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Advanced Motion Control: From Classical PID to Nonlinear Adaptive Robust Control

(Plenary Paper)

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***Abstract*—The ever increasingly stringent performance requirements of modern mechanical systems have forced control engineers to look beyond traditional linear control theory for more advanced nonlinear controllers. During the past decade, a mathematically rigorous nonlinear adaptive robust control (ARC) theory has been developed and has been experimentally demonstrated achieving significant performance improvement in a number of motion control applications. This plenary paper first uses a simple motion control problem as an example to bring out the conceptual connection and nonlinear extension of the widely used PID controller structure to the developed ARC approach. Through this example, some of the key underlying working mechanisms of the ARC theory can be grasped easily. The paper then highlights how major issues in the precision motion control can be handled systematically and effectively with the ARC framework. The issues considered include (i) large variations of physical parameters of a system; (ii) unknown nonlinearities such as cogging and ripple forces of linear motors; (iii) dynamic uncertain nonlinearities with non-uniformly detectable unmeasured internal states (e.g., friction described by dynamic models in high precision motion controls); and (iv) control input saturation due to limited capacity of physical actuators. The precision motion control of a linear motor driven high-speed/high-acceleration industrial gantry is used as a case study and comparative experimental results are presented to illustrate the achievable performance and limitations of various ARC controllers in implementation.**

I. Introduction

Modern mechanical systems such as microelectronics manufacturing equipment, robot manipulators, and automatic inspection machines are often required to operate at high speeds in order to yield high productivity. At the same time, as the society moves into the era of micro and nano-technology, these systems are also required to have motion accuracy in the micro or nano-meter range. These industrial trends put an urgent demand on *performance oriented* advanced controls that can effectively handle (i) nonlinearities due to the strong nonlinear coupling effect of multiple-degree-of-freedom (MDOF) mechanical systems during high-speed operations, and (ii) the unavoidable uncertainties associated with modeling of any physical system. The modeling uncertainties may come from either (i) parameters of the system model which may depend on operating conditions and may not be precisely known in advance (e.g., inertia of an object grasped by a robot), or (ii) nonlinearities which cannot be modeled exactly (e.g., nonlinear friction characteristics) or terms which change with time randomly (e.g., external disturbances); the former is referred to as *parametric uncertainties* and the latter as *uncertain nonlinearities* in this paper. To meet these ever increasingly stringent control performance requirements, control engineers have been forced to look beyond traditional linear control theory for more advanced nonlinear controllers which can deal with various nonlinearities and model uncertainties directly. There has been an exponential growth in nonlinear control research during the past two decades, with major advances and breakthroughs reported in both the nonlinear robust control (NRC) area [1]–[9] as well as the adaptive or robust adaptive control (RAC) area [10]–[17], along with systematic nonlinear control design methodologies such as the backstepping technique [17], [18].

During the past decade, a mathematically rigorous nonlinear adaptive robust control (ARC) theory has also been developed to lay a solid foundation for the design of a new generation of controllers which will help industry build modern machines of great performance and high intelligence [19]–[29] . The developed ARC theory bridges the gap between two of the main control research areas - robust adaptive controls (RAC) and nonlinear robust controls (NRC). Traditionally, those two research areas have been presented as competing control design approaches, with each having its own benefits and limitations. By integrating the fundamentally different working mechanisms of the two approaches, the developed ARC theory is able to preserve the theoretical performance results of both design approaches while overcoming their well-known practical performance limitations. The developed ARC theory

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has also been applied to the design of various intelligent and precision industrial mechatronic systems, including electro-hydraulic actuator driven large-scale mechanical systems [30]–[32], energy-saving control via novel programmable valves [33]–[35], machine tools [36], robot manipulators [37], [38], electrical motor driven mechanical systems with control accuracy down to sub-micrometer levels (e.g.,linear motor driven positioning stages [20], [39]–[41] and ultra-high density hard disk drives [42], [43]), and piezo-electrical actuator driven nano-positioning systems for nanotechnology applications [44], [45]. The results from this comprehensive set of applied research projects have shown ARC to be a powerful approach to the control of a wide variety of mechatronic systems. Other researchers have demonstrated the effectiveness of adaptive robust control in various applications as well (e.g., the control of pneumatic muscles driven parallel manipulators [46]–[48], hard disk drives [49], [50], active suspension systems [51], [52], and vehicles [53]).

The theoretical breakthrough and the significant performance improvement of the proposed adaptive robust control (ARC) in various implementations make the approach an ideal choice for industrial applications demanding stringent performance. In addition, the indirect adaptive robust controls (IARC) [54] and the integrated direct/indirect adaptive robust controls (DIARC) [23] also enable accurate on-line parameter estimations in actual implementation [55]. This by-product of the approach – accurate parameter and nonlinearity estimations – makes possible to add intelligent features such as the automated on-board modeling [34] and the fault detection and prognosis [56]. It is thus beneficial for motion control engineers to get familiar with this advanced nonlinear control design methodology and to learn how the method can be used to build intelligent and yet precision motion systems. For this purpose, instead of seeking mathematical rigor, this paper will focus on physical interpretation of the underlying working mechanisms of the ARC theory and use simple examples to bring out the unique features of different types of ARC controllers. In particular, the precision motion control of a linear motor driven high-speed/high-acceleration industrial gantry will be used as a case study and comparative experimental results are presented to illustrate the achievable performance and limitations of various ARC controllers in implementation.

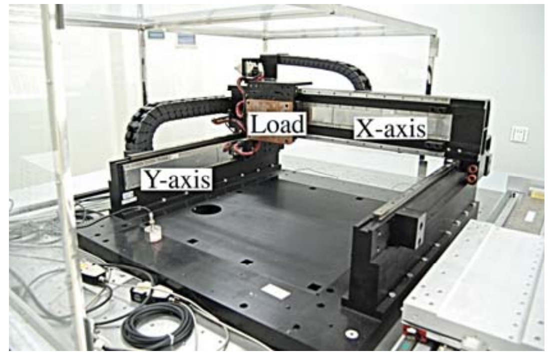

Fig. 1. A linear motor driven biaxial industrial gantry system

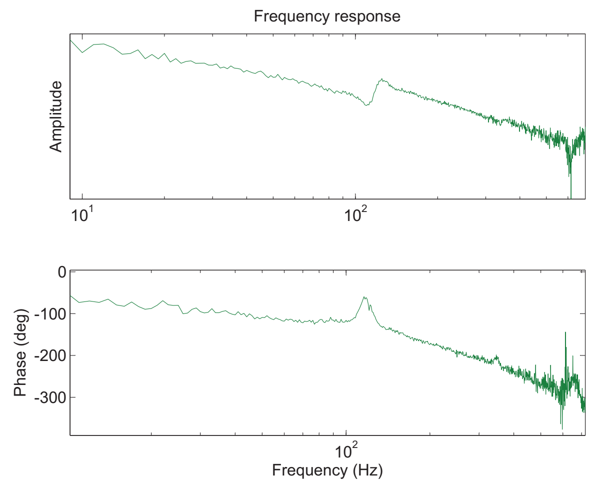

Fig. 2. Identified frequency response of X-axis

II. Typical Issues in Precision Motion Control

To study fundamental problems associated with high-speed/high-acceleration motion control of iron-core linear motor drive systems, a two-axis X-Y Anorad HERC-510-510-AA1-B-CC2 gantry by Rockwell Automation has been set up at Zhejiang University as a test-bed. As shown in Fig. 1, the two axes of the X-Y stage are mounted orthogonally with X-axis on top of Y-axis. The resolution of the encoders is 0.5 μm after quadrature. The velocity signal is obtained by the difference of two consecutive position measurements. This section uses the control of this system as an example to bring out some of the major issues to be addressed in precision motion control. To avoid duplication, in the following, only experimental results for X-axis are presented.

The specifications given by the manufacturer indicate that the electrical dynamics of the current amplifiers for the linear motors have bandwidths in KHz range. To verify this and to determine an appropriate dynamic model that should be used for control design, a an input signal which is sum of 1000 sinusoidal signals ranging from 1 Hz to 5 KHz with a sampling frequency of 10 KHz is applied to the system. The obtained experimental data set is then processed using the standard system identification toolbox in MATLAB to obtain the estimated frequency response of the X-axis from the input voltage to the output velocity. As seen from the identified frequency response in Fig. 2, the system has a mechanical resonance mode a slightly above 100Hz. Therefore, within the frequency range of 100Hz, only the rigid body dynamics of the stage need to be considered and, if the targeted closed-loop bandwidth is substantially below 100 Hz, all mechanical resonance modes and the electrical dynamics could be neglected. In this case, a suitable dynamic model of the X-axis linear motor stage would be

$$M\ddot{x} = u - B\dot{x} - F_l, \quad F_l = F_f + F_r + F_d, \tag{1}$$

where x represents the position of the inertia load along the

X-axis stage, M is the normalized[1] mass of the inertia load plus the coil assembly, u is the input voltage to the motor, B is the equivalent viscous friction coefficient of the stage, F_l is the normalized lumped effect of all other forces acting on the stage including the Coulomb friction force F_f, the cogging force F_r, and any external disturbance F_d (e.g. cutting force in machining). In the above model, the magnetic saturation effect is ignored for simplicity, as the working range of the positioning stage is normally restricted to the linear region of the generated torque to the current command to the amplifier. Should the magnetic saturation effect become an issue, the nonlinear models of magnetic saturation presented in [57], [58] may be used to come out a better description of the input behavior and similar analysis as presented in the paper can be carried out.

