Nonlinear Adaptive Robust Observer Design for a Class of Nonlinear Systems [1]

Phanindra Garimella   Bin Yao +

School of Mechanical Engineering
Purdue University
West Lafayette, IN 47907, USA
+ Email: byao@ecn.purdue.edu

Abstract

To be practical, observers have to function not only in the ideal situation of having a perfect assumed model structure, but also in the presence of some degrees of model uncertainties. The adaptive observer designs in the literature have a major drawback in that they may destabilize in the presence of uncertain nonlinearities which could occur in real systems. In this paper, based on the recently proposed adaptive robust control (ARC), robust filter structures and controlled parameter adaptation are effectively integrated to deal with some typical model uncertainties in a physical system to improve the state estimation properties of an observer. Theoretically, the proposed adaptive robust observer (ARO) is shown to possess the Input-to-State-Practically Stable (ISpS) property. Practically, explicit on-line monitoring of certain persistence of excitation conditions is used as well to obtain better and robust parameter estimates to further improve the state estimation accuracy in implementation. Experimental results are obtained to verify the effectiveness of the proposed ARO design.

1 Introduction

Observers are dynamical systems that act as state estimators and are used in a wide range of applications such as in the implementation of advanced controllers which do not have the entire state vector for feedback, model based fault detection and isolation [1, 2] and virtual sensing [3].

Some of the major difficulties in the design of *practical* observers for most physical systems include: (i) the inherent nonlinear dynamics, and (ii) model uncertainties. The model uncertainties can be due to either constant or slowing changing unknown quantities such as unknown physical parameters or fast changing unknown quantities such as external disturbances and un-modelled nonlinearities.

The extensive research work in the field of linear systems has led to the development of extensive tools for the design of observers for linear systems. The difficulty posed by the lack of knowledge of systems parameters was addressed by the design of adaptive observers which estimate both the states and parameters of a system. An extensive survey of the design of adaptive observers was done in [4]. Unfortunately, these observers can become unstable in the presence of unmodeled dynamics and uncertain nonlinearities.

In the presence of hard nonlinearities such as coulomb friction and saturation which cannot be linearized, observer designs based on the linearized model do not perform adequately and have a limited range of operation. Hence, the design of observers for the state estimation of uncertain nonlinear systems remains an active area of research. An extensive survey of the various observer designs for nonlinear systems was done in [5–8].

It should be noted that most of the observer designs in the literature deal with certain specific classes of nonlinear systems for which co-ordinate transformations can be found to transform the system into an equivalent observable form. The complexity in the nature of nonlinear systems forces us to come up with designs based on performance requirements. Most methods deal with observer designs for systems linear in the unmeasured state. In the case of adaptive observers for uncertain nonlinear systems, as with other adaptive designs, the parameter convergence can occur only when certain persistence of excitation conditions are satisfied.

This paper presents a novel nonlinear adaptive robust observer (ARO) that estimates both the unmeasurable states and unknown but constant parameters for a class of systems which can be characterized as being in the parametric semi-strict feedback form. A co-ordinate transformation is first used to convert the given nonlinear system into an equivalent observable canonical form. A novel observer design philosophy that is based on the recently developed adaptive robust control (ARC) theory [9] is employed to handle typical model uncertainties in a physical system effectively. Theoretically, it is shown that the proposed ARO is Input-State-Practically-Stable (ISpS), and the state and parameter estimates are guaranteed to be bounded even in the presence

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of uncertain nonlinearities such as bounded disturbances. Practically, explicit on-line monitoring of certain persistent excitation conditions is used to improve the parameter and state estimation process as well. Experimental results obtained on the velocity estimation of hydraulic cylinders using pressure measurement only verify the practicality and effectiveness of the proposed ARO framework.

2 Problem Statement and Issues to be Addressed

A goal in many practical applications is to combine *a priori* knowledge of the physical system with experimental data to estimate the states and parameters of the system. In this paper an adaptive robust observer is designed that estimates the states and parameters of a class of nonlinear systems. Specifically system dynamics in the following form are considered in this paper:

$$\begin{aligned} \dot{\eta} &= F_\eta(x,u)\theta + G_\eta(x)\eta + \Delta_\eta \\ \dot{x}_i &= \theta^T F_{x_i}(x,u) + \phi_i^T(x,u,\theta)\eta + \Delta_{x_i} \\ y &= x \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in \mathcal{R}^n$ is a vector of states that can be measured, $\eta \in \mathcal{R}^\eta$ is a vector of states that are unmeasurable and $\theta \in \mathcal{R}^p$ is a vector of constant but unknown parameters that also need to be estimated. $F_\eta(x,u) \in \mathcal{R}^{\eta \times p}$, $G_\eta \in \mathcal{R}^{\eta\times\eta}$, $F_{x_i}(x,u) \in \mathcal{R}^p$ and $\phi_i(x,u,\theta) \in \mathcal{R}^\eta$ are matrices or vectors of known smooth functions which are used to describe the nominal model of the system. Δ_η and Δ_{x_i} represent the lumped unknown nonlinear functions such as disturbances and modelling errors. The following practical assumptions are made:

