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# A Globally Stable High Performance Adaptive Robust Control Algorithm with Input Saturation for Precision Motion Control of Linear Motor Drive System

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*Abstract*— **This paper focuses on the synthesis of nonlinear adaptive robust controller with saturated actuator authority for a linear motor drive system, which is subject to parametric uncertainties, unmodeled nonlinearities and input disturbances as well. Global stability is achieved by breaking down the overall uncertainties to state-linearly-dependent uncertainties (such as viscous friction) and bounded nonlinearities (such as coulomb friction, cogging force and etc.) and treating them with different strategies. Furthermore, a guaranteed transient performance and final tracking accuracy can be obtained by incorporating the well-developed adaptive robust controller and effective parameter identifier. Asymptotic output tracking is also achievable in the presence of parametric uncertainties only. Meanwhile, the choice of design parameters is easier to make with the controller designed based on the original states which have physical meanings than the existing control structure with saturation designed in a transformed coordinate.**

## I. INTRODUCTION

All actuators of physical devices are subject to amplitude saturation. Although in some applications it may be possible to ignore this fact, the reliable operation and acceptable performance of most control systems must be assessed in the light of actuator saturation [1]. Lots of research works have been done under the strategy of taking into account the saturation nonlinearities at the stage of the controller design to stabilize the system. It was proved in [2] that global stabilization is not achievable using linear feedback laws for linear systems subject to input saturation. A low-and-high gain method was used in [3] and [4] to provide semi-global stability as well as performance such as disturbance rejection, robustness and so on for asymptotically null controllable linear systems. For the same class of systems, robust control techniques such as H-infinity was employed in [5] to extend the stability from semi-global to global yet with the difficulty to obtain a closed form expression of the solution for high-dimensional systems. A novel control structure was proposed in [6] to ensure the global asymptotic stability for a chain of integrators of arbitrary order by using linear coordinate transformation and multiple sigmoidal functions.

In all those works, it was assumed that the systems of concern are linear and exactly known, which is not the case for most physical systems in reality. Some nonlinear factors such as friction affect system behavior significantly and are rather difficulty to model precisely. There are always discrepancies between the model for controller design and the real system. It is not unusual that some of the system parameters are unknown or their values may vary from time to time. Unpredictable external disturbances also affect the system performance. Therefore, it is of practical importance to take these issues into account when attacking the actuator saturation problem. [7] combined the wise use of saturation functions proposed in [6] with the adaptive robust control (ARC) strategy proposed in [8] and achieved stability and high performance for a chain of integrators subject to matched parameter uncertainty, unmodeled nonlinearities and external disturbance.

In this paper, a new control structure based on the backstepping design [9] and the ARC strategy is proposed. Essentially, a bounded virtual control law is designed to ensure the boundedness and convergence for the error signal during each step except the last one and the actual control input is designed at the last step, consisting of a model compensation term whose bound is calculable due to the pre-known information of the system and a robust control term represented by a saturation function. Two bounds add up equal to the saturation level of the actuator. Thus, the resulting controlled system could be pushed to its physical limit with the guaranteed global stability even in the presence of unmatched nonlinear uncertainties.

The proposed control law is applied on a positioning system driven by linear motors for the purpose of illustration while its generalization to higher order systems is under the investigation. The same equipment set has also been used to test the design in [7]. A coordinate transformation was employed there and all new states are the linear combination of old ones with big coefficients except for $x_n$ whose coefficient is 1. Therefore any unmatched uncertainty or disturbance would be amplified significantly whereas the control input may not have enough effort to dominate that. Furthermore, the design is overly conservative due to the multiple addition of the model uncertainties in obtaining the linear working regions and constraints on the level of control effort.

The interest in this new approach is to provide a more practical solution to the saturated actuator problem with certain type of systems subject to matched uncertainties and unmatched ones as well. The needed control effort is calculated on the original states and there is no need to consider uncertainties for clean channels. The choice of the

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design parameters are also more straightforward due to the physical meaning of the original states.

