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Adaptive Robust Control of Linear Motor Systems with Dynamic Friction Compensation Using Modified LuGre Model

Lu Lu, Bin Yao, Qingfeng Wang, and Zheng Chen

***Abstract*— LuGre model has been widely used in dynamic friction modeling and compensation. However, there are some practical difficulties when applying it to systems experiencing large range of motion speeds such as the linear motor drive system studied in the paper. This paper first details the digital implementation problems of the LuGre model based dynamic friction compensation. A modified model is then presented to overcome those shortcomings. The proposed model is equivalent to LuGre model at low speed, and the static friction model at high speed, with a smooth transition between them. A discontinuous projection based adaptive robust controller (ARC) is then constructed, which explicitly incorporates the proposed modified dynamic friction model for a better friction compensation. Nonlinear observers are built to estimate the unmeasurable internal state of the dynamic friction model. Online parameter adaptation is utilized to reduce the effect of various parametric uncertainties while certain robust control laws are synthesized to effectively handle various modeling uncertainties for a guaranteed robust performance. The proposed controller is also implemented on an industrial linear motor driven gantry system, along with controllers with the traditional static friction compensation and LuGre model compensation. Extensive comparative experimental results have been obtained, revealing the instability when using the traditional LuGre model for dynamic friction compensation at high speed experiments and the improved tracking accuracy when using the proposed modified dynamic friction model. The results validate the effectiveness of the proposed approach in practical applications.**

***Index Terms*— Dynamic friction; LuGre Model; Motion Control; Linear Motor; Adaptive Robust Control**

I. INTRODUCTION

Friction modeling and compensation has been a research topic which has been studied extensively but still full of interesting problems due to its practical significance and the complex behavior of friction [7]. It has been well known that to have accurate motion control accuracy at low speed movement, friction cannot be simply modeled as a static nonlinear function of velocity alone but rather a *dynamic* function of velocity and displacement. Thus, during the past decade, significant efforts have been devoted to solving the difficulties in modeling and compensation of dynamic friction with various types of models proposed [2], [1], [4]. Among them, the so called LuGre model by Canudas de Wit *et al.* [4] can describe major features of dynamic friction, including presliding displacement, varying break-away force and Stribeck effect.

Due to its relatively simpler form and yet powerful enough to be able to simulate major dynamic friction behaviors, LuGre model has been widely used in control with dynamic friction compensation [6], [9]. Although many good application results have been reported, some practical problems are also discovered, especially when applying the LuGre model to systems experiencing large ranges of motion speeds such as the linear motor drive system studied in this paper. Namely, the traditional LuGre model could become very stiff when the velocity is large. This leads to some unavoidable implementation problems since dynamic friction compensation can only be implemented digitally due to its highly nonlinear characteristics. For example, it has been reported in [5] that the observer dynamics to recover the unmeasurable internal state of the LuGre model could become unstable at high speed motions.

On the other hand, no matter how accurate the mathematical models of dynamic friction is, it is impossible to capture the entire nonlinear behaviors of actual friction and to have a perfect friction compensation. So advanced control techniques have to be used in parallel with appropriate selection of dynamic friction models for effective friction compensation. The idea of adaptive robust control (ARC) [15], [16], [12] incorporates the merits of deterministic robust control (DRC) and adaptive control (AC), which guarantees certain robust performance in presence of uncertainties while having a controlled robust learning process for better control performance. In [10], the proposed ARC strategy has also been applied and tested on an *epoxy* core linear motor without dynamic friction compensations.

In this paper, we first revisit the LuGre model and discuss the digital implementation problems when using the model for dynamic friction compensation. Based on the analysis, a modified version of LuGre model is proposed for dynamic friction compensation to overcome those shortcomings. We then construct a discontinuous projection based adaptive robust controller (ARC), which explicitly incorporates the proposed modified dynamic friction model. It is shown theoretically that the proposed algorithm guarantees certain robust performance in presence of various uncertainties while

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having the perfect trajectory tracking capability when the system has parametric uncertainties only. The proposed ARC algorithm, along with ARC algorithms with friction compensations using the LuGre model and the static friction model respectively, are also tested on a linear motor driven industrial gantry system. Comparative experimental results are presented to illustrate the effectiveness of the proposed modified LuGre model in practical applications and the excellent tracking performance of the proposed ARC algorithm.

