Advanced Motion Control: An Adaptive Robust Control Framework

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Abstract— **The rapid advances in microelectronics and microprocessor technologies during the past decades have made the physical integration of mechanical systems, various sensors, and computer based control implementation platform rather affordable and a standard choice for any modern precision machines. Such a hardware configuration enables the control of the overall system to be constructed in the same way as what a human brain normally does - seamless integration of the fast reaction (or instantaneous feedback reaction) to immediate feedback information and the slow learning utilizing large amount of stored past information that is available in the computer based control systems. The theoretically solid nonlinear adaptive robust control (ARC) theory that has been developed recently well reflects such an intuitive integrated design philosophy of human brains, and has been experimentally demonstrated achieving better control performance than existing nonlinear robust controls (e.g., sliding mode controls) or nonlinear adaptive controls in a number of motion control applications. This paper is to expose motion control engineers to the essences of such an advanced nonlinear control design methodology. Some recent ARC research results are discussed as well. The precision motion control of a linear motor driven high-speed/high-acceleration X-Y positioning stage is used as a case study and comparative experimental results are presented to illustrate the performance and practical benefits that can be achieved by the proposed ARC approach in implementation.**

I. Introduction

As the society steps into the era of micro and nano-technology, modern mechanical systems are often required to produce motion accuracy down to micro and/or nano-meter range. At the same time, these machines also have to operate at high speeds to yield high productivity. Such an increasingly tight control performance requirement puts a great challenge to control community and forces control engineers to look beyond traditional linear control theory for more advanced model-based nonlinear controllers to have a better handling of the avoidable nonlinear effects (e.g., the nonlinear Coulomb friction and the highly coupled nonlinear dynamics of robot manipulators) for high performance. There has been an exponential growth in nonlinear control research during the past decades, with major advances and breakthroughs reported in both the nonlinear deterministic robust control (DRC) area [1], [2], [3], [4] and adaptive control area [5], [6], [7]. Systematic and constructive nonlinear control design methodologies such as the backstepping technique [8] have been proposed.

Recently, a new approach, adaptive robust control (ARC) [9], [10], [11], [12], was developed to preserve the advantages of both adaptive control and DRC while overcoming their practical performance limitations for a reasonably large class of nonlinear systems. Specifically, the following categories of adaptive robust controllers have been developed: (i) the smooth projection based full state feedback ARC designs [10], (ii) the discontinuous projection based full state feedback ARC design [9], [13] that has a more stable parameter adaptation process for a better performance in implementation, (iii) the desired compensation ARC controllers [14] that reduce the effect of measurement noise and have a faster adaptation rate in implementation to improve overall tracking performance, (iv) the saturated adaptive robust controller (SARC) [15] developed for uncertain nonlinear systems in the "chain-of-integrator" form in the presence of practical constraint of control input saturation, (v) the partial state feedback ARC scheme [16] that incorporates a nonlinear observer to recover the unmeasured states associated with the dynamic uncertainties for better performance, (vi) the output feedback ARC schemes [17], [18] that need the output measurement sensor only, (vii) the indirect adaptive robust control (IARC) designs [19] that, in addition to good control performance, achieve the secondary goal of having as accurate parameter estimates as possible for purposes such as machine health monitoring and prognostic, (ix) the neural network adaptive robust controls [20], [21] that incorporates the universal approximation capability of neural networks in learning general nonlinearities into the ARC designs to enlarge the applicable systems of the proposed ARC theory, and (x) the adaptive robust repetitive controls [22] that utilizes repetitive learning for applications having repetitive tasks.

The proposed ARC approach has also been applied to the control of precision mechanical systems driven by rotary [23] or linear electro-magnetic motors with different physical characteristics [24], [25], [26], [27], and the electro-hydraulic systems [28], [29], [30], [31], [32], [33] in various specific applications. Extensive comparative experimental results have been obtained to verify the effectiveness of the proposed ARC approach and the significant improvement in the tracking accuracy of motion over the existing methods.

The theoretical breakthrough and the significant performance improvement of the proposed ARC in various implementations make the approach an ideal choice for industrial applications demanding stringent performance.