## II. PROBLEM FORMULATION AND PRACTICAL ASSUMPTIONS

The linear motor used as a case study in this paper is a current-controlled three-phase epoxy core motor, which drives a linear positioning stage supported by recirculating bearings. The governing equation could be written as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ M\dot{x}_2 &= -Bx_2 - F_{sc}sgn(x_2) + d(t) + K_f u \end{aligned} \tag{1}$$

where $x_1$ and $x_2$ represent the stage position and velocity respectively, $M$ is the inertia of the payload plus the coil assembly, $u$ is the control voltage with an input gain of $K_f$, $B$ and $F_{sc}$ represent two major friction coefficients, viscous and coulomb respectively, $d(t)$ represents the lumped unmodeled force and external disturbance.

The above system is subject to unknown parameters due to the variations of the payload, friction coefficients, and the uncertainty of the lumped model dynamics and external disturbance. However, since the mass term $M$ is unlikely to vary (once the payload is fixed) and easy to calculate in advance compared to other three parameters, it is justifiable to estimate it offline and treat it as a known parameter. The low frequency component of the lumped uncertainties is modeled as a unknown constant. The input gain $K_f$ could be calculated from the data provided by the manufacture. Therefore the second equation above could be rewritten as:

$$\begin{aligned} \dot{x}_2 &= -B_m x_2 - F_{scm}sgn(x_2) + d_{0m} + \Delta + K_f u \\ &= \varphi^T(x)\theta + \Delta + \overline{u} \end{aligned} \tag{2}$$

where $\varphi(x) = [-x_2, -sgn(x_2), 1]^T$ is the regressor, $\theta = [B_m, F_{scm}, d_{0m}]^T$ is the vector of unknown parameters, $\Delta = d_m(t) - d_{0m}$ is the lumped disturbance and unmodeled dynamics (high frequency component), and $\overline{u} = K_f * u/M$ whose limits can be backed out from the physical input saturation level.

Before the design procedure goes, the following assumptions are made for the practical system, which could be regarded as pre-known information.

**Assumption 1:** The extent of the parameter variations and normal uncertain nonlinearities are known, i.e.,

$$B \in [B_l, B_u], F_{sc} \in [F_{scl}, F_{scu}], |d(t)| \leq \delta_d$$

where $B_l$, $B_u$, $F_{scl}$, $F_{scu}$, $\delta_d$ are known. With the inertia $M$ estimated for the linear motor, the corresponding bounds for $B_m$, $F_{scm}$, $d_{0m}$ are 8, 15, 2, 5, and $\pm 10$. This assumption leads to the second one.

**Assumption 2:** The total effect of parameter estimates error and lumped unmodeled dynamics are bounded, i.e.,

$$|-\varphi^T(x)\tilde{\theta} + \Delta| \leq h,$$

where $\tilde{\theta} = \hat{\theta} - \theta$, $\hat{\theta}$ is the parameter estimation.

**Remark 1** These two assumptions are reasonable and practical for the linear motor system working under a normal condition, i.e. the controller is powerful enough to overcome the external disturbance of normal level and the bounded model uncertainty plus the model mismatch. However, Assumption 2 needs a little modification to make our case since this paper is mainly focused on the issue of global stability. In other words, the saturated control law we've designed should be able to establish stability no matter where the system states are. A big but short external disturbance such as a strike on the positioning stage could make the states get far beyond the normal working range, so that Assumption 2 won't be practical since the first element of $\varphi(x)$ is $-x_2$. The adjustment of Assumption 2 would be made in the next part.

## III. SATURATED ADAPTIVE ROBUST CONTROL

### A. Saturated ARC Controller Design

The desired trajectory the linear motor is supposed to follow is preset, i.e. $x_{1d}$, $\dot{x}_{1d}$ and $\ddot{x}_{1d}$ are known and we want to design a control law to make the tracking error as small as possible while simultaneously taking into account the physical actuator limit to avoid any disaster caused by control saturation.

Back-stepping design introduced in [9] is employed.