Assumption 1 *The extent of the parametric uncertainties and uncertain nonlinearities are known. Specifically,*

$$\begin{aligned} \theta &\in \Omega_\theta = \theta : \theta_{min} < \theta < \theta_{max} \\ \Delta_\eta &\in \Omega_{\Delta_\eta} = \Delta_\eta : |\Delta_\eta| \le \delta_\eta \\ \Delta_{x_i} &\in \Omega_{\Delta_{x_i}} = \Delta_{x_i} : |\Delta_{x_i}| \le \delta_{x_i} \end{aligned} \tag{2}$$

where θ_{min}, θ_{max}, δ_η and δ_{x_i} are known. ($|\cdot|$) denotes the usual Euclidean norm.

Assumption 2 *There exists a vector of functions $\omega(x,\theta_\omega) \in \mathcal{R}^\eta$ that can be linearly parameterized by a set of unknown parameters $\theta_\omega \in \mathcal{R}^{p_\omega}$ and satisfies*

$$\frac{\partial\omega}{\partial x_i}\phi_i^T(x,u,\theta) = \psi_i(x,u) \tag{3}$$

where $\psi_i(x,u)$ is a matrix of known functions of x and u and independent of θ.

Assumption 3 *There exists a co-ordinate transformation of the form*

$$\xi = \eta - \omega(x,\theta_\omega) \tag{4}$$

where $\omega(x,\theta_\omega)$ is a vector of design functions of the measured states x and the set of unknown parameters θ_ω as defined in assumption 2 such that the matrix

$$A_\xi(x,u) = G_\eta(x) - \Sigma_{i=1}^n \psi_i(x,u) \tag{5}$$

is exponentially stable (i.e., $A^T + A$ is a uniformly negative definite matrix).

The objective is to design an observer such that the estimated state vector $\hat{\eta}$ is as close as possible to the true state η in spite of the parametric uncertainties and the uncertain nonlinearities. Specifically, the observer should exhibit the following desirable properties:

1. The observer is Input-to-State-Practically stable (ISPS) with the plant states and inputs of the actual system as inputs and the observer states as the states.
2. In the absence of uncertain nonlinearities i.e., when $\Delta_\eta = \Delta_{x_i} = 0$, the observer estimates of the transformed states converge to their true values.
3. When certain persistence of excitation conditions are satisfied and $\Delta_\eta = \Delta_{x_i} = 0$, both the parameter estimates $\hat{\theta}$ and the state estimates $\hat{\eta}$ asymptotically converge to their true values.

In the following, an adaptive robust observer (ARO) framework is presented to solve the above observer design problem. The framework is based on the recently developed adaptive robust control (ARC) philosophy [9] that emphasizes the use of both *robust filter structures* and em controlled parameter adaptation in dealing with typical model uncertainties in a physical system. Specifically, robust filter structures are used to reduce the effect of various model uncertainties as much as possible while controlled parameter adaptation is used to reduce the model uncertainty for an improved state estimation accuracy.

3 Definitions and Mathematical Preliminaries

3.1 Projection Mapping and ISPS Stability

Let $\hat{\theta}(t)$ denote the estimate of the parameter θ and $\tilde{\theta}$ the estimation error (i.e., $\tilde{\theta} = \hat{\theta}(t) - \theta$). Defining the discontinuous projection as:

Definition 1 *Let Ω_θ be a convex set with the interior of the set denoted by $\mathring{\Omega}_\theta$ and its boundary by $\partial\Omega_\theta$. Let $n_{\hat{\theta}}$ be the unit outward normal at $\hat{\theta} \in \partial\Omega_\theta$. The standard projection mapping [10] is:*

$$Proj_{\hat{\theta}}(\zeta) = \begin{cases} \zeta, & if \quad \hat{\theta} \in \mathring{\Omega}_\theta \ \ or \ n_{\hat{\theta}}^T\zeta \le 0 \\ (I - \Gamma\frac{n_{\hat{\theta}} n_{\hat{\theta}}^T}{n_{\hat{\theta}}^T \Gamma n_{\hat{\theta}}})\zeta, & \hat{\theta} \in \partial\Omega_\theta \ and \ n_{\hat{\theta}}^T\zeta > 0 \end{cases} \tag{6}$$

where $\zeta \in \mathcal{R}^p$ is any function and $\Gamma(t) \in \mathcal{R}^{p\times p}$ can be any time-varying positive definite symmetric matrix.