II. LUGRE MODEL AND ITS MODIFICATION

Among all the existing dynamic friction models, LuGre model is a very popular one, especially favored by control engineers. It is proposed by Canudas de Wit *et al.* in [4]. Theories and experiments have shown that LuGre model is able to capture most friction phenomena that are of interests to feedback controls. With this model, the friction is given by

$$f = \sigma_0 z + \sigma_1 \dot{z} + \alpha_2 v \quad (1)$$

$$\dot{z} = v - \frac{|v|}{g(v)} z \quad (2)$$

$$g(v) = \alpha_0 + \alpha_1 e^{-(v/v_s)^2} \quad (3)$$

where z represents the unmeasurable internal friction state, σ_0, σ_1, α_2 are unknown friction force parameters that can be physically explained as the stiffness, the damping coefficient of bristles, and viscous friction coefficient. The function $g(v)$ is positive and it describes the Stribeck effect: $\sigma_0\alpha_0$ and $\sigma_0(\alpha_0+\alpha_1)$ represent the levels of the Coulomb friction and stiction force respectively, and v_s is the Stribeck velocity.

In [8], the passivity of the LuGre model is discussed, a decreasing $\sigma_1(v)$ with respect to velocity is shown to be necessary. It has also been proved in [8] that the LuGre model is passive if $\sigma_1(v) < \frac{4\sigma_0 g(v)}{|v|}$. Thus we explicitly denote $\sigma_1(v)$ as $\bar{\sigma}_1 h(v)$, where $h(v)$ is a decay function with respect to velocity, satisfying the above conditions for passivity. With this modification, (1) can be rewritten as:

$$f = \sigma_0 z + \bar{\sigma}_1 h(v) \dot{z} + \alpha_2 v \quad (4)$$

However, this model still has some implementation problems. Namely, in order to compensate for the dynamic friction using LuGre model, it is necessary to build observers to estimate the unknown internal state z. With LuGre model, the observer dynamics would be of the form of

$$\dot{\hat{z}} = v - \frac{|v|}{g(v)} \hat{z} + \gamma\tau \quad (5)$$

where γ represents the observer gain and τ is the observer error correction function to be selected. Since the observer dynamics (5) are highly nonlinear, the only way to implement it is through microprocessors using the discretized version of (5) assuming certain sampling rate. With the digital implementation of (5), to avoid instability due to discretization with a finite sampling rate, it is necessary that the equivalent gain $\frac{|v|}{g(v)}$ in (5) is not too high. In [5], it is shown that if the velocity exceeds a critical value which is proportionally related to the sampling rate, digital implementation of the above observer dynamics will become unstable.

On the other hand, the dynamic friction effect is noticeable only when the relative velocity is low. For high speed motions, it is enough to use the following traditional static friction model:

$$f = F_c \mathrm{sgn}(v) + F_v v \quad (6)$$

With all these facts in mind, it is worth noting that Ref. [3] also briefly mentioned the possibility of stopping the integration of z and using its steady-state value $\hat{z}_{ss} = F_c \mathrm{sgn}(v) + \alpha_2 v$ when the speed is above certain critical value. But this rather simplistic modification may result in discontinuous internal state estimation when the speed changes between high and low ranges. In addition, no experimental results have been provided to validate such a modification. In the following, a modified LuGre model will be proposed, which is essentially equivalent to LuGre model (4) at low speeds, and the static friction model (6) at high speeds, with a smooth transition between these two models from low speeds to high speeds. Specifically, the proposed model has the form of

$$f = \sigma_0 s(|v|) z + \bar{\sigma}_1 h(v) \dot{z} + F_c \mathrm{sgn}(v)[1 - s(|v|)] + \alpha_2 v \quad (7)$$

$$\dot{z} = s(|v|)(v - \tfrac{|v|}{g(v)} z) \quad (8)$$

$$g(v) = \alpha_0 + \alpha_1 e^{-(v/v_s)^2} \quad (9)$$

where $s(|v|)$ is a non-increasing continuous function of $|v|$ with the following properties:

P1: $s(|v|) = 1$ if $|v| < l_1$ and $s(|v|) = 0$ if $|v| > l_2$, in which $l_2 > l_1 > 0$.

