Integrated Direct/Indirect Adaptive Robust Control of SISO Nonlinear Systems in Semi-Strict Feedback Form [1]

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Abstract

The paper focuses on the synthesis of adaptive robust controllers that achieve not only excellent output tracking performance but also accurate parameter estimations for secondary purposes such as machine health monitoring and prognostics. Such an objective is accomplished through an intelligent integration of the output tracking performance oriented direct adaptive robust control (DARC) design with the recently proposed accurate parameter estimation based indirect adaptive robust control (IARC) design. SISO nonlinear systems transformable to semi-strict feedback forms are considered. Theoretically, regardless of the specific estimation algorithm to be used, certain guaranteed transient performance and final tracking accuracy are achieved even in the presence of uncertain nonlinearities–a desirable feature in applications. In addition, the theoretical performance of adaptive designs–asymptotic output tracking in the presence of parametric uncertainties only–is preserved. The construction of physical parameter estimation law is based on the actual system dynamics and totally independent from the design of underline robust control law, which allows various estimation algorithms having better parameter convergence properties and practical modifications such as the on-line explicit monitoring of signal excitation levels to be used to significantly improve the accuracy of the resulting parameter estimates in implementation.

1 Introduction

In [1, 2], an adaptive robust control (ARC) approach is presented to systematically construct performance oriented control laws for nonlinear systems transformable to semi-strict feedback forms. The approach has been applied to several applications and comparative experimental results have demonstrated the substantially improved performance of the ARC approach in implementation [3, 4, 5, 6].

The underline parameter adaptation law in ARC controllers in [1, 2] are based on the direct adaptive control designs [7] including the tuning function based adaptive backstepping [8], in which the adaptive control law and parameter adaptation law are synthesized simultaneously to meet the sole objective of reducing the output tracking error. Although excellent output tracking performance is achieved, the parameter estimates of such a direct ARC design hardly converge to their true values in implementation [3, 4, 5, 6], even when the desired trajectory might be persistently exciting and of large signal. The problem is mainly caused by the use of the gradient type parameter adaptation law with certain actual tracking errors as driving signals; the actual tracking errors of a well designed direct ARC law in implementation are normally extremely small and near the measurement resolution, and, thus, are more prone to be corrupted by other factors such as the sampling delay and noise that have been neglected when synthesizing the parameter adaptation law.

To overcome the poor parameter estimates of the direct ARC designs [2], an indirect adaptive robust control design has recently been developed [9], in which the construction of parameter estimation law is totally separated from the design of underline robust control law. As a result, various estimation algorithms having better parameter convergence properties (e.g., the least squares type method) can be used. Furthermore, on-line explicit monitoring of signal excitation level can be employed in implementation to significantly improve the accuracy of parameter estimates. Because of these algorithm improvements, the resulting parameter estimates are normally accurate enough to be used for secondary purposes such as machine health monitoring and prognostics, which are becoming ever-increasingly important for practical industrial applications.

In [10], the IARC in [9] is applied to the precision control of an epoxy core linear motor with an improved estimation model, and is experimentally compared with the direct ARC (DARC) designs in [5]. The comparative experimental results show that, although the proposed IARC design has a much better accuracy of parameter estimates than the direct ARC [5], the output tracking performances of IARC are not as good as those of DARC [5], especially during the transient periods. A more detailed thorough analysis reveals that the poorer tracking performance of IARC is caused by the loss of dynamic compensation type fast adaptation that is inherited in the DARC designs. To overcome this loss

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of tracking performance problem of IARC, in this paper, an integrated direct/indirect ARC (DIARC) design framework will be developed for SISO nonlinear systems transformable to semi-strict feedback forms. The design not only uses the same adaptation process as in the IARC design [9] for accurate estimation of physical parameters, but also introduces dynamic compensation type fast adaptation to achieve a better transient performance. Consequently, the resulting DIARC controller is able to achieve better tracking performances than either IARC or DARC while having the same level of accurate physical parameter estimates as in IARC.

