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A Case Study for Adaptive Robust Precision Motion Control of Systems Preceded by Unknown Dead-zones with Comparative Experiments

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Abstract—**The recently proposed integrated direct/indirect adaptive robust controller (DIARC) for a class of nonlinear systems with unknown input dead-zones is combined with desired trajectory compensation to achieve asymptotic stability with excellent tracking performance. The algorithm is tested on a linear motor drive system preceded by a simulated non-symmetric dead-zone which is practically supposed to be unknown. Certain guaranteed robust transient performance and final tracking accuracy are achieved even when the overall system may be subjected to parametric uncertainties, time-varying disturbances and other uncertain nonlinearities. Signal noise that affects the adaptation function is alleviated by replacing the noisy state signal with the desired state feedback. Furthermore, asymptotic output tracking is achieved when there is unknown dead-zone nonlinearity only. Comparative experimental results obtained validate the necessity of dead-zone compensation and the high-effectiveness nature of the proposed approach as well.** [1]

I. Introduction

A great deal of effort has been devoted to addressing the issue of dead-zone. Specifically, an adaptive dead-zone inverse was proposed in [1] but only bounded output tracking errors are achieved. In [2] and [3], fuzzy-logic and neural network were used to provide alternative interpretations to the basis functions needed for adaptive dead-zone inversions [1] but with no essential improvement on theoretically achievable results, i.e., only bounded output tracking errors are obtained. In [4], [5], [6], the dead-zone was modeled as a combination of a linear input with either an unknown constant gain for symmetric dead-zones or time-varying unknown gain for non-symmetric ones and a bounded disturbance-like term. With this formulation, traditional robust adaptive control design techniques can be conveniently applied to achieve bounded output tracking errors without explicitly exploring the detailed dead-zone characteristics rather than the fact that the dead-zone effect can always be treated as a bounded input disturbance. In [7], smooth basis functions as opposed to the discontinuous ones in [1] were explored to provide some approximate inversions of the dead-zone nonlinearities.

In [8], an integrated direct/indirect adaptive robust control (DIARC) scheme has been developed for a class of SISO uncertain nonlinear systems preceded by unknown dead-zone nonlinearity. Asymptotic output tracking has been achieved even in the presence of unknown dead-zone nonlinearity, a theoretic result which cannot be attained in all previous researches on adaptive dead-zone compensation. Furthermore, the proposed DIARC algorithm also achieves certain guaranteed robust transient performance and final tracking accuracy even when the overall system may be subjected to other uncertain nonlinearities and time-varying disturbances.

In this paper, the proposed integrated DIARC in [8] is combined with desired trajectory compensation to achieve asymptotic stability with excellent tracking performance. This controller has several implementation merits such as less on-line computation time, reduced effect of measurement noise, and a faster adaptation rate [9]. The algorithm is tested on a linear motor drive system preceded by a simulated non-symmetric dead-zone which is practically supposed to be unknown. Certain guaranteed robust transient performance and final tracking accuracy are achieved even when the overall system may be subjected to parametric uncertainties, time-varying disturbances and other uncertain nonlinearities. Furthermore, asymptotic output tracking is achieved when there are unknown dead-zone nonlinearities only. Comparative experimental results show that the DIARC with the proposed adaptive dead-zone compensation achieves almost the same best tracking performance as that DIARC with complete dead-zone compensation achieves. The comparative results also verify the necessity of dead-zone compensation if high-performance tracking is of major concern. Overall, the high-performance tracking results obtained from the experiments validate the effectiveness of the proposed DIARC strategy.

II. PROBLEM STATEMENT

A. System Model

Neglecting fast electrical dynamics, the dynamics of a linear motor preceded by unknown dead-zone can be described as

$$\begin{aligned} \ddot{x} &= Ew(t) - B\dot{x} - AS_f(\dot{x}) + f_u \\ y &= x(t), \quad w(t) = D(v(t)) \end{aligned} \tag{1}$$

where x represents the position of the inertia load, $v(t)$ is the input to the unknown dead-zone, $w(t)$ represents the control input voltage to the motor, and $y(t)$ is the output from the

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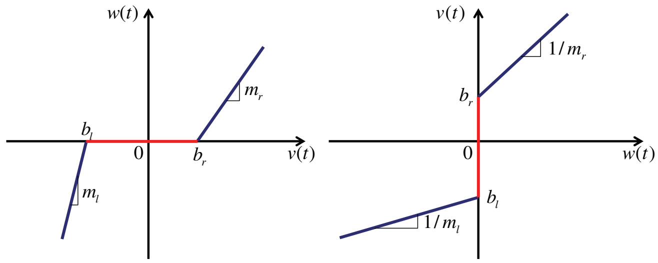