Let $y_r(t)$ be the reference motion trajectory, which is assumed to be known, with bounded derivatives up to the third order. The objective is to synthesize a control input u such that the output $y = x$ follows $y_r(t)$ with a prescribed transient performance and a guaranteed steady-state tracking accuracy. It is easy to see that major hurdles in achieving such a control objective include (i) large variation of system parameters (e.g., the mass of the inertia load); (ii) uncertain nonlinear cogging force F_r; (iii) uncertain nonlinear Coulomb friction F_f, which may not be adequately modeled as a static nonlinear function of the velocity $\dot{x}$ only [59], [60]; (iv) input saturation as u is typically limited within certain range, i.e., $|u| \leq u_M$ for some known constant u_M; and (v) the neglected high frequency dynamics such as the mechanical resonance modes seen in Fig. 2. In following sections, various strategies will be presented to handle these hurdles using the ARC theory.

III. A Simplistic Direct Adaptive Robust Controller

This section considers a simplified motion control problem in which the system parameters M and B in (1) are assumed to be known perfectly and the stage does not interact with external environment (i.e., $F_d = 0$). For this scenario, a simplistic direct adaptive robust controller (DARC) will be developed and compared with the widely used PID controller structure, from which some of the key underlying working mechanisms of the developed ARC theory can be grasped relatively easily.

Though the cogging force F_r and the Coulomb friction F_f are nonlinear and may not be known exactly in reality, they are always bounded and their bounds are normally known. For example, the cogging force and the Coulomb friction of the X-axis of the gantry studied in this paper are within 10 N (or 0.145 Volt in terms of input voltage) and 14N (or 0.2 Volt in terms of input voltage) respectively. Thus, though the lumped force F_l in (1) may not be known, it is bounded with known bounds, i.e.,

$$|F_l| < F_M \tag{2}$$

[1]Normalized with respect to the unit voltage of the control input.

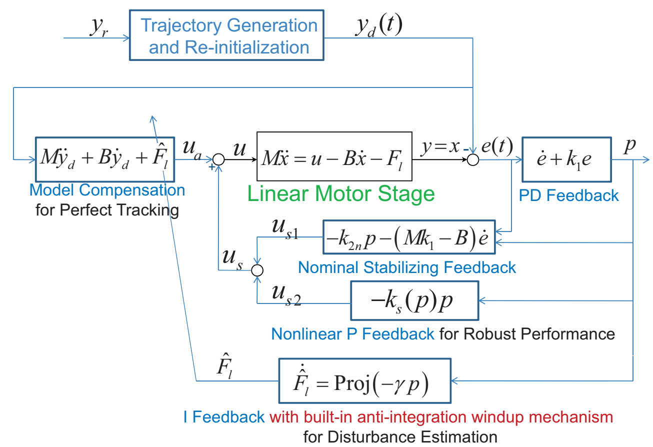

Fig. 3. A simplistic direct adaptive robust controller

where F_M is a known constant (e.g., for the X-axis, one can set $F_M = 0.345$ Volt). With this information, a DARC shown in Fig. 3 can be constructed. Specifically, let $y_d(t)$ be a desired motion trajectory to be tracked by the stage. For the stage to be able to track $y_d(t)$ perfectly, it is necessary that the initial conditions are matched, i.e.,

$$y_d(0) = x(0), \quad \dot{y}_d(0) = \dot{x}(0) \tag{3}$$

and the ideal control input for perfect tracking, determined by the stage dynamics (1), is within the actuator limit, i.e.,

$$\forall t, \ |u_d(t)| \leq u_M, \quad u_d = M\ddot{y}_d(t) + B\dot{y}_d(t) + F_l \tag{4}$$

In general, the reference command $y_r(t)$ provided by the user may not satisfy either the initial value matching conditions (3) (e.g., the usual step input type reference command) or the actuator limitation (4) and other limits (e.g., the acceleration and the velocity of the reference command exceed the physical limits of the stage). Through proper trajectory generation algorithm with the initial value of $y_d(t)$ chosen according to (3), one can always generate a feasible desired trajectory $y_d(t)$ that converges to the reference command $y_r(t)$ with a prescribed transient response since there are no uncertainties involved in this trajectory planing process [28], [29].

In reality, due to model uncertainties, even with a feasible trajectory $y_d(t)$, the ideal control action for perfect tracking of $y_d(t)$ in (4) cannot be generated. Thus, the best one can do is to use the estimated values of unknown terms to obtain a control action which is close to the ideal action. This is done by the model compensation u_a shown in Fig. 3, in which $\hat{F}_l$ represents the estimated value of the actual lumped model uncertainty F_l. However, with this approximate model compensation u_a, perfect tracking may not be achieved anymore and certain feedback action is needed to keep the output tracking error $e = y - y_d$ within the allowed tolerance band. Noting that the actual system dynamics (1) is of second order and has matched uncertainties only (i.e., the uncertainty F_l and the input u are present in the same channel in the system dynamics), one can simplify the feedback control design significantly by regulating the quantity p shown in Fig. 3 instead of the output tracking error $e = y - y_d$. p essentially represents a proportional plus derivative (PD) feedback of output tracking error and is given by

$$p = \dot{e} + k_1 e = \dot{y} - \dot{y}_{eq}, \quad \dot{y}_{eq} = \dot{y}_d - k_1 e, \tag{5}$$

where k_1 is a positive feedback gain. It is easy to see that if p is kept small or is regulated to converge to zero exponentially, then the output tracking error e will be small or converge to zero exponentially since the transfer function from p to e, $G_p(s) = \frac{e(s)}{p(s)} = \frac{1}{s+k_1}$, is stable. In this sense, controlling the output tracking error e is the same as regulating p. The control input and the model uncertainty are related to p through a first-order dynamics given by

$$M\dot{p} = M\ddot{y} - M\ddot{y}_{eq} = u - B\dot{y} - F_l - M\ddot{y}_{eq} \tag{6}$$

where $\ddot{y}_{eq} = \ddot{y}_d - k_1\dot{e}$. Thus, the ARC control law shown in Fig. 3 can be obtained

$$\begin{aligned} u = u_a + u_s, \quad & u_a = M\ddot{y}_d + B\dot{y}_d + \hat{F}_l \\ u_s = u_{s1} + u_{s2}, \quad & u_{s1} = -k_{2n}p - (Mk_1 - B)\dot{e} \\ & u_{s2} = -k_s(p)p \end{aligned} \tag{7}$$

where k_{2n} is any positive constant and $k_s(p)$ is a nonlinear gain. With this control law, the following error dynamics are obtained

$$M\dot{p} + k_{2n}p = u_{s2} + \tilde{F}_l \tag{8}$$

where $\tilde{F}_l = \hat{F}_l - F_l$ represents the estimation error of the lumped model uncertainties. It is thus clear that, in the absence of model uncertainties, i.e., $\tilde{F}_l = 0$, the closed-loop system is stable without u_{s2}. In this sense, u_{s1} represents a nominal stabilizing feedback. The purpose of introducing additional feedback action u_{s2} is to achieve a guaranteed robust performance when model uncertainties exist. Such an objective can be accomplished if u_{s2} satisfies the following conditions

$$\begin{aligned} &\text{i} \quad p u_{s2} \le 0 \\ &\text{ii} \quad p\{u_{s2} + \tilde{F}_l\} \le \varepsilon \end{aligned} \tag{9}$$

where ε is a design parameter. Essentially, i of (9) ensures that u_{s2} is dissipating in nature and ii of (9) shows that u_{s2} is synthesized to dominate the model uncertainties $\tilde{F}_l$ for robust stability and certain guaranteed robust performance quantified by ε.

Theoretically, as long as the estimation error of the lumped model uncertainties $\tilde{F}_l$ is bounded by a known function of states, as shown in [27]–[29], there always exists a continuous or sufficiently smooth u_{s2} such that (9) is satisfied for any ε, which could be arbitrarily small. If a fixed estimate of $\hat{F}_l = \hat{F}_{l0}$ is used as in the traditional nonlinear robust controls, noting (2), $|\tilde{F}_l| < |\hat{F}_{l0}| + F_M$. Thus $\tilde{F}_l$ would be uniformly bounded and a robust feedback u_{s2} satisfying (9) can be found. However, with such a fixed estimate, the estimation error $\tilde{F}_l$ always exists, which leads to non-zero steady-state output tracking error even when the lumped model uncertainties F_l are constant. To overcome this performance limitation, on-line

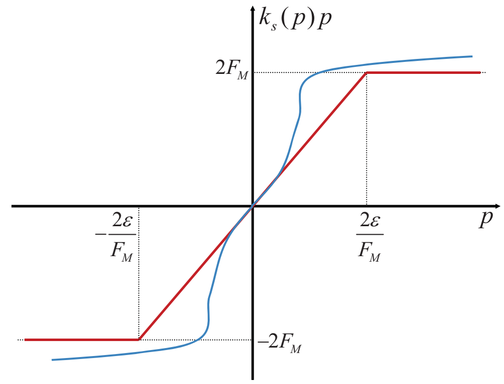

Fig. 4. Nonlinear robust feedback design

estimation of F_l should be used. For unknown but constant F_l, when the standard gradient type parameter adaptation law is used, the resulting on-line estimation $\hat{F}_l(t)$ would be updated by

$$\dot{\hat{F}}_l(t) = -\gamma p \quad \text{or} \quad \hat{F}_l(t) = \hat{F}_l(0) - \gamma \int_0^t p(\tau)d\tau \tag{10}$$

where γ is any positive adaptation rate. However, such a parameter adaptation law does not guarantee that the on-line estimate $\hat{F}_l(t)$ will be bounded by a known function. Without a bounded $\hat{F}_l(t)$, the existence of the robust feedback u_{s2} satisfying (9) is in question, which is the reason why the traditional adaptive controls do not have a guaranteed transient response. Thus, one of the key elements of the ARC design [19], [24] is to use the practically available prior process information to construct projection type adaptation law for a controlled learning process in general. The resulting parameter adaptation is shown in Fig. 3, in which Proj $(\cdot)$ represents the standard projection mapping used in robust adaptive controls [10], [17] and can be implemented as [27]

$$\dot{\hat{F}}_l(t) = \begin{cases} 0, & \text{if} \quad \hat{F}_l \ge F_M \text{ and } p \le 0 \\ 0, & \quad \hat{F}_l \le -F_M \text{ and } p \ge 0 \\ -\gamma p, & \text{else} \end{cases} \tag{11}$$