In the above, l_1 and l_2 are the cutoff velocities to be selected based on the particular characteristics of system studied and the sampling rate of digital implementation. The essence of this modified LuGre model is to make the internal dynamics stops to update when the velocity is high enough. This solves the instability problem of the original LuGre model in digital implementation. Different from [3], we do not force z to be its static value at high speeds. Thus the estimation of z will be continuous. Furthermore, with the proposed model, it is easy to find that the following desirable properties hold (the proofs are omitted due to space limit but can be obtained from the authors):

Property 1: With the initial internal state chosen such that $|z(0)| \le \alpha_0 + \alpha_1$, the internal states of the modified model (7) to (9) are always bounded above by the same upper bound, i.e., $|z(t)| \le \alpha_0 + \alpha_1, \ \forall t \ge 0$.

Property 2: The mapping from v to f is dissipative if $\bar{\sigma}_1 h(v) < \frac{4\sigma_0 g(v)}{|v|}$

Property 3: When $|v| > l_2$, then the proposed model simplifies into the static friction model given by (6), and when $|v| < l_1$, the model is the exactly the same as the LuGre model of (1) to (3).

For any constant speed v, the steady state friction can be obtained easily by letting $\dot{z} = 0$ in (8):

$$f_{ss} = \{\sigma_0 s(|v|) g(v) + F_c[1 - s(|v|)]\} \mathrm{sgn}(v) + \alpha_2 v \quad (10)$$

Another good property of this model is that F_c can be different from $\sigma_0\alpha_0$. Additionally, $\alpha_2 v$ can also be replaced by $\alpha_{21}vs(|v|)+\alpha_{22}v[1-s(|v|)]$, which makes the viscous term different at high speed from the low speed. As such, the description of friction at low and high speeds are completely separated, with a smooth transition region. This gives us greater flexibility in fitting the friction measurement data over a large range of motion speeds. In the following, for simplicity, we still use the term $\alpha_2 v$, but an extension as discussed above can be worked out easily as well.

III. PROBLEM FORMULATION

The paper uses a three phase iron core linear motor system as a case study. The amplifier has a current loop having a bandwidth higher than $1kHz$. Thus the electrical dynamics can be ignored. Input saturation is also ignored since we do not operate the linear motor with extremely high accelerations. With these simplifications, the linear motor dynamics can be captured well by a second-order system given by

$$\dot{x}_1 = x_2 \quad (11)$$

$$m\dot{x}_2 = u - f + \bar{\Delta} \quad (12)$$

where $x = [x_1 \quad x_2]^T$ represents the state vector consisting of the position and velocity. m denotes the inertia of the system normalized with respect to the control input unit of voltages, $u(t)$ is the control input, f represents the normalized friction, and $\bar{\Delta}$ represents the lumped unknown nonlinear functions including the friction modeling errors and the external disturbances. Here the effect of cogging forces is not explicitly modeled and is lumped into the lumped uncertainties term $\bar{\Delta}$. Using the technique as in [14], the effect of cogging forces can be incorporated easily into the proposed control algorithm as done in some of the experimental results detailed later.

With the modified LuGre model proposed in the previous section, the overall system dynamics are thus given by

$$\dot{z} = s(|x_2|)\left[x_2 - \frac{|x_2|}{g(x_2)}z\right] \quad (13)$$

$$\dot{x}_1 = x_2 \quad (14)$$

$$\begin{aligned} m\dot{x}_2 = \; & u - \sigma_0 s(|x_2|)z \\ & -\bar{\sigma}_1 h(x_2)s(|x_2|)(x_2 - \frac{|x_2|}{g(x_2)}z) \\ & -F_c \mathrm{sgn}(x_2)[1-s(|x_2|)] - \alpha_2 x_2 + \bar{\Delta} \quad (15) \end{aligned}$$

Let $y_d(t)$ be the desired motion trajectory, which is assumed to be known, bounded, with bounded derivatives up to the second order. With the assumed dynamic friction model having known structural information but unknown friction parameters (i.e., shape functions $g(x_2)$, $h(x_2)$ and $s(|x_2|)$ are assumed to be known but the model parameters $\sigma_0, \bar{\sigma}_1, F_c$, and α_2 are unknown), the objective is to synthesize a bounded control input u such that the actual position x_1 tracks $y_d(t)$ as closely as possible in spite of various model uncertainties.