Lemma 1 *By using the projection type adaptation law given by*

$$\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\zeta), \quad \hat{\theta}(0) \in \Omega_\theta \tag{7}$$

it can be shown [11] that the projection mapping in equation (6) has the following desirable properties

P1. The parameter estimates are always within the known closed set $\bar{\Omega}_\theta$, i.e., $\hat{\theta}(t) \in \bar{\Omega}_\theta, \forall t$.

P2. If the true parameters are within the known convex set Ω_θ, then for any adaptation function τ and $\Gamma(t) > 0$,

$$\tilde{\theta}^T(\Gamma^{-1}Proj_{\hat{\theta}}(\Gamma\tau) - \tau) \le 0, \forall \tau, \Gamma(t), and\ \theta \in \Omega_\theta, \tag{8}$$

Definition 2 *A system $\dot{x} = f(x,u)$ is Input to State Practically Stable (ISpS) if there exists a class $\mathcal{KL}$ function β, a class $\mathcal{K}$ function γ, and a non-negative constant d such that, for any initial condition $x(0)$ and each input $u \in \mathcal{L}_\infty[0,t)$, the corresponding solution $x(t)$ satisfies*

$$|x(t)| \le \beta(|x(0)|,t) + \gamma(\|u(t)\|) + d, \quad \forall t \ge 0 \tag{9}$$

where $u(t)$ is the truncated function of u at t and $\|\cdot\|$ represents the $\mathcal{L}_\infty$ supremum norm.

3.2 Mathematical Preliminaries

If $A \in \mathcal{R}^{n\times m}$, $B \in \mathcal{R}^{r\times s}$ then, the Kronecker product as defined in [12] is,

$$A \bigotimes B = \begin{pmatrix} A_{11}B & A_{12}B & \dots & A_{1n}B \\ A_{21}B & A_{22}B & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ A_{m1}B & A_{m2}B & \dots & A_{mn}B \end{pmatrix} = matrix[A_{ij}B] \tag{10}$$

where $A \bigotimes B \in \mathcal{R}^{nr\times ms}$.

Lemma 2 *Consider the following matrix product $ABCD$ where $A \in \mathcal{R}^{\eta\times p_\omega}$, $B \in \mathcal{R}^{p_\omega\times 1}$, $C \in \mathcal{R}^{1\times p}$ and $D \in \mathcal{R}^{p\times 1}$. Then*

$$ABCD = (A \bigotimes D^T)\cdot(B \bigotimes C^T) \tag{11}$$

Proof: The proof follows from the definition.

4 Adaptive Robust Observer

Since the η subsystem is not measurable, a nonlinear adaptive robust observer will be designed to provide estimates of the η subsystem. Motivated by the research work in [13,14], a transformation of co-ordinates is introduced. Define a vector

$$\xi = \eta - \omega(x,\theta_\omega) \tag{12}$$

where $\omega(x,\theta_\omega)$ is the vector of design functions satisfying Assumptions 2 and 3. Its derivative is computed as

$$\begin{aligned} \dot{\xi} &= \dot{\eta} - \dot{\omega}(x,\theta_\omega) \\ &= (F_\eta(x,u)\theta + G_\eta\eta + \Delta_\eta) - \Sigma_{i=1}^n \frac{\partial\omega}{\partial x}\dot{x}_i \qquad (13)\\ &= \begin{array}{c}(F_\eta(x,u)\theta - \Sigma_{i=1}^n \frac{\partial\omega}{\partial x_i}\theta^T F_{x_i}) + (G_\eta - \Sigma_{i=1}^n \psi_i(x,u))\eta \\ +(\Delta_\eta - \Sigma_{i=1}^n \frac{\partial\omega}{\partial x_i}\Delta_{x_i})\end{array} \end{aligned}$$

