adaptation can be utilized to reduce the effect of the possible large nominal disturbances. Estimation errors that come from initial state estimates and uncompensated disturbances are effectively dealt with via certain robust feedback at each step of the ARC backstepping design. The resulting controller achieves a guaranteed output tracking transient performance and a prescribed final tracking accuracy. In the presence of parametric uncertainties only, asymptotic output tracking is also achieved. The scheme is implemented on a precision epoxy core linear motor. Experimental results are presented to illustrate the effectiveness and the achievable control performance of the proposed scheme. © 2001 Elsevier Science Ltd. All rights reserved.

*Keywords:* Linear motors; Output feedback; Robust control; Adaptive control; Motion control; Precision manufacturing

## 1. Introduction

Modern mechanical systems, such as machine tools, semiconductor manufacturing equipment, and automatic inspection machines, often require high-speed, high-accuracy linear motions. These linear motions are usually realized using rotary motors with mechanical transmission mechanisms such as reduction gears and lead screw. Such mechanical transmissions not only significantly reduce linear motion speed and dynamic response, but also introduce backlash, large frictional and inertial loads, and structural flexibility. Backlash and structural flexibility physically limit the accuracy that any control system can achieve. As an alternative, direct drive linear motors, which eliminate the use of mechanical transmissions, show promise for widespread use in high performance positioning systems.

Direct drive linear motor systems gain high-speed, high-accuracy potential by eliminating mechanical transmissions. However, they also lose the advantage of using mechanical transmissions—gear reduction reduces the effect of model uncertainties such as parameter variations (e.g., uncertain payloads) and external disturbance (e.g., cutting forces in machining). Furthermore, certain types of linear motors (e.g., iron core linear motors) are subjected to significant force ripple (Braembussche, Swevers, Van Brussel, & Vanherck, 1996). These uncertain nonlinearities are directly transmitted to the load and have significant effects on the motion of the load. Thus, in order for a linear motor system to be able to function and to deliver its high performance potential, a controller

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* Corresponding author. Tel.: + 1-765-494-7746; fax: + 1-765-494-0539.

*E-mail addresses:* xul@ecn.purdue.edu (L. Xu), byao@ecn.purdue.edu (B. Yao).


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which can achieve the required high accuracy in spite of various parametric uncertainties and uncertain nonlinear effects, has to be employed.

A great deal of effort has been devoted to solving the difficulties in controlling linear motor systems (Braembussche et al., 1996; Alter & Tsao, 1996, 1994; Komada, Ishida, Ohnishi, & Hori, 1991; Egami & Tsuchiya, 1995; Otten, Vries, Amerongen, Rankers, & Gaal, 1997; Yao & Xu, 1999). Alter and Tsao (1996) presented a comprehensive design approach for the control of linear-motor-driven machine tool axes. $H_\infty$ optimal feedback control was used to provide high dynamic stiffness to external disturbances (e.g., cutting forces in machining). Feedforward was also introduced in Alter and Tsao (1994) to improve tracking performance. Practically, $H_\infty$ design may be conservative for high-speed/high-accuracy tracking control and there is no systematic way to translate practical information about plant uncertainty and modeling inaccuracy into quantitative terms that allow the application of $H_\infty$ techniques. In Komada et al. (1991), a disturbance compensation method based on disturbance observer (DOB) (Ohnishi, Shibata, & Murakami, 1996) was proposed to make a linear motor system robust to model uncertainties. It was shown both theoretically and experimentally by Yao, Al-Majed, and Tomizuka (1997) that DOB design may not handle discontinuous disturbances such as Coulomb friction well and cannot deal with large extent of parametric uncertainties. To reduce the nonlinear effect of force ripple, in Braembussche et al. (1996), feedforward compensation terms, which were based on an off-line experimentally identified force ripple model, were added to a position controller. Since not all magnets in a linear motor and not all linear motors of the same type are identical, feedforward compensation based on the off-line identified model may be too sensitive and costly to be useful. In Otten et al. (1997), a neural-network-based learning feedforward controller was proposed to reduce positional inaccuracy due to reproducible ripple forces or any other reproducible and slowly varying disturbances over different runs of the same desired trajectory (or repetitive tasks). However, overall closed-loop stability was not guaranteed. In fact, it was observed in Otten et al. (1997) that instability may occur at high-speed movements. Furthermore, the learning process may take too long to be useful due to the use of a small adaptation rate for stability. In Yao and Xu (1999), under the assumption that the full state of the system is measured, the idea of adaptive robust control (ARC) (Yao & Tomizuka, 1996, 1997b) was generalized to provide a theoretic framework for the high performance motion control of an iron core linear motor. The controller took into account the effect of model uncertainties coming from the inertia load, friction, force ripple and electrical parameters, etc. In particular, based on the structure of the motor model, on-line parameter adaptation was utilized to reduce the effect of parametric uncertainties while the uncompensated uncertain nonlinearities were handled effectively via certain robust control laws for high performance. As a result, time-consuming and costly rigorous offline identification of friction and ripple forces was avoided without sacrificing tracking performance. In Xu and Yao (2000a,b), the proposed ARC algorithm (Yao & Xu, 1999) was applied on an epoxy core linear motor. To reduce the effect of measurement noise, a desired compensation ARC algorithm in which the regressor was calculated by reference trajectory information was also presented and implemented.

The ARC schemes in Xu and Yao (2000a,b) used velocity feedback. However, most linear motors systems do not equip velocity sensors due to their special structure. In practice, the velocity signal is usually obtained by the backward difference of the position signal, which is very noisy and limits the overall performance. It is thus of practical significance to see if one can construct ARC controllers based on the position measurement only, which is the focus of the paper. An output feedback ARC scheme is constructed for a linear motor subjected to both parametric uncertainties and bounded disturbances. Since only the output signal is available for measurement, a Kreisselmeier observer (Kreisselmeier, 1977) is first designed to provide exponentially convergent estimates of the unmeasurable states. This observer has an extended filter structure so that on-line parameter adaptation can be utilized to reduce the effect of the possible large nominal disturbance, which is very important from the view point of application (Yao et al., 1997). The destabilizing effect of the estimation errors is effectively dealt with using robust feedback at each step of the design procedure. The resulting controller achieves a guaranteed transient performance and a prescribed final tracking accuracy. In the presence of parametric uncertainties only, asymptotic output tracking is also achieved. Finally, the proposed scheme, as well as a PID controller, is implemented on an epoxy core linear motor. Comparative experimental results are presented to justify the validity of the ARC algorithm.

The paper is organized as follows. Problem formulation and dynamic models are presented in Section 2. The proposed ARC controller is shown in Section 3. Experimental setup and comparative experimental results are presented in Section 4, and conclusions are drawn in Section 5.