IV. ADAPTIVE ROBUST CONTROL (ARC)

A. Notations and Assumptions

Throughout the paper, the following notations will be used: $\bullet_{\min}$ for the minimum value of $\bullet$, $\bullet_{\max}$ for the maximum value of $\bullet$, and the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors. $\hat{\bullet}$ denotes the estimate of $\bullet$, $\tilde{\bullet} = \hat{\bullet} - \bullet$ denotes the estimation error.

We define a parameter set as $\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^T = [m, \sigma_0, \bar{\sigma}_1, F_c, \alpha_2, -\bar{\Delta}_0]^T$ in which $\bar{\Delta}_0$ can be thought as the constant nominal value of the lumped uncertainties $\bar{\Delta}$ in (15). Denote the time-varying portion of $\bar{\Delta}$ as $\tilde{\bar{\Delta}} = \bar{\Delta} - \bar{\Delta}_0$. Then equation (15) can be re-written as

$$\begin{aligned} \theta_1 \dot{x}_2 = \; & u - \theta_2 s(|x_2|)z - \theta_3 h(x_2)s(|x_2|)(x_2 - \frac{|x_2|}{g(x_2)}z) \\ & -\theta_4 \mathrm{sgn}(x_2)[1-s(|x_2|)] - \theta_5 x_2 - \theta_6 + \tilde{\bar{\Delta}} \quad (16) \end{aligned}$$

To design an adaptive robust controller, the following reasonable and practical assumptions are made:

Assumption 1: The extent of parametric uncertainties is known, i.e.,

$$\theta \in \Omega \triangleq \{\theta : \quad 0 < \theta_{\min} < \theta < \theta_{\max}\}, \quad (17)$$

where $\theta_{\min} = [\theta_{1\min}, \cdots, \theta_{6\min}]^T$ and $\theta_{\max} = [\theta_{1\max}, \cdots, \theta_{6\max}]^T$ are known.

Assumption 2: The uncertain nonlinearity $\tilde{\bar{\Delta}}$ is bounded by a known function $\delta(x,t)$ multiplied by an unknown but bounded time-varying disturbance $d(t)$, i.e.,

$$\tilde{\bar{\Delta}} \in \Omega_{\tilde{\bar{\Delta}}} \triangleq \{\tilde{\bar{\Delta}} : \quad |\tilde{\bar{\Delta}}(x,z,u,t)| \le \delta(x,t)d(t)\}. \quad (18)$$

B. Adaptive Robust Control Law Synthesis

Following the adaptive robust control (ARC) in [11], the control law is developed as follows. Let $e(t) = x_1(t) - y_d(t)$ be the position tracking error. Define a switching-function-like variable p as:

$$p = \dot{e} + k_1 e = x_2 - x_{2eq}, \quad x_{2eq} \triangleq \dot{y}_d - k_1 e, \quad (19)$$

where $k_1 > 0$ is a feedback gain. If p can be made small or converge to zero, then the tracking error e will be small or converge to zero since $G(s) = \frac{e(s)}{p(s)} = \frac{1}{s+k_1}$ is a stable transfer function. Taking the derivative of p give:

$$\begin{aligned} \theta_1 \dot{p} = \; & u - \theta_1 \dot{x}_{2eq} - \theta_2 s(|x_2|)z \\ & -\theta_3 h(x_2)s(|x_2|)(x_2 - \tfrac{|x_2|}{g(x_2)}z) \\ & -\theta_4 \mathrm{sgn}(x_2)[1-s(|x_2|)] - \theta_5 x_2 - \theta_6 + \tilde{\bar{\Delta}} \end{aligned} \quad (20)$$

where $\dot{x}_{2eq} \triangleq \ddot{y}_d - k_1 \dot{e}$. We construct two projection-type observers to estimate the unknown internal state z

$$\begin{aligned} \dot{\hat{z}}_1 &= Proj_{\hat{z}_1}\left\{s(|x_2|)\left[x_2 - \tfrac{|x_2|}{g(x_2)}\hat{z}_1 - \gamma_1 p\right]\right\} \\ \dot{\hat{z}}_2 &= Proj_{\hat{z}_2}\left\{s(|x_2|)\left[x_2 - \tfrac{|x_2|}{g(x_2)}\hat{z}_2 + \gamma_2 \tfrac{h(x_2)|x_2|}{g(x_2)}p\right]\right\} \end{aligned} \quad (21)$$

where the projection mapping is defined as

$$Proj_{\hat{\zeta}}(\bullet) = \begin{cases} 0 & \text{if } \hat{\zeta} = \zeta_{max} \text{ and } \bullet > 0 \\ 0 & \text{if } \hat{\zeta} = \zeta_{min} \text{ and } \bullet < 0 \\ \bullet & \text{otherwise} \end{cases} \quad (22)$$