It is clear that such a projection type adaptation law with $|\hat{F}_l(0)| < F_M$ always guarantees that $|\hat{F}_l(t)| \le F_M, \forall t$. Thus, noting (2), $|\tilde{F}_l(t)| < 2F_M, \forall t$. With this known bound on $|\tilde{F}_l(t)|$, it can be verified that any nonlinear proportional feedback of p shown in Fig. 4 will satisfy (11) as long as the non-negative nonlinear gain $k_s(p) \ge 0$ is chosen such that

$$\begin{aligned} k_s(p) \ge \tfrac{F_M^2}{\varepsilon}, \quad & \text{when } |p| \le \tfrac{2\varepsilon}{F_M} \\ k_s(p)|p| \ge 2F_M, \quad & \text{when } |p| > \tfrac{2\varepsilon}{F_M} \end{aligned} \tag{12}$$

Such a nonlinear robust feedback, with gains chosen to satisfy (12) with equal sign is shown by the red straight-line segments in Fig. 4 and any function above this red straight-line segments in the first quadrant (e.g., the curve in blue color) satisfies (12). With this DARC controller, the following theoretical performance can be obtained [29]:

Theorem 1: With the DARC law (7) and estimation of the lumped uncertainties given by (11), the following results hold:

A. In general, all signals of the closed-loop (CL) system are bounded and the output tracking is guaranteed to have a prescribed transient and steady-state performance in the sense that the output tracking error is bounded above in magnitude by a known function which exponentially converges to the ball of $\{e(\infty) : |e(\infty)| < \frac{1}{k_1}\sqrt{\frac{\varepsilon}{k_{2n}}}\}$, with a converging rate no less than $min\{k_1, k_{2n}/M\}$. In particular,

$$p^2(t) \leq \exp\left(-\frac{2k_{2n}}{M}t\right)\left[p^2(0) - \frac{\varepsilon}{k_{2n}}\right] + \frac{\varepsilon}{k_{2n}} \qquad (13)$$

B. When the lumped model uncertainty F_l becomes constant after a finite time, in addition to the results stated in A, the on-line estimate $\hat{F}_l(t)$ asymptotically converges to F_l and the output tracking error asymptotically converges to zero or zero steady-state error is obtained. △

Some of the underlying working mechanisms of the developed ARC theory can now be illustrated through various connections of the above DARC controller to the widely used PID controller structures as follows:

A. Connection to PD Controllers and Role of Fast Robust Feedback

Let us first consider the situation when the adaptation loop in Fig. 4 is turned off, i.e., let $\gamma = 0$ or $\hat{F}_l = \hat{F}_{l0}$ where $\hat{F}_{l0}$ is a constant. In such a case, the proposed DARC becomes a **nonlinear robust control (NRC) law** as detailed in [27], [28] and the general theoretical robust performance results stated in A of Theorem 1 still hold true. To understand the fundamental working mechanism of such a design, let us carefully examine the resulting CL error dynamics (8), which is rewritten here as

$$M\dot{p} + (k_{2n} + k_s(p))\,p = \tilde{F}_{l0}, \quad \tilde{F}_{l0} = \hat{F}_{l0} - F_l \qquad (14)$$

With $|\hat{F}_{l0}| < F_M$, using (2), we can write that $|\tilde{F}_{l0}(t)| < 2F_M, \forall t$. It is thus easy to show that p in (14) reaches to the region of $\{p : \ |p| \leq \frac{2\varepsilon}{F_M}\}$ in a finite time and remains inside the region thereafter. So let us focus on the behavior of the closed-loop system inside the region $\{p : \ |p| \leq \frac{2\varepsilon}{F_M}\}$, which is described by [2]

$$\dot{p} + \lambda_p p = \frac{1}{M}\tilde{F}_{l0}, \quad \lambda_p = \frac{1}{M}\left[k_{2n} + \frac{F_M^2}{\varepsilon}\right] \qquad (15)$$

This is a stable first order dynamics from the compensation error of the lumped model uncertainty $\tilde{F}_{l0}$ to p. Noting (5), the overall closed-loop transfer function (CLTF) from the lumped model uncertainty F_l to the output tracking error e is

$$G_{CL}(s) = \frac{e(s)}{-F_l(s)} = \frac{1/M}{(s+k_1)(s+\lambda_p)} \qquad (16)$$

which has a static gain of $G_{CL}(0) = \frac{1}{Mk_1\lambda_p} = \frac{1}{k_1[k_{2n}+F_M^2/\varepsilon]}$ and two stable first-order poles at $-k_1$ and $-\lambda_p$ respectively. By decreasing the design parameter ε, the static gain $G_{CL}(0)$ is decreased and the Bode magnitude plot of $G_{CL}(s)$ is lowered across all frequencies. This indicates that stronger attenuation of the lumped model uncertainty F_l is achieved at *all* frequencies. Furthermore, by using larger feedback gains k_1 and k_{2n}, the two CL poles can be made arbitrarily fast as well. As a result, theoretically, arbitrarily good attenuation of model uncertainties with arbitrarily fast response is obtained with such a nonlinear robust controller if there are no other neglected dynamics. These results are not so surprising if one realizes that, during the working region of $\{p : \ |p| \leq \frac{2\varepsilon}{F_M}\}$, the above nonlinear robust feedback control law is essentially of similar structure as the classical PD feedback controller, which can arbitrarily place the resulting two CL poles when the plant dynamics is of second-order like the one described by (1). Specifically, within the working region, noting (5), the robust feedback control law u_s can be re-written as

$$\begin{aligned} u_s = & -\left[k_{2n} + \tfrac{F_M^2}{\varepsilon}\right]p - (Mk_1 - B)\dot{e} \\ = & -(M\lambda_p + Mk_1 - B)\,\dot{e} - M\lambda_p k_1 e \end{aligned} \qquad (17)$$

which is essentially a PD feedback controller of the output tracking error e with a controller transfer function of

$$C(s) = -\frac{u_s(s)}{e(s)} = (M\lambda_p + Mk_1 - B)\,s + M\lambda_p k_1 \qquad (18)$$

The second-order system described by (1) has a plant transfer function of $P(s) = \frac{y(s)}{u(s)} = \frac{1}{Ms^2+Bs}$. For this system, as shown in Fig. 6, it is easy to verify that the PD controller (18) places the two CL poles at $-k_1$ and $-\lambda_p$ respectively. With a CL system as illustrated in Fig. 5 which can be made as fast as one wants, it is no wonder that arbitrarily good disturbance rejection performance can be obtained.

With the above analysis, it is also easy to see that the fundamental working mechanism of traditional robust controls relies in their use of **fast robust feedback**. It uses proper feedback structure such that all CL poles can be made well-behaved and can be pushed fast enough to obtain a sufficiently high bandwidth closed-loop system with respect to the model compensation error caused by various uncertainties. For example, with a PD feedback in (18), both CL poles in (18) are real, and thus no lightly damped CL poles exist. Furthermore, they can be placed arbitrarily left in the complex plane. By doing so, the effect of model uncertainties could be sufficiently attenuated to meet certain performance requirements. In this process, no attempt is made to reduce the model compensation error due to uncertainties (e.g., $\tilde{F}_{l0}$ in (14)).

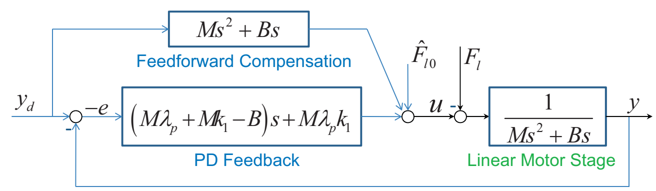

Fig. 5. A PD feedback with model based feedforward compensation

[2]For simplicity, assume that the nonlinear robust feedback u_{s2} is chosen according to (12) with equal signs (i.e., the red-line shown in Fig. 4)

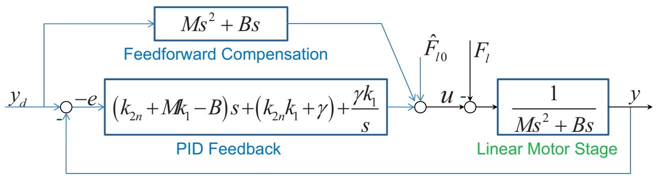

Fig. 6. A PID feedback with model based feedforward compensation

B. Connection to PID Controllers and Role of Slow Learning

Let us now consider the situation when the nonlinear robust feedback u_{s2} in Fig. 4 is turned off, i.e.,

$$\begin{aligned} u = u_a + u_s, \quad u_a = M\ddot{y}_d + B\dot{y}_d + \hat{F}_l \\ u_s = u_{s1} = -k_{2n}p - (Mk_1 - B)\dot{e} \end{aligned} \tag{19}$$

with $\hat{F}_l$ updated by (11). In such a case, the proposed DARC becomes a **robust adaptive control (RAC) law** as detailed in [27], [28]. The general theoretical robust performance results stated in A of Theorem 1 is not valid any more but the steady-state performance result in B of Theorem 1 still holds true. To understand the fundamental working mechanism of such a design, let us first consider the situation when no projection is used in the adaptation law (11), i.e., the standard parameter adaptation law (10) used in the adaptive controls. The resulting control law now becomes a standard **adaptive control (AC) law** and, noting (5) and (10), can be re-written in terms of feedback of the output tracking error e as

$$\begin{aligned} &u = u_f + u_e, \quad u_f = M\ddot{y}_d + B\dot{y}_d + \hat{F}_l(0), \\ &u_e = -k_{2n}p - (Mk_1 - B)\dot{e} - \gamma \int_0^t p(\tau)d\tau \\ &\quad = -\left(k_{2n} + Mk_1 - B\right)\dot{e} - \left(k_{2n}k_1 + \gamma\right)e - \gamma k_1 \int_0^t e d\tau \end{aligned} \tag{20}$$