Figure 1. (1) A dead-zone model (2) A perfect dead-zone inverse

motor. E is the motor input gain, B is the equivalent viscous friction coefficient, $AS_f(\dot{x})$ represents the nonlinear Coulomb friction, in which the amplitude A may be unknown but the continuous shape function $S_f(\dot{x})$ is known, and f_u represents the lumped uncertain nonlinearities including various model approximation errors and external disturbances. The nonlinearity $D(v(t))$ is described as a dead-zone characteristic shown in Fig. 1-(1). With input $v(t)$ and output $w(t)$, the dead-zone can be represented as in [1] as follows:

$$w(t) = D(v(t)) = \begin{cases} m_r v(t) - m_r b_r & for\ \ v(t) \geq b_r \\ 0 & for\ \ b_l < v(t) < b_r \\ m_l v(t) - m_l b_l & for\ \ v(t) \leq b_l \end{cases} \quad (2)$$

where the parameters m_r, m_l, b_r, and b_l are constants and stand for the right slope, left slope, right break-point and left break-point of the dead-zone respectively. Let $E = 1$ for that its effect can be considered in the unknown slope m_r and m_l. For notation simplicity, let $\theta \in R^6$ be the vector of all unknown constant parameters, i.e., $\theta = [B, A, m_r, m_r b_r, m_l, m_l b_l]^T$. $X = [x(t), \dot{x}(t)]$ is the state vector, $X_d = [x_d(t), \dot{x}_d(t)]$ is the desired trajectory vector. The control objective is to design a control law $v(t)$ to ensure that all closed-loop signals are bounded and the plant state vector X tracks the specified desired trajectory X_d asymptotically with certain guaranteed transient responses under the following practical assumptions. [2]

Assumption 1: The dead-zone output $w(t)$ is not available for measurement and the dead-zone parameters are unknown, but their signs are known as: $m_r > 0$, $m_l > 0$, $b_r > 0$, $b_l < 0$.

Assumption 2: The unknown parameter vector θ is within a known bounded convex set Ω_θ. Without loss of generality, it is assumed $\forall \theta \in \Omega_\theta$, $B_{min} \leq B \leq B_{max}$, $A_{min} \leq A \leq A_{max}$, and $0 < m_{rmin} \leq m_r \leq m_{rmax}$, $0 < m_{lmin} \leq m_l \leq m_{lmax}$, $0 < (m_r b_r)_{min} \leq m_r b_r \leq (m_r b_r)_{max}$, $(m_l b_l)_{min} \leq m_l b_l \leq (m_l b_l)_{max} < 0$, where B_{min}, B_{max}, A_{min}, A_{max},m_{rmin}, m_{rmax}, m_{lmin}, m_{lmax}, $(m_r b_r)_{min}$, $(m_r b_r)_{max}$, $(m_l b_l)_{min}$, and $(m_l b_l)_{max}$ are all known constants.

Assumption 3: The nonlinearity f_u can be bounded by

$$|f_u| \leq \delta(X) f_d(t) \quad (3)$$

where $\delta(X)$ is a known function and $f_d(t)$ is an unknown but bounded time-varying function.

[2]The following nomenclature is used throughout this paper: $\bullet_{min}$ and $\bullet_{max}$are the minimum value and maximum value of $\bullet(t)$ for all t respectively, $\widehat{\bullet}$ denotes the estimate of $\bullet$, $\widetilde{\bullet} = \widehat{\bullet} - \bullet$ denotes the estimation error, e.g., $\widetilde{\theta} = \widehat{\theta} - \theta$, $\bullet_i$ is the i-th component of the vector $\bullet$.

B. Dead-zone Compensation

The essence of compensating dead-zone effect is to employ a perfect dead-zone inverse function $v(t) = D_I(w(t))$ shown in Fig. 1-(2) such that $D(D_I(w(t))) = w(t), \forall w(t)$. The following dead-zone inverse will be used [8]

$$v(t) = \kappa_+(w_d)\frac{w_d(t) + \widehat{(m_r b_r)}}{\widehat{m_r}} + \kappa_-(w_d)\frac{w_d(t) + \widehat{(m_l b_l)}}{\widehat{m_l}} \quad (4)$$

where $\widehat{m_r}$, $\widehat{(m_r b_r)}$, $\widehat{m_l}$, $\widehat{(m_l b_l)}$ are the estimates of m_r, $m_r b_r$, m_l, $m_l b_l$ respectively, w_d is the desired control signal which would achieve the stated control objective when there is no dead-zone effect. And $\kappa_+(w_d)$ and $\kappa_-(w_d)$ are defined by

$$\kappa_+(w_d)(t) = \begin{cases} 1 & if\ \ (w_d(t) > 0\ \ or\ (w_d(t) = 0\& \\ & |v(t_-) - \widehat{b_l}| \geq |v(t_-) - \widehat{b_r}|)) \\ 0 & else \end{cases} \quad (5)$$

$$\kappa_-(w_d)(t) = \begin{cases} 1 & if\ \ (w_d(t) < 0\ \ or\ (w_d(t) = 0\& \\ & |v(t_-) - \widehat{b_l}| < |v(t_-) - \widehat{b_r}|)) \\ 0 & else \end{cases} \quad (6)$$

where $\widehat{b_r} = \frac{\widehat{(m_r b_r)}}{\widehat{m_r}}$ and $\widehat{b_l} = \frac{\widehat{(m_l b_l)}}{\widehat{m_l}}$. The proposed DIARC in section III will use projection type adaptation law for all parameter estimates. With those types of parameter adaptation law and the Assumption 2, the dead-zone parameter estimates $\widehat{m_r}$, $\widehat{(m_r b_r)}$, $\widehat{m_l}$, and $\widehat{(m_l b_l)}$ are guaranteed to be within their known bounded regions all the time. Thus, in the following, it is implicitly assumed that $\widehat{b_r}(t) > 0$ and $\widehat{b_l}(t) < 0, \forall t$. With these facts in mind, the following lemma can be obtained.