The observation bounds are set as $z_{1max} = z_{2max} = \alpha_0 + \alpha_1$, $z_{1min} = z_{2min} = -\alpha_0 - \alpha_1$, which corresponds to the physical bounds of internal state. In order to estimate unknown parameters, we use the following on-line adaptation law

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\tau), \quad \tau = \varphi p, \tag{23}$$

where $\varphi^T = [-\dot{x}_{2eq}, \ -s(|x_2|)\hat{z_1}, \ -h(x_2)s(|x_2|)(x_2 - \frac{|x_2|}{g(x_2)}\hat{z_2}), \ -\text{sgn}(x_2)[1-s(|x_2|)], \ -x_2, \ -1]$ and $\Gamma > 0$ is a diagonal matrix. With the above observers and the parameter adaptation law, the following adaptive robust control law is proposed:

$$\begin{aligned} u &= u_a + u_s, \\ & u_a = -\hat{\theta}^T\varphi, \\ & u_s = u_{s1} + u_{s2}, \quad u_{s1} = -k_{s1}p \end{aligned} \tag{24}$$

In (24), u_a is the model compensation term. u_s is a robust control law, in which u_{s1} is used to stabilize the nominal system and u_{s2} is a robust feedback term used to attenuate the effect of various model uncertainties. u_{s2} must satisfy two conditions

i. $pu_{s2} \le 0$
ii. $p\left[u_{s2} - \tilde{\theta}^T\varphi + \theta_2 s(|x_2|)\tilde{z_1} - \theta_3 h(x_2)s(|x_2|)\frac{|x_2|}{g(x_2)}\tilde{z_2} + \tilde{\tilde{\Delta}}\right] \le \varepsilon_0 + \varepsilon_1\|d\|_\infty$

(25)

where ε_0 and ε_1 are two design parameters which may be arbitrarily small.

Theorem 1: If the ARC law (24) is applied, then

A. In general, all signals are bounded. Furthermore, the positive definite function V_s defined by

$$V_s = \frac{1}{2}mp^2 \tag{26}$$

is bounded above by

$$V_s \le \exp(-\lambda_V t)V_s(0) + \frac{\varepsilon_0 + \varepsilon_1\|d\|_\infty}{\lambda_V}[1 - \exp(-\lambda t)], \tag{27}$$

where $\lambda_V = 2k_{s1}/\theta_{1max}$.

B. If after a finite time t_0, there exist parametric uncertainties only (i.e., $\tilde{\tilde{\Delta}} = 0, \forall t \ge t_0$), then, in addition to results in A, zero final tracking error is also achieved, i.e, $e \longrightarrow 0$ and $p \longrightarrow 0$ as $t \longrightarrow \infty$.

The proof of the theorem is omitted due to space limit and can be obtained from the authors.

V. EXPERIMENTAL RESULTS

A. System Setup

In the Precision Mechatronics Lab at Zhejiang University, a two-axes commercial Anorad HERC-510-510-AA1-B-CC2 Gantry by Rockwell Automation has been set-up. The gantry has two built-in linear encoders providing each axis a position measurement resolution of $0.5\mu m$. To study dynamic friction and its compensation at low speed motions, a Renishaw RLE10-SX-XC laser position measurement system with a laser encoder compensation kit RCU10-11ABZ is used. The external laser interferometer system provides a direct measurement of load position with a resolution of $20nm$. The entire system is controlled through a dSPACE DS1103 controller board.

B. System Identification

Experiments have been conducted on the upper X-axis. Off-line parameter identification is carried out at high speed first, in which the proposed dynamic friction model simplifies into (6) and the system dynamics is the same with that in [13]. It is found that the nominal value of m is $0.12volt/m/sec^2$, and the value of F_c is $0.15volt$.