At the same time, the by-product of the approach – accurate parameter and nonlinearity estimations – makes adding intelligent features such as prognostic to the system possible. It is thus beneficial for motion control engineers to get exposed to such an advanced nonlinear control design methodology and to master how the method can be used to build intelligent and yet precision mechatronic systems, which is the main objective of the paper.

II. Adaptive Robust Control Framework

To avoid getting bugged down to the technical design complexity, in this section, the tracking control of a simple first order nonlinear systems with uncertainties will be used to illustrate the advantages and limitations of different types of adaptive robust controls.

A. Direct Adaptive Robust Control (DARC)

The simplest ARC design is the direct ARC designs presented in [12], [9], in which learning laws such as parameter adaptations are synthesized along with the control law to achieve the sole purpose of reducing output tracking error as illustrated below. For simplicity, consider the first-order nonlinear system described by

$$\dot{x} = f(x,t) + u, \qquad f = \varphi^T(x)\theta + \Delta(x,t) \tag{1}$$

where $x, u \in R$, and f is an unknown nonlinear function. In general, f can be approximated by a group of known basis functions $\varphi(x) \in R^p$ with unknown weights $\theta \in R^p$, and the approximation error is denoted by the unknown nonlinear function $\Delta(x,t)$. The objective is to let x track its desired trajectory $x_d(t)$ as closely as possible. The following reasonable and practical assumption is made, which is satisfied by most applications:

A1 . The extent of parametric uncertainties and uncertain nonlinearities is known, i.e.,

$$\begin{array}{lcl} \theta & \in & \Omega_\theta \stackrel{\Delta}{=} \{\theta : \ \theta_{min} < \theta < \theta_{max} \ \} \\ \Delta & \in & \Omega_\Delta \stackrel{\Delta}{=} \{\Delta : \ \|\Delta(x,t)\| \le \delta(x,t) \ \} \end{array} \tag{2}$$

where $\theta_{min}, \ \theta_{max}$ and $\delta(x,t)$ are known. $\diamondsuit$

Under Assumption A1, the discontinuous projection based ARC design [9] can be applied to solve the robust tracking control problem for (1). Specifically, the parameter estimate $\hat{\theta}$ is updated through a parameter adaptation law having the form given by

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\tau) \tag{3}$$

where Γ is any symmetric positive definite (s.p.d.) adaptation rate matrix, τ is an adaptation function to be specified later, and the projection mapping $Proj_{\hat{\theta}}(\bullet)$ is defined by (for simplicity, assume that Γ is a diagonal matrix in the following)

$$Proj_{\hat{\theta}}(\bullet) = \begin{cases} 0 & if \quad \begin{cases} \hat{\theta}_i = \hat{\theta}_{imax} & and \quad \bullet > 0 \\ \hat{\theta}_i = \hat{\theta}_{imin} & and \quad \bullet < 0 \end{cases} \\ \bullet & \qquad otherwise \end{cases} \tag{4}$$

It can be shown [12] that the projection mapping has the following nice properties

$$\begin{array}{ll} \text{P1} & \hat{\theta} \in \bar{\Omega}_\theta = \{\hat{\theta} : \ \theta_{min} \le \hat{\theta} \le \theta_{max}\} \\ \text{P2} & \tilde{\theta}^T(\Gamma^{-1} Proj_{\hat{\theta}}(\Gamma\bullet) - \bullet) \le 0, \quad \forall \bullet \end{array} \tag{5}$$