Define $z_1 = x_1 - x_{1d}$ as the tracking error, $\alpha_1$ as the bounded virtual control law designed for $z_1$ dynamics, which is $\dot{z}_1 = x_2 - \dot{x}_{1d}$, also define $z_2 = x_2 - \alpha_1$, then $z_1$ dynamics becomes:

$$\dot{z}_1 = z_2 + \alpha_1 - \dot{x}_{1d} \tag{3}$$

The adaptive robust control law for $\alpha_1$ is proposed as:

$$\alpha_1 = \alpha_{1a} + \alpha_{1s}, \alpha_{1a} = \dot{x}_{1d}, \alpha_{1s} = -\sigma_1(z_1) \tag{4}$$

where $\sigma_1(z_1)$ is a saturation function and will be described in detail later. Then we have

$$\dot{z}_1 = z_2 - \sigma_1(z_1) \tag{5}$$

$\sigma_1(z_1)$ is designed to be a smooth (first order differentiable) saturation function with respect to $z_1$ and have four properties as follows:

(i) If $|z_1| < L_{11}$, then $\sigma_1(z_1) = k_1 z_1$

(ii) $z_1\sigma_1 > 0, \forall z_1 \neq 0$.

(iii) $|\sigma_1(z_1)| \leq M_1, \forall z_1 \in \mathbb{R}$.

(iv) $|\frac{\partial \sigma_1}{\partial z_1}| \leq k_1$ if $|z_1| < L_{12}$, and $|\frac{\partial \sigma_1}{\partial z_1}| = 0$ if $|z_1| \geq L_{12}$.

Graphically, this function could be drawn as Fig. 1 and $L_{11}$, $L_{12}$, $k_1$, $M_1$ are the design parameters. Now, let's look at the dynamics of $z_2$:

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = \varphi^T(x)\theta + \Delta + \overline{u} - \ddot{x}_{1d} + \frac{\partial \sigma_1}{\partial z_1}(z_2 - \sigma_1) \tag{6}$$

Design $\overline{u} = \overline{u}_a + \overline{u}_s$ as model compensation and robust term respectively. The idea is to use $\overline{u}_a$ to compensate the known model dynamics and $\overline{u}_s$ to fight against the model mismatch plus the disturbance. Moreover, both $\overline{u}_a$ and $\overline{u}_s$ should be designed bounded to make sure that $\overline{u}$ stays within its limitation.

As we have discussed earlier, $\varphi(x)$ cannot be assumed to be bounded. However, if we take a look at each element

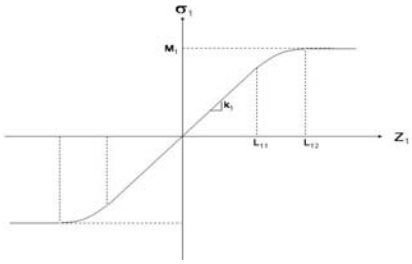


Fig. 1. Saturated robust control term for $z_1$

of $\varphi(x)$, we notice that it can be divided into two parts: $-x_2 = -z_2 - \alpha_1$ and $[-sgn(x_2), 1]^T$. The former part would go unbounded under a sudden and strong disturbance. However it also acts as a damping to help stabilize the system, therefore it could be left uncompensated as long as $-\alpha_1$ is compensated, which is bounded. The latter part is highly nonlinear, but is bounded always. Therefore, it could be taken care of by the model compensation term $\overline{u}_a$ as follows:

$$\overline{u}_a = \hat{B}_m\alpha_1 - [-sgn(x_2), 1]\cdot[\hat{F}_{scm}, \hat{d}_m]^T + \ddot{x}_{1d} + \frac{\partial \sigma_1}{\partial z_1}\sigma_1 \quad (7)$$

Let's denote the new bounded regressor $[-\alpha_1, -sgn(x_2), 1]^T$ as $\varphi_b$ and the bound of $\overline{u}_a$ is easy to estimate with Assumption 1, known desired trajectory and property (iii) and (iv) of $\sigma_1$.

Besides the control law, we also design a projection-type parameter estimate algorithm to provide $\hat{\theta}$ as follows:

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\varphi_b z_2) \quad (8)$$

$$Proj_{\hat{\theta}}(\bullet_i) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{max} \text{ and } \bullet_i > 0 \\ 0 & \text{if } \hat{\theta}_i = \theta_{min} \text{ and } \bullet_i < 0 \\ \bullet_i & \text{otherwise} \end{cases} \quad (9)$$

and make sure that the following desirable properties always hold.

P1. The parameter estimates are always within the known bound at any time instant $t$.

P2. $\tilde{\theta}^T(\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\varphi_b z_2) - \varphi_b z_2) \leq 0$.

Note, this law cannot guarantee that the parameter estimates would converge to their true value.