Since our model is a modified version of LuGre model, we follow the same procedures as in [4] to estimate the dynamic friction parameters. The Stribeck curve is shown in Fig. 1, from which we get $\sigma_0 g(x_2) = 0.1236 + 0.0861e^{-|x_2/0.0022|}$ and $\alpha_2 = 0.166$. It is observed that the Stribeck effect is evident during motion with speeds less than $0.08m/sec$, and beyond that the traditional static friction model describes the static friction curve well. Thus we set the l_1 to be $0.08m/sec$ and l_2 to be $0.1m/sec$. Further study shows that using a sampling rate of $5kHz$, the observer dynamics is marginally stable at $0.11m/sec$ when the original LuGre model are used to construct observers like (5). So setting l_2 to be a little less than this critical velocity is reasonable. The part of $s(|x_2|)$ in $[l_1 \ l_2]$ and $[-l_2 \ -l_1]$ is simply chosen as a line section. Higher order functions can also be used when smoothness is concerned.

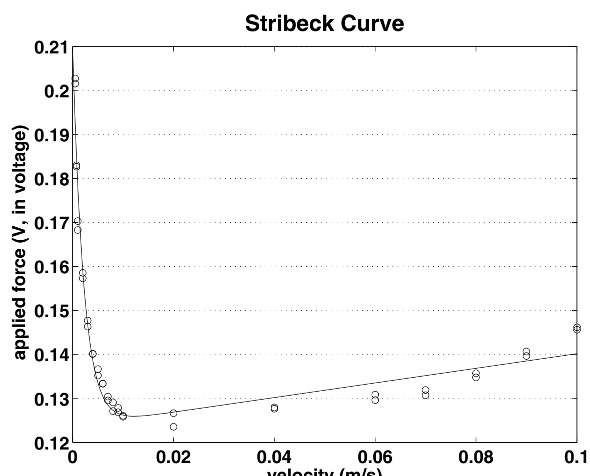

Fig. 1. Stribeck Curve

The function of $h(x_2)$ is chosen to be $h(x_2) = \frac{0.00013}{0.00013+|x_2|}$, which satisfies the passivity condition stated in (2). To obtain σ_0 and $\bar{\sigma}_1$, we operate the system around zero velocity, give it a step input and measure the output response. With these experiments, we get $\sigma_0 = 7000$ and $\bar{\sigma}_1 = 1176$.

C. Comparative Experimental Results

The control system is implemented using a dSPACE DS1103 controller board. The controller executes programs at a sampling frequency $f_s = 5kHz$.

In the following, we compare two algorithms - the adaptive robust control (ARC) with the proposed modified LuGre model based dynamic friction compensation and the adaptive robust control with static friction compensation as done in [13] - for two different types of trajectories, to show the improved performance after using the proposed model and corresponding compensation algorithms.

Low Speed Motions: The ARCs with different friction compensations are first compared for motions of really low speed. The desired trajectory represents a point-to-point movement, with a maximum velocity of $0.0002m/s$ and a maximum acceleration of $0.0002m/s^2$ and a traveling distance of $0.001m$, as shown in Fig. 2. Due to the small travel distance, the cogging force effect is negligible in this case and not explicitly accounted for in the ARC controllers. For dynamic friction compensation with the proposed modified LuGre model, the bounds of the parameter variations in experiments are chosen as: $\theta_{min} = [0.1,\ 4000,\ 500,\ 0.1,\ 0,\ -0.5]^T$ and $\theta_{max} = [0.2,\ 10000,\ 1500,\ 0.3,\ 0.5,\ 0.5]^T$. The gains of the controller are chosen as: $k_1 = 250$, $k_{s1} = 500$. The adaptation rates are set as $\Gamma = \mathbf{diag}\{1,\ 2.5\times10^{10},\ 2.5\times10^{8},\ 100,\ 10,\ 1000\}$ and $\gamma_1 = \gamma_2 = 0.2$. The initial parameter estimates are chosen as: $\hat{\theta}(0) = [0.12, 7000, 1176, 0.15, 0.166, 0]^T$, $\hat{z_1} = 0$ and $\hat{z_2} = 0$. For static friction compensation, all the bounds and controller parameters are the same as in the dynamic friction compensation case, except the parameters related to internal states of dynamic friction model.