The ARC control law consists of two parts given by

$$\begin{array}{ll} u = u_f + u_s, & u_f = \dot{x}_d(t) - \varphi^T\hat{\theta} \\ & u_s = u_{s1} + u_{s2}, \qquad u_{s1} = -kz \end{array} \tag{6}$$

where $z = x - x_d$ is the tracking error. In (6), u_f is the adjustable model compensation needed for achieving perfect tracking, and u_s is the robust control law consisting of two parts: u_{s1} is used to stabilize the nominal system, which is a simple proportional feedback in this case; and u_{s2} is a robust feedback used to attenuate the effect of model uncertainties, which is synthesized to satisfy the following two constraints

$$\begin{array}{ll} \text{i} & z[-\varphi^T\tilde{\theta} + \Delta(x,t) + u_{s2}] \le \varepsilon \\ \text{ii} & z u_{s2} \le 0 \end{array} \tag{7}$$

where ε is a positive design parameter representing the attenuation level of the model uncertainties that one would like to have. In (7), condition i is used to represent the fact that u_{s2} is synthesized to dominate the model uncertainties coming from both the parametric uncertainties and uncertain nonlinearities to achieve a guaranteed level of attenuation ε, and the passive-like constraint ii is imposed to make sure that introducing u_{s2} does not interfere with the nominal identification process of parameter adaptation. The specific forms of u_{s2} satisfying constraints like (7) can be found in ARC designs in [11], [10], [9].

Theorem 1: [9] If the adaptation function in (3) is chosen as

$$\tau = \varphi(x)z \tag{8}$$

then, the ARC law (6) with the parameter adaptation law (8) guarantees that

A. In general, all signals are bounded and the tracking error is bounded by

$$|z|^2 \le exp(-2kt)|z(0)|^2 + \tfrac{\varepsilon}{k}[1 - exp(-2kt)] \tag{9}$$

i.e., the tracking error exponentially decays to a ball. The exponential converging rate $2k$ and the size of the final tracking error ($|z(\infty)| \le \sqrt{\frac{\varepsilon}{k}}$) can be freely adjusted by the controller parameters ε and k in a *known* form.

B. If after a finite time, there exist parametric uncertainties only (i.e., $\Delta(x,t) = 0, \ \forall t \ge t_0$), then, in addition to the results in A, asymptotic tracking or zero final tracking error is achieved, i.e, $z \longrightarrow 0$ as $t \longrightarrow \infty$. Furthermore, if the desired trajectory satisfies the following persistent excitation (PE) condition

$$\int_t^{t+T} \varphi(x_d(\nu))\varphi^T(x_d(\nu))d\nu \ge \varepsilon_p I_p \qquad \forall t \ge t_0 \tag{10}$$

where T, t_0 and ε_p are some positive scalars, then, the estimated parameter $\hat{\theta}$ converges to their true values as well (i.e., $\tilde{\theta} \longrightarrow 0$ when $t \longrightarrow \infty$). $\triangle$

Remark 1: In the absence of parameter adaptation (i.e., $\Gamma = 0$), the proposed ARC law reduces to a deterministic robust control (DRC) law and Result A of Theorem 1 still holds. Therefore, the adaptation loop can be switched off at any time without affecting the stability and the guaranteed output tracking transient performance. However, such a control law does not discriminate the difference between parametric uncertainties and uncertain nonlinearities and results in a conservative design since Result B of Theorem 1 is lost. As for adaptive control [8], the proposed ARC uses certain coordination mechanisms (e.g., the discontinuous projection mapping used in (3)) and robust feedback control u_s to achieve a guaranteed output tracking transient performance even in the presence of uncertain nonlinearities (A of Theorem 1) while without losing its nominal performance (B of Theorem 1). $\triangle$

B. Desired Compensation ARC (DCARC)

In the DARC design presented in the above subsection, the regressor $\varphi(x)$ in the model compensation u_f in (6) and the parameter adaptation function (8) depends on the state x. Such an adaptation structure may have some potential implementation problems. Firstly, the regressor $\varphi(x)$ has to be calculated on-line based on the actual measurement of the state x. Thus, the effect of measurement noise may be severe, and a slow adaptation rate may have to be used, which in turn reduces the effect of parameter adaptation. Secondly, despite that the intention of introducing u_f is for model compensation, because of $\varphi(x)$, u_f depends on the actual feedback of the state also. Although theoretically the effect of this added implicit feedback loop has been considered in the robust control law design as seen from condition i of (7), practically, there still exists certain interactions between the model compensation u_f and the robust control u_s. This may complicate the design of the robust control law and the controller gain tuning process in implementation. To by pass these problems, the desired compensation ARC (DCARC) [14] can be used as explained in the following.