Lemma 1: [8] With the dead-zone inverse (4), the error between the actual dead-zone output w by (2) and the desired output w_d can be parametrized as

$$\begin{aligned} w - w_d = &\widetilde{(m_r b_r)}\kappa_+(w_d) + \widetilde{(m_l b_l)}\kappa_-(w_d) \\ &-\frac{w_d + \widehat{(m_r b_r)}}{\widehat{m_r}}\widetilde{m_r}\kappa_+(w_d) - \frac{w_d + \widehat{(m_l b_l)}}{\widehat{m_l}}\widetilde{m_l}\kappa_-(w_d) + d(t) \end{aligned} \quad (7)$$

where $d(t)$ is a function defined as

$$\begin{cases} 0 & if\ \ \kappa_+(w_d) = 1\ \&\ v(t) \geq b_r \\ m_r b_r - \frac{w_d + \widehat{(m_r b_r)}}{\widehat{m_r}} m_r & if\ \ \kappa_+(w_d) = 1\ \&\ 0 < v < b_r \\ m_l b_l - \frac{w_d + \widehat{(m_l b_l)}}{\widehat{m_l}} m_l & if\ \ \kappa_-(w_d) = 1\ \&\ b_l < v < 0 \\ 0 & if\ \ \kappa_-(w_d) = 1\ \&\ v(t) \leq b_l \end{cases} \quad (8)$$

and bounded above by

$$\begin{cases} \left|(m_r b_r)_{max} - \frac{m_{rmin}(m_r b_r)_{min}}{m_{rmax}}\right| & if\ \ \kappa_+(w_d) = 1 \\ \left|(m_l b_l)_{min} - \frac{m_{lmin}(m_l b_l)_{max}}{m_{lmax}}\right| & if\ \ \kappa_-(w_d) = 1 \end{cases} \quad (9)$$

In addition, $d(t) \in L_2[0,\infty)$ when $\widetilde{\theta} \in L_2[0,\infty)$, and $d(t) \longrightarrow 0$ when $\widetilde{\theta}(t) \longrightarrow 0$ as $t \longrightarrow \infty$.

III. Integrated Direct/Indirect Adaptive Robust Controller (DIARC) Synthesis

In this section, the DIARC strategy [10] will be combined with desired trajectory compensation for synthesizing $w_d(t)$ and an on-line parameter adaptation algorithm with conditional monitoring for the parameter estimates $\widehat{\theta}$.

A. Projection Type Adaptation Law Structure

As in [10], [11], the first step is to use a projection mapping to keep the parameter estimates within the known bounded set Ω_θ as in [11]. Suppose that the parameter estimate $\widehat{\theta}$ is updated by the following projection type adaptation law [12]:

$$\dot{\widehat{\theta}} = Proj_{\widehat{\theta}}(\Gamma\tau),\ \widehat{\theta}(0) \in \Omega_\theta \tag{10}$$

where τ is any estimation function and $\Gamma(t) > 0$ is any continuously differentiable positive symmetric adaptation rate matrix. With this adaptation law structure, the following desirable properties hold [12]:

(P1) The parameter estimates are always within the known bounded set Ω_θ, i.e., $\widehat{\theta} \in \Omega_\theta, \forall t$. Thus, from Assumption 2, $\forall t$, $B_{min} \le \widehat{B}(t) \le B_{max}$, $A_{min} \le \widehat{A}(t) \le A_{max}$, $0 < m_{rmin} \le \widehat{m_r}(t) \le m_{rmax}$, $0 < m_{lmin} \le \widehat{m_l}(t) \le m_{lmax}$, $0 < (m_r b_r)_{min} \le \widehat{(m_r b_r)}(t) \le (m_r b_r)_{max}$, and $(m_l b_l)_{min} \le \widehat{(m_l b_l)}(t) \le (m_l b_l)_{max} < 0$.