For simplicity, let

$$A_\xi(x,u) = (G_\eta(x) - \Sigma_{i=1}^n \psi_i(x,u)) \tag{14}$$

and

$$\Delta_\xi(x,u) = (\Delta_\eta - \Sigma_{i=1}^n \frac{\partial\omega}{\partial x_i}\Delta_{x_i}) \tag{15}$$

Substituting (14) and (15) into (13), we have

$$\dot{\xi} = (F_\eta(x,u)\theta - \Sigma_{i=1}^n \frac{\partial\omega}{\partial x_i}\theta^T F_{x_i}(x,u)) + A_\xi(x,u)\eta + \Delta_\xi \tag{16}$$

Substituting (12) into (16) and utilizing the assumption that $\omega(x,\theta_\omega)$ can be linearly parametrized in terms of θ_ω, i.e., $\omega(x,\theta_\omega) = \sigma(x)\theta_\omega$ for some matrix $\sigma(x) \in R^{\eta\times p_\omega}$, we have

$$\begin{aligned} \dot{\xi} &= A_\xi(x,u)\xi + A_\xi(x,u)\sigma(x)\theta_\omega + \\ &\quad F_\eta(x,u)\theta - \Sigma_{i=1}^n \frac{\partial\sigma}{\partial x_i}\theta_\omega\theta^T F_{x_i}(x,u) + \Delta_\xi \end{aligned} \tag{17}$$

Using (11) we have

$$\begin{aligned} \Sigma_{i=1}^n \frac{\partial\sigma}{\partial x_i}\theta_\omega\theta^T F_{x_i}(x,u) &= \Sigma_{i=1}^n (\frac{\partial\sigma}{\partial x_i} \bigotimes F_{x_i}^T(x,u))(\theta_\omega \bigotimes \theta) \\ &= \Sigma_{i=1}^n \varphi_i^T(x,u)\theta_{new} \end{aligned} \tag{18}$$

where $\varphi_i^T(x,u) \in \mathcal{R}^{\eta\times pp_\omega}$ and $\theta_{new} \in \mathcal{R}^{pp_\omega\times 1}$. Substituting (18) into (17) we get,

$$\begin{aligned} \dot{\xi} &= A_\xi(x,u)\xi + A_\xi(x,u)\sigma(x)\theta_\omega + F_\eta(x,u)\theta \\ &\quad -\Sigma_{i=1}^n \varphi_i^T(x,u)\theta_{new} + \Delta_\xi \end{aligned} \tag{19}$$

If θ and θ_ω were known, we would design a nonlinear observer

$$\begin{aligned} \dot{\hat{\xi}} &= A_\xi(x,u)\hat{\xi} + A_\xi(x,u)\sigma(x)\theta_\omega + F_\eta(x,u)\theta - \Sigma_{i=1}^n \varphi_i^T(x,u)\theta_{new} \\ &= A_\xi(x,u)\hat{\xi} + \chi_\eta(x,u)\theta_\omega + F_\eta(x,u)\theta - \Sigma_{i=1}^n \varphi_i^T(x,u)\theta_{new} \end{aligned} \tag{20}$$

where $\chi_\eta(x,u) = A_\xi(x,u)\sigma(x) \in \mathcal{R}^{\eta\times p_\omega}$. Then, the state estimation error $\tilde{\xi} = \hat{\xi} - \xi$ would be governed by the following dynamic system

$$\dot{\tilde{\xi}} = A_\xi(x,u)\tilde{\xi} - \Delta_\xi \tag{21}$$

Since θ and θ_ω are not known, the observer in equation (20) is not implementable but it provides motivation for the design of the following nonlinear filters:

$$\dot{\tau}_{\theta_j} = A_\xi(x,u)\tau_{\theta_j} + F_{\eta j}(x,u) \tag{22}$$

$$\dot{\tau}_{\theta_{\omega_j}} = A_\xi(x,u)\tau_{\theta_{\omega_j}} + \chi_{\eta j}(x,u) \tag{23}$$

$$\dot{\tau}_j = A_\xi(x,u)\tau_j - \Sigma_{i=1}^n \varphi_{ij}^T(x,u) \tag{24}$$