## 2. Problem formulation and dynamic models

The linear motor considered here is a current-controlled three-phase epoxy core motor driving a linear positioning stage supported by recirculating bearings. To fulfill the high performance requirements, the model is obtained to include most nonlinear effects like friction and force ripple. In the derivation of the model, the current dynamics is neglected in comparison to the mechanical dynamics due to the much faster electric response. The mathematical model of the system can be described by the following equations:

$$M\ddot{q} = u - F(q,\dot{q}),$$
$$\mathrm{F}(q,\dot{q}) = F_{\mathrm{f}} + F_{\mathrm{r}} - F_d, \tag{1}$$

where $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$ represent the position, velocity and acceleration of the inertia load, respectively, $M$ is the normalized[1] mass of the inertia load plus the coil assembly, $u$ is the input voltage to the motor, $F$ is the normalized lumped effect of uncertain nonlinearities such as friction $F_{\mathrm{f}}$, ripple force $F_{\mathrm{r}}$ and external disturbance $F_d$ (e.g. cutting force in machining). While there have been many friction models proposed (Armstrong-Hèlouvry, Dupont, & Canudas de Wit, 1994), a simple and often adequate approach is to regard the friction force as a static nonlinear function of the velocity, i.e., $F_{\mathrm{f}}(\dot{q})$, which is given by

$$F_{\mathrm{f}}(\dot{q}) = B\dot{q} + F_{\mathrm{fn}}(\dot{q}), \tag{2}$$

where $B$ is the equivalent viscous coefficient of the system, $F_{\mathrm{fn}}$ is the nonlinear friction term that can be modeled as (Armstrong-Hèlouvry et al., 1994)

$$F_{\mathrm{fn}}(\dot{q}) = -[f_{\mathrm{c}} + (f_{\mathrm{s}} - f_{\mathrm{c}})\mathrm{e}^{-|\dot{q}/\dot{q}_{\mathrm{s}}|^{\xi}}]\mathrm{sgn}(\dot{q}), \tag{3}$$

where $f_{\mathrm{s}}$ is the level of stiction, $f_{\mathrm{c}}$ is the level of Coulomb friction, and $\dot{q}_{\mathrm{s}}$ and $\xi$ are empirical parameters used to describe the Stribeck effect. Thus, considering (2), one can rewrite (1) as

$$M\ddot{q} = u - B\dot{q} - F_{\mathrm{fn}}(\dot{q}) + \Delta, \tag{4}$$

where $\Delta \triangleq F_d - F_{\mathrm{r}}$ represents the lumped disturbance.

Let $q_{\mathrm{r}}(t)$ be the reference motion trajectory, which is assumed to be known, bounded with bounded derivatives up to the second order. The control objective is to synthesize a control input $u$ such that the output $q(t)$ tracks $q_{\mathrm{r}}(t)$ as closely as possible in spite of various model uncertainties.

## 3. Adaptive robust control of linear motor systems

### 3.1. Friction compensation

A simple but effective method for overcoming problems due to friction is to introduce a cancellation term for the friction force. Since the nonlinearity $F_{\mathrm{fn}}$ depends on the velocity $\dot{q}$ which is not measurable, the friction compensation scheme developed in Lee and Tomizuka (1996) cannot be applied directly to achieve our objective. In order to bypass the difficulty, in the following, the "*estimated friction force*" $\hat{F}_{\mathrm{fn}}(\dot{q}_d)$ will be used to approximate $F_{\mathrm{fn}}(\dot{q})$, where $q_d$ is the desired trajectory to be tracked by $q$. The approximation error $\tilde{F}_{\mathrm{fn}} = \hat{F}_{\mathrm{fn}}(\dot{q}_d) - F_{\mathrm{fn}}(\dot{q})$ will be treated as disturbance. In other words, the control input $u(t)$ becomes

$$u(t) = u^*(t) + \hat{F}_{\mathrm{fn}}(\dot{q}_d), \tag{5}$$

where $u^*$ is the output of an adaptive robust controller yet to be designed. Substituting (5) into (4), one obtains

$$M\ddot{q} = u^*(t) - B\dot{q} + d, \tag{6}$$

where $d \triangleq \Delta + \tilde{F}_{\mathrm{fn}}$.

In general, the system is subject to parametric uncertainties due to the variations of $M$, $B$, and the nominal value of the lumped disturbance $d$, $d_{\mathrm{n}}$. Define the unknown parameter set $\theta = [\theta_1, \theta_2, \theta_3]$ as $\theta_1 = 1/M$, $\theta_2 = B/M$, and $\theta_3 = d_{\mathrm{n}}/M$.

A state space realization of the plant (6), which is linearly parameterized in terms of $\theta$, is thus given by

$$\dot{x}_1 = x_2 - \theta_2 x_1,$$
$$\dot{x}_2 = \theta_1 u^* + \theta_3 + \tilde{d},$$
$$y = x_1, \tag{7}$$

where $x_1$ is one state of the second order system that represents the position $q$, $x_2$ is the other state that is not measurable, $y$ is the output, and $\tilde{d} = (d - d_{\mathrm{n}})/M$.

For simplicity, in the following, the following notations are used: $\bullet_i$ for the $i$th component of the vector $\bullet$, $\bullet_{\min}$ for the minimum value of $\bullet$, and $\bullet_{\max}$ for the maximum value of $\bullet$. The operation $\leqslant$ for two vectors is performed in terms of the corresponding elements of the vectors. The following practical assumption is made:

**Assumption 1.** *The extent of the parametric uncertainties and uncertain nonlinearities is known, i.e.,*

$$\theta \in \Omega_\theta \triangleq \{\theta\colon \theta_{\min} \leqslant \theta \leqslant \theta_{\max}\},$$
$$\tilde{d} \in \Omega_d \triangleq \{\tilde{d}\colon |\tilde{d}| \leqslant \delta_d\}, \tag{8}$$

*where*

$$\theta_{\min} = [\theta_{1\min}, \ldots, \theta_{3\min}]^{\mathrm{T}}, \quad \theta_{\max} = [\theta_{1\max}, \ldots, \theta_{3\max}]^{\mathrm{T}},$$

*and* $\delta_d$ *are known.*

### 3.2. State estimation

Since only the output $y$ is available for measurement, a nonlinear observer is first built to provide an exponentially convergent estimate of the unmeasurable state $x_2$. The design model (7) can be rewritten as

$$\dot{x} = A_0 x + (k - e_1\theta_2)y + e_2\theta_3 + e_2\theta_1 u^* + e_2\tilde{d},$$
$$y = x_1, \tag{9}$$

[1] Normalized with respect to the unit input voltage.

where $x = [x_1, x_2]^T$, $e_1$ and $e_2$ denote the standard basis vectors in $\mathbb{R}^2$ and

$$A_0 = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix}, \quad k = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}. \tag{10}$$

Then, by suitably choosing $k$, the observer matrix $A_0$ will be stable. Thus, there exists a symmetric positive definite (s.p.d.) matrix $P$ such that

$$PA_0 + A_0^T P = -I, \quad P = P^T > 0. \tag{11}$$

Following the design procedure of Krstic, Kanellakopoulos, and Kokotovic (1995), one can define the following filters:

$$\begin{aligned} \dot{\xi}_2 &= A_0 \xi_2 + ky, \\ \dot{\xi}_1 &= A_0 \xi_1 + e_1 y, \\ \dot{\upsilon} &= A_0 \upsilon + e_2 u^*, \\ \dot{\psi} &= A_0 \psi + e_2. \end{aligned} \tag{12}$$

Notice that the last equation of (12) is introduced so that parameter adaptation can be used to reduce the parametric uncertainties coming from $\theta_3$. The state estimates can thus be represented by

$$\hat{x} = \xi_2 - \theta_2 \xi_1 + \theta_1 \upsilon + \theta_3 \psi. \tag{13}$$

Let $\varepsilon_x = x - \hat{x}$ be the estimation error, from (9), (12) and (13), it can be verified that the observer error dynamics is given by

$$\dot{\varepsilon}_x = A_0 \varepsilon_x + e_2 \tilde{d}. \tag{14}$$

The solution of Eq. (14) can be written as $\varepsilon_x = \varepsilon + \varepsilon_u$, where $\varepsilon$ is the zero input response satisfying $\dot{\varepsilon} = A_0 \varepsilon$ and

$$\varepsilon_u = \int_0^t e^{A_0(t-\tau)} e_2 \tilde{d}(y, \tau)\, d\tau, \quad t \geqslant 0, \tag{15}$$

is the zero state response. Noting Assumption 1 and the fact that matrix $A_0$ is stable, one has

$$\varepsilon_u \in \Omega_\varepsilon \triangleq \{\varepsilon_u \colon |\varepsilon_u(t)| \leqslant \delta_\varepsilon(t)\}, \tag{16}$$

where $\delta_\varepsilon(t)$ is known. In the following controller design, $\varepsilon$ and $\varepsilon_u$ will be treated as disturbances and accounted for using different robust control functions at each step of the design to achieve a guaranteed final tracking accuracy.