$z_2$ dynamics becomes:

$$\dot{z}_2 = -B_m z_2 - \varphi_b^T\tilde{\theta} + \Delta + \overline{u}_s + \frac{\partial \sigma_1}{\partial z_1}z_2 \quad (10)$$

Now since we have made $\varphi_b$ bounded, Assumption 2 modified as follows becomes acceptable.

**Assumption 2 (modified):** $|-\varphi_b^T(x)\tilde{\theta} + \Delta| \leq h$,where $h$ is the bound of the total effect of model mismatch plus the unmodeled uncertainty under the normal working environment and could be estimated based on the prior known information.

As we can see from (10), the robust term $\overline{u}_s$ needs to overcome the term coming from the first channel $\frac{\partial \sigma_1}{\partial z_1}z_2$ plus the bounded model mismatch in order to make $z_2$ converge or at least, bounded.

In order to actively take into account the actuator saturation problem when the control law is designed, again we use another saturation function $\sigma_2(z_2)$ to construct $\overline{u}_s$. Let $\overline{u}_s = -\sigma_2(z_2)$, where $\sigma_2(z_2)$ has the properties similar as those that $\sigma_1(z_1)$ has. Notice this is the last channel of the system and $\overline{u}_s$ is a part of the real control input, therefore $\sigma_2(z_2)$ only needs to be continuous instead of smooth. Fig. 2 shows what $\sigma_2(z_2)$ looks like and the design parameters are $L_{21}$, $k_{21}$, $L_{22}$, $k_{22}$. The complete form of control input is as follows:

$$\begin{aligned} \overline{u} &= \hat{B}_m\alpha_1 - [-sgn(x_2), 1]\cdot[\hat{F}_{scm}, \hat{d}_m]^T \\ &+ \ddot{x}_{1d} + \tfrac{\partial \sigma_1}{\partial z_1}\sigma_1 - \sigma_2(z_2) \end{aligned} \quad (11)$$

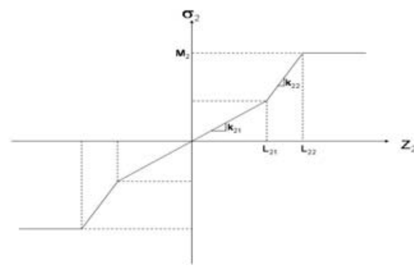


Fig. 2. Saturated robust control term for $z_2$

**Remark 2** Notice that $\sigma_2(z_2)$ has two regions with different gain plus the saturated part whereas $\sigma_1(z_1)$ only has one region of linear gain. This is due to the model mismatch and uncertainties that come into $z_2$ dynamics only. The region with moderate gain $k_{21}$ represents the normal operation of the system and is determined to achieve better trade-off between the noise rejection and modeling error plus disturbance attenuation. When $|z_2|$ is between $L_{21}$ and $L_{22}$, for example during the transient, more aggressive gain $k_{22}$ is employed to improve the disturbance rejection and system performance. This high gain $k_{22}$ could also be designed as some complicated nonlinear function to make more improvement. When some emergency happens, such as a sudden strike on the positioning stage, which overpowers the limited control authority and drags system states far away from the normal operation region, i.e.$|z_2| \gg L_{22}$, even small gain could make the actuator saturated and the system may end up unstable. Therefore, under that condition, $\sigma_2$ is designed to be a constant, the biggest control authority allowed, and ensure to pull all states back to the operation region once the huge disturbance disappears.

*B. Proof of globally stability*

Combine (5) and (10), the error dynamics can be rewritten as follows:

$$\begin{aligned} \dot{z}_1 &= z_2 - \sigma_1(z_1) \\ \dot{z}_2 &= -B_m z_2 - \varphi_b^T(x)\tilde{\theta} + \Delta + \tfrac{\partial \sigma_1}{\partial z_1}z_2 - \sigma_2(z_2) \end{aligned} \quad (12)$$

Before the evolution of $z_1$ and $z_2$ is analyzed in detail, some conditions need to be set for design parameters and explained as follows:

(a) $k_{22} > k_{21} > k_1$, both aggressive and moderate gains have to be greater than the linear gain for $z_1$ to overpower the term $\frac{\partial \sigma_1}{\partial z_1} z_2$.

(b) $M_1 > L_{22}$ and $k_1 L_{11} > L_{21}$, once $z_2$ is bounded within a pre-set range, $z_1$ is guaranteed to reduce and be bounded accordingly.