(P2)

$$\widetilde{\theta}_b^T(\Gamma^{-1} Proj_{\widehat{\theta}}(\Gamma\tau) - \tau) \le 0,\ \forall\tau \tag{11}$$

B. Integrated Direct/Indirect ARC Law

With the use of the projection type adaptation law structure (10), the parameter estimates are bounded with known bounds, regardless of the estimation function τ to be used. In the following, this property will be used to synthesize an integrated DIARC control law for the system (1) which achieves a guaranteed transient and steady-state output tracking accuracy in general. Define

$$s(t) = \dot{\widetilde{x}}(t) + \lambda\widetilde{x}(t) \quad \text{with} \quad \lambda > 0 \tag{12}$$

Combining (1), (7) and (12), we get

$$\begin{aligned}\dot{s}(t) &= \lambda\dot{\widetilde{x}}(t) - \ddot{x}_d(t) - B\dot{x} - AS_f(\dot{x}) + w(t) + f_u \\ &= \lambda\dot{\widetilde{x}}(t) - \ddot{x}_d(t) + \varphi\theta_p + w_d(t) + \widetilde{(m_r b_r)}\kappa_+(w_d) \\ &\quad + \widetilde{(m_l b_l)}\kappa_-(w_d) - \tfrac{w_d + \widehat{(m_r b_r)}}{\widehat{m_r}}\widetilde{m_r}\kappa_+(w_d) \\ &\quad - \tfrac{w_d + \widehat{(m_l b_l)}}{\widehat{m_l}}\widetilde{m_l}\kappa_-(w_d) + d(t) + f_u\end{aligned} \tag{13}$$

where $\varphi = [-\dot{x}, -S_f(\dot{x})]$, $\theta_p = [B, A]^T$. The following DIARC function is proposed to design w_d as follows:

$$\begin{aligned}&w_d = w_{da} + w_{ds}, w_{da} = w_{da1} + w_{da2}, w_{ds} = w_{ds1} + w_{ds2}, \\ &w_{da1} = -(\lambda\dot{\widetilde{x}}(t) - \ddot{x}_d(t)) - \varphi_d\widehat{\theta}_p, w_{da2} = -\widehat{d}_c, w_{ds1} = -k_{s1}s\end{aligned} \tag{14}$$

In (14), w_{da1} represents the adjustable model compensation with the physical parameter estimates $\widehat{\theta}_p$ updated using an on-line adaptation algorithm with condition monitoring to be detailed in subsection III-C, $\varphi_d = [-\dot{x}_d, -S_f(\dot{x}_d)]$ is the regressor that depends on the desired trajectory $X_d(t)$ only and thus is free of measurement noise effect. w_{da2} is a model compensation term similar to the fast dynamic compensation type model compensation used in the DARC designs [11], in which $\widehat{d}_c$ can be thought as the estimate of the low frequency component of the lumped model uncertainties. w_{ds} represents the robust control term in which w_{ds1} is a simple proportional feedback to stabilize the nominal system and w_{ds2} is a robust feedback term used to attenuate the effect of various model uncertainties for a guaranteed robust control performance in general. k_{s1} is a nonlinear gain large enough so that

$$\mathbf{Q_1} = \begin{bmatrix} k^\star k_{s1} - k_2 + B + Ag & -\frac{1}{2}\lambda(B + Ag + \lambda) \\ -\frac{1}{2}\lambda(B + Ag + \lambda) & \frac{1}{2}\lambda^3 \end{bmatrix} > 0 \tag{15}$$

where g is defined by $S_f(\dot{x}) - S_f(\dot{x}_d) = g(\dot{x}, t)\dot{\widetilde{x}}$, k_2 is any positive constant gain, $k^\star = \kappa_+(w_d)\frac{m_r}{\widehat{m_r}} + \kappa_-(w_d)\frac{m_l}{\widehat{m_l}}$ which has a value of 1 in the absence of dead-zone slopes estimation error. Substituting (14) into (13),

$$\begin{aligned}\dot{s}(t) &= (-B - Ag)\dot{\widetilde{x}} - \varphi_d\widetilde{\theta}_p + d(t) + f_u \\ &\quad + [\tfrac{m_r}{\widehat{m_r}}(w_{ds} + w_{da2}) - \tfrac{\widetilde{m_r}}{\widehat{m_r}}w_{da1} + \widetilde{(m_r b_r)} - \widetilde{m_r}\tfrac{\widehat{(m_r b_r)}}{\widehat{m_r}}]\kappa_+(w_d) \\ &\quad + [\tfrac{m_l}{\widehat{m_l}}(w_{ds} + w_{da2}) - \tfrac{\widetilde{m_l}}{\widehat{m_l}}w_{da1} + \widetilde{(m_l b_l)} - \widetilde{m_l}\tfrac{\widehat{(m_l b_l)}}{\widehat{m_l}}]\kappa_-(w_d)\end{aligned} \tag{16}$$

Define a constant d_c and time varying $\triangle^\star(t)$ such that

$$\begin{aligned}d_c + \triangle^\star(t) &= -\varphi_d\widetilde{\theta}_p + d(t) + f_u - [\tfrac{\widetilde{m_r}}{\widehat{m_r}}\kappa_+(w_d) \\ &\quad + \tfrac{\widetilde{m_l}}{\widehat{m_l}}\kappa_-(w_d)]w_{da1} + [\widetilde{(m_r b_r)} - \widetilde{m_r}\tfrac{\widehat{(m_r b_r)}}{\widehat{m_r}}]\kappa_+(w_d) \\ &\quad + [\widetilde{(m_l b_l)} - \widetilde{m_l}\tfrac{\widehat{(m_l b_l)}}{\widehat{m_l}}]\kappa_-(w_d) = \vartheta\psi(t) + d(t) + f_u\end{aligned} \tag{17}$$