**Remark 1.** The $\xi$ and $\upsilon$ variables in (12) can be obtained from the algebraic expressions (Krstic et al., 1995)

$$\begin{aligned} \xi_2 &= -A_0^2 \eta, \\ \xi_1 &= A_0 \eta, \\ \upsilon &= \lambda, \end{aligned} \tag{17}$$

where $\eta$ and $\lambda$ are the states of the following two-dimensional filters

$$\begin{aligned} \dot{\eta} &= A_0 \eta + e_2 y, \\ \dot{\lambda} &= A_0 \lambda + e_2 u^*. \end{aligned} \tag{18}$$

### 3.3. Parameter projection

Let $\hat{\theta}$ denote the estimate of $\theta$ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta} = \hat{\theta} - \theta$). In view of (8), the following adaptation law with discontinuous projection modification can be used:

$$\dot{\hat{\theta}} = \mathrm{Proj}_{\hat{\theta}}(\Gamma \tau), \tag{19}$$

where $\Gamma > 0$ is a diagonal matrix, $\tau$ is an adaptation function to be synthesized later. The projection mapping $\mathrm{Proj}_{\hat{\theta}}(\bullet) = [\mathrm{Proj}_{\hat{\theta}_1}(\bullet_1), \ldots, \mathrm{Proj}_{\hat{\theta}_p}(\bullet_p)]^T$ is defined in Yao and Tomizuka (1996) and Sastry and Bodson (1989) as

$$\mathrm{Proj}_{\hat{\theta}_i}(\bullet_i) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{i\max} \text{ and } \bullet_i > 0, \\ 0 & \text{if } \hat{\theta}_i = \theta_{i\min} \text{ and } \bullet_i < 0, \\ \bullet_i & \text{otherwise.} \end{cases} \tag{20}$$

It can be shown (Yao and Tomizuka, 1996) that for any adaptation function $\tau$, the projection mapping defined in (20) guarantees

$$\begin{aligned} &\text{P1} \quad \hat{\theta} \in \Omega_\theta = \{\hat{\theta} \colon \theta_{\min} \leqslant \hat{\theta} \leqslant \theta_{\max}\}, \\ &\text{P2} \quad \tilde{\theta}^T(\Gamma^{-1} \mathrm{Proj}_{\hat{\theta}}(\Gamma \tau) - \tau) \leqslant 0, \quad \forall \tau. \end{aligned} \tag{21}$$

### 3.4. Controller design

The design combines the adaptive backstepping design in Krstic et al. (1995) with the ARC design procedure in Yao (1997). In the following, the unmeasurable state of the system is replaced by its estimate and the estimation error is dealt with at each step via robust feedback to achieve a guaranteed robust performance. The plant is of relative degree 2, and the design is in two steps.

*Step* 1: Define the output tracking error as $z_1 = y - q_d$. From (7), the derivative of $z_1$ is

$$\dot{z}_1 = x_2 - \theta_2 y - \dot{q}_d. \tag{22}$$

From (13), the unmeasurable state $x_2$ can be expressed as

$$x_2 = \xi_{2,2} + \theta_1 \upsilon_2 - \theta_2 \xi_{1,2} + \theta_3 \psi_2 + \varepsilon_{x2}, \tag{23}$$

where $\varepsilon_{x2} = \varepsilon_2 + \varepsilon_{u2}$ is the estimation error of $x_2$. Substituting (23) into (22), one obtains

$$\begin{aligned} \dot{z}_1 &= \xi_{2,2} + \theta_1 \upsilon_2 - \theta_2(\xi_{1,2} + y) + \theta_3 \psi_2 - \dot{q}_d + \bar{\Delta}_1, \\ \bar{\Delta}_1 &\triangleq \varepsilon_2 + \varepsilon_{u2}. \end{aligned} \tag{24}$$

If the filter state $\upsilon_2$ were the actual control input, one can synthesize for it a virtual control law $\alpha_1$ which consists of two terms given by

$$\alpha_1 = \alpha_{1a} + \alpha_{1s},$$

$$\alpha_{1a} = -\frac{1}{\hat{\theta}_1}\{\xi_{2,2} - \hat{\theta}_2(\xi_{1,2} + y) + \hat{\theta}_3\psi_2 - \dot{q}_d\}, \tag{25}$$

where $\alpha_{1a}$ is the adjustable model compensation, and $\alpha_{1s}$ is a robust control law to be synthesized later. Let $z_2 = v_2 - \alpha_1$ denote the input discrepancy. Substituting (25) into (24) and simplifying the resulting expression, one obtains

$$\dot{z}_1 = \theta_1(z_2 + \alpha_{1s}) - \tilde{\theta}^{\mathrm{T}}\phi_1 + \bar{\Delta}_1, \tag{26}$$

where $\phi_1 \triangleq [\alpha_{1a}, -(\xi_{1,2} + y), \psi_2]^{\mathrm{T}}$.

In Krstic et al. (1995), it needs to incorporate the *tuning functions* in the construction of control functions. Here, due to the use of discontinuous projection (20), the adaptation law (19) is discontinuous and thus *cannot* be used in the control law design at each step since backstepping design requires that the control function synthesized at each step be sufficiently smooth in order to obtain its partial derivatives. In the following, it will be shown that this design difficulty can be overcome by strengthening the robust control law design. The robust control function $\alpha_{1s}$ consists of three terms given by

$$\alpha_{1s} = \alpha_{1s1} + \alpha_{1s2} + \alpha_{1s3}, \quad \alpha_{1s1} = -\frac{1}{\theta_{1\,\min}}k_{1s}z_1, \tag{27}$$

where $\alpha_{1s2}$ and $\alpha_{1s3}$ are robust control functions designed in the following and $k_{1s}$ is any nonlinear feedback gain satisfying

$$k_{1s} \geqslant g_1 + \|C_{\phi 1}\Gamma\phi_1\|^2, \quad g_1 > 0, \tag{28}$$

in which $C_{\phi 1}$ is a positive definite constant diagonal matrix to be specified later. Substituting (27) into (26), one obtains

$$\dot{z}_1 = \theta_1 z_2 - \frac{\theta_1}{\theta_{1\,\min}}k_{1s}z_1 + \theta_1(\alpha_{1s2} + \alpha_{1s3}) - \tilde{\theta}^{\mathrm{T}}\phi_1 + \bar{\Delta}_1. \tag{29}$$

Define a positive semi-definite (p.s.d.) function $V_1$ as

$$V_1 = \tfrac{1}{2}w_1 z_1^2, \tag{30}$$

where $w_1 > 0$ is a weighting factor. From (29), its time derivative satisfies

$$\dot{V}_1 \leqslant \theta_1 w_1 z_1 z_2 - w_1 k_{1s} z_1^2 + w_1 z_1(\theta_1\alpha_{1s2} - \tilde{\theta}^{\mathrm{T}}\phi_1) + w_1 z_1(\theta_1\alpha_{1s3} + \bar{\Delta}_1). \tag{31}$$

From Assumption 1, it follows that

$$\|\tilde{\theta}^{\mathrm{T}}\phi_1\| \leqslant \|\theta_{\mathrm{M}}\|\,\|\phi_1\|, \tag{32}$$

where $\theta_{\mathrm{M}} = \theta_{\max} - \theta_{\min}$. Thus $\|\tilde{\theta}^{\mathrm{T}}\phi_1\|$ is bounded by a known function, which ensures that there exists a robust control function $\alpha_{1s2}$ satisfying the following conditions (Yao, 1997):

$$\text{Condition i: } z_1\{\theta_1\alpha_{1s2} - \tilde{\theta}^{\mathrm{T}}\phi_1\} \leqslant \epsilon_{11},$$

$$\text{Condition ii: } z_1\alpha_{1s2} \leqslant 0, \tag{33}$$

where $\epsilon_{11}$ is a positive design parameter which can be arbitrarily small. Essentially, condition i of (33) shows that $\alpha_{1s2}$ is synthesized to dominate the model uncertainties coming from parametric uncertainties $\tilde{\theta}$ with the level of control accuracy being measured by the design parameter $\epsilon_{11}$, and condition ii is to make sure that $\alpha_{1s2}$ is dissipating in nature so that it does not interfere with the functionality of the adaptive control part $\alpha_{1a}$.