(c) $h < M_2 - k_1 M_1$, for the design problem to be meaningful, the level of modeling error plus disturbance has to be within the limits of control authority.

(d) $M_2 \leq \bar{u}_{bd} - \bar{u}_{abd}$, $\bar{u}_{bd}$ corresponds to the physical actuator limitation, $\bar{u}_{abd}$ is the bound on the model compensation control part $\bar{u}_a$. This condition implies that a trade-off has to be made between the system robustness and the aggressive trajectory it follows.

**Remark 3** The above are just the minimum requirements that the control parameters have to satisfy in order to make the system globally stable. Deeper research is needed to explore the flexibility of choosing controller parameters given the above constraints to optimize the achievable performance.

**Theorem 1** *With the proposed controller (4)(11) satisfying conditions (a)-(d), all signals are bounded. Furthermore, $z_1$ and $z_2$ reduce to a preset region in a finite time and stay within thereafter.*

**Proof.** Due to the special properties of the saturation function designed for $\sigma_1$ and $\sigma_2$, it's more convenient to carry out the proof by dividing the $z_1$-$z_2$ plane into several regions and analyzing the travelling time from one region to another. As a result, the upper bound of the reaching time from any arbitrary initial condition to the convergence region which could be preset as a performance specification could then be estimated.

As shown in Fig. 3, the entire $z_1$-$z_2$ plane is divided into four regions marked as $\Omega_1$-$\Omega_4$, on which little black arrows show the phase portrait and big hollow arrows indicate the travelling from one region to another with the reaching time marked on. For the sake of simplicity, only upper half plane is taken for analysis and one can easily find out that the corresponding region of the lower half plane follows the same way in the evolution process. Meanwhile,

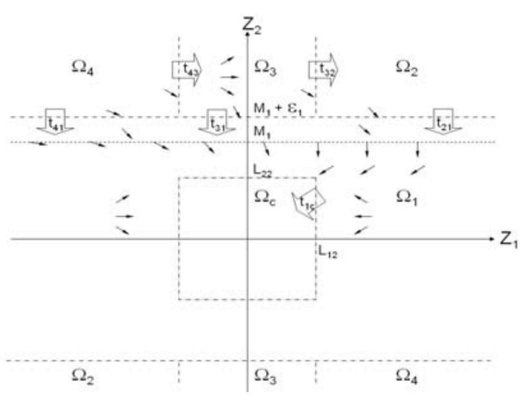


Fig. 3. $z_1$-$z_2$ plane

according to condition (b) and (c), there exist $\varepsilon_1$,$\varepsilon_2$ and $\varepsilon_3$, all positive, such that $h + k_1(M_1 + \varepsilon_1) + \varepsilon_2 = M_2$ and $L_{22} + \varepsilon_3 = M_1$. Consequently, $\sigma_2^{-1}(h + k_1(M_1 + \varepsilon_1) + \varepsilon_2) = L_{22}$ and $\sigma_1^{-1}(L_{22} + \varepsilon_3) = L_{12}$. The convergence region is set as $\Omega_c = \{z_1, z_2 : |z_1| \leq L_{12}, |z_2| \leq L_{22}\}$.

Region 1: $\Omega_1 = \{z_1(0) arbitrary, z_2(0) \leq M_1 + \varepsilon_1\}$ which contains the convergence region. In order to estimate the reaching time from any point in $\Omega_1$ to $\Omega_c$, first let's consider the situation that $L_{22} \leq z_2(0) \leq M_1 + \varepsilon_1$ and we have the following inequality according to the error dynamics:

$$\begin{array}{rcl} \dot{z}_2 & \leq & -B_m z_2 + h + k_1 z_2 - \sigma_2(z_2) \\ & \leq & h + k_1(M_1 + \varepsilon_1) - M_2 \\ & \leq & -\varepsilon_2 \end{array} \quad (13)$$

Therefore the upper bound of the reaching time for $z_2$ is obtained

$$t_{1c,2} \leq \max\{0, \frac{z_2(0) - L_{22}}{\varepsilon_2}\} \quad (14)$$

Once $z_2$ enters the convergence region, the upper bound of the reaching time for $z_1$ can be calculated similarly. If $z_1 > L_{12}$ then

$$\dot{z}_1 \leq L_{22} - \sigma_1(z_1) \leq L_{22} - M_1 \leq -\varepsilon_3 \quad (15)$$