2 Problem Formulation

The paper considers the class of uncertain nonlinear systems transformable to the semi-strict feedback form with unknown input gains described by

$$\begin{array}{rcl} \dot{x}_1 & = & b_1 x_2 + \varphi_1(x_1,t)^T\theta + \Delta_1(\bar{x}_n,t) \\ & \vdots & \\ \dot{x}_n & = & b_n u + \varphi_n(\bar{x}_n,t)^T\theta + \Delta_n(\bar{x}_n,t) \end{array} \tag{1}$$

where x_1 is the system output, $\bar{x}_i = [x_1,...,x_i]^T$ is the vector of the first i states, b_i is the unknown input gain of the i-th channel, $\theta = [\theta_1,...,\theta_{p_\theta}]^T$ represents the vector of other unknown parameters, $\Delta_i(\bar{x}_n,t)$ is the uncertain nonlinearity in the i^{th} channel, and u is the control input. For notation simplicity, let $\theta_b \in R^p$ be the vector of all unknown parameters, i.e., $\theta_b = [\theta^T, b_1,\ldots,b_n]^T$. The following nomenclature is used throughout this paper: $\hat{\bullet}$ is used to denote the estimate of $\bullet$, $\tilde{\bullet}$ is used to denote the parameter estimation error of $\bullet$, e.g., $\tilde{\theta} = \hat{\theta} - \theta$, $\bullet_i$ is the i^{th} component of the vector $\bullet$, $\bar{\bullet}_i$ is a column vector of the first i components of $\bullet$, e.g., $\bar{b}_i = [b_1,b_2,...,b_i]^T$, $\bullet_{max}$ and $\bullet_{min}$ are the maximum and minimum value of $\bullet(t)$ for all t respectively. The following practical assumptions are made:

Assumption 1 *The unknown parameter vector θ_b is within a known bounded convex set Ω_{θ_b}. Furthermore, within Ω_{θ_b}, the input gains $b_i, i = 1,\ldots,n$, are of known signs and nonzero. Without loss of generality, it is assumed that $\forall\theta_b \in \Omega_{\theta_b}$, $\theta_{imin} \le \theta_i \le \theta_{imax}$ and $0 < b_{imin} \le b_i \le b_{imax}, i = 1,\ldots,n$, where θ_{imin}, θ_{imax}, b_{imin}, and b_{imax} are some known constants.*

Assumption 2 *The uncertain nonlinearity $\Delta_i(\bar{x}_n,t)$ can be bounded by*

$$|\Delta_i(\bar{x}_n,t)| \le \delta_i(\bar{x}_i)d_i(t), \quad \forall i \tag{2}$$

where $\delta_i(\bar{x}_i)$ is a known positive function, and $d_i(t)$ is an unknown but bounded positive time-varying function.

The integrated ARC problem is stated as that of, under Assumptions 1 and 2, synthesizing a control law for the input u and a parameter estimation law for the unknown parameter vector θ_b so that: (i) all the signals of the resulting closed system are bounded, (ii) the output x_1 tracks the desired output trajectory $x_{1d}(t)$ with a guaranteed transient performance and final tracking accuracy, (iii) in the presence of parametric uncertainties only (i.e., $\Delta_i = 0$, $\forall i$), the output tracking error $z_1 = x_1 - x_{1d}(t)$ converges to zero asymptotically, and (iv) effective parameter estimation algorithms such as the least square method and practical modifications are sought to have accurate on-line estimates of physical parameters θ_b.

3 Integrated Direct/Indirect Adaptive Robust Control

In this section, an integrated DIARC scheme will be developed for the system (1). As in the IARC in [9], the first step is to use a rate-limited projection type adaptation law structure to achieve a controlled learning or adaptation process as detailed in the following.

3.1 Rated Limited Projection Type Adaptation Law Structure

One of the key elements of the ARC design [1, 2] is to use the practical available prior process information to construct projection type adaptation law for a controlled learning process even in the presence of disturbances. As in [1, 11], the widely used projection mapping $Proj_{\hat{\theta}_b}(\bullet)$ will be used to keep the parameter estimates within the known bounded set $\bar{\Omega}_{\theta_b}$, the closure of the set Ω_{θ_b}. The standard projection mapping is [12, 8]:

$$Proj_{\hat{\theta}_b}(\zeta) = \begin{cases} \zeta, & \text{if} \quad \hat{\theta}_b \in \mathring{\Omega}_{\theta_b} \text{ or } n_{\hat{\theta}_b}^T\zeta \le 0 \\ \left(I - \Gamma\frac{n_{\hat{\theta}_b}n_{\hat{\theta}_b}^T}{n_{\hat{\theta}_b}^T\Gamma n_{\hat{\theta}_b}}\right)\zeta & \quad \hat{\theta}_b \in \partial\Omega_{\theta_b} \text{ and } n_{\hat{\theta}_b}^T\zeta > 0 \end{cases} \tag{3}$$

where $\zeta \in R^p$ is any function and $\Gamma(t) \in R^{p\times p}$ can be any time-varying positive definite symmetric matrix. In (3), $\mathring{\Omega}_{\theta_b}$ and $\partial\Omega_{\theta_b}$ denote the interior and the boundary of Ω_{θ_b} respectively, and $n_{\hat{\theta}_b}$ represents the outward unit normal vector at $\hat{\theta}_b \in \partial\Omega_{\theta_b}$.