Similarly, from Assumption 1, (24) and (16), it follows that

$$|\bar{\Delta}_1| \leqslant \bar{\delta}_1(t) \triangleq |\varepsilon_2| + \delta_{\varepsilon 2}(t). \tag{34}$$

Note that $\bar{\delta}_1$ is an unknown but bounded function. In principle, the same strategy as in (33) can be used to design a robust control function $\alpha_{1s3}$ to handle the effect of $\bar{\Delta}_1$. However, since the bound of $\bar{\Delta}_1$ is unknown, the level of control accuracy cannot be pre-specified and thus results in a robust control function $\alpha_{1s3}$ (Yao & Tomizuka, 1997a) which satisfies more relaxed conditions than (33)

$$\text{Condition i: } z_1(\theta_1\alpha_{1s3} + \bar{\Delta}_1) \leqslant \epsilon_{12}\bar{\delta}_1^2,$$

$$\text{Condition ii: } z_1\alpha_{1s3} \leqslant 0 \tag{35}$$

with the level of attenuation $\epsilon_{12}$ being a design parameter (Yao & Tomizuka, 1997a), which can be arbitrarily small.

**Remark 2.** One smooth example of $\alpha_{1s2}$ satisfying (33) can be found in the following way. Let $h_1$ be any smooth bounding function satisfying

$$h_1 \geqslant \|\theta_{\mathrm{M}}\|^2\|\phi_1\|^2. \tag{36}$$

Then, $\alpha_{1s2}$ can be chosen as (Yao & Tomizuka, 1997b; Yao, 1997)

$$\alpha_{1s2} = -\frac{h_1}{4\theta_{1\,\min}\epsilon_{11}}z_1. \tag{37}$$

An example of $\alpha_{1s3}$ satisfying (35) is given by Yao and Tomizuka (1997a)

$$\alpha_{1s3} = -\frac{1}{4\theta_{1\,\min}\epsilon_{12}}z_1. \tag{38}$$

Other smooth or continuous examples of the needed robust control functions satisfying conditions (33) and (35) can be worked out in the same way as in Yao (1997) and Yao and Tomizuka (1997a,b).

*Step* 2: From (25), (27) and (17), it is easy to check that $\alpha_1$ is a function of $y$, $t$, $\eta$, $\psi_2$ and $\hat{\theta}$. Thus, the derivative of $\alpha_1$ is given by

$$\dot{\alpha}_1 = \dot{\alpha}_{1c} + \dot{\alpha}_{1u},$$
$$\dot{\alpha}_{1c} = \frac{\partial \alpha_1}{\partial y}(\xi_{2,2} + \hat{\theta}^{\mathrm{T}}\omega) + \frac{\partial \alpha_1}{\partial \eta}\dot{\eta} + \frac{\partial \alpha_1}{\partial \psi_2}\dot{\psi}_2 + \frac{\partial \alpha_1}{\partial t},$$
$$\dot{\alpha}_{1u} = \frac{\partial \alpha_1}{\partial y}(-\tilde{\theta}^{\mathrm{T}}\omega + \bar{\Delta}_1) + \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}}, \quad (39)$$

where $\omega^{\mathrm{T}} = [v_2, -(\xi_{1,2} + y), \psi_2]$. In (39), by replacing $\dot{\eta}$ and $\dot{\psi}$ in terms of their expressions in (18) and (12) respectively, $\dot{\alpha}_{1c}$ is calculable and can be used in the design of control functions, but $\dot{\alpha}_{1u}$ cannot due to various uncertainties. Therefore, $\dot{\alpha}_{1u}$ has to be dealt with via robust feedback in this step of design. From (12) and (39), the derivative of $z_2 = v_2 - \alpha_1$ can be expressed as

$$\dot{z}_2 = u^* - k_2 v_1 - \dot{\alpha}_{1c} - \dot{\alpha}_{1u}. \quad (40)$$

Consider an augmented p.s.d. function $V_2$ given by

$$V_2 = V_1 + \tfrac{1}{2} w_2 z_2^2, \quad w_2 > 0. \quad (41)$$

From (31) and (40), it follows that

$$\dot{V}_2 \leqslant \theta_1 w_1 z_1 z_2 + \dot{V}_1|_{\alpha_1} + w_2 z_2 \dot{z}_2$$
$$= \dot{V}_1|_{\alpha_1} + w_2 z_2 \left\{ \frac{w_1}{w_2}\theta_1 z_1 + u^* - k_2 v_1 - \dot{\alpha}_{1c} - \dot{\alpha}_{1u} \right\}, \quad (42)$$

where $\dot{V}_1|_{\alpha_1}$ is a shorthand notation for $\dot{V}_1$ when $v_2 = \alpha_1$ (or $z_2 = 0$). Similar to (25), the actual ARC control $u^*$ consists of two parts given by

$$u^* = u_{\mathrm{a}}^* + u_{\mathrm{s}}^*,$$
$$u_{\mathrm{a}}^* = -\frac{w_1}{w_2}\hat{\theta}_1 z_1 + k_2 v_1 + \dot{\alpha}_{1c},$$
$$u_{\mathrm{s}}^* = u_{\mathrm{s}1}^* + u_{\mathrm{s}2}^* + u_{\mathrm{s}3}^*, \quad (43)$$
$$u_{\mathrm{s}1}^* = -k_{2\mathrm{s}} z_2, \quad k_{2\mathrm{s}} \geqslant g_2 + \left\| \frac{\partial \alpha_1}{\partial \hat{\theta}} C_{\theta 2} \right\|^2 + \| C_{\phi 2} \Gamma \phi_2 \|^2,$$

where $g_2 > 0$ is a constant, $C_{\theta 2}$ and $C_{\phi 2}$ are positive definite constant diagonal matrices, $u_{\mathrm{s}2}^*$ and $u_{\mathrm{s}3}^*$ are robust control functions to be chosen later. Substituting (43) and (39) into (42) and using similar techniques as in (26), one obtains

$$\dot{V}_2 \leqslant \dot{V}_1|_{\alpha_1} - w_2 k_{2\mathrm{s}} z_2^2 + w_2 z_2 (u_{\mathrm{s}2}^* - \tilde{\theta}^{\mathrm{T}}\phi_2)$$
$$+ w_2 z_2 (u_{\mathrm{s}3}^* + \bar{\Delta}_2) - w_2 z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}}, \quad (44)$$

in which

$$\phi_2 = e_1^* \frac{w_1}{w_2} z_1 - \frac{\partial \alpha_1}{\partial y}\omega,$$
$$\bar{\Delta}_2 = -\frac{\partial \alpha_1}{\partial y}\bar{\Delta}_1, \quad (45)$$

where $e_1^*$ denotes the first basis vector in $\mathbb{R}^3$. Noting (34), one has $|\bar{\Delta}_2| \leqslant |\partial \alpha_1 / \partial y| \bar{\delta}_1$. Similar to (33) and (35), the robust control functions $u_{\mathrm{s}2}^*$ and $u_{\mathrm{s}3}^*$ are chosen to satisfy

$$\text{Condition i: } z_2 (u_{\mathrm{s}2}^* - \tilde{\theta}^{\mathrm{T}}\phi_2) \leqslant \epsilon_{21},$$
$$\text{Condition ii: } z_2 u_{\mathrm{s}j}^* \leqslant 0, \quad j = 2,3, \quad (46)$$
$$\text{Condition iii: } z_2 (u_{\mathrm{s}3}^* + \bar{\Delta}_2) \leqslant \epsilon_{22} \bar{\delta}_1^{\ 2},$$