This is essentially a PID feedback with a controller transfer function of

$$\frac{u_e(s)}{-e(s)} = \left(k_{2n} + Mk_1 - B\right)s + \left(k_{2n}k_1 + \gamma\right) + \gamma k_1 \frac{1}{s} \tag{21}$$

The resulting CL system is shown in Fig. 6 and has a CLTF from $-F_l$ to e given by

$$G_{CL}(s) = \frac{e(s)}{-F_l(s)} = \frac{s}{(s+k_1)(Ms^2+k_{2n}s+\gamma)} \tag{22}$$

which is stable and has three CL poles at

$$p_{CL1} = -k_1, \quad p_{CL2,3} = -\frac{k_{2n}}{2M} \pm \sqrt{\left(\frac{k_{2n}}{2M}\right)^2 - \frac{\gamma}{M}} \tag{23}$$

Noting that the CLTF in (22) contains a differentiator due to the integration feedback in the controller, it is no surprise that such a standard adaptive control law can achieve zero steady-state error when the lumped model uncertainty F_l is constant, which agrees with the result in B of Theorem 1.

Some of the major concerns of practicing engineers about the traditional adaptive controls are that the transient response of an adaptive system is in general not known and it is normally very hard to tune adaptation rate to obtain a reasonably fast transient response. Such problems can be seen quite easily from the above simple analysis as well. Specifically, traditional adaptive controls typically focus on the use of parameter adaptation (or on-line learning) only to deal with model uncertainties, not so much on the proper selection of underlying controller structure and the use of fast robust feedback. For example, the self-tuning regulator (STR) [11] and the model reference adaptive control (MRAC) [13] simply use the standard pole-placement design method as the underlying control law structure with the desired nominal CL poles chosen based on the reference model for command following. If the same design philosophy is used in the design and tuning of the robust adaptive control law (19), noting that the two nominal CL poles[3] are at $-k_1$ and $-k_{2n}/M$, the feedback gain k_{2n} and k_1 would be chosen as $k_{2n} = M\omega_d$ and $k_1 = \omega_d$ respectively. Here, ω_d represents the desired bandwidth for command following and is normally set at a reasonable value to avoid actuator saturation. With this selection of feedback gains for the underlying control law, the subsequent tuning of adaptation rate γ for good transient response and fast convergence of tracking errors to zero becomes rather difficult. As seen from (23), with a pre-determined value of $k_{2n} = M\omega_d$, when a adaptation rate of $\gamma > \frac{M}{4}\omega_d^2$, two of the CL poles would become under-damped. Thus, when a large adaptation rate is chosen, lightly damped CL poles will appear and the system will exhibit large amount of oscillations during the transient period. To avoid this poor transient response problem, typically, a rather small adaptation rate is typically used in the traditional adaptive controls [11], [13], which significantly limits the usefulness of on-line adaptation.

Through the above analysis, it is seen that the fundamental working mechanism of traditional adaptive controls lies in their ability to make full use of the structural information of model uncertainties (e.g., the lumped model uncertainty F_l is assumed to be constant in the development of adaptation law (11)) so that certain **slow learning or adaptation** can be constructed to obtain better estimates of various unknown but constant or slowly changing variables. By doing so, the effect of model uncertainties could be reduced or eliminated and a better steady-state tracking performance can be obtained.

C. Integration of Fast Robust Feedback and On-line Learning

The preceding analysis shows that traditional nonlinear robust control (NRC) and robust adaptive control (RAC) use feedback in essentially two distinct ways when dealing with model uncertainties. Using either one of them alone has certain obvious practical performance limitations. Specifically, almost all physical systems have neglected high-frequency dynamics (e.g., the neglected mechanical resonance mode around 100 Hz shown in Fig.2). Thus, there always exist certain practical limits on how high the CL bandwidth can be pushed with respect to the model compensation error. Consequently, as one would expect when only a PD controller is used in practice, with NRC, some steady-state tracking errors are

[3]poles of the closed-loop system when the robust adaptive control law (19) is used assuming a perfect estimation of all unknown quantities, i.e., assuming $\hat{F}_l = F_l$

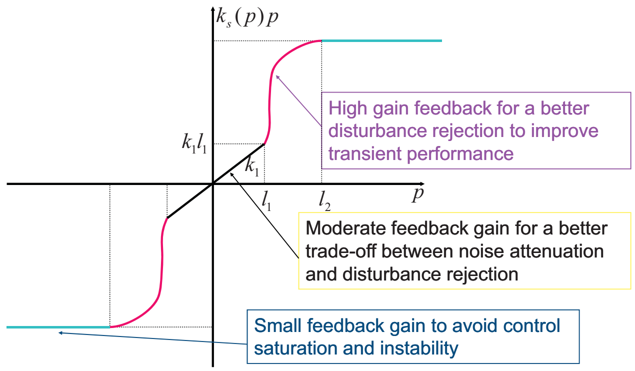

Fig. 7. A nonlinear feedback for improved transient performance and better trade-off in meeting conflicting requirements

always present. On the other hand, as one can only learn things which do not change much during the learning period, and since it takes time to learn, totally relying on the use of on-line learning and adaptation to gain performance, like what is done in RAC, will never lead to a system that can respond fast and deliver consistent control performance when reality differs from what is assumed in the construction of learning law (e.g., the lumped model uncertainty F_l is not constant). Because of these distinct benefits and practical limitations of NRC and RAC, they have been traditionally presented as competing control design approaches [9], [17]. Instead, a better way to control systems with uncertainties is to effectively integrate these two distinct but complementary philosophy of using feedback information, which is exactly what the developed adaptive robust control (ARC) theory has accomplished. In this sense, the developed ARC controllers can be thought as a well-designed PID controller, focusing on **the use of both PD and I feedback** as well as **their interactions for effective integration**. For example, the DARC controller shown in Fig.3 has features of both NRC and RAC while possible negative interaction of using both feedback mechanisms on the CL stability is avoided through the use of projection type parameter adaptation law (11) and the explicit use of the known bound on the resulting model compensation error in the synthesis of nonlinear robust feedback (12) as shown in Fig.4.

D. Benefits of Nonlinear Feedback Designs

Departing from traditional linear control theory, the ARC theory also emphasizes effective use of nonlinear feedback to make better trad-offs while meeting various conflicting performance requirements. As pointed out in [36], the use of nonlinear projection mapping in the integral type adaptation law (11) essentially functions as a built-in anti-integration windup mechanism. Another example, shown in Fig.7, is explained below.

Rewriting the tracking error dynamics (8) as

$$M\dot{p} + (k_{2n} + k_s(p))\, p = \tilde{F}_l \tag{24}$$

it can be considered as a nonlinear filter with the tracking error index p being the output and the model compensation error $\tilde{F}_l$ as the input. During the region of $|p| \leq l_2$ shown in Fig.7, the nonlinear robust feedback k_s is large, which leads to a fast first-order dynamics (24). As such, p would converge very quickly to its steady-state response and can be approximated reasonably well by $\frac{1}{k_{2n}+k_s}\tilde{F}_l$. Thus, in general, larger the k_s is, smaller the the steady-state tracking error would be. On the other hand, a too large k_s will lead to severe control input chattering in implementation due to unavoidable measurement noises. Thus, a trade-off will have to be made in determining an appropriate k_s to meet the dual objectives of having a smaller steady-state tracking error while keeping the degree of the control input chattering within certain limits. This is done by using a moderate feedback gain of k_1 for k_s when p is near zero, since this region represents most of the experimental execution period where the model compensation error $\tilde{F}_l$ is reduced to within the level of $|\tilde{F}_l| \leq (k_{2n} + k_1) l_1 \approx k_1 l_1$ after the initial adaptation period. On other other hand, during the initial period when the adaptation has not come into effect and the model compensation error may be larger than $k_1 l_1$, p will be in the region $l_1 < |p| \leq l_2$. With k_s larger than k_1 used in this region, a stronger attenuation of $\tilde{F}_l$ is achieved, which will lead to a transient error less than what it would have been if the same moderate gain k_1 is used. One may argue that this may lead to severe control input chattering problem as well. However, it should be noted that, unlike the traditional nonlinear robust control where $\tilde{F}_l$ always stays large due to the use of fixed model compensation, the proposed ARC actively uses adaptation to reduce $\tilde{F}_l$. As a result, $\tilde{F}_l$ will be large only during the initial period of adaptation. Thus, the elevated control input chattering problem caused by the use of larger feedback gain during the transient should not be a serious issue.

Finally, consider the rare situation when p is very large (e.g., emergency situations caused by extremely large disturbances), a high gain feedback will easily lead to severe controller windup or integrator windup problem and cause instability even when large disturbances are removed. As such, the main objective in these emergency cases would be to avoid controller windup so that the system can regain stability quickly once the large disturbances disappear. The saturation type robust feedback, shown in Fig.7, still guarantees global stability once the system is free of large disturbances, and thus avoids the common controller windup problem.