where $\vartheta = [\widetilde{B}, \widetilde{A}, \frac{\widetilde{m_r}}{\widehat{m_r}}\kappa_+(w_d) + \frac{\widetilde{m_l}}{\widehat{m_l}}\kappa_-(w_d), [\widetilde{(m_r b_r)} - \widetilde{m_r}\frac{\widehat{(m_r b_r)}}{\widehat{m_r}}]\kappa_+(w_d) + [\widetilde{(m_l b_l)} - \widetilde{m_l}\frac{\widehat{(m_l b_l)}}{\widehat{m_l}}]\kappa_-(w_d)]$, $\psi(t) = -[-\dot{x}_d, -S_f(\dot{x}_d), w_{da1}, -1]^T$. Conceptually, (17) lumps the original system uncertain nonlinearity f_u with the model uncertainties due to physical parameter estimation errors, and divides it into the static component (or low frequency component in reality) d_c and the high frequency components $\triangle^\star(t)$. The low frequency component d_c will be compensated through fast adaptation similar to those in DARC designs [11] and DIARC designs [10] as follows.

Let d_{cM} be any pre-set bound and use this bound to construct the projection type adaptation law for $\widehat{d}_c$:

$$\dot{\widehat{d}}_c = Proj_{\widehat{d}_c}(\gamma s) = \begin{cases} 0 & if \quad |\widehat{d}_c(t)| = d_{cM} \ \&\ \widehat{d}_c s > 0 \\ \gamma s & else \end{cases} \tag{18}$$

with $\gamma > 0$ and $\widehat{d}_c(0) = 0$. Such an adaptation law guarantees $|\widehat{d}_c(t)| \le d_{cM}$, $\forall t$. Substituting (17) to (16) and noting (14), $\dot{s}(t)$ can be written as

$$\begin{aligned}\dot{s}(t) &= (-B - Ag)\dot{\widetilde{x}} + k^\star(w_{ds} + w_{da2}) + d_c + \triangle^\star(t) \\ &= (-B - Ag)\dot{\widetilde{x}} + k^\star w_{ds1} + k^\star w_{ds2} - \widetilde{d}_c \\ &\quad + (1 - k^\star)\widehat{d}_c + \triangle^\star(t)\end{aligned} \tag{19}$$

Noting Assumptions 2 and 3, and (P1), there exists a w_{ds2} such that the following two conditions are satisfied:

$$\begin{aligned}&(i) \quad s w_{ds2} \le 0 \\ &(ii) \quad s[k^\star w_{ds2} - \widetilde{d}_c + (1 - k^\star)\widehat{d}_c + \triangle^\star(t)] \le \varepsilon + \varepsilon_d f_d^2\end{aligned} \tag{20}$$

where ε and ε_d are design parameters which can be arbitrarily small. Essentially, (ii) of (20) shows that w_{ds2} is synthesized to dominate the model uncertainties coming from both parametric uncertainties and uncertain nonlinearities, and (i) is to guarantee that w_{ds2} is dissipative in nature so that it does not interfere with the functionality of the adaptive control part w_{da}.

Remark 1: One example of w_{ds2} satisfying (20) can be found in the following way. Let h be any function satisfying

$$h \geq |k^{\star}_{max}||w_{da2}| + ||\vartheta_{max}||||\psi|| + |d(t)|_{max} \tag{21}$$

where $k^{\star}_{max} = max\{\frac{m_{rmax}}{m_{rmin}}, \frac{m_{lmax}}{m_{lmin}}\}$, $\vartheta_{1max} = B_{max} - B_{min}$, $\vartheta_{2max} = A_{max} - A_{min}$, $\vartheta_{3max} = max\{\frac{m_{rmax}-m_{rmin}}{m_{rmin}}, \frac{m_{lmax}-m_{lmin}}{m_{lmin}}\}$, $\vartheta_{4max} = max\{(m_r b_r)_{max} - (m_r b_r)_{min} + (m_{rmax} - m_{rmin})\frac{(m_r b_r)_{max}}{m_{rmin}}, (m_l b_l)_{max} - (m_l b_l)_{min} - (m_{lmax} - m_{lmin})\frac{(m_l b_l)_{min}}{m_{lmin}}\}$, $|d(t)|_{max} = max\{|(m_r b_r)_{max} - m_{rmin}\frac{(m_r b_r)_{min}}{m_{rmax}}|, |(m_l b_l)_{min} - m_{lmin}\frac{(m_l b_l)_{max}}{m_{lmax}}|\}$. Then, w_{ds2} can be chosen as

$$w_{ds2} = -\left[\frac{1}{4\varepsilon k^{\star}_{min}}h^2 + \frac{1}{4\varepsilon_d k^{\star}_{min}}\delta^2(X)\right]s \tag{22}$$

where $k^{\star}_{min} = min\{\frac{m_{rmin}}{m_{rmax}}, \frac{m_{lmin}}{m_{lmax}}\}$. It is easy to show that the above choice of w_{ds2} does satisfy (20) [10], [13].