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where $F_{\eta j}$, $\chi_{\eta j}(x,u)$ and φ_{ij}^T represent the jth column of F_η, χ_η and φ_i^T matrices. The state estimate can thus, be represented by

$$\begin{aligned}\hat{\xi} &= \Sigma_{i=1}^{p}\tau_{\theta_j}\theta_j + \Sigma_{i=1}^{p_\omega}\tau_{\theta_{\omega_j}}\theta_{\omega_i} + \Sigma_{k=1}^{pp_\omega}\tau_k\theta_{new_k} \\ &= \tau_\theta\theta + \tau_{\theta_\omega}\theta_\omega + \tau\theta_{new} \end{aligned} \tag{25}$$

where $\tau_\theta \in \mathcal{R}^{\eta\times p}$, $\tau_{\theta_\omega} \in \mathcal{R}^{\eta\times p_\omega}$ and $\tau \in \mathcal{R}^{\eta\times pp_\omega}$. From (22), (23), (24) and (25), it can be verified that the observer error dynamics are still represented by (21). Therefore, the equivalent expression for the unmeasurable state η is

$$\begin{aligned}\eta &= \tau_\theta\theta + \tau_{\theta_\omega}\theta_\omega + \tau\theta_{new} + \omega(x,\theta_\omega) - \tilde{\xi} \\ &= \tau_\theta\theta + \tau_{\theta_\omega}\theta_\omega + \tau\theta_{new} + \sigma(x)\theta_\omega - \tilde{\xi}\end{aligned} \tag{26}$$

But since information about θ and θ_ω is not available, we need to use the parameter estimates for the estimation of the unmeasured states η i.e.,

$$\begin{aligned}\hat{\eta} &= \tau_\theta\hat{\theta} + (\tau_{\theta_\omega} + \sigma(x))\hat{\theta}_\omega + \tau\hat{\theta}_{new} \\ &= \Upsilon^T\hat{\theta}_\Upsilon\end{aligned} \tag{27}$$

where $\Upsilon^T = [\tau_\theta, (\tau_{\theta_\omega} + \sigma(x)), \tau] \in \mathcal{R}^{\eta\times(p+p_\omega+pp_\omega)}$ and $\hat{\theta}_\Upsilon = [\hat{\theta}, \hat{\theta}_\omega, \hat{\theta}_{new}]^T \in \mathcal{R}^{p+p_\omega+pp_\omega}$.

Now an adaptation law needs to be designed to estimate the system parameters so that these estimates can be used in the implementation of the adaptive robust observer. Consider the dynamics of the x_i subsystem in (1), using (26), the equation for the x_i dynamics can be rewritten in the following form:

$$\dot{x}_i = \theta^T F_{x_i}(x,u) + \phi_i^T(x,u,\theta)(\Upsilon^T\theta_\Upsilon - \tilde{\xi}) + \Delta_{x_i} \tag{28}$$

Utilizing the fact that $\phi_i^T(x,u,\theta)$ is linear in terms of θ, i.e., $\phi_i^T(x,u,\theta) = \theta^T\delta_i(x,u)$, the equivalent x_i dynamics can be written as

$$\begin{aligned}\dot{x}_i &= \theta^T F_{x_i}(x,u) + \theta^T\delta_i(x,u)(\Upsilon^T\theta_\Upsilon - \tilde{\xi}) + \Delta_{x_i} \\ &= \theta^T F_{x_i}(x,u) + \theta^T\delta_i(x,u)\Upsilon^T\theta_\Upsilon - \theta^T\delta_i(x,u)\tilde{\xi} + \Delta_{x_i} \\ &= \theta^T F_{x_i}(x,u) + \theta^T\Xi(x,u)\theta_\Upsilon - \theta^T\delta_i(x,u)\tilde{\xi} + \Delta_{x_i}\end{aligned} \tag{29}$$

Then,

$$\begin{aligned}\theta^T\delta_i(x,u)\Upsilon^T\theta_\Upsilon &= \theta^T\Xi(x,u)\theta_\Upsilon \\ &= (\theta_\Upsilon^T \bigotimes \theta^T)\Lambda_i(x,u) \\ &= \theta_\Lambda^T\Lambda_i(x,u)\end{aligned} \tag{30}$$

where $\Lambda_i(x,u) \in \mathcal{R}^{p(p+p_\omega+pp_\omega)}$ is a vector of the elements of the matrix $\Xi(x,u)$ and $\theta_\Lambda \in \mathcal{R}^{p(p+p_\omega+pp_\omega)}$ is a vector of the unknown parameters.