IV. Direct ARC of Linear Motors with Large Parameter Variations and Unknown Nonlinearities

In the previous section, the inertia and damping of linear motors are assumed to be known exactly. In addition, other than the fact that the lumped model uncertainty F_l is assumed to be constant so that its low frequency component can be compensated for using integral type on-line adaptation, no other structural information about F_l is utilized. In reality, parameters like the inertia M may experience large variations due to different working conditions. One may also

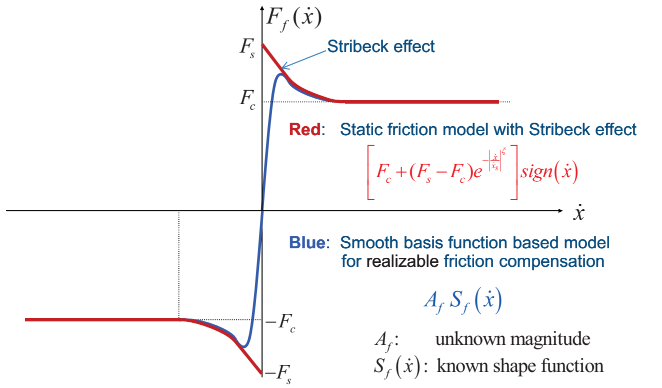

Fig. 8. Discontinuous static friction model and its smooth approximation for realizable friction compensation

know additional structural information about various model uncertainties and it would make sense for us to make full use of those information to synthesize better learning laws. This section shows how the developed ARC theory can be used to systematically handle these issues.

To simplify the problem somewhat, yet without losing generality, this section only shows how the usual static nonlinear Coulomb friction model can be used to obtain a better estimate of the lumped model uncertainty F_l. The technique of how to make full use of the fact that cogging force of iron core linear motors is mostly periodic function of the position x with a known period of pitch between magnets is studied in [61]. To approximate the unknown cogging force, sufficiently large number of sinusoidal functions of the position x are used as basis functions with weights adapted on-line. The additional aperiodic part of the cogging forces with respect to position is also accounted for by using B-spline functions in [41].

For most applications, it might be sufficient to consider Coulomb friction as a static function of velocity $\dot{x}$ only, which is shown in Fig.8 by the red line. Note that the static friction model is discontinuous (in fact, not defined) at zero velocity. If such a model is used for friction compensation, it would be very sensitive to measurement noise and quantization errors at zero velocity and may excite the neglected high frequency dynamics. In addition, due to the electrical dynamics, the sudden jump of friction compensation required by the model when velocity changes direction can never be realized as well. Thus, in practice, it is better to use a continuous or sufficiently smooth shape function with a slope around zero velocity being large enough to capture the essential characteristics of the static friction model while not being overly large that produce a friction compensation command which can never be delivered by the actuator. Such a model for realizable friction compensation is shown in Fig.8 by the blue line and described by $\bar{F}_f = A_f S_f(\dot{x})$, in which the amplitude A_f could be unknown and $S_f(\dot{x})$ is a known continuous or smooth function having the same characteristics as the static friction model when the velocity is out of the very low velocity region. With this friction modeling, the linear motor dynamics (1) can be re-written as:

$$M\ddot{x} \quad = \quad u - B\dot{x} - A_f S_f(\dot{x}) + d, \tag{25}$$

where $d = -\left(F_f - \bar{F}_f\right) - F_r - F_d$. Let the unknown parameter set be $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]$ where $\theta_1 = M$, $\theta_2 = B$, $\theta_3 = A_f$ and $\theta_4 = d_n$, which represents the effect of unknown mass, viscous damping coefficient, Coulomb friction magnitude, and the nominal value of the lumped uncertain nonlinearities d respectively. With the two state variables x_1 and x_2 being the position and velocity respectively, the state space representation of (25) can be linearly parameterized in terms of θ as

$$\dot{x}_1 \quad = \quad x_2, \tag{26}$$

$$\theta_1 \dot{x}_2 \quad = \quad u - \theta_2 x_2 - \theta_3 S_f + \theta_4 + \tilde{d}, \tag{27}$$

where $\tilde{d} = d - d_n$. The following practical assumptions are made:

Assumption 1: The unknown parameter vector θ is within a known bounded convex set Ω_θ. Without loss of generality, it is assumed that $\forall \theta \in \Omega_\theta$, $\theta_{imin} \le \theta_i \le \theta_{imax}, i = 1, \ldots, 4$, where θ_{imin} and θ_{imax} are some known constants.

Assumption 2: The uncertain nonlinearity $\tilde{d}$ is bounded, i.e.,

$$\tilde{d} \in \Omega_d = \{\ \tilde{d} : \quad |\tilde{d}| \le \delta_d\ \} \tag{28}$$

where $\delta_d(t)$ is a known bounded function.

The following nomenclature is used throughout this paper: $\hat{\bullet}$ is used to denote the estimate of $\bullet$, $\tilde{\bullet}$ is used to denote the parameter estimation error of $\bullet$, e.g., $\tilde{\theta} = \hat{\theta} - \theta$, $\bullet_i$ is the i^{th} component of the vector $\bullet$, $\bullet_{max}$ and $\bullet_{min}$ are the maximum and minimum value of $\bullet(t)$ for all t respectively, and the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors.

A. Projection Type Adaptation Law

Let $\hat{\theta}$ denote the estimate of θ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta} = \hat{\theta} - \theta$). One of the key elements of the ARC design [19], [24] is to use the practical available process information to construct the projection type adaptation law for a controlled learning process even in the presence of disturbances. As in [19], [27], the widely used projection mapping $Proj_{\hat{\theta}}(\bullet)$ will be used to keep the parameter estimates within the known bounded set $\bar{\Omega}_\theta$, the closure of the set Ω_θ. The standard projection mapping is [10], [17]:

$$Proj_{\hat{\theta}}(\zeta) = \begin{cases} \zeta, & \text{if} \quad \hat{\theta} \in \mathring{\Omega}_\theta \text{ or } n_{\hat{\theta}}^T \zeta \le 0 \\ \left(I - \Gamma \frac{n_{\hat{\theta}} n_{\hat{\theta}}^T}{n_{\hat{\theta}}^T \Gamma n_{\hat{\theta}}}\right) \zeta, & \hat{\theta} \in \partial\Omega_\theta \text{ and } n_{\hat{\theta}}^T \zeta > 0 \end{cases} \tag{29}$$

where $\zeta \in R^m$, $\Gamma(t) \in R^{m \times m}$, m is the dimension of parameter vector θ, $\mathring{\Omega}_\theta$ and $\partial\Omega_\theta$ denote the interior and the boundary of Ω_θ respectively, and $n_{\hat{\theta}}$ represents the outward unit normal vector at $\hat{\theta} \in \partial\Omega_\theta$. It is proven in [27] that the following lemma holds:

Lemma 1: Suppose that the parameter estimate $\hat{\theta}$ is updated using the following projection type adaptation law:

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}\left(\Gamma\tau\right), \qquad \hat{\theta}(0) \in \Omega_\theta \tag{30}$$

where τ is any adaptation function and $\Gamma(t) > 0$ is any continuously differentiable positive symmetric adaptation rate matrix. With this adaptation law, the following desirable properties hold:

P1. The parameter estimates are always within the known bounded set $\bar{\Omega}_\theta$, i.e., $\hat{\theta}(t) \in \bar{\Omega}_\theta, \ \forall t$. Thus, from Assumption 1, $\forall t, \ \ \theta_{imin} \leq \hat{\theta}_i(t) \leq \theta_{imax}, i = 1, \ldots, 4$.

P2.

$$\tilde{\theta}^T \left(\Gamma^{-1} Proj_{\hat{\theta}}\left(\Gamma\tau\right) - \tau\right) \leq 0, \quad \forall \tau \tag{31}$$

△

Property P2 enables one to show that the use of projection modification to the traditional integral type adaptation law in (30) does not interfere with the perfect learning capability of the original integral type adaptation law.

B. Desired Compensation DARC Law

With the projection type adaptation law (30), a direct adaptive robust control law is synthesized in [40] for the system (25) that achieves a guaranteed transient and final tracking accuracy. Furthermore, to reduce the effect of measurement noises, desired compensation ARC (DCARC) [19] is used. The resulting DCARC control law has the following form:

$$u \ = \ u_a + u_s, \ \ u_a = -\varphi_d^T \hat{\theta}, \tag{32}$$

where u_a is the adjustable model compensation needed for achieving perfect tracking, $\varphi_d^T = [-\ddot{y}_d, -\dot{y}_d, -S_f(\dot{y}_d), 1]$ is the regressor that depends on the reference trajectory $y_d(t)$ only and thus is free of measurement noise effect, and u_s is a robust control function having the form of

$$u_s = u_{s1} + u_{s2}, \quad u_{s1} = -k_{s1} p, \tag{33}$$

where p is defined in (5), k_{s1} is a nonlinear gain large enough such that the matrix A_1 defined below is positive definite

$$A_1 = \begin{bmatrix} k_{s1} - k_2 - \theta_1 k_1 + \theta_2 + \theta_3 g & -\frac{1}{2}k_1(\theta_2 + \theta_3 g) \\ -\frac{1}{2}k_1(\theta_2 + \theta_3 g) & \frac{1}{2}\theta_1 k_1^3 \end{bmatrix} \tag{34}$$

in which g is defined by $S_f(x_2) - S_f(\dot{y}_d) = g(x_2, t)\dot{e}$, and u_{s2} is synthesized to satisfy the following robust performance conditions

$$\begin{array}{ll} \text{i} & p\{u_{s2} - \varphi_d^T \tilde{\theta} + \tilde{d}\} \leq \ \varepsilon \\ \text{ii} & p u_{s2} \leq 0 \end{array} \tag{35}$$

in which ε is a design parameter. With this DARC control law, it is shown in [40] that the tracking error dynamics are

$$\theta_1 \dot{p} \ = \ \underbrace{u_{s1} + (\theta_1 k_1 - \theta_2 - \theta_3 g)\dot{e}} + \underbrace{u_{s2} - \varphi_d^T \tilde{\theta} + \tilde{d}} \tag{36}$$

and the following theoretical performance holds:

Theorem 2: With the projection type adaptation law (30) and an adaptation function of $\tau = \varphi_d p$, the DCARC law (32) guarantees that

A. In general, all signals are bounded. Furthermore, the positive definite function V_s defined by

$$V_s = \frac{1}{2}\theta_1 p^2 + \frac{1}{2}\theta_1 k_1^2 e^2 \tag{37}$$

is bounded above by

$$V_s \leq \exp(-\lambda t) V_s(0) + \frac{\varepsilon}{\lambda}[1 - \exp(-\lambda t)], \tag{38}$$

where $\lambda = \min\{2k_2/\theta_{1max}, k_1\}$.