where $\epsilon_{21}$ and $\epsilon_{22}$ are positive design parameters which can be arbitrarily small. As in Remark 2, examples of $u_{\mathrm{s}2}^*$ and $u_{\mathrm{s}3}^*$ satisfying (46) are given by

$$u_{\mathrm{s}2}^* = -\frac{h_2}{4\epsilon_{21}} z_2,$$
$$u_{\mathrm{s}3}^* = -\frac{1}{4\epsilon_{22}} \left( \frac{\partial \alpha_1}{\partial y} \right)^2 z_2, \quad (47)$$

in which $h_2$ is any smooth bounding function satisfying

$$h_2 \geqslant \|\theta_{\mathrm{M}}\|^2 \|\phi_2\|^2. \quad (48)$$

From (31) and (44), the derivative of $V_2$ satisfies the following inequality:

$$\dot{V}_2 \leqslant -\sum_{j=1}^{2} w_j k_{js} z_j^2 + w_1 z_1 \{\theta_1 \alpha_{1s2} - \tilde{\theta}^{\mathrm{T}}\phi_1\}$$
$$+ w_1 z_1 \{\theta_1 \alpha_{1s3} + \bar{\Delta}_1\} + w_2 z_2 \{u_{\mathrm{s}2}^* - \tilde{\theta}^{\mathrm{T}}\phi_2\}$$
$$+ w_2 z_2 \{u_{\mathrm{s}3}^* + \bar{\Delta}_2\} - w_2 \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} z_2. \quad (49)$$

**Theorem 1.** *Let the parameter estimates be updated by the adaptation law* (19) *in which* $\tau$ *is chosen as*

$$\tau = \sum_{j=1}^{2} w_j \phi_j z_j. \quad (50)$$

*If the controller parameters* $C_{\theta 2}$ *and* $C_{\phi k}$, $k = 1,2$, *are chosen such that* $c_{\phi k r}^2 \geqslant w_k w_2 / 2 c_{\theta 2 r}^2$ *where* $c_{\theta 2 r}$ *and* $c_{\phi k r}$ *are the rth elements of* $C_{\theta 2}$ *and* $C_{\phi k}$, *respectively. Then, the control law* (43) *guarantees that*

A. *In general, the control input and all internal signals are bounded. Furthermore,* $V_2$ *is bounded above by*

$$V_2(t) \leqslant \exp(-\lambda_v t) V_2(0) + \int_0^t \exp(-\lambda_v (t-\tau))(\bar{\epsilon}_{21} + \bar{\epsilon}_{22} \bar{\delta}_1^{\ 2}(\tau))\, \mathrm{d}\tau$$
$$\leqslant \exp(-\lambda_v t) V_2(0) + \frac{\bar{\epsilon}_{21} + \bar{\epsilon}_{22} \|\bar{\delta}_1\|_\infty^2}{\lambda_v} \times [1 - \exp(-\lambda_v t)], \quad (51)$$

*where* $\lambda_v = 2\min\{g_1, g_2\}$, $\bar{\epsilon}_{21} = \sum_{j=1}^{2} w_j \epsilon_{j1}$, $\bar{\epsilon}_{22} = \sum_{j=1}^{2} w_j \epsilon_{j2}$, *and* $\|\bar{\delta}_1\|_\infty$ *stands for the infinity norm of* $\bar{\delta}_1$.

B. *If after a finite time* $t_0$, $\tilde{d} = 0$, *i.e., in the presence of parametric uncertainties only, then, in addition to results in A, asymptotic output tracking (or zero final tracking error) is also achieved.*

Proof of the theorem is given in the appendix.

**Remark 3.** Results in A of Theorem 1 indicate that the proposed controller has an exponentially converging transient performance with the exponentially converging rate $\lambda_v$ and the final tracking error being able to be adjusted via certain controller parameters freely in a known form; it is seen from (51) that $\lambda_v$ can be made arbitrarily large, and $\bar{\epsilon}_{21} + \bar{\epsilon}_{22}\|\bar{\delta}_1\|_\infty^2/\lambda_v$, the bound of $V_2(\infty)$ (an index of the final tracking errors), can be made arbitrarily small by increasing the feedback gains $g_i$ and/or decreasing the controller parameters $\epsilon_{i1}$ and $\epsilon_{i2}$. Theoretically, this result is what a well-designed robust controller can achieve. In fact, when the parameter adaptation law (19) is switched off, the proposed ARC law becomes a deterministic robust control law and results in A of Theorem 1 remain valid (Yao & Tomizuka, 1996, 1997b).

B of Theorem 1 implies that the effect of parametric uncertainties is eliminated via parameter adaptation law and an improved tracking performance—asymptotic tracking—is achieved without using any discontinuous or infinite gain feedback. Theoretically, result B is what a well-designed adaptive controller can achieve.

**Remark 4.** It is seen from (51) that the transient tracking error is affected by the initial value $V_2(0)$. To further reduce the transient tracking error, the idea of filter initialization (Yao & Tomizuka, 1997b; Krstic et al., 1995) can be used to render $V_2(0) = 0$.

## 4. Comparative experiments

### 4.1. Experimental setup

To test the proposed nonlinear ARC strategy and study fundamental problems associated with high-speed/high-acceleration/high-accuracy motion control of linear motor drive systems, a two-axis positioning stage is set up as a test-bed. As shown in Fig. 1, the test-bed consists of four major components: a precision $X$–$Y$ stage with two integrated linear drive motors, two linear encoders, a servo controller, and a host PC. The two axes of the $X$–$Y$ stage are mounted orthogonally on a horizontal plane with the $Y$-axis on top of the $X$-axis. A particular feature of the setup is that the two linear motors are of different type: the $Y$-axis is driven by an Anorad LEM-S-3-S linear motor (epoxy core) and the $X$-axis is driven by an Anorad LCK-S-1 linear motor (iron core). They represent the two most commonly used linear motors and have different characteristics. The resolution of the encoders is 1 μm after quadrature. In the experiments, only the $Y$-axis is used.

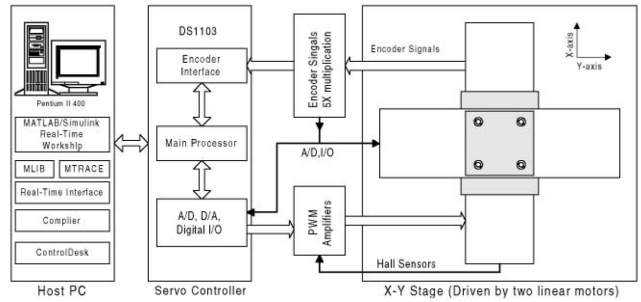


Fig. 1. Experimental setup.