Hence, using equation (30), the dynamics of the x_i channel can be written as,

$$\begin{aligned}\dot{x}_i &= \theta^T F_{x_i}(x,u) + \theta_\Lambda^T\Lambda_i(x,u) - \theta^T\delta_i(x,u)\tilde{\xi} + \Delta_{x_i} \\ &= \Theta(x,u)\theta_0 - \theta^T\delta_i(x,u)\tilde{\xi} + \Delta_{x_i}\end{aligned} \tag{31}$$

where $\theta_0^T = [\theta^T, \theta_\Lambda^T] \in \mathcal{R}^{1\times(p+p(p+p_\omega+pp_\omega))}$ and $\Theta^T(x,u) = [F_{x_i}(x,u), \Lambda_i(x,u)] \in \mathcal{R}^{(p+p(p+p_\omega+pp_\omega))}$.

The dynamics (31) is linear in terms of unknown parameter vector θ_0, from which parameter estimation can be constructed. To by-pass the need for the derivatives of the measured states, the following filters are proposed:

$$\dot{\Omega}^T = A\Omega^T + \Theta(x,u) \tag{32}$$

$$\dot{\Omega}_0 = A(\Omega_0 + x_i) \tag{33}$$

where A is any exponentially stable matrix to be specified later, $\Omega \in \mathcal{R}^{1\times(p+p(p+p_\omega+pp_\omega))}$ and $\Omega_0 \in \mathcal{R}^1$. Now define

$$z = x_i + \Omega_0 \tag{34}$$

which is calculable. By substituting equations (31) and (33) into the derivative of (34),

$$\dot{z} = Az + \Theta(x,u)\theta_0 - \theta^T\delta_i(x,u)\tilde{\xi} + \Delta_{x_i} \tag{35}$$

Let $\varepsilon = x_i + \Omega_0 - \Omega^T\theta_0$, then z can be written as

$$z = \Omega^T\theta_0 + \varepsilon \tag{36}$$

where ε is governed by

$$\dot{\varepsilon} = A\varepsilon - \theta^T\delta_i(x,u)\tilde{\xi} + \Delta_{x_i} \tag{37}$$

As the last two terms in (37) can be bounded by known nonlinear functions, a nonlinear filter matrix A can then be constructed to guarantee that the error dynamics (37) are stable. Now define the estimate of z as

$$\hat{z} = \Omega^T\hat{\theta}_0 \tag{38}$$

and define the prediction error as $e = \hat{z} - z$. By doing so,

$$e = \Omega^T\tilde{\theta}_0 - \varepsilon \tag{39}$$

which is linearly parameterized in terms of the parameter estimation error $\tilde{\theta}_0$ with an additional term that exponentially converges to zero in the absence of disturbances (i.e., $\Delta_\eta = \Delta_{x_i} = 0$). Because the prediction error is in the static form, various standard estimation algorithms can be used. With the least squares estimation algorithm, the resulting adaptation law is given by,

$$\dot{\hat{\theta}}_0 = Proj_{\hat{\theta}_0}\left(-\Gamma\frac{\Omega e}{1 + \nu Trace(\Omega^T\Gamma\Omega)}\right) \tag{40}$$

where $\Gamma(t)$ is the adaptation rate matrix updated by,

$$\dot{\Gamma} = \frac{\alpha\Gamma - \Gamma\Omega\Omega^T\Gamma}{1 + \nu Trace(\Omega^T\Gamma\Omega)}, \Gamma(0) = \Gamma^T(0) > 0 \tag{41}$$

in which the normalization factor ν and the forgetting factor α are non-negative constants, with $\nu = 0$ leading to unnormalized algorithm.

With the above ARO design, the observer estimation error of η is given by

$$\tilde{\eta} = \hat{\eta} - \eta = \Upsilon^T\tilde{\theta}_\Upsilon + \tilde{\xi} \tag{42}$$

5 Performance Results

The following qualitative results hold for the ARO defined by the equations (27) and (40).

1. In the presence of uncertain nonlinearities, the signals from the parameter estimator of the ARO given by equation (40) and the state estimator given by equation (27) are bounded, and the ARO given by equations (27) and (40) is ISpS.
2. In the absence of uncertain nonlinearities, i.e., $\Delta_{x_i} = \Delta_\eta = 0$, if the parameters are updated only when certain persistence of excitation conditions are satisfied, then the parameter and state estimates converge to their true values.

These results are formally summarized in the following lemmas:

Lemma 3 *With the observer in (27) and the projection type adaptation law in (6), the parameter estimation error and the estimation error* $\tilde{\xi}$ *are always bounded, i.e.,* $\tilde{\theta}_0 \in \mathcal{L}_\infty[0,\infty)$, *and* $\tilde{\xi} \in \mathcal{L}_\infty[0,\infty)$.