If $z_1 < -L_{12}$ then

$$\dot{z}_1 \geq L_{22} - (-\sigma_1(z_1)) \geq L_{22} + M_1 \geq \varepsilon_3 \quad (16)$$

As a result,

$$t_{1c,1} \leq \max\{0, \frac{|z_1(t_{1c,2})| - L_{12}}{\varepsilon_3}\} \quad (17)$$

Combine (14) and (17) then the upper bound of the reaching time from $\Omega_1$ to $\Omega_c$ is obtained

$$\begin{array}{rcl} t_{1c} & = & t_{1c,2} + t_{1c,1} \\ & \leq & \max\{0, \frac{z_2(0) - L_{22}}{\varepsilon_2}\} + \max\{0, \frac{|z_1(t_{1c,2})| - L_{12}}{\varepsilon_3}\} \end{array} \quad (18)$$

Region 2: $\Omega_2 = \{z_1(0) > L_{12}, z_2(0) > M_1 + \varepsilon_1\}$, where $\frac{\partial \sigma_1}{\partial z_1} = 0$ and any trajectory starting from this region will get into $\Omega_1$ and then $\Omega_c$ after finite time. $z_2$ dynamics becomes

$$\dot{z}_2 \leq -B_m z_2 + h - M_2 \leq -(M_2 - h) \quad (19)$$

Then the upper bound of the reaching time from $\Omega_2$ to $\Omega_1$ is obtained as

$$t_{21} \leq \frac{z_2(0) - (M_1 + \varepsilon_1)}{M_2 - h} \quad (20)$$

Region 3: $\Omega_3 = \{|z_1(0)| \leq L_{12}, z_2(0) > M_1 + \varepsilon_1\}$. The trajectory starting from this region would enter either $\Omega_1$ or $\Omega_2$, whichever comes first. At this point it is difficult to make certain how $z_2$ evolves due to the unknown modelling error, but the lower bound of $\dot{z}_1$ is calculable, so is the upper bound of the reaching time from $\Omega_3$ to $\Omega_2$.

$$\dot{z}_1 \geq M_1 + \varepsilon_1 - M_1 \geq \varepsilon_1, t_{32} \leq \frac{L_{12} - z_1(0)}{\varepsilon_1} \quad (21)$$

If necessary the trajectory would enter $\Omega_1$ within less time than $t_{32}$, so it is safe to estimate $t_{31} \leq t_{32} \leq \frac{L_{12} - z_1(0)}{\varepsilon_1}$.

Region 4: $\Omega_4 = \{z_1(0) < -L_{12}, z_2(0) > M_1 + \varepsilon_1\}$, where $\frac{\partial \sigma_1}{\partial z_1} = 0$. The trajectory starting from this region would enter either $\Omega_1$ or $\Omega_3$ depending on the initial states and the corresponding reaching time could be calculated as follows.

$$\Omega_4 \longrightarrow \Omega_1 : \dot{z}_2 \leq h - M_2, t_{41} \leq \frac{z_2(0)-(M_1+\varepsilon_1)}{M_2-h} \tag{22}$$

$$\Omega_4 \longrightarrow \Omega_3 : \dot{z}_1 \geq M_1 + \varepsilon_1 + M_1, t_{43} \leq \frac{-z_1(0)-L_{12}}{2M_1+\varepsilon_1} \tag{23}$$

$\Omega_1$-$\Omega_4$ with their lower half plane counterparts cover the whole plane and the reaching time from any point to the convergence region could be calculated based on the above discussion. The global stability is thus proved.

**Remark 4** The convergence region $\Omega_c$ is a very conservative estimate for the steady state and the transient performance (reaching time) could be improved by better choices of the design parameters. The extent of the reduction of $z_1$ and $z_2$ depends on the instant value of $\varphi_b^T(x)\tilde{\theta} - \Delta$. In detail, if a better estimation law is designed and the parameter estimates error would approach zero or very small value, then it is likely that the amplitude of $z_2$ would reduce even within $L_{21}$ and $z_1$ within $L_{11}$ accordingly, which means the system operates in its desired working region.