B. If after a finite time t_0, there exist parametric uncertainties only (i.e., $\tilde{d} = 0, \ \forall t \geq t_0$), then, in addition to results in A, zero final tracking error is also achieved, i.e, $e \longrightarrow 0$ and $p \longrightarrow 0$ as $t \longrightarrow \infty$. Furthermore, if the desired trajectory satisfies the following persistent exciting (PE) condition

$$\exists \ T, t_0, \varepsilon_p > 0 \ \ \text{s.t.} \ \ \int_t^{t+T} \varphi_d(\nu)\varphi_d(\nu)^T d\nu \geq \varepsilon_p I_p, \ \ \forall t \geq t_0 \tag{39}$$

then, the parameter estimates $\hat{\theta}$ asymptotically converge to their true values (i.e., $\tilde{\theta} \rightarrow 0$ when $t \rightarrow \infty$). △

V. Indirect ARC of Linear Motors

The underlying parameter adaptation law (30) in DARC is based on direct adaptive control designs, in which the control law and the parameter adaptation law are synthesized simultaneously through certain stability criteria to meet the sole objective of reducing the output tracking error. Such a design normally leads to a controller whose dynamic order is as low as the number of unknown parameters to be adapted while achieving excellent output tracking performance [24], [29], [62]. However, the direct approach has the drawback that parameter estimates normally do not converge or even approach their true values fast enough as observed in actual use [25], [30]. Such a poor convergence of parameter estimates with DARC designs is mainly due to: (i) *a gradient type adaptation law (30) with a constant adaptation rate matrix* Γ *that does not convergence as well as the least squares type*; (ii) *the adaptation function* τ *is driven by the actual tracking error index p, which is very small in implementation for a well designed direct ARC law. Thus it is more prone to be corrupted by factors that were neglected during synthesis of the parameter adaptation law such as the sampling delay and noise*; (iii) *the PE condition (39) needed for parameter convergence cannot always be met during operation*. These practical limitations make it almost impossible for the resulting parameter estimates to be used for secondary purposes that require more reliable and accurate on-line parameter estimates such as machine component health monitoring and prognosis. If more accurate parameter estimates are needed, then the indirect adaptive robust control (IARC) design presented in [54] can be used. It completely separates the construction of parameter estimation law from the design of the underlying robust control law as reviewed below.

A. Projection Type Adaptation Law with Rate Limits

In order to achieve a complete separation of estimator design and robust control law design, in addition to the projection-type parameter adaptation law (30), it is also necessary to use the preset adaptation rate limits for a controlled estimation process [54]. For this purpose, for any $\zeta \in R^m$, define a saturation function as:

$$sat_{\dot{\theta}_M}(\zeta) = s_0\zeta, \qquad s_0 = \begin{cases} 1, & \|\zeta\| \le \dot{\theta}_M \\ \frac{\dot{\theta}_M}{\|\zeta\|}, & \|\zeta\| > \dot{\theta}_M \end{cases} \tag{40}$$

where $\dot{\theta}_M$ is a pre-set rate limit. It can be verified that the following lemma holds [54]:

Lemma 2: Suppose that the parameter estimate $\hat{\theta}$ is updated using the following projection type adaptation law with a pre-set rate limit $\dot{\theta}_M$:

$$\dot{\hat{\theta}} = sat_{\dot{\theta}_M}\left(Proj_{\hat{\theta}}(\Gamma\tau)\right), \qquad \hat{\theta}(0) \in \Omega_\theta \tag{41}$$

where τ is any adaptation function and $\Gamma(t) > 0$ is any continuously differentiable positive symmetric adaptation rate matrix. With this adaptation law, the following desirable properties hold:

P1. The parameter estimates are always within the known bounded set $\bar{\Omega}_\theta$, i.e., $\hat{\theta}(t) \in \bar{\Omega}_\theta, \ \forall t$. Thus, from Assumption 1, $\forall t, \ \ \theta_{imin} \le \hat{\theta}_i(t) \le \theta_{imax}, i = 1, \ldots, 4$.

P2.

$$\tilde{\theta}^T\left(\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\tau) - \tau\right) \le 0, \quad \forall\tau \tag{42}$$

P3. The parameter update rate is uniformly bounded by $\|\dot{\hat{\theta}}(t)\| \le \dot{\theta}_M, \ \forall t$ △

B. Desired Compensation ARC Law

With the use of the projection type adaptation law with rate limit (41), the parameter estimates and their derivatives are bounded with known bounds, regardless of the estimation function τ to be used. As such, the same design techniques as in the direct ARC design can be used to construct a desired compensation ARC function that achieves a guaranteed transient and final tracking accuracy, independent of the specific identifier to be used later. The resulting control function is of the desired compensation type having the same form as (32) with the same form of u_s as in (33). The only difference is that the regressor used in the parameter adaptation function of the proposed IARC will be based on the actual system dynamics for better parameter estimates, i.e., $\varphi = [-\ddot{y}, -\dot{y}, -S_f(\dot{y}), 1]^T$, instead of the desired regressor vector φ_d as in DARC. To address the effect of this change of adaptation law design, in addition to choosing k_{s1} large enough so that $A_1 > 0$ as in DARC, it is also required that $A_2 > 0$ where A_2 is defined as

$$\begin{bmatrix} \frac{\theta_1}{\hat{\theta}_1}(k_{s1} + \hat{\theta}_2 + \hat{\theta}_3 g) - k_2 - \theta_1 k_1 & -\frac{1}{2}\frac{\theta_1}{\hat{\theta}_1}(\hat{\theta}_2 + \hat{\theta}_3 g)k_1 \\ -\frac{1}{2}\frac{\theta_1}{\hat{\theta}_1}(\hat{\theta}_2 + \hat{\theta}_3 g)k_1 & \frac{1}{2}\theta_1 k_1^3 \end{bmatrix} \tag{43}$$

With this ARC control law, following the same derivations as in [40], the following theorem can be obtained.

Theorem 3: Consider the ARC law (32) with the projection type adaptation law with rate limits (41), in which τ could be any adaptation function. Then, the same robust performance results as in A of Theorem 2 can be obtained.

C. Indirect Parameter Estimation Algorithms

In the above subsection, an ARC law which can admit any estimation function τ has been constructed and a guaranteed transient and final tracking performance is achieved even in the presence of uncertain nonlinearities. Thus, the reminder of the IARC design is to construct suitable estimation functions τ so that an improved final tracking accuracy–asymptotic tracking or zero final tracking error in the presence of parametric uncertainties only–can be obtained with an emphasis on good parameter estimation process as well. As such, in this subsection, it is assumed the system is absence of uncertain nonlinearities, i.e., let $\tilde{d} = 0$ in (27).

Let $H_f(s)$ be the transfer function of a stable filter with a relative degree 1, e.g., $H_f(s) = \frac{1}{\tau_f s+1}$. Then, when $\tilde{d} = 0$, applying the filter to both sides of (27), one obtains

$$\theta_1\ddot{y}_f = u_f - \theta_2\dot{y}_f - \theta_3 S_{ff} + \theta_4 1_f \tag{44}$$

where $\dot{y}_f, y_f, u_f, S_{ff}$, and 1_f represent the filtered output speed, position, input, the shape function, and 1 respectively, i.e., $\dot{y}_f(t) = H_f(p)[\dot{y}(t)], y_f(t) = H_f(p)[y(t)], u_f = H_f(p)[u(t)], S_{ff} = H_f(p)[S_f(\dot{y}(t))]$, and $1_f = H_f(p)[1]$, and $\ddot{y}_f$ is obtained from the filter of the output speed as $H_f(s)$ has a relative degree of 1. From (44), a linear regression model can be obtained as

$$u_f = -\varphi_f^T\theta \tag{45}$$

where the regressor is $\varphi_f^T = [-\ddot{y}_f, -\dot{y}_f, -S_{ff}, 1_f]$. Thus, by defining the prediction output and the prediction error as

$$\begin{aligned} \hat{u}_f &= -\varphi_f^T\hat{\theta} \\ \epsilon &= \hat{u}_f - u_f \end{aligned} \tag{46}$$

one obtains the following prediction error model

$$\epsilon = -\varphi_f^T\tilde{\theta} \tag{47}$$

With this static linear regression model, various estimation algorithms can be used to identify unknown parameters, of which the gradient estimation algorithm and the least squares estimation algorithm [12], [17] are given below.

1) Gradient Estimator: With the gradient type estimation algorithm, the resulting adaptation law is given by (41), in which Γ can be chosen as a constant positive diagonal matrix, i.e., $\Gamma = diag[\gamma_1, ..., \gamma_4]$, and τ is defined as

$$\tau = \frac{1}{1 + \nu\|\varphi_f\|^2}\varphi_f\epsilon, \qquad \nu \ge 0 \tag{48}$$

where by allowing $\nu = 0$, one encompasses unnormalized adaptation function.