Theorem 1: When the DIARC control law (14) with the dead-zone inverse (4) and the projection type adaptation law (10) is applied, regardless the estimation function τ to be used, in general, all signals in the resulting closed loop system are bounded and the output tracking is guaranteed to have a prescribed transient performance and final tracking accuracy in the sense that the tracking error index s is bounded by

$$V_s \leq e^{-\lambda_v t}V_s^2 + \frac{\varepsilon + \varepsilon_d f^2_{dmax}}{\lambda_v}[1 - e^{-\lambda_v t}] \tag{23}$$

where $V_s = \frac{1}{2}s^2 + \frac{1}{2}\lambda^2\tilde{x}^2$, $\lambda_v = min\{2k_2, \lambda\}$ and f_{dmax} represents the L_∞ norm of the bounded time function $f_d(t)$.

C. Parameter adaptation with On-line Condition Monitoring

The reminder of this part is to construct suitable parameter adaptation algorithms so that an improved final tracking accuracy - asymptotic output tracking can be obtained in the absence of uncertain nonlinearities (i.e., assuming $f_u = 0$ in (1)) with an emphasis on having a good parameter estimation process as well. In the following, we will make full use of the fact that the dead-zone can be perfectly linearly parameterized during the working regions of $v \geq b_r$ or $v \leq b_l$ and the on-line condition monitoring to construct specific estimation functions.

Lemma 2: [8] Define a positive constant H as $H = max\{(m_r b_r)_{max} - (m_r b_r)_{min},\ (m_l b_l)_{max} - (m_l b_l)_{min}\}$. Then, when $|w_d(t)| \geq H$, the dead-zone input $v(t)$ by the proposed dead-zone inverse (4) would satisfy $v(t) \geq b_r$ or $v(t) \leq b_l$.

For notation simplicity, define

$$\chi_+(\bullet) = \begin{cases} 1 & if\ \bullet > 0 \\ 0 & if\ else \end{cases}, \qquad \chi_-(\bullet) = \begin{cases} 1 & if\ \bullet < 0 \\ 0 & if\ else \end{cases} \tag{24}$$

and $\chi(w_d(t)) = \chi_+(w_d(t) - H) + \chi_-(w_d(t) + H)$. Then, noting Lemma 2 and multiplying the plant dynamics (1) by $\chi(w_d(t))$, we have

$$\begin{aligned} \chi(w_d(t))\ddot{x} = {} & \chi(w_d(t))\left(-Bx - AS_f(\dot{x})\right) \\ & + \chi_+(w_d(t) - H)(m_r v(t) - m_r b_r) \\ & + \chi_-(w_d(t) + H)(m_l v(t) - m_l b_l) \end{aligned} \tag{25}$$

which is linearly parameterized by the system parameters. The following linear regression model for parameter estimation is then obtained

$$y_\chi(t) = F^T(t)\theta \tag{26}$$

where

$$\begin{aligned} & y_\chi(t) = \chi(w_d(t))\ddot{x}, \\ & F^T(k) = [\chi(w_d(t))(-Bx), \chi(w_d(t))(-AS_f(\dot{x})), \\ & \quad v(t)\chi_+(w_d(t) - H),\ -\chi_+(w_d(t) - H), \\ & \quad v(t)\chi_-(w_d(t) + H),\ -\chi_-(w_d(t) + H)], \\ & \theta = [B, A, m_r, m_r b_r, m_l, m_l b_l]^T \end{aligned} \tag{27}$$

Note that (26) is in the standard regression form. With this static model, various estimation algorithms can be used to identify unknown parameters, of which the least square estimation algorithm [14] is given below to estimate the values of B, A, m_r, $m_r b_r$, m_l and $m_l b_l$.

Let $H_f(s)$ be the transfer function of any filter with a relative degree larger than or equal to 1 (e.g., $H_f(s) = 1/(\tau_f s + 1)$). Then, applying the filter to both side of (26), we obtains

$$y_{\chi f}(t) = F_f^T(t)\theta \tag{28}$$

Applying the least-square estimation algorithm with forgetting and discrete version of the projection (10), the following formula are obtained to update the parameter estimates as the parameter adaptation algorithm [14]:

$$\begin{aligned} & \varsigma = F_f^T\hat{\theta} - y_{\chi f} = F_f^T\tilde{\theta} \\ & \tau = -\frac{1}{1 + \upsilon tr\{F_f^T\Gamma F_f\}}F_f\varsigma \\ & \dot{\Gamma} = \begin{cases} \kappa\Gamma - \frac{1}{1 + \upsilon tr\{F_f^T\Gamma F_f\}}\Gamma F_f F_f^T\Gamma, & \text{if } \lambda_{max}(\Gamma(t)) \leq \rho_M \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{29}$$

in which ς is called the prediction error, $\kappa \geq 0$ is the forgetting factor, ρ_M is the pre-set upper bound for $\|\Gamma(t)\|$, $\upsilon \geq 0$ with $\upsilon = 0$ leading to the unnormalized algorithm. With these practical modifications, $\Gamma(t) \leq \rho_M I$, $\forall t$. The following lemma summarizes the properties of these estimators [14]:

Lemma 3: [8] With the least squares type adaptation law with projection (29), in the absence of uncertain nonlinearities (i.e., $f_u = 0$ in (1)), in general, $\hat{\theta}(t) \in \Omega_\theta, \forall t$. In addition, if the following persistent excitation (PE) condition is satisfied:

$$\int_t^{t+T} F_f F_f^T d\tau \geq \kappa_p I_p,\ for\ some\ \kappa_p > 0\ and\ T > 0 \tag{30}$$

then, the physical parameter estimate $\hat{\theta}$ converge to their true values (i.e., $\tilde{\theta} \to 0$ as $t \to \infty$) and $\tilde{\theta} \in L_2[0, \infty)$.