Standard least-squares identification is performed to obtain the parameters of the $Y$-axis. The nominal values of $M$ is 0.027 V/m/s$^2$, which is equivalent to $\theta_1 = 37$. To test the learning capability of the proposed ARC algorithm, a 9.1 kg load is mounted on the motor in the experiments and the identified values of the parameters are

$$\theta_1 = 10, \quad \theta_2 = 2.73. \tag{52}$$

The bounds of the parameter variations are chosen as

$$\begin{aligned} \theta_{\min} &= [8.3, 2.4, -50]^{\mathrm{T}}, \\ \theta_{\max} &= [50, 17.5, 50]^{\mathrm{T}}. \end{aligned} \tag{53}$$

### 4.2. Performance index

As in Yao et al. (1997), the follow performance indexes will be used to measure the quality of the control algorithm:

- $\|e\|_{\mathrm{rms}} = \sqrt{(1/T_{\mathrm{f}})\int_0^{T_{\mathrm{f}}}|e|^2\,\mathrm{d}t}$, the rms of the tracking error, is used as an objective numerical measure of *average tracking performance* for an entire error curve $e(t)$, where $T_{\mathrm{f}}$ represents the total running time;
- $e_{\mathrm{M}} = \max_t \{|e(t)|\}$, the maximum absolute value of the tracking error, is used as an index of measure of *transient performance*;
- $e_{\mathrm{f}} = \max_{T_{\mathrm{f}}-2 \leqslant t \leqslant T_{\mathrm{f}}} \{|e(t)|\}$, the maximum absolute value of the tracking error during the last 2 s, is used as an index of measure of *final tracking accuracy*;
- $\|u\|_{\mathrm{rms}} = \sqrt{1/T_{\mathrm{f}}\int_0^{T_{\mathrm{f}}}|u|^2\,\mathrm{d}t}$, the rms of the control input, is used to evaluate the amount of *control effort*;
- $c_u = \|\Delta u\|_{\mathrm{rms}}/\|u\|_{\mathrm{rms}}$, the normalized control variations, is used to measure the *degree of control chattering*, where

$$\|\Delta u\|_{\mathrm{rms}} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}|u(j\Delta T) - u((j-1)\Delta T)|^2}. \tag{54}$$

is the rms of the control input increments.

### 4.3. Comparative experimental results

Experiments are performed with the $Y$-axis. The control system is implemented using a dSPACE DS1103 controller board. The controller executes programs at a sampling frequency $f_s = 2.5$ kHz. The following two controllers are compared:

*PID*: PID control with feedforward compensation—consider the linear motor system described by (4), and assume that the following variables are available (either measured or computed) for control implementation; $q(t), q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$. The velocity signal $\dot{q}(t)$ is obtained by the difference of two consecutive position measurements. If the parameters and nonlinear friction term of (4) are known, the control objective can be achieved with the following PID control law:

$$u = M\ddot{q}_d(t) + B\dot{q}(t) + F_{\mathrm{fn}}(\dot{q}) - K_p e - K_i \int e\,\mathrm{d}t - K_d\dot{e}, \tag{55}$$

where $e \triangleq q - q_d$. Closing the loop by applying (55) to (4) easily leads to the closed-loop characteristic equation

$$s^3 + \frac{K_d}{M}s^2 + \frac{K_p}{M}s + \frac{K_i}{M} = 0. \tag{56}$$

By placing the closed-loop poles at desired locations, the design parameters $K_p$, $K_i$ and $K_d$ can thus be determined. In the experiments, since $M$ and $B$ are unknown parameters, instead of using (55) the following control law is used:

$$u = \hat{M}(0)\ddot{q}_d(t) + \hat{B}(0)\dot{q}(t) + \hat{F}_{\mathrm{fn}}(\dot{q}) - K_p e - K_i \int e\,\mathrm{d}t - K_d\dot{e}, \tag{57}$$

where $\hat{M}(0)$ and $\hat{B}(0)$ are the fixed parameter estimates chosen as 0.05 and 0.24, respectively. $\hat{F}_{\mathrm{fn}}(\dot{q})$ is the friction compensation term which depends on the velocity $\dot{q}$ and is chosen as $(0.2/\pi)\arctan(900\dot{q})$. By placing all the three closed-loop poles at $-300$ when $M = M_{\min} = 0.02$, one obtains $K_p = 5.4\times10^3$, $K_i = 5.4\times10^5$ and $K_d = 18$.

*ARC*: Adaptive robust control—the output feedback ARC law proposed in Section 3. All the roots of the observer polynomials are placed at $s = -200$ which leads to $k_1 = 400$ and $k_2 = 4\times10^4$. The controller parameters are: $w_1 = 1$, $g_1 = 600$, $\epsilon_{11} = \epsilon_{12} = 5\times10^{-5}$, $C_{\phi1} = 10^{-5}\cdot\mathbf{diag}[5, 0.5, 10]$; $w_2 = 0.1$, $g_2 = 850$, $\epsilon_{21} = 1\times10^{-3}$, $\epsilon_{22} = 1$, $C_{\phi2} = C_{\phi1}$, $C_{\theta2} = 10^5 I_3$. The adaptation rate is $\Gamma = 10^5\cdot\mathbf{diag}[5,3,20]$. The estimated friction compensation term $\hat{F}_{\mathrm{fn}}(\dot{q}_d)$ is chosen as $(0.2/\pi)\arctan(900\dot{q}_d)$. The initial parameter estimates are chosen as $\hat{\theta}(0) = [30,10,0]^{\mathrm{T}}$.

To test the tracking performance of the proposed algorithm, the following two typical reference trajectories are considered.

*Case* 1: Tracking a sinusoidal trajectory $y_r = 0.05\sin(4t)$. Comparative experiments are run for tracking a sinusoidal trajectory. The desired trajectory is generated by a stable second order system:

$$\ddot{q}_d + \beta_1\dot{q}_d + \beta_2 q_d = \ddot{q}_{\mathrm{r}} + \beta_1\dot{q}_{\mathrm{r}} + \beta_2 q_{\mathrm{r}}, \tag{58}$$

where $\beta_1 = 100$ and $\beta_2 = 2500$. By choosing $q_d(0) = y(0)$ and setting all remaining filter initial conditions to zero (i.e., $\dot{q}_d(0) = 0$, $\ddot{q}_d(0) = 0$, $\lambda(0) = 0$, $\eta(0) = 0$ and $\psi(0) = 0$), one has $V_2(0) = 0$ for an improved transient performance as explained in Remark 4. The following test sets are performed:

*Set* 1: To test the nominal tracking performance of the controllers, the motor is run without payload, which is equivalent to $\theta_1 = 37$;

*Set* 2: To test the performance robustness of the algorithms to parameter variations, a 9.1 kg payload is mounted on the motor, which is equivalent to $\theta_1 = 10$;

*Set* 3: A large step disturbance (a simulated 0.5 V electrical signal) is added at $t = 2.2$ s and removed at $t = 7.2$ s to test the performance robustness of each controller to disturbance.

The experimental results in terms of performance indexes are given in Table 1. As seen from the table, in terms of performance indexes $e_{\mathrm{M}}$ and $e_{\mathrm{F}}$, PID performs poorly for all three sets, but with a slightly lesser degree of control input chattering. One may argue that the performance of PID control can be further improved by increasing the feedback gains. However, in practice,

Table 1
Experimental results

| Controller | Set 1 | | Set 2 | | Set 3 | |
|---|---|---|---|---|---|---|
| | PID | ARC | PID | ARC | PID | ARC |
| $e_{\mathrm{M}}$ (μm) | 51.2 | 14.4 | 156 | 113 | 85.8 | 55.9 |
| $e_{\mathrm{F}}$ (μm) | 15.1 | 6.88 | 21.2 | 6.71 | 16.1 | 6.30 |
| $\|\|e\|\|_{\mathrm{rms}}$ (μm) | 2.99 | 1.88 | 8.04 | 4.32 | 4.35 | 3.26 |
| $\|\|u\|\|_{\mathrm{rms}}$ (V) | 0.19 | 0.20 | 0.21 | 0.21 | 0.40 | 0.42 |
| $\|\|\Delta u\|\|_{\mathrm{rms}}$ (V) | 0.05 | 0.06 | 0.06 | 0.06 | 0.04 | 0.08 |
| $c_u$ | 0.28 | 0.32 | 0.28 | 0.30 | 0.11 | 0.19 |

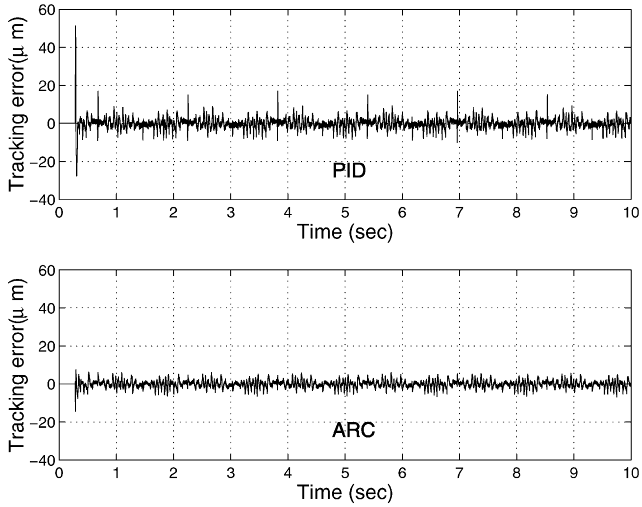


Fig. 2 . Tracking errors for sinusoidal trajectory without load.