2) Least Squares Estimator: When the least squares type estimation algorithm with exponential forgetting [12] is used, the resulting adaptation law is given by (41), in which $\Gamma(t)$ is updated by

$$\dot{\Gamma} = \alpha\Gamma - \frac{1}{1 + \nu\varphi_f^T\Gamma\varphi_f}\Gamma\varphi_f\varphi_f^T\Gamma, \quad \Gamma(0) = \Gamma^T(0) > 0 \tag{49}$$

where $\nu \geq 0$ with $\nu = 0$ leading to the unnormalized algorithm, α is the forgetting factor, and τ is defined as

$$\tau = \frac{1}{1+\nu\varphi_f^T\Gamma\varphi_f}\varphi_f\epsilon \tag{50}$$

In practice, the above least square estimator may lead to estimator windup (i.e., $\lambda_{max}(\Gamma(t)) \longrightarrow \infty$) when the regressor is not persistently exciting. To prevent this estimator windup and to take into account the effect of the rate-limited adaptation law (41), (49) is modified to

$$\dot{\Gamma} = \begin{cases} \alpha\Gamma - \frac{1}{1+\nu\varphi_f^T\Gamma\varphi_f}\Gamma\varphi_f\varphi_f^T\Gamma, \\ \quad \text{if } \lambda_{max}(\Gamma(t)) \leq \rho_M \text{and} \|Proj_{\hat{\theta}}(\Gamma\tau)\| \leq \dot{\theta}_M \\ 0, \quad\quad \text{otherwise} \end{cases} \tag{51}$$

where ρ_M is the pre-set upper bound for $\|\Gamma(t)\|$. With these practical modifications, $\Gamma(t) \leq \rho_M I, \ \forall t$. The following lemma and theorem summarize the properties of these estimators and the theoretical performance which can be achieved [54]:

Lemma 3: When the rate-limited projection type adaptation law (41) with either the gradient estimator (48) or the least squares estimator (50) is used, the following results hold:

$$\tilde{\theta} \in \mathcal{L}_\infty[0,\infty) \tag{52}$$
$$\epsilon \in \mathcal{L}_2[0,\infty) \cap \mathcal{L}_\infty[0,\infty) \tag{53}$$
$$\dot{\hat{\theta}} \in \mathcal{L}_2[0,\infty) \cap \mathcal{L}_\infty[0,\infty) \tag{54}$$

Theorem 4: In the presence of parametric uncertainties only, i.e., $\tilde{d} = 0$, by using the control law (32) and the adaptation law (41) with either the gradient type estimation function (48) or the least squares type estimation function (50), in addition to the robust performance results stated in Theorem 3, an improved steady-state tracking performance – asymptotic tracking – is also achieved, i.e., $e \to 0$ as $t \to \infty$. Furthermore, when the PE condition (39) is satisfied, the parameter estimates converge to their true values. △

VI. Integrated Direct/Indirect ARC of Linear Motors

As shown in the comparative experimental results later, compared to the rather fast response of the actual tracking error dynamics (36), the parameter adaptation in the IARC design (41) is relatively slower. Thus significant transient tracking error and non-zero steady-state error may exhibit due to the parameter estimation error $\tilde{\theta}$. While the direct ARC (DARC) design does not necessarily produce good parameter estimates, it has been observed in practice that DARC normally has a better tracking performance than IARC. A thorough analysis reveals that the relatively poorer IARC tracking performance is caused by the loss of dynamic compensation type fast adaptation that is inherent in DARC designs. To overcome this IARC problem, an integrated direct/indirect ARC (DIARC) design framework is developed in [23]. The design not only uses the same IARC adaptation process for accurate estimation of physical parameters, but also introduces dynamic compensation type fast adaptation to achieve better transient and steady-state performance. Such an integrated ARC design is reviewed below.

The proposed DIARC control law has the form of

$$u = u_a + u_s, \quad u_a = u_{a1} + u_{a2}, \quad u_{a1} = -\varphi_d^T\hat{\theta}, \tag{55}$$

where u_{a1} is the same adjustable model compensation as in the IARC law (32) with the same parameter estimation algorithm (41) for $\hat{\theta}$, u_{a2} is a fast dynamic compensation term to be synthesized later, and u_s is a robust control function having the same form as (33) with the same u_{s1} as in IARC but different robust performance conditions for u_{s2}, which will be detailed later. Substituting (55) into (27), corresponding to (36), the new tracking error dynamics are

$$\theta_1\dot{p} = \underbrace{u_{s1} + (\theta_1k_1 - \theta_2 - \theta_3 g)\dot{e}} + u_{a2} + u_{s2} \underbrace{-\varphi_d^T\tilde{\theta} + \tilde{d}} \tag{56}$$

Define a constant d_0 and time varying function $\tilde{d}^*(t)$ such that

$$d_0 + \tilde{d}^*(t) = -\varphi_d^T\tilde{\theta} + \tilde{d} \tag{57}$$

Conceptually, (57) lumps the disturbance and the model uncertainties due to parameter estimation error together and divides it into the low frequency component d_0 and the higher frequency components, $\tilde{d}^*(t)$, so that the low frequency component d_0 can be compensated for through the fast adaptation of direct ARC design as follows. Substituting (57) into (56),

$$\theta_1\dot{p} = \underbrace{u_{s1} + (\theta_1k_1 - \theta_2 - \theta_3 g)\dot{e}} + u_{a2} + u_{s2} + d_0 + \tilde{d}^*(t) \tag{58}$$

Choose the fast compensation term u_{a2} as

$$u_{a2} = -\hat{d}_0 \tag{59}$$

where $\hat{d}_0$ represents the estimate of d_0 updated by

$$\dot{\hat{d}}_0 = Proj_{\hat{d}_0}(\gamma_d \frac{1}{\hat{\theta}_1} p), \qquad |\hat{d}_0(0)| \leq \hat{d}_{max} \tag{60}$$

in which $\hat{d}_{max}$ is a pres-set bound for $\hat{d}_0(t)$. As in DARC in section 3, the projection mapping in (60) guarantees that $|\hat{d}_0(t)| \leq \hat{d}_{max}, \forall t$. Substituting (59) into (58),

$$\theta_1\dot{p} = \underbrace{u_{s1} + (\theta_1k_1 - \theta_2 - \theta_3 g)\dot{e}} + u_{s2} - \tilde{d}_0 + \tilde{d}^*(t) \tag{61}$$

Similar to (35), the robust feedback u_{s2} is now chosen to satisfy the following robust performance conditions:

$$\begin{array}{ll} \text{i} & p\{u_{s2} - \tilde{d}_0 + \tilde{d}^*(t)\} \leq \varepsilon \\ \text{ii} & pu_{s2} \leq 0 \end{array} \tag{62}$$

Theorem 5: With the same parameter estimation algorithm for $\hat{\theta}$ as in IARC in section 4 and $\hat{d}_0$ updated by (60), the DIARC law (55) achieves the same theoretical performance results as DARC in Theorem 2. △

VII. ARC of Linear Motors with Dynamic Friction Compensation

It has been well known that to have high accuracy of motion control at low speed movement, friction cannot be simply modeled as a static nonlinear function of velocity alone but rather as a *dynamic* function of velocity and displacement. Thus, during the past decade, significant efforts have been devoted to solving the difficulties in modeling and compensation of dynamic friction with various types of models proposed [59], [63], [64]. Among them, the LuGre model by Canudas de Wit *et al.* [59] can describe major features of dynamic friction, including presliding displacement, varying break-away force and Stribeck effect. Due to its relatively simpler form and its ability to simulate major dynamic friction behaviors, LuGre model has been widely used in controls with dynamic friction compensation [65], [66]. Although many application results have been reported [67], some practical problems are also discovered, especially when applying the LuGre model to systems experiencing large ranges of motion speeds such as the linear motor drive system studied in this paper. Namely, the traditional LuGre model could become very stiff when the velocity is large. This leads to some unavoidable implementation problems since dynamic friction compensation can only be implemented digitally due to its highly nonlinear characteristics. For example, it has been reported in [68] that the observer dynamics to recover the unmeasurable internal states of the LuGre model could become unstable at high speed motions.

In [69], the LuGre model was first revisited to reveal the cause for the digital implementation problems when using the model for dynamic friction compensation. Based on the analysis, a modified version of LuGre model was proposed for dynamic friction compensation, in which the estimation of internal states is automatically stopped at high speed movements to by-pass the instability problem of the LuGre model based observer dynamics. A continuous function is designed to make a continuous transition from the LuGre model based low speed dynamic friction compensation to the static friction model based high speed friction compensation. The ARC strategy is then utilized along with the proposed modified LuGre model based dynamic friction compensation to achieve accurate trajectory tracking for both low-speed and high-speed movements. The proposed ARC algorithm, along with ARC algorithms with friction compensations using the LuGre model and the static friction model respectively, are tested on the linear motor system in section II. Comparative experimental results were also presented to illustrate the effectiveness of the proposed modified LuGre model based dynamic friction compensation in practical applications and the excellent tracking performance of the proposed ARC algorithm.

VIII. Globally Stable Saturated ARC of Linear Motors with Input Saturation

All actuators of physical devices are subject to amplitude saturation. While it may be possible to ignore input saturation in some applications, reliable operation and acceptable performance of most control systems must be assessed in light of actuator saturation. Significant amount of research has been done to stabilize the system while taking into account the saturation nonlinearities at the controller design stage. However, most of the research assume that the systems of concern are linear and exactly known, which is not the case for most physical systems in reality. Therefore, it is of practical importance to take model uncertainties and external disturbances into account when attacking the actuator saturation problem. In [70], the judicious use of saturation functions in [71] is combined with the ARC strategy to achieve both global stability and high performance for a chain of integrators subject to matched parametric uncertainty and uncertain nonlinearities. However, like the saturated controller in [71], the design is based on a set of transformed coordinates, in which the effect of model uncertainties immediately shows up at the beginning step of the backstepping-like controller design, even though the model uncertainties studied are matched uncertainties. In other words, the actual model uncertainties has been "amplified" n times to be accommodated by the control input where n represents the order of the system. Thus, with a limited control authority, the extent of model uncertainties that the resulting controller can handle is quite restricted, leading to a conservative overall design. In [20], [72], a new saturated control structure based on the back-stepping design and the ARC strategy was proposed. The control law is designed to ensure fast error convergence during normal working conditions while globally stabilizing the system for a much larger class of modeling uncertainties than those considered in [70]. Essentially, a bounded virtual control law is designed to ensure the boundedness and convergence of the error signal at each step. The actual control input comes from the design at the final step and consists of a model compensation term and a local-high-gain-but-globally-saturated robust control term like the one shown in Fig.7. The bound on the model compensation can be calculated from the pre-known information of the system and the desired trajectory to be tracked. The nonlinear robust control is used to meet the dual objective of (i) maintaining global stability with limited control efforts during rare emergencies when the tracking errors are large and (ii) having a high CL bandwidth for high performance during normal running conditions when the errors are small. Experimental results were presented to verify these claims as well.