For simplicity, denote the desired regressor as $\varphi_d(t) = \varphi(x_d(t))$. Let the regressor error be $\tilde{\varphi} = \varphi(x) - \varphi_d$. Noting that θ is unknown but bounded as assumed in (2), there exists a known function $\delta_\phi(x,t)$ such that

$$|\tilde{\varphi}^T\theta| = |\varphi(x)^T\theta - \varphi(x_d)^T\theta| \leq \delta_\phi(x,t)|z| \tag{11}$$

The desired compensation ARC law and the adaptation function have the same forms as (6) and (8) respectively but with the desired regressor $\varphi_d(t)$ and a strengthened robust control u_s, which are given by

$$\begin{aligned} u = u_f + u_s, \quad & u_f = \dot{x}_d(t) - \varphi_d^T(t)\hat{\theta} \\ & u_s = u_{s1} + u_{s2}, \quad u_{s1} = -k_{s1} z \\ \tau = \varphi_d(t) z & \end{aligned} \tag{12}$$

where k_{s1} can be any nonlinear gain satisfying

$$k_{s1} \geq k + \delta_\phi(x,t) \tag{13}$$

and u_{s2} is required to satisfy constraints similar to (7) with the constraint i modified to

$$\text{i.} \qquad z[-\varphi_d^T\tilde{\theta} + \Delta(x,t) + u_{s2}] \leq \varepsilon \tag{14}$$

Theorem 2: [14] If the DCARC law (12) is applied, the same results as stated in Theorem 1 are achieved. $\triangle$

C. Indirect Adaptive Robust Control (IARC)

The underline parameter adaptation law (8) in DARC and DCARC in the above two subsections are based on the direct adaptive control designs [34], in which the adaptive control law and parameter adaptation law are synthesized simultaneously to meet the sole objective of reducing the output tracking error. Such a design normally leads to a controller whose dynamic order is as low as the number of unknown parameters to be adapted while achieving excellent output tracking performance as done in [11], [10], [9]. However, the direct approach also has the drawback that the design of adaptive control law and the parameter estimation law cannot be separated and the choice of the parameter estimation law is limited to the gradient type with certain actual tracking errors as driving signals. It is well known that the gradient type of estimation law may not have as good parameter convergence properties as other types of parameter estimation laws (e.g., the least square method). Furthermore, although the desired trajectory might be persistently exciting and of large signal, for a well designed direct adaptive control law, the actual tracking errors in implementation are normally very small, and thus are more prone to be corrupted by other factors such as the sampling delay and noise that have been neglected when synthesizing the parameter adaptation law. As a result, in implementation, the parameter estimates in the direct adaptive control are normally not accurate enough to be used for secondary purposes such as prognostics and machine component health monitoring, even when the desired trajectory is persistently exciting enough.

For the applications that need accurate parameter estimates for other secondary purposes in addition to the good output tracking performance, the indirect adaptive robust control design presented in [19] can be used, which completely separates the construction of parameter estimation law from the design of underline robust control law as illustrated as follows.

One of the key elements of the ARC design [12], [10] is to use the practical available prior process information to construct projection type adaptation law for a controlled learning process even in the presence of disturbances. In the above ARC designs, the discontinuous projection mapping (4) is used for its simplicity to ease implementation. However, theoretically, such a discontinuous projection mapping is valid only for diagonal adaptation rate matrix Γ, which is not a problem for the above direct ARC designs that use gradient type adaptation laws only. For the indirect ARC introduced below, as the least square type adaptation law will be used to achieve better convergency of parameter estimations, the adaptation rate matrix will be time-varying and non-diagonal. As such, the standard projection mapping $Proj_{\hat{\theta}}(\bullet)$ in the adaptive control [35], [8] should be used to keep the parameter estimates within the known bounded set $\bar{\Omega}_\theta$, the closure of the set Ω_θ. The expression of $Proj_{\hat{\theta}}(\Gamma\tau)$ is