Now that it is proved that all signals are bounded after finite time, let's demonstrate that asymptotic tracking is achievable in the presence of parameter estimation error only, i.e., $\Delta = 0$.

**Theorem 2** *With the proposed controller (4)(11) satisfying conditions (a)-(d) and the adaptation law (8)(9), asymptotic output tracking is achieved if the system is only subject to parametric uncertainty, i.e.,* $\Delta = 0, \forall t$.

**Proof.** Introduce a positive semi-definite function $V = \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}$. In the case of $\Delta = 0$, with property 2 of the parameter estimation law, $\dot{V}$ becomes:

$$\begin{array}{rcl} \dot{V} & = & z_2\dot{z}_2 + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \\ & = & z_2(-B_m z_2 - \varphi_b^T(x)\tilde{\theta} + \frac{\partial\sigma_1}{\partial z_1}z_2 - \sigma_2(z_2)) \\ & + & \tilde{\theta}^T\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\varphi_b z_2) \\ & \leq & -B_m z_2^2 - (z_2\sigma_2(z_2) - \frac{\partial\sigma_1}{\partial z_1}z_2^2) \end{array} \tag{24}$$

which is negative semi-definite once $z_1$ and $z_2$ evolve to $\Omega_c$ as proved. As a result, $z_2$ converges to zero asymptotically and so would $z_1$ according to (5). Hence, asymptotic output tracking is achieved.

## IV. HARDWARE EXPERIMENTS

### A. System Setup

The linear motor system under study is set up as Fig. 4, which is the same as in [10]. The control algorithm is designed and tested on the Y-axis of the stage. The normalized mass (with respect to control input $u$) could be calculated from the parameter sheet provided by the manufacture. Offline estimation by least square method is also conducted to verify the accuracy of that number, which is 0.12. The corresponding bound on $\bar{u}$ is 103.7. The bounds of the variation for other system parameters are set as in Assumption 1. The sampling frequency is set at 2.5 kHz during the experiments and the resolution of the position encoder is $1\mu m$.

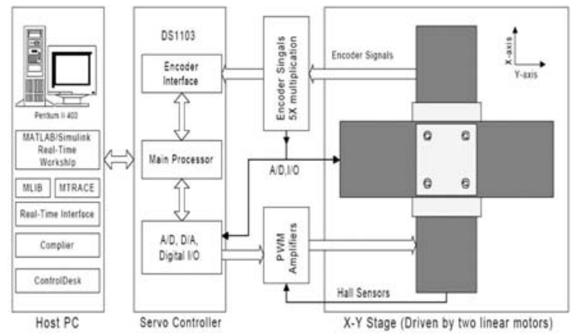


Fig. 4. Experiment setup of linear motor system

### B. Design Parameters

The choice of the control parameters is quite important not only to the system stability but also to the improvement of the transient and steady state performance. The guideline of setting those values is conditions (a)-(d) mentioned before. Also, there are other relevant issues. For example $L_{11}$, the linear region of position error should be set to satisfy the desired working requirement; linear gain $k_1$ relates to the convergence rate of $z_1$ and the steady state error, and etc.

The control parameters are designed as follows, $L_{11} = 100\mu m$, $L_{12} = 140\mu m$, $k_1 = 400$, $M_1 = 1.2k_1L_{11}$, $L_{21} = 0.5M_1$, $k_{21} = k_1 + 600$,$k_{22} = k_{21} + 200$, $M_2 = 0.95(\bar{u}_{bd} - \bar{u}_{abd})$, $L_{22} = (M_{22} - k_{21}L_{21})/k_{22} + L_{21}$.

### C. Experimental Results

The desired trajectory is a point-to-point movement,with distance $0.4m$, maximum velocity $1m/s$ and maximum acceleration $12m/s^2$ as shown in [10]. In implementation, the system is at rest and there is no control input during the first about 20 seconds.

The experimental results with the proposed saturated ARC are given in Fig. 5 and Fig. 6. It can be seen that the position error converges pretty quickly in Fig. 5 as well as the steady state error is within $1\mu m$. Both transient and steady state performances could be improved by employing a better parameter estimation algorithm such as recursive least square method [9].