D. Asymptotic Output Tracking

Theorem 2: In the absence of uncertain nonlinearities (i.e., assuming $f_u = 0$ in (1)), when the PE condition (30) is satisfied, an improved steady-state tracking performance – asymptotic output tracking – is also achieved, i.e., $\tilde{X}(t) \to 0$ and $s \to 0$ as $t \to \infty$.

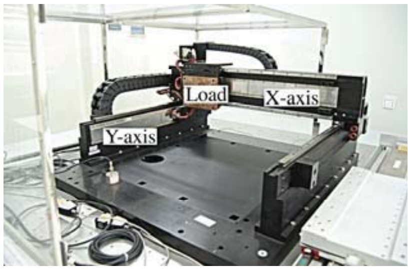

Figure 2. A biaxial linear motor driven gantry system

IV. Comparative Experiments

The proposed DIARC control algorithm is implemented on the Y-axis of a precision X-Y Anorad HERC-510-510-AA1-B-CC2 gantry driven by LC-50-200 iron core linear motors as shown in Figure2. The position sensors of the gantry are two linear encoders with a resolution of $0.5\mu m$ after quadrature. The velocity signal is obtained by the difference of two consecutive position measurements. Standard off-line least-square identification is performed to obtain the parameters of the Y-axis. To test the learning capability of the proposed DIARC algorithms, a 5kg load is mounted on the motor in experiments and the identified values of the parameters are $B = 0.8/s,\ A = 0.23m/s,\ E = 1.55m/s^2/V,\ f_N = 0m/s^2$ where f_N is the nominal value of f_u. And $w(t)$ is the output of a dead-zone described by:

$$w = D(v) = \begin{cases} 0.9(v-1.2) & for\ v \geq 1.2 \\ 0 & for\ -1 < v < 1.2 \\ 1(v-(-1)) & for\ v \leq -1 \end{cases} \tag{31}$$

Adding the effect of $E = 1.55m/s^2$ into the dead-zone, we can get that the approximate accurate values are $\theta = [0.8, 0, 23, 1.395, 1.674, 1.55, -1.55]^T$. As the simulated dead-zone is supposed to be unknown, we set the nominal values to be $\theta_N = [0.8, 0, 23, 1.425, 1.65, 1.5, -1.5]^T$. The bounds of the parametric variations are chosen as

$$\begin{aligned} \theta_{min} &= [0.7, 0.1, 1.3, 1.4, 1.4, -1.8]^T \\ \theta_{max} &= [1, 0.4, 1.6, 1.9, 1.8, -1.4]^T \end{aligned} \tag{32}$$

Assuming $x_d = 0.21 + 0.08sin(\pi t - \pi/2) + 0.07sin(0.8\pi t - \pi/2) + 0.06sin(1.2\pi t - \pi/2)$. Initial values of plant states are set as $X(0) = [0, 0]^T$. The control algorithms are implemented using a dSPACE DS1103 controller board. The controller executes programs at a sampling period of $T_s = 0.2ms$, resulting in a velocity measurement resolution of $0.0025m/sec$. The following three control algorithms for the system (1) are implemented and compared:

C1: The proposed DIARC adapting the estimates of physical parameters B, A, E, f_N with ignoring the existence of the unknown dead-zone (31).

C2: The proposed DIARC presented in section II and III.

C3: The proposed DIARC in which only B, A, E, f_N are updated based on the proposed on-line parameter adaptation algorithm and the dead-zone (31) effect is compensated by (4) with actual dead-zone parameter values assuming the dead-zone parameters are known.

In C2, w_{ds2} in (14) is given in section III-B. Theoretically, we should use the form of $w_{ds2} = -k_{s2}s$ with k_{s2} being a nonlinear proportional feedback gain as given in (22) to satisfy the robust performance requirement (20) globally. In implementation, a large enough constant feedback gain k_{s2} is used instead to simplify the resulting control law as done in [15]. With this simplification, noting (14), we choose $w_{ds} = -k_s s$ in the experiments where k_s represents the summation of k_{s1} and k_{s2} and is chosen as: $k_s = 200$. The constant λ in (12) is $\lambda = 200$. The constant adaptation rate γ in (18) is $\gamma = 1000$ and the bound of $\widehat{d_c}$ is $d_{cM} = 2$. The initial values£º $\widehat{\theta}(0) = [0.8, 0.23, 1.425, 1.65, 1.5, -1.5]^T$, $\Gamma(0) = 1000I_6$. The forgetting factor is $\alpha = 0.02$. The constant H in Lemma 2 is $H = 0.7$. For a fair comparison, the controller parameters and the initial values in case C1 and C3 are chosen the same as in C2 when they have the same meaning (e.g., w_{ds} in C1 and C3 are chosen the same as in C2). The initial values of the adaptation matrix Γ in (29) for C1 and C3 is $\Gamma(0) = 1000I_4$ for that the dead-zone parameters are not adapted, consequently there are B, A, E, f_N need to be estimated and their initial values are $[0.8, 0.23, 1.55, 0]^T$. In C3, the dead-zone (31) are known and can be completely compensated by its inverse (4).