$$\begin{cases} \Gamma\tau, & \text{if} \quad \hat{\theta} \in \overset{\circ}{\Omega}_\theta \quad \text{or} \quad n_{\hat{\theta}}^T \Gamma\tau \leq 0 \\ \left(I - \Gamma \frac{n_{\hat{\theta}} n_{\hat{\theta}}^T}{n_{\hat{\theta}}^T \Gamma n_{\hat{\theta}}}\right) \Gamma\tau & \hat{\theta} \in \partial\Omega_\theta \quad \text{and} \quad n_{\hat{\theta}}^T \Gamma\tau > 0 \end{cases} \tag{15}$$

where $\Gamma(t)$ can be any time-varying positive definite symmetric matrix. In (15), $\overset{\circ}{\Omega}_\theta$ and $\partial\Omega_\theta$ denote the interior and the boundary of Ω_θ respectively, and $n_{\hat{\theta}}$ represents the outward unit normal vector at $\hat{\theta} \in \partial\Omega_\theta$. Such a projection mapping has the same nice properties as in (5).

With the use of the projection type adaptation law structure (15), the parameter estimates are bounded within known bounds, regardless of the estimation function τ to be used. As a result, the same adaptive robust control law as in the direct ARC designs (i.e., (6) and (7)) can be used to achieve a guaranteed transient and final output tracking accuracy, independent of the specific identifier to be used later. Thus, the reminder of the IARC design is to construct suitable estimation functions τ so that an improved final tracking accuracy– zero final tracking error in the presence of parametric uncertainties only–can be obtained with an emphasis on good parameter estimation process as well. For this purpose, it is assumed that the system is absence of uncertain nonlinearities, i.e., $\Delta = 0$ in (1). Using any filters with a stable transfer function $H_f(s)$ having relative degree no less than 1, the filtered system dynamics is obtained as

$$\dot{x}_f = \varphi_f^T \theta + u_f \tag{16}$$

where $x_f = H_f[x]$, $\varphi_f = H_f[\varphi(x)]$, and $u_f = H_f[u]$ are the filter output, regressor, and input respectively. Define the estimation output and its estimate as

$$y = \dot{x}_f - u_f, \qquad \hat{y} = \varphi_f^T \hat{\theta} \tag{17}$$

With the calculable prediction error defined as $\epsilon = \hat{y} - y$, the resulting static prediction error model is linearly parameterized in terms of parameter estimation error $\tilde{\theta}$ as

$$\epsilon = \varphi_f^T \tilde{\theta} \tag{18}$$

Various estimation algorithms can then be used to identify unknown parameters [19]. For example, when the least squares type estimation algorithm with co-variance resetting [36] and exponential forgetting [6] is used, the resulting adaptation law is given by (15), in which $\Gamma(t)$ is updated by

$$\dot{\Gamma} = \alpha\Gamma - \Gamma \frac{\varphi_f \varphi_f^T}{1 + \nu \varphi_f^T \Gamma \varphi_f} \Gamma, \quad \Gamma(t_r^+) = \rho_0 I, \quad \nu \geq 0 \tag{19}$$

where $\nu = 0$ leads to the unnormalized algorithm, and τ is defined as

$$\tau = -\frac{\varphi_f \epsilon}{1 + \nu \varphi_f^T \Gamma \varphi_f} \tag{20}$$

In (19), α is the forgetting factor, t_r is the covariance resetting time, i.e., the time when $\lambda_{min}(\Gamma(t)) = \rho_1$ where ρ_1 is a pre-set lower limit for $\Gamma(t)$ satisfying $0 < \rho_1 < \rho_0$.

With the above estimator and the adaptive robust control law, it is shown in [19] that the same theoretical output tracing performance results as in DARC in Theorem 1 are achieved.

D. Integrated Direct/Indirect Adaptive Robust Control (DIARC)

The above IARC design overcome the poor parameter estimates of the DARC designs through the complete separation of the parameter estimation law design from the underline robust control law. In addition, on-line explicit monitoring of signal excitation level can be employed in implementation to significantly improve the accuracy of parameter estimates. Because of these algorithm improvements, the resulting parameter estimates are normally accurate enough to be used for secondary purposes such as machine health monitoring and prognostics.