Proof: From the properties of the projection mapping in (6), it is seen that $\hat{\theta}_0 \in \mathcal{L}_\infty[0,\infty)$. Hence, the parameter estimation error also $\tilde{\theta}_0 \in \mathcal{L}_\infty[0,\infty)$.

Since, $A_\xi(x,u)$ is assumed to be stable, there exist two positive definite matrices $P > 0$ and $Q > 0$ such that $A_\xi^T P + PA_\xi = -Q$.

Consider the following Lyapunov function $V_{\tilde{\xi}} = \tilde{\xi}^T P \tilde{\xi}$, then $\lambda_{min}(P)|\tilde{\xi}|^2 \leq V_{\tilde{\xi}}(\tilde{\xi}) \leq \lambda_{max}(P)|\tilde{\xi}|^2$. Looking at equation (21), we have

$$\begin{aligned} \dot{V}_{\tilde{\xi}} &= \dot{\tilde{\xi}}^T P\tilde{\xi} + \tilde{\xi}^T P \dot{\tilde{\xi}} \\ &= \dot{\tilde{\xi}}^T (A_\xi^T(x,u)P + PA_\xi(x,u))\tilde{\xi} - \Delta_\xi^T P\tilde{\xi} - \dot{\tilde{\xi}}^T P\Delta_\xi \\ &\leq -\lambda_{min}(Q)|\tilde{\xi}|^2 - 2\Delta_\xi^T P\tilde{\xi} \qquad (43) \\ &\leq -\lambda_{min}(Q)|\tilde{\xi}|^2 + 2\lambda_{max}(P)(\delta_\eta + \Sigma_{i=1}^n |L|\delta_{x_i})|\tilde{\xi}| \\ &\leq -c_{\tilde{\xi}}\lambda_{max}(P)|\tilde{\xi}|^2 + \frac{\lambda_{max}^2(P)(\delta_\eta + \Sigma_{i=1}^n|L|\delta_{x_i})^2}{\lambda_{min}(Q) - c_{\tilde{\xi}}\lambda_{max}(P)} \end{aligned}$$

where $c_{\tilde{\xi}} > 0$ is any positive constant satisfying $c_{\tilde{\xi}} \leq \frac{\lambda_{min}(Q)}{\lambda_{max}(P)}$. Since $(\delta_\eta + \Sigma_{i=1}^n |L|\delta_{x_i})$ is a function of the measured states and t only and is bounded with respect to t, there exists a class $\mathcal{K}_\infty$ function $\gamma(x)$ and a positive constant d such that $\gamma(x) + d \geq \frac{(\lambda_{max}^2(P))(\delta_\eta + \Sigma_{i=1}^n|L|\delta_{x_i})^2}{\lambda_{min}(Q) - c_{\tilde{\xi}}\lambda_{max}(P)}$. Thus, we have that,

$$\dot{V}_{\tilde{\xi}} \leq -c_{\tilde{\xi}} V_{\tilde{\xi}}(\tilde{\xi}) + \gamma(x) + d \qquad (44)$$

Hence, the system with $\tilde{\xi}$ as the state and x as the input is ISpS. □

Lemma 4 *The systems of filters in (32) and (33) is ISpS with the inputs being the measured states x and the control input u and the states being the filter outputs* Ω_0 *and* Ω.

Proof: As the filter matrix used in equations (32) and (33) is stable, there exist $P > 0$ and $Q > 0$ such that $A^T P + PA = -Q$.

Consider the following Lyapunov function

$$V = Tr(\Omega P \Omega^T) \qquad (45)$$

where P is a positive definite matrix.

Then the derivative of the Lyapunov function (45) using equations (32) and (33) is given as

$$\begin{aligned} \dot{V} &= Tr((\dot{\Omega}P\Omega^T) + (\Omega P \dot{\Omega}^T)) \\ &= Tr((\Omega(A^T P + PA)\Omega^T) + \Theta^T P \Omega^T + \Omega P \Theta) \qquad (46) \end{aligned}$$

Hence, the derivative of the Lyapunov equation is

$$\begin{aligned} \dot{V} &= -Tr(\Omega Q \Omega^T) + Tr(\Theta^T P \Omega^T + \Omega P \Theta) \\ &\leq \frac{-N_1}{\lambda_{max}(P)} Tr(\Omega P \Omega^T) + kTr(\frac{\Theta^T P \Omega^T}{k} + \frac{\Omega P \Theta}{k} - \Omega P \Omega^T) \\ &= \frac{-N_1}{\lambda_{max}(P)} V + kTr(-(\Omega - \frac{\Theta^T}{k})P(\Omega - \frac{\Theta^T}{k})^T) + Tr(\frac{\Theta P \Theta^T}{k}) \end{aligned}$$

where N, N_1, k are positive scalars which satisfy $\lambda_{min}(Q) > N > 0$ and $N = N_1 + k\lambda_{max}(P)$.