The experimental results in terms of all performance indexes after running for one period are given in Table I. Overall, C2 and C3 achieve almost the same tracking performances which are better than C1. The experimental results for all three cases are shown in Figures 3–6. Specifically, Fig. 3 shows the tracking errors of all three cases. It is also seen that steady-state output tracking errors of C2 and C3 are almost the same and in the order of $10^{-6}m$, much less than that of C1 which is in the order of $10^{-5}m$. These results agree with the prediction by theory as it is shown in section III that the proposed DIARC in C2 is able to achieve asymptotic convergence of dead-zone parameter estimates and perfect dead-zone compensation when the estimates converge. This is also verified by the history of dead-zone parameter estimates shown in Fig. 4. The estimate values $\widehat{B} = 0.8, \widehat{A} = 0.23, \widehat{E} = 1.58$ shown in Fig. 5 are quite accordant with their off-line identified values, and the estimate values $\widehat{\theta} = [0.82, 0.225, 1.42, 1.66, 1.5, -1.49]^T$ shown in Fig. 4 are accordant with the approximate accurate value of θ. Fig. 6 shows the control inputs $v(t)$ of all three cases, which show the chattering phenomena especially the sudden jumps due to the dead-zone inverse (4) when the inputs are near zero. Overall, the improved steady-state error of C2 over C1 validates the necessity of dead-zone compensation. And the almost same asymptotically converging steady-state tracking performance of C2 and C3 validates the perfect compensation of unknown dead-zones by the proposed algorithm.

Table I
Performances Indexes

Controller	C1	C2	C3
$\|\|\tilde{x}\|\|_2(\mu m)$	23.41	2.70	2.64
$\|\|\tilde{x}\|\|_\infty(\mu m)$	84.04	9.95	9.96

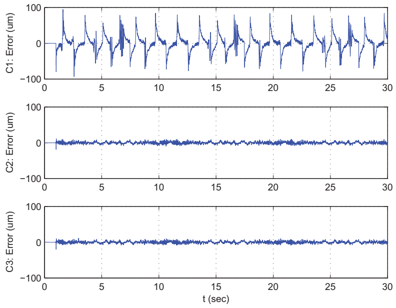

Figure 3. Output tracking errors of DIARCs with dead-zone

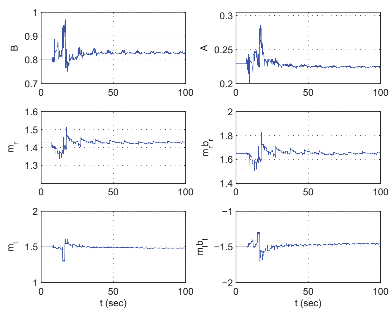

Figure 4. Parameter estimates of DIARC in C2

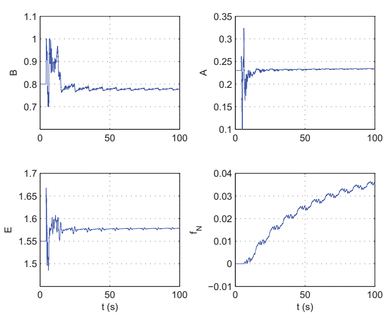

Figure 5. Parameter estimates of DIARC in C3

V. Conclusion

In this paper, an integrated direct/indirect desired compensation adaptive robust control scheme has been developed for electrical drive systems having unknown dead-zone effects. Theoretically, certain guaranteed robust transient performance and steady-state tracking accuracy are achieved even when the overall system may be subjected to parametric uncertainties, time-varying disturbances and other uncertain nonlinearities. Furthermore, zero steady-state output tracking error is achieved when the system is subjected to unknown parameters and unknown dead-zone nonlinearity only. Comparative experimental results are obtained on a linear motor drive system

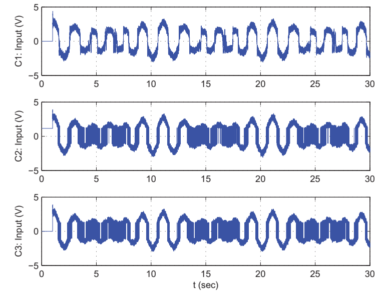

Figure 6. Control input history of DIARCs with dead-zone

preceded by simulated unknown dead-zone nonlinearity, which validate the effectiveness of the proposed dead-zone compensation scheme. The excellent output tracking performances obtained in the experiments also verify the high performance nature of the proposed DIARC strategy.

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