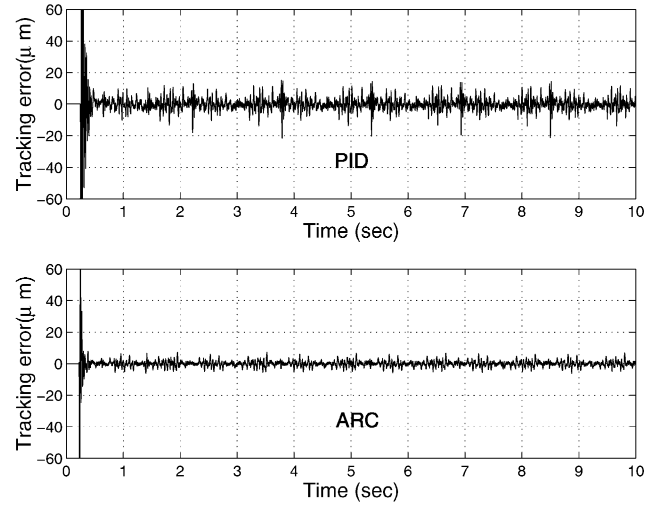


Fig. 3. Tracking errors for sinusoidal trajectory with load.

feedback gains have upper limits because the bandwidth of every physical system is finite. To verify this claim, the closed-loop poles of the PID controller are placed at $-320$ instead of $-300$, which is translated into PID gains of $K_p = 6144$, $K_i = 655360$ and $K_d = 19.2$. With these gains, the closed-loop system is found to be unstable in the experiments. This indicates that the closed-loop bandwidth that a PID controller can achieve in implementation has been pushed almost to its limit and not much further performance improvement can be expected from PID controllers. Thus, in order to realize the high-acceleration/high-speed/high-accuracy potential of a linear motor system, a PID controller even with feedforward compensation may not be sufficient.

For Set 1, the tracking errors are given in Fig. 2. It shows that the ARC controller achieves very good nominal tracking performance. For Set 2, the tracking errors are given in Fig. 3 (The tracking errors are chopped off.). It shows the ARC controller achieves good tracking performance in spite of the change of inertia load. The tracking errors for Set 3 are given in Fig. 4. As seen from the figures, the added large disturbance does not affect the performance of ARC much, except for the spike when the sudden change of the disturbance occurs. This result illustrates the performance robustness of the ARC design.

*Case* 2: High-acceleration/high-speed point-to-point motion trajectory (without load). A fast point-to-point motion trajectory with high-acceleration/deceleration, which runs back and forth several times, is shown in Fig. 5. The trajectory has a maximum velocity of $v_{\max} = 1$ m/s and a maximum acceleration of $a_{\max} = 12$ m/s$^2$. The tracking errors of PID and ARC are shown in Fig. 6. As seen, the proposed ARC has a much better performance than PID. Furthermore, during the zero velocity portion of motion, the ARC tracking error is within $\pm 1$ μm.

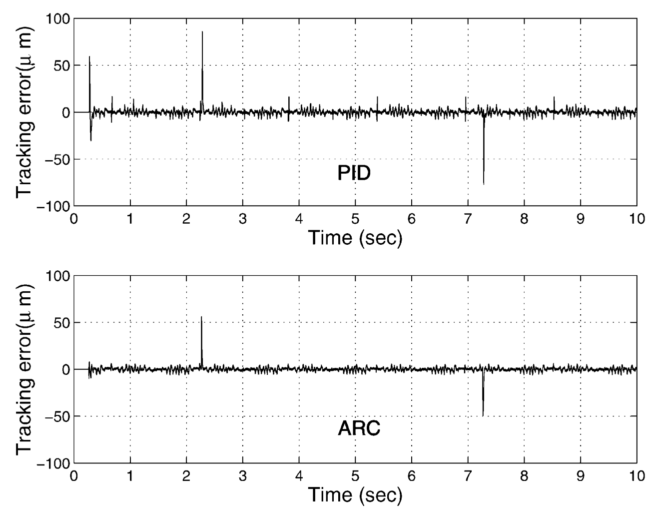


Fig. 4 . Tracking errors for sinusoidal trajectory with disturbance.

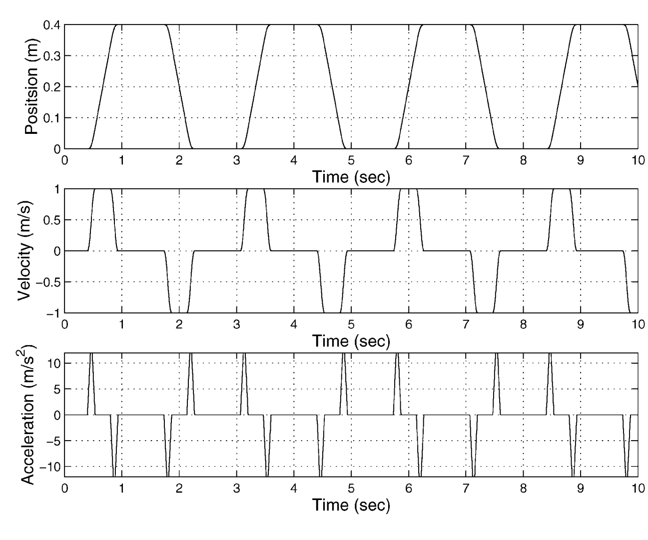


Fig. 5. High-acceleration/high-speed point-to-point motion trajectory.

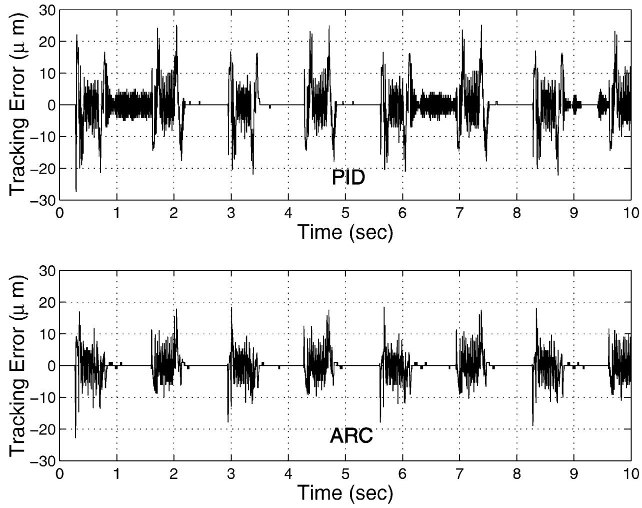


Fig. 6. Tracking errors for high-acceleration/high-speed point-to-point motion trajectory.

## 5. Conclusions

In this paper, an output feedback ARC scheme based on discontinuous projection method has been developed for high performance robust motion control of linear motors. The proposed controllers take into account the effect of model uncertainties coming from the inertia load, friction force, force ripple and bounded external disturbances. The proposed controller uses on-line parameter adaptation to compensate for the effect of nonlinear disturbances that can be modeled. The uncompensated disturbances and the estimation errors of the unmeasurable states are effectively handled via certain robust feedback to achieve a robust performance. The resulting controller achieves a guaranteed transient performance and a prescribed final tracking accuracy in the presence of both parametric uncertainties and bounded disturbances. In the presence of parametric uncertainties only, asymptotic output tracking is achieved without using an infinite fast switching control law or an infinite-gain feedback. Comparative experimental results are obtained for the motion control of an epoxy core linear motor. Experimental results illustrate the high performance of the proposed ARC strategy.