As shown in the comparative experimental results [37], though the proposed IARC design has a much better accuracy of parameter estimates than the direct ARC, the output tracking performances of IARC are not as good as those of DARC, especially during the transient periods. A more detailed thorough analysis reveals that the poorer tracking performance of IARC is caused by the loss of dynamic compensation type fast adaptation that is inherited in the DARC designs. To overcome this loss of tracking performance problem of IARC, an integrated direct/indirect ARC (DIARC) design framework is developed in [38]. The design not only uses the same adaptation process as in the IARC design [19] for accurate estimation of physical parameters, but also introduces dynamic compensation type fast adaptation to achieve a better transient performance as illustrated below.

For (1), the resulting DIARC law is:

$$\begin{aligned} &u = u_a + u_s, \quad u_a = u_{a1} + u_{a2}, \quad u_s = u_{s1} + u_{s2}, \\ &\quad u_{a1} = -\varphi^T \hat{\theta} + \dot{x}_d(t), \quad u_{a2} = -\hat{d}_c \\ &\quad u_{s1} = -k_{s1} z, \quad u_{s2} = -k_{s2}(x,t) z \end{aligned} \tag{21}$$

In (21), u_{a1} represents the usual model compensation with the physical parameter estimates $\hat{\theta}(t)$ updated using the same indirect parameter estimator as in the above IARC design, u_{a2} is a model compensation term similar to the fast dynamic compensation type model compensation used in the DARC design, in which $\hat{d}_c$ can be thought as the estimate of the low frequency component of the lumped model uncertainties defined later. From (1) and (21), the error equation is obtained as

$$\dot{z} = u_s + u_{a2} - \varphi^T \tilde{\theta} + \Delta \tag{22}$$

Define a constant d_c and time varying $\Delta^*(t)$ such that

$$d_c + \Delta^*(t) = -\varphi^T \tilde{\theta} + \Delta \tag{23}$$

Conceptually, (23) lumps the disturbances and the model uncertainties due to physical parameter estimation error together and divides it into the static component (or low frequency component in reality) d_c and the high frequency components $\Delta^*(t)$, so that the low frequency component

d_c can be compensated through fast adaptation similar to those in the above direct ARC design as follows.

Let d_{cM} be any pre-set bound and use this bound to construct the following projection type adaptation law for $\hat{d}_c(t)$

$$\dot{\hat{d}}_c = \begin{cases} 0 & \text{if} \quad |\hat{d}_c| = d_{cM} \text{ and } \hat{d}_c z > 0 \\ \gamma_d z & \quad \text{else} \end{cases} \tag{24}$$

with $\gamma_d > 0$ and $|\hat{d}_c(0)| \leq d_{cM}$. Such an adaptation law guarantees that $|\hat{d}_c(t)| \leq d_{cM}, \forall t$.

Substituting (23) into (22) and noting (21),

$$\begin{aligned} \dot{z} &= u_s + u_{a2} + d_c + \Delta^*(t) \\ &= u_{s1} + \left[u_{s2} - \tilde{d}_c + \Delta^*(t) \right] \end{aligned} \tag{25}$$

Due to the use of projection type adaptation law, all estimation errors are bounded within known bounds. As such, the same as in the DARC, it can be shown that, as long as the nonlinear feedback gain k_{s2} is chosen large enough, the same robust performance condition as (7) can be satisfied

$$z \left[u_{s2} - \tilde{d}_c + \Delta^* \right] \leq \varepsilon_c \tag{26}$$

With the above estimator and the adaptive robust control law, it can be shown [38] that theoretically the same output tracing performance results as in DARC in Theorem 1 are achieved.

III. Precision Motion Control of Linear Motor Drive Systems

All the proposed ARC designs have been applied to the precision motion control of a linear motor drive system [24]. The details on how the ARC control laws are implemented are given in [37]. This section only gives some typical experimental results for illustration purpose.