Fig. 6 shows the tracking error and control effort under a 1V constant disturbance adding to the physical control input $u$. It is about fifteen percents of the total control authority, which represents the normal working environment. With Matlab program, the disturbance is introduced to the system at about 24th second and lasts 10 seconds. And the system performance adjusts well due to the parameter adaptation mechanism. The parameter estimates are not shown here because the estimation law is not good enough to identify the system parameters accurately.

Due to the hardware restriction, the scenario of the stage enduring a strong but short disturbance is illustrated by Matlab simulation results Fig. 7. This disturbance is set at 15V, greater than the physical control limit, and last only 0.1 seconds, which is more than enough to drag the system states far away. System stability is preserved although it takes rather long time for the tracking error to decrease. Moreover, the tracking error finally converges to $1\mu m$, the resolution level.

The above results help verify that the proposed saturated ARC can achieve high performance under normal working environment while ensuring global stability.

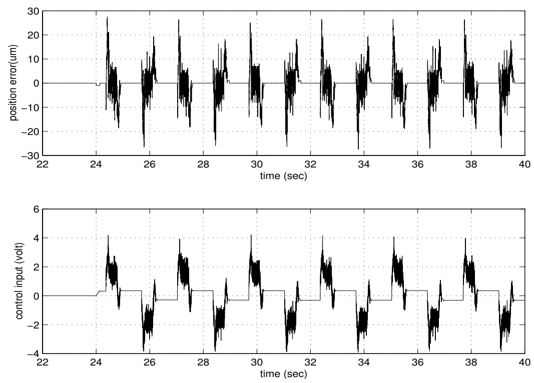


Fig. 5. Position error and control input (w/o disturbance)

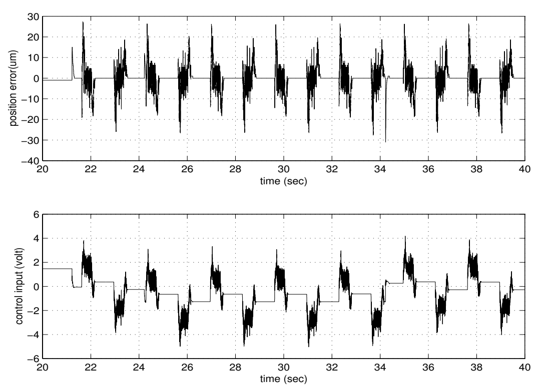


Fig. 6. Position error and control input (w/1V disturbance)

## V. CONCLUSIONS AND FUTURE WORKS

### A. Conclusions

In this paper, a saturated ARC algorithm is designed to guarantee global stability as well as excellent tracking performance with the linear motor system subject to limited control authority, model uncertainties and external disturbance. Such a goal is accomplished by taking the actuator saturation problem into account at the design stage and employing saturation function in the control algorithm. An identification law of projection type is applied to provide bounded parameter estimates and attain asymptotic tracking performance in the presence of parametric uncertainties only. Simulation and hardware experiments are conducted to confirm the study.

### B. Future Works

There are lots of future works that could be done based on this paper. For example, the choice of those design parameters is not optimal yet. One may apply some optimization techniques such as linear matrix inequality to get better performance with the existing control limit and working environment. Furthermore, other identification law may be used to get accurate parameter estimates for smaller tracking error and secondary purposes such as machine prognostics and health monitoring. The last but not the least, it is of great interest to investigate the possibility to apply the proposed control structure on higher order systems such as a chain of integrators with disturbance and unknown parameters existing in the last channel.

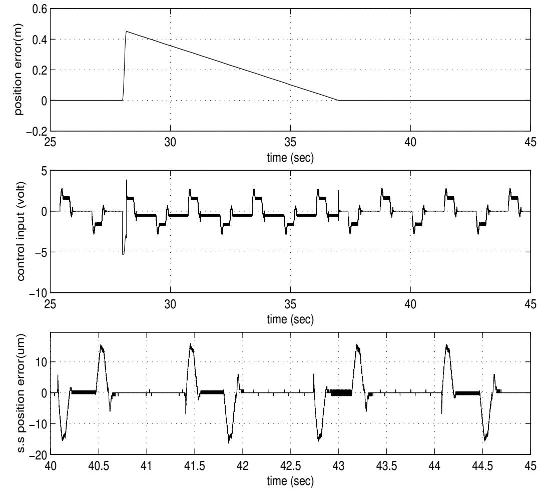


Fig. 7. Position error and control input (w/15V disturbance)

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