To achieve a complete separation of estimator design and adaptive robust control law design, in addition to the use of projection type parameter adaption law as in the direct ARC [11], it is also necessary to preset a limit on the adaptation rate for a controlled estimation process. For this purpose, for any $\zeta \in R^p$, define a saturation function as:

$$sat_{\dot{\theta}_M}(\zeta) = s_0\zeta, \quad s_0 = \begin{cases} 1, & \|\zeta\| \le \dot{\theta}_M \\ \frac{\dot{\theta}_M}{\|\zeta\|}, & \|\zeta\| > \dot{\theta}_M \end{cases} \tag{4}$$

where $\dot{\theta}_M$ is a pre-set rate limit.

Lemma 1 *[9] Suppose that the parameter estimate $\hat{\theta}_b$ is updated using the following rate-limited projection type adaptation law with a pre-set rate limit $\dot{\theta}_M$:*

$$\dot{\hat{\theta}}_b = sat_{\dot{\theta}_M}\left(Proj_{\hat{\theta}_b}(\Gamma\tau)\right), \quad \hat{\theta}_b(0) \in \Omega_{\theta_b} \tag{5}$$

where τ is any estimation function and $\Gamma(t) > 0$ is any continuously differentiable positive symmetric adaptation rate matrix. With this adaptation law structure, the following desirable properties hold:

P1. The parameter estimates are always within the known bounded set $\bar{\Omega}_{\theta_b}$, i.e., $\hat{\theta}_b(t) \in \bar{\Omega}_{\theta_b}$, $\forall t$. Thus, from Assumption 1, $\forall t$, $\theta_{imin} \le \hat{\theta}_i(t) \le \theta_{imax}$ and $0 < b_{imin} \le \hat{b}_i(t) \le b_{imax}$, $i = 1, \dots, n$.

P2.

$$\tilde{\theta}_b^T \left(\Gamma^{-1} Proj_{\hat{\theta}_b}(\Gamma\tau)) - \tau \right) \le 0, \quad \forall \tau \tag{6}$$

P3. The parameter adaptation rate is uniformly bounded by $\|\dot{\hat{\theta}}_b(t)\| \le \dot{\theta}_M$, $\forall t$ $\triangle$

3.2 Integrated Direct/Indirect ARC Law

With the use of the rate limited projection type adaptation law structure (5), the parameter estimates and their derivatives are bounded with known bounds, regardless of the estimation function τ to be used. In the following, this property will be used to synthesize an integrated DIARC law for the system (1) that achieves a guaranteed transient and final tracking accuracy in general, independent of the specific identifier to be used later. As the system (1) has unmatched model uncertainties, backstepping design [8] is used as in [2]:

Step 1:
Let us rewrite the first equation of (1) as

$$\dot{x}_1 = b_1 x_2 + \phi_1(x_1,t)^T \theta + \breve{\Delta}_1(\bar{x}_n, t) \tag{7}$$

where $\phi_1(x_1,t) = \varphi_1(x_1,t)$ and $\breve{\Delta}_1(\bar{x}_n,t) = \Delta_1(\bar{x}_n,t)$. Let $\breve{\delta}_1(x_1) = \delta_1(x_1)$ and $\breve{d}_1(t) = d_1(t)$. Then $\left|\breve{\Delta}_1(\bar{x}_n,t)\right| \le \breve{\delta}_1(x_1)\breve{d}_1(t)$. The step 1 is to synthesize a virtual DIARC control function α_1 for x_2 so that the output tracking error $z_1 = x_1 - x_{1d}$ converges to zero or some small values with a guaranteed transient and final tracking accuracy when the input mismatch $z_2 = x_2 - \alpha_1$ is zero. The following virtual DIARC function is suggested:

$$\begin{array}{l} \alpha_1(x_1, \hat{\theta}, \hat{b}_1, \hat{d}_{c1}, t) = \alpha_{1a} + \alpha_{1s} \\ \alpha_{1a} = \alpha_{1a1} + \alpha_{1a2}, \quad \alpha_{1a1} = \frac{1}{\hat{b}_1}\left[-\phi_1^T\hat{\theta} + \dot{x}_{1d}(t)\right], \quad \alpha_{1a2} = -\frac{1}{\hat{b}_1}\hat{d}_{c1} \\ \alpha_{1s} = \alpha_{1s1} + \alpha_{1s2}, \quad \alpha_{1s1} = -k_{1s1}(x_1,t)z_1, \quad \alpha_{1s2} = -k_{1s2}(x_1,t)z_1 \end{array} \tag{8}$$