The tracking errors of the two ARC controllers are shown in Fig.3. As seen from the plots, during transient periods when the velocity changes directions during the back-and-forth point-to-point motions, the tracking error peaks shown in the upper figure for the ARC with static friction compensation only almost disappear after using the proposed modified LuGre model based dynamic friction compensation. The proposed dynamic friction compensation based ARC achieves a maximum tracking error about $700nm$, while the static friction compensation based ARC has a maximum tracking error around $2\mu m$.

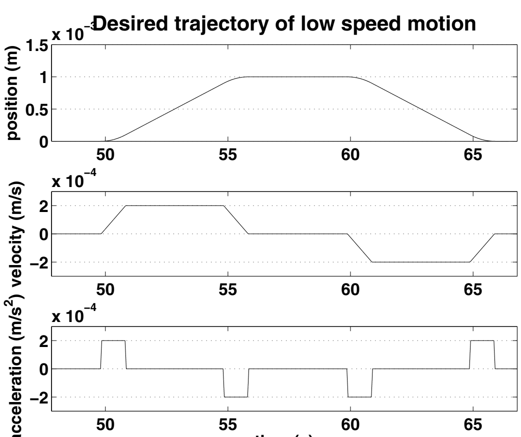

Fig. 2. Desired trajectory, low speed ($v_{max} = 0.0002m/s$)

High Speed Motions: The ARCs with different friction compensations are then compared for motions of high speed. The desired trajectory is also a point-point one similar to that of low speed, but with a maximum velocity of $0.3m/sec$, a maximum acceleration of $5m/sec^2$, and a traveling distance

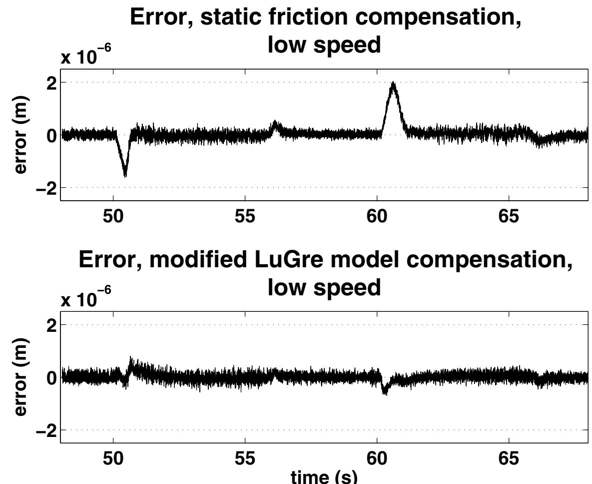

Fig. 3. Error compare, low speed ($v_{max} = 0.0002m/s$)

of $0.4m$. Due to the large travel distance, the cogging force effect should be considered as the Anorad gantry is powered by iron-core linear motors. Cogging force compensation terms are added into the proposed algorithms as in [14].

For the proposed dynamic friction compensation, the bounds of the parameter variations, the initial parameter estimates and the controller gains (k_1 and k_{s1}) are chosen to be the same as in low speed experiment. The adaptation rates are set as $\Gamma = \mathbf{diag}\{1,\ 2.5\times10^{10},\ 10000,\ 100,\ 10,\ 2000\}$ and $\gamma_1 = \gamma_2 = 0.2$. For static friction compensation, all the bounds and controller parameters are the same as in the dynamic friction compensation case, except the parameters related to internal states of dynamic friction model.

To verify the implementation problems of the original LuGre model based observer designs in high speed motions, we also implemented the proposed ARC with LuGre model based dynamic friction compensation, i.e., assuming $s(|v|) = 1$ for all velocity in the proposed ARC controller. As shown in Fig. 4, the estimation of internal states quickly becomes unstable due to the digital implementation and an inadequate sampling rate.

The tracking errors of the ARCs with the proposed dynamic friction compensation and the traditional static friction compensation are plotted in Fig. 5. It can be seen that the transient tracking errors have been significantly reduced with the proposed dynamic friction compensation; the maximum tracking error is reduced by half from around $14\mu m$ to $7\mu m$. The estimates of internal state z shown in Fig. 6 over a single back-and-forth movement reveal a well-behaved observer. All these results validate the effectiveness of the proposed dynamic friction model and compensation.