## Appendix

**Proof of Theorem 1.** From (49), (33), (35) and (46), it follows that:

$$\dot V_2 \leqslant \sum_{j=1}^{2} w_j\left\{ -\left(g_j + \left\|\frac{\partial\alpha_{j-1}}{\partial\hat\theta}C_{\theta j}\right\|^2 + \|C_{\phi j}\Gamma\phi_j\|^2\right)z_j^2 + \varepsilon_{j1} + \varepsilon_{j2}\bar\delta_1{}^2\right\} - w_2\frac{\partial\alpha_1}{\partial\hat\theta}\dot{\hat\theta}z_2, \tag{A.1}$$

in which the fact that $\partial\alpha_0/\partial\hat\theta = 0$ is used in the above concise description of $\dot V_2$ via summation. By completion of square,

$$-\frac{\partial\alpha_1}{\partial\hat\theta}\dot{\hat\theta}z_2 \leqslant |z_2|\left|\frac{\partial\alpha_1}{\partial\hat\theta}C_{\theta 2}C_{\theta 2}^{-1}\dot{\hat\theta}\right| \leqslant \left\|\frac{\partial\alpha_1}{\partial\hat\theta}C_{\theta 2}\right\|^2 z_2^2 + \frac{1}{4}\|C_{\theta 2}^{-1}\dot{\hat\theta}\|^2. \tag{A.2}$$

Noting that $C_{\theta 2}^{-1}$ and $\Gamma$ are diagonal matrices, from (19) and (A.2), one obtains

$$\|C_{\theta 2}^{-1}\dot{\hat\theta}\|^2 = \|C_{\theta 2}^{-1}\,\mathrm{Proj}_{\hat\theta}(\Gamma\tau)\|^2 \leqslant \|C_{\theta 2}^{-1}\Gamma\tau\|^2 \leqslant \left(\sum_{k=1}^{2}\|C_{\theta 2}^{-1}\Gamma w_k\phi_k z_k\|\right)^2 \leqslant 2\left(\sum_{k=1}^{2}\|C_{\theta 2}^{-1}\Gamma\phi_k\|^2 w_k^2 z_k^2\right). \tag{A.3}$$

Thus, if $C_{\theta 2}$ and $C_{\phi k}$ satisfy the conditions in the theorem, from (A.2) and (A.3), it follows that:

$$-w_2\frac{\partial\alpha_1}{\partial\hat\theta}\dot{\hat\theta}z_2 \leqslant w_2\left\|\frac{\partial\alpha_1}{\partial\hat\theta}C_{\theta 2}\right\|^2 z_2^2 + \frac{1}{2}w_2\sum_{k=1}^{2}\|C_{\theta 2}^{-1}\Gamma\phi_k\|^2 w_k^2 z_k^2 \leqslant w_2\left\|\frac{\partial\alpha_1}{\partial\hat\theta}C_{\theta 2}\right\|^2 z_j^2 + \sum_{k=1}^{2} w_k\|C_{\phi k}\Gamma\phi_k\|^2 z_k^2. \tag{A.4}$$

From (A.4) and (A.1), it can be verified that

$$\dot V_2 \leqslant \sum_{j=1}^{2} w_j g_j z_j^2 + \sum_{j=1}^{2} w_j(\epsilon_{j1} + \epsilon_{j2}\bar\delta_1^2) \leqslant -\lambda_v V_2 + \bar\epsilon_{21} + \bar\epsilon_{22}\bar\delta_1^2, \tag{A.5}$$

which leads to (51). Following the standard adaptive control arguments as in Krstic et al. (1995), it can be proved that all internal signals are bounded. A of Theorem 1 is thus proved.

The following is to prove B of the theorem. In the presence of parametric uncertainties only (i.e., $\tilde d = 0$), from (15) and (24), $\bar\Delta_1 = \varepsilon_2$. With the robust control functions given by (38) and (47), it is easy to check that $|z_1(\theta_1\alpha_{1s3} + \bar\Delta_1)| \leqslant \epsilon_{12}\varepsilon_2^2$ and $|z_2(u_{s3}^* + \bar\Delta_2)| \leqslant \epsilon_{22}\varepsilon_2^2$. Thus, noting (50) and condition ii of (33), (35) and (46), from (49) and (A.4), one has

$$\dot V_2 \leqslant \sum_{j=1}^{2}(-w_j\tilde\theta^{\mathrm T}\phi_j z_j - w_j g_j z_j^2 + w_j\epsilon_{j2}\varepsilon_2^2) = -\sum_{j=1}^{2} w_j g_j z_j^2 - \tilde\theta^{\mathrm T}\tau + \bar\epsilon_{22}\varepsilon_2^2. \tag{A.6}$$

Define a new p.d. function $V_\theta$ as

$$V_\theta = V_2 + \tfrac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} + \gamma\varepsilon^{\mathrm{T}}P\varepsilon, \tag{A.7}$$

where $\gamma \geqslant \bar{\epsilon}_{22}$. Noting P2 of (21) and the fact that $\dot{\varepsilon} = A_0\varepsilon$, from (A.7) and (11), the derivative of $V_\theta$ satisfies

$$\dot{V}_\theta \leqslant -\sum_{j=1}^{2} w_j g_j z_j^2 - \tilde{\theta}^{\mathrm{T}}\tau + \bar{\epsilon}_{22}\varepsilon_2^2 + \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\theta}} - \gamma\|\varepsilon\|^2$$

$$\leqslant -\sum_{j=1}^{2} w_j g_j z_j^2. \tag{A.8}$$

Therefore, $z \in L_2^2$. It is also easy to check that $\dot{z}$ is bounded. So, $z \to 0$ as $t \to \infty$ by the Barbalat's lemma, which leads to B of Theorem 1. □

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**Li Xu** received his B.S. degree from the University of Electronic Science and Technology of China in 1991, his M.S. degree from Tsinghua University, China, in 1997. He is currently a Ph.D. student in the School of Mechanical Engineering at Purdue University, West Lafayette. His research interests include coordinated control of electro-mechanical systems, adaptive robust control, and nonlinear observer design. Mr. Xu was the recipient of the Best Student Paper Award at the 2000 IEEE Conference on Control Applications (CCA), and a finalist for the ASME Dynamic System and Control Division Student Best Paper Award at the 2000 International Mechanical Engineering Congress and Exposition (IMECE).

**Bin Yao** received his Ph.D. degree in Mechanical Engineering from the University of California at Berkeley in February 1996. Since 1996, he has been an Assistant Professor in the school of Mechanical Engineering at Purdue University.

Dr. Yao's research focuses on the development of a general framework for the design of high performance adaptive robust control algorithms. Integration of the approach with actual sensor properties, physical properties, and control-oriented modeling has been carried out for different applications such as the control of electro-hydraulic systems, linear motors, machine tools, and robot manipulators. He experimentally verified the approach on the precision control of various electro-mechanical/hydraulic systems. Dr. Yao is the recipient of the 1997 Caterpillar Engineering Young Faculty Development Fund for his work on the electro-hydraulic control, and the 1998 National Science Foundation CAREER award for his work on the engineering synthesis of high performance adaptive robust controllers for mechanical systems and manufacturing processes. His research interests include design and coordinated control of intelligent high performance electro-mechanical/electro-hydraulic systems, optimal adaptive and robust control, nonlinear observer design and neural networks for virtual sensing, modeling, fault detection, diagnostics, and adaptive fault-tolerant control, and data fusion.