A typical high-speed/high-acceleration motion trajectory for the pick-and-place operations in industry is used in all experiments. The desired trajectory has a movement of $0.4m$ with a maximum speed of $1m/s$ and an acceleration of $12m/sec^2$. The experimental results in terms of the quantitative indexes defined in [24] are given in Table 1 with time history given in Figs.1-2.

TABLE I

	without load			with load		
controller	D	I	DI	D	I	DI
e_M (μm)	10.4	13.0	10.7	18.4	14.9	10.7
e_F (μm)	10.4	12.7	9.2	10.8	12.7	9.3
$L_2[e]$ (μm)	1.84	3.32	1.66	1.64	3.36	1.76
$L_2[u]$ (V)	0.28	0.29	0.28	0.45	0.46	0.46
$L_2[\Delta u]$ (V)	0.10	0.11	0.11	0.10	0.10	0.10
c_u	0.34	0.38	0.39	0.21	0.23	0.22

As seen from these results, the tracking errors of all the controllers are very small, which are within $20\mu m$ over the entire run. For both no load and load cases, the parameter estimates of IARC and DIARC algorithms are better than that of DARC, especially the inertial load (not shown) and the friction estimates (Fig.2 and 3). However, the tracking performances of DARC and DIARC controllers are better than that of IARC as seen from Fig.1. Overall, DIARC achieves the best tracking performance while having more robust parameter estimation process and accurate parameter estimates than DARC.

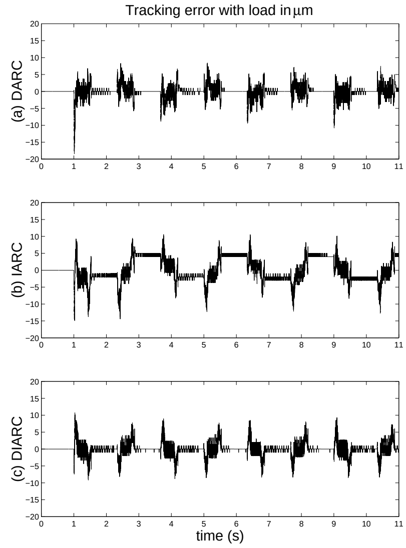

Fig. 1. Tracking error for (a)DARC, (b)IARC, (c)DIARC with load

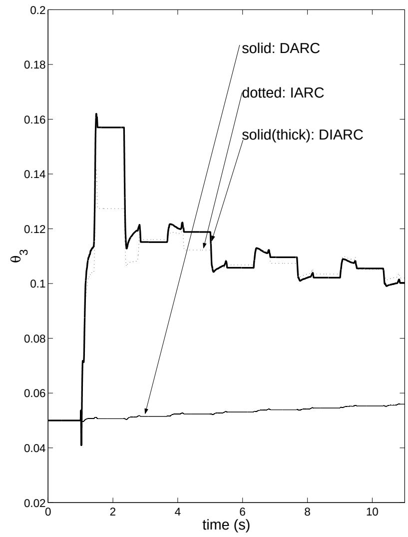

Fig. 2. $\hat{\theta}_3$ for (a)DARC, (b)IARC, (c)DIARC without load

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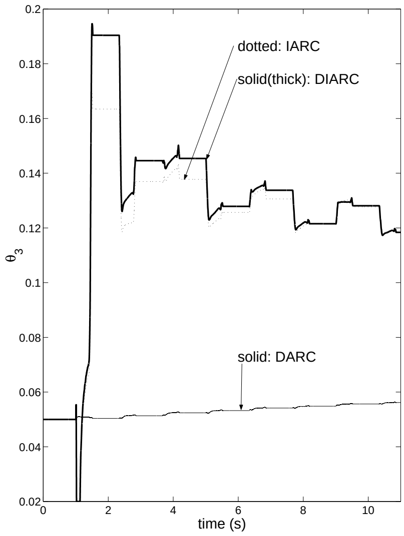

Fig. 3. $\hat{\theta}_3$ for (a)DARC, (b)IARC, (c)DIARC with load

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