In (8), α_{1a1} represents the usual model compensation with the physical parameter estimates $\hat{\theta}_b(t)$ updated later using an indirect parameter estimator as in the IARC design [9], α_{1a2} is a model compensation term similar to the fast dynamic compensation type model compensation used in the DARC designs [2, 11], in which $\hat{d}_{c1}$ can be thought as the estimate of the low frequency component of the lumped model uncertainties defined later, α_{1s} represents the robust control term with the nonlinear feedback gains k_{1s1} and k_{1s2} specified in the following.

From (7) and (8), noting $z_2 = x_2 - \alpha_1$, the first error equation is obtained as

$$\dot{z}_1 = b_1 z_2 + b_1\alpha_{1s} + b_1\alpha_{1a2} - \tilde{b}_1\alpha_{1a1} - \phi_1^T\tilde{\theta} + \breve{\Delta}_1 \tag{9}$$

Define a constant d_{c1} and time varying $\Delta_1^*(t)$ such that

$$d_{c1} + \Delta_1^*(t) = -\tilde{b}_1\alpha_{1a1} - \phi_1^T\tilde{\theta} + \breve{\Delta}_1 \tag{10}$$

Conceptually, (10) lumps the disturbances and the model uncertainties due to physical parameter estimation error together and divides it into the static component (or low frequency component in reality) d_{c1} and the high frequency components $\Delta_1^*(t)$, so that the low frequency component d_{c1} can be compensated through fast adaptation similar to those in the direct ARC designs [2, 11] as follows.

Let d_{c1M} be any pre-set bound and use this bound to construct the following projection type adaptation law for $\hat{d}_{c1}(t)$

$$\begin{array}{ll} \dot{\hat{d}}_{c1} & = Proj_{\hat{d}_{c1}}(\gamma_{d1}z_1) \\ & \triangleq \begin{cases} 0 & \text{if} \quad |\hat{d}_{c1}(t)| = d_{c1M} \ \text{and} \ \hat{d}_{c1}(t)z_1 > 0 \\ \gamma_{d1}z_1 & \text{else} \end{cases} \end{array} \tag{11}$$

with $\gamma_{d1} > 0$ and $|\hat{d}_{c1}(0)| \le d_{c1M}$. Such an adaptation law guarantees that $|\hat{d}_{c1}(t)| \le d_{c1M}, \forall t$.

Substituting (10) into (9) and noting (8),

$$\begin{array}{l} \dot{z}_1 = b_1 z_2 + b_1\alpha_{1s} + b_1\alpha_{1a2} + d_{c1} + \Delta_1^*(t) \\ \quad = b_1 z_2 + b_1\alpha_{1s1} + \left[b_1\alpha_{1s2} - \tilde{b}_1\alpha_{1a2} - \tilde{d}_{c1} + \Delta_1^*(t)\right] \end{array} \tag{12}$$

In the above, due to the use of projection type adaptation law, all estimation errors are bounded within known bounds. As such, the same as in the DARC in [2, 11], it can be shown that, as long as the nonlinear feedback gain k_{1s2} is chosen large enough, the following robust performance condition can be satisfied

$$z_1\left[b_1\alpha_{1s2} - \tilde{b}_1\alpha_{1a2} - \tilde{d}_{c1} + \Delta_1^*\right] \le \varepsilon_{c1} + \varepsilon_{d1}\breve{d}_1^2 \tag{13}$$

where ε_{c1} and ε_{d1} are constant design parameters that can be thought as the theoretical indexes for the attenuation level of model uncertainties as seen later. The theoretical lower bounds for k_{1s2} for (13) to be satisfied can be worked out in the same way as in [2, 11]. It can be easily verified that the derivative of $V_1 = 1/2z_1^2$ is

$$\dot{V}_1 = b_1 z_1 z_2 + z_1 b_1\alpha_{1s1} + z_1[b_1\alpha_{1s2} - \tilde{b}_1\alpha_{1a2} - \tilde{d}_{c1} + \Delta_1^*] \tag{14}$$

$$= -b_1 k_{1s1} z_1^2 + b_1 z_1 z_2 + \varepsilon_{c1} + \varepsilon_{d1}\breve{d}_1^2 \tag{15}$$

It is thus obvious that if z_2 is bounded, the output tracking error z_1 would be bounded. Thus, the next step is to make sure that z_2 converges to zero or some small values.