VI. CONCLUSIONS

A. Conclusions

In this paper, practical digital implementation problems with existing LuGre model and its variations for dynamic friction modeling and compensation are discussed and experimentally verified. A modified version of LuGre was then proposed to solve those implementation problems. An

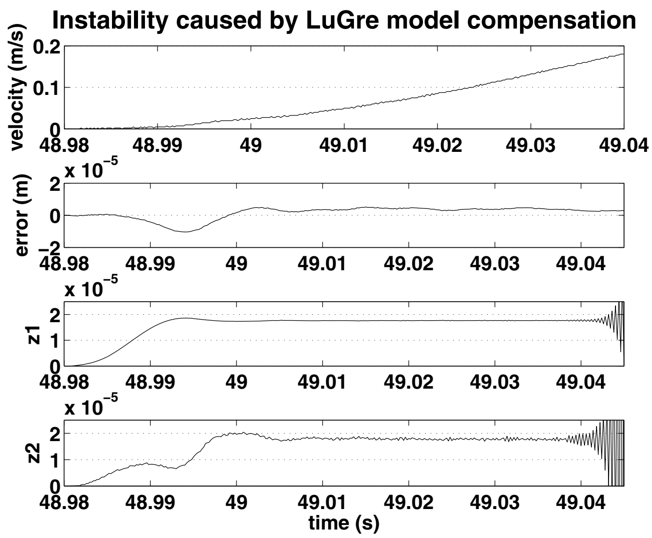

Fig. 4. Instability caused by using LuGre model compensation in high speed ($v_{max} = 0.3m/s$)

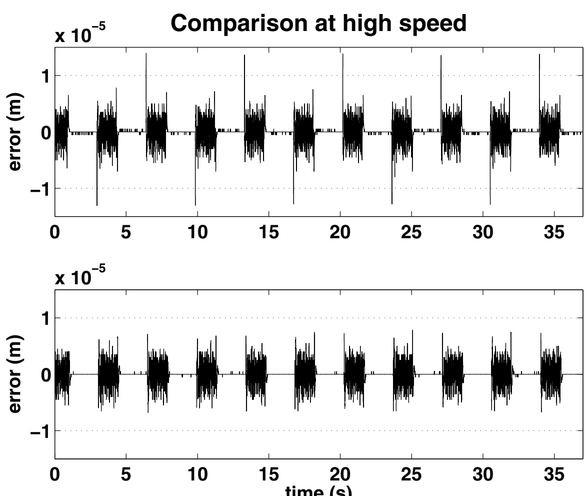

Fig. 5. Error compare, high speed ($v_{max} = 0.3m/s$)

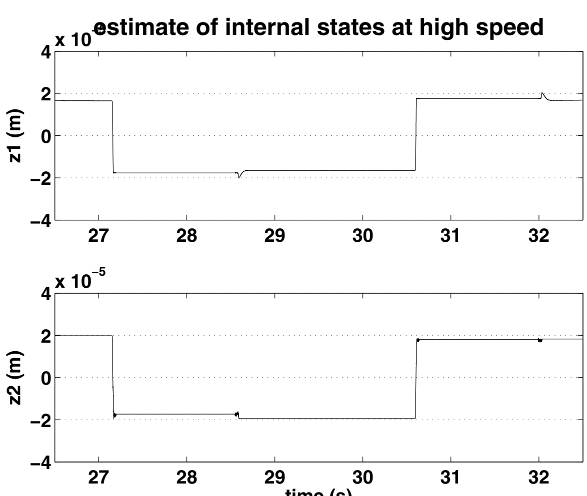

Fig. 6. Estimate of z, high speed ($v_{max} = 0.3m/s$)

adaptive robust control (ARC) algorithm with friction compensation using the proposed dynamic friction model was also developed with rigorous closed-loop stability and performance robustness proofs. The proposed ARC algorithm was also implemented on a linear motor driven industrial gantry system and experimentally compared with the previously presented ARC algorithms with static friction compensation. Comparative experimental results obtained have revealed the substantially improved tracking performance of the proposed ARC with dynamic friction compensation at both low and high speed motions, while without the instability problem of the LuGre model based dynamic friction compensation at high speeds.

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