IX. Comparative Experiments

A. Experiment Setup

All control algorithms have been implemented on the linear motor system in section II using a dSPACE DS1103 controller board. The controller executes programs at a sampling frequency of $f_s = 5\ kHz$, which results in a velocity measurement resolution of $0.0025\ m/sec$. Standard least-square identification is performed to obtain the nominal values of parameters; the nominal value of M is 0.12 $(V/m/s^2)$. To test the learning capability of the proposed ARC algorithms, a 5 kg load is also mounted on the motor and the identified values of parameters with this load are $\theta_1 = 0.19$ $(V/m/s^2)$, $\theta_2 = 0.2$ $(V/m/s)$, and $\theta_3 = 0.3$ (V). The bounds of the parameter variations are chosen as:

$$\begin{aligned} \theta_{min} &= [0.08,\ 0.08,\ 0.1,\ -0.5]^T \\ \theta_{max} &= [0.25,\ 0.3,\ 0.5,\ 0.5]^T \end{aligned}$$

The initial parameter estimates of $\theta_0 = [0.1\ 0.2\ 0.1\ 0.0]^T$ are used for all experiments. The following three control algorithms are compared: (1) **DARC**, the Direct Adaptive Robust Control (DARC) law; (2) **IARC**, the Indirect Adaptive Robust Control with the least square type estimation algorithm; and (3) **DIARC**, the Direct/Indirect Adaptive Robust Control (DIARC) with the least square type estimation algorithm.

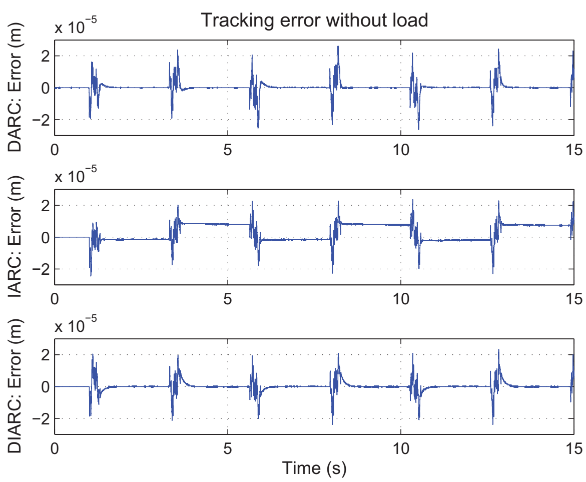

Fig. 9. Tracking error for (a)DARC, (b)IARC, (c)DIARC with no load

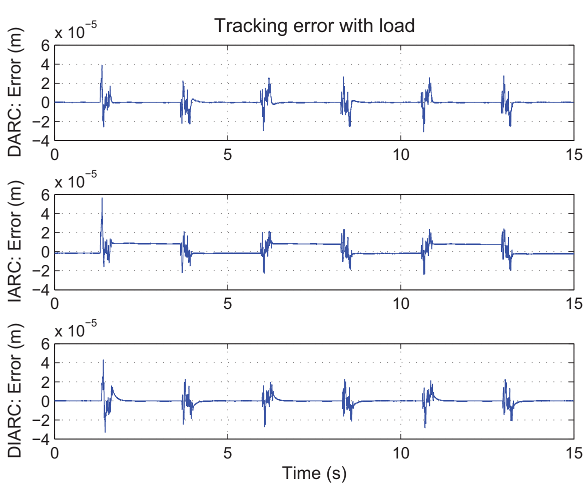

Fig. 10. Tracking error for (a)DARC, (b)IARC, (c)DIARC with load

To test the tracking performance of the proposed algorithms, as in [40], a typical high-speed/high acceleration motion trajectory for the pick-and-place operations in industry is used in all experiments. The desired trajectory corresponds to a movement of 0.4 m with a maximum speed of 2 m/s and an acceleration of 30 m/s^2. The experimental results in terms

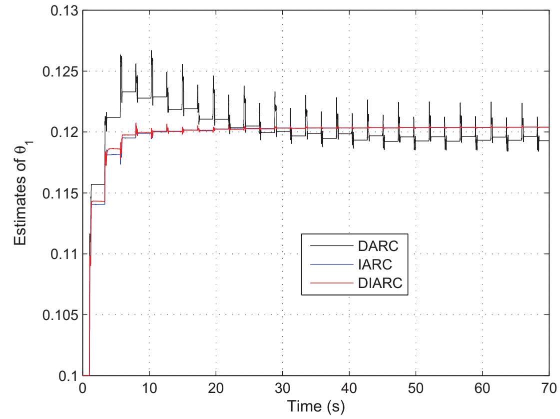

Fig. 11. $\hat{\theta}_1$ for (a)DARC, (b)IARC, (c)DIARC with no load

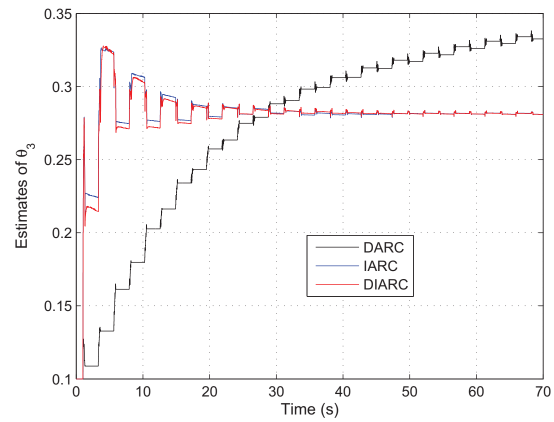

Fig. 12. $\hat{\theta}_3$ for (a)DARC, (b)IARC, (c)DIARC with no load

of time history are given in Figs.9-14. As seen from these results, the tracking errors of all three ARC controllers are very small, which are within 20 micrometers even during the high acceleration and deceleration periods. The tracking errors with and without load are almost the same, indicating the strong performance robustness of ARC designs to parameter variations. It is also seen that IARC and DIARC do have better parameter estimates than DARC, but IARC exhibits non-zero steady-state tracking errors. Overall, DIARC achieves the best tracking performance while having a more robust parameter estimation process and more accurate parameter estimates than DARC. All these agree with previous theoretical analysis.

X. Conclusion

The paper gives an overview on how the recently developed nonlinear adaptive robust control (ARC) can be used to effectively and systematically address some of the major issues in precision motion control. Control of a linear motor drive system is used as a case study and comparative experimental results are presented to demonstrate the achievable performance and limitations of various ARC designs. The case study makes it easier to understand the fundamental design principles of ARC theory. In some senses, the proposed ARC

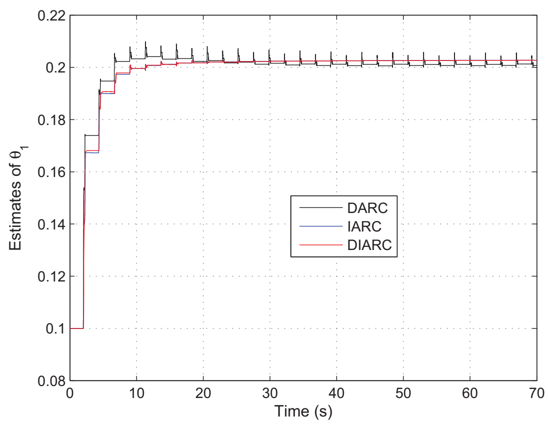

Fig. 13. $\hat{\theta}_1$ for (a)DARC, (b)IARC, (c)DIARC with load

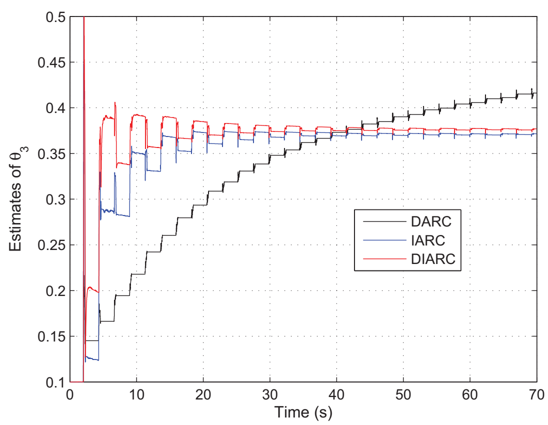

Fig. 14. $\hat{\theta}_3$ for (a)DARC, (b)IARC, (c)DIARC with load

could be thought as the modern version of classic PID design. The adjustable nonlinear model compensation with on-line parameter adaptation or other learning tools used in ARC is a refined version of traditional feedforward design with integral feedback. The emphasis on the use of nonlinear robust control law having targeted locally high gain but globally low gain feedback is the key to build a closed-loop system that locally has sufficiently high loop gain to meet the stringent disturbance rejection or model uncertainty attenuation requirement while avoiding the severe control input saturation problem associated with the traditional linear high-gain feedback for global stability.

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