Since the regressor is bounded,

$$\dot{V} \leq \frac{-N_1}{\lambda_{max}(P)} V + \gamma(|x,u|) + d \qquad (47)$$

Hence, the system with x and u as the inputs and Ω as the state is ISpS. □

Theorem 1 *The Adaptive Robust Observer given by equations (27) and (40) is ISpS with the measured states x and the control input u as the inputs and the observer estimation error* $\tilde{\eta}$ *given (42).*

Proof: The prediction error for the observer is given by the static relation (39). From Lemma 3, $\tilde{\theta}_0 \in \mathcal{L}_\infty[0,\infty)$ and the system with $\tilde{\xi}$ as a state is ISpS. Also, utilizing Lemma 4, the set of filters with Ω as a state is ISpS. Hence, the ARO is ISpS with x and u as inputs and $\tilde{\eta}$ as the state. □

Theorem 2 *In the absence of uncertain nonlinearities, i.e.,* $\Delta_\eta = \Delta_{x_i} = 0$, *the parameter estimation error* $\tilde{\theta}_0 \to 0$ *if the following persistent excitation (PE) is satisfied:*

$$\exists T, \alpha > 0, s.t. \int_t^{t+T} \Omega(\tau)\Omega^T(\tau) d\tau \geq \alpha I, \forall t \qquad (48)$$

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Proof: In the absence of uncertain nonlinearities i.e., $\Delta_\eta = \Delta_{x_i} = 0$,

$$\dot{\tilde{\xi}} = A_\xi(x,u)\tilde{\xi} \quad (49)$$

$$\dot{\varepsilon} = A\varepsilon - \phi_i(x,u,\theta)\tilde{\xi} \quad (50)$$

It is thus clear that both $\tilde{\xi}$ and ε exponentially converge to zero. Thus, using standard adaptive design techniques, we can show that the least squares type projection adaptation law guarantees that the prediction error $\varepsilon \to 0$ as $t \to \infty$. From (40), $\dot{\hat{\theta}}_0 \to 0$ as $t \to \infty$.

As $e = \Omega^T\tilde{\theta}_0 - \varepsilon$, one has that $\Omega^T\tilde{\theta}_0 \to 0$ as $t \to \infty$. Hence, for any T,

$$\int_t^{t+T} \tilde{\theta}_0^T \Omega\Omega^T \tilde{\theta}_0 d\tau \to 0, ast \to \infty \quad (51)$$

Using the mean value theorem,

$$\tilde{\theta}_0(\tau) = \tilde{\theta}_0(t) + \dot{\tilde{\theta}}_0(\mu)(\tau - t) \quad (52)$$

Thus, noting that $\dot{\tilde{\theta}}_0 \to 0$ as $t \to \infty$, (51) and (52) lead to

$$\tilde{\theta}_0^T(t)[\int_t^{t+T} \Omega\Omega^T d\tau]\tilde{\theta}_0(t) \to 0 \quad (53)$$

Thus when the PE condition (48) is satisfied,

$$\alpha\|\tilde{\theta}_0(t)\|^2 \leq \tilde{\theta}_0^T(t)[\int_t^{t+T} \Omega\Omega^T d\tau]\tilde{\theta}_0(t) \quad (54)$$

From (53), $\tilde{\theta}_0 \to 0$. □

6 Conclusions

An adaptive robust observer (ARO) design has been presented for a class of parametric semi-strict feedback nonlinear systems that are linear in terms of the unmeasured states. It has been shown that using a robust filter structure along with controlled parameter adaptation enables us to effectively combat the effect of various model uncertainties including both unknown but constant plant parameters and bounded fast-changing unknown nonlinearities. Theoretically, the proposed ARO guarantees bounded estimates of the states and parameters even in the presence of bounded fast-changing unknown nonlinearities, and the asymptotically converging estimates of the transformed states in the presence of parametric uncertainties only. Practically, the experimental results obtained on the velocity estimates of an electro-hydraulic system using pressure measurements only [15] have demonstrated the effectiveness of the proposed ARO design in industrial applications.

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