Step i
The derivation of the remaining design steps follows the similar procedure as in the first design step. Namely, at each step i, a virtual DIARC law $\alpha_i(\bar{x}_i, \hat{\theta}, \hat{\bar{b}}_i, \hat{d}_{ci}, t)$ is designed in order that x_i tracks its desired virtual DIARC law $\alpha_{i-1}(\bar{x}_{i-1}, \hat{\theta}, \hat{\bar{b}}_{i-1}, \hat{d}_{c(i-1)}, t)$ that was synthesized in step $i-1$ with a desired transient performance. The virtual DIARC control function, along with the error equation and an augmented non-negative function with its derivative, are explicitly given in the following lemma.

Lemma 2 *For each $i \le n$, let $z_i = x_i - \alpha_{i-1}$. Using the notations of $\alpha_0(t) = x_{1d}(t)$ and $b_0 = 0$, one can recursively define the following terms for step i from the previous steps:*

$$\phi_i(\bar{x}_i, \hat{\theta}, \hat{\bar{b}}_{i-1}, t) \quad = \quad \varphi_i(\bar{x}_i, t) - \sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_l}\varphi_l \tag{16}$$

$$\breve{\Delta}_i(\bar{x}_n, \hat{\theta}, \hat{\bar{b}}_{i-1}, t) = \Delta_i(\bar{x}_n, t) - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l} \Delta_l \quad (17)$$

with $\breve{\Delta}_i$ bounded by

$$|\breve{\Delta}_i(\bar{x}_n, \hat{\theta}, \hat{\bar{b}}_{i-1}, t)| \le \breve{\delta}_i(\bar{x}_i, t) \breve{d}_i(t) \quad (18)$$

where $\breve{\delta}_i$ is any smooth function satisfying $\breve{\delta}_i \ge i\left[\max\left\{\delta_i, max_{l=1,\ldots,i-1}\{|\frac{\partial \alpha_{i-1}}{\partial x_l}|\delta_l\}\right\}\right]$ and $\breve{d}_i(t) = max_{l=1,\ldots,i}\{d_l(t)\}$.

Define d_{ci}, the static component, and Δ_i^*, the time-varying component, of the lumped model compensation error at step i as

$$d_{ci} + \Delta_i^*(t) = -\phi_i^T \tilde{\theta} + \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l} \tilde{b}_l x_{l+1} - \tilde{b}_i \alpha_{ia1} + \breve{\Delta}_i \quad (19)$$

where α_{ia1} is the usual model compensation defined below. Choose the following virtual DIARC control function for x_{i+1}

$$\begin{array}{l} \alpha_i(\bar{x}_i, \hat{\theta}, \hat{\bar{b}}_i, \hat{d}_{ci}, t) = \alpha_{ia} + \alpha_{is}, \quad \alpha_{ia} = \alpha_{ia1} + \alpha_{ia2} \\ \alpha_{ia1} = \frac{1}{\hat{b}_i}\left[\sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l} \hat{b}_l x_{l+1} + \frac{\partial \alpha_{i-1}}{\partial t} - \phi_i^T \hat{\theta}\right], \quad \alpha_{ia2} = -\frac{1}{\hat{b}_i}\hat{d}_{ci} \\ \alpha_{is} = \alpha_{is1} + \alpha_{is2}, \quad \alpha_{is1} = -k_{is1}(\bar{x}_i, t) z_i, \quad \alpha_{is2} = -k_{is2}(\bar{x}_i, t) z_i \end{array} \quad (20)$$

where $\hat{d}_{ci}$ is the estimate of d_{ci} updated by

$$\dot{\hat{d}}_{ci} = Proj_{\hat{d}_{ci}}(\gamma_{di} z_i) \triangleq \left\{ \begin{array}{lll} 0 & if & |\hat{d}_{ci}(t)| = d_{ciM} \;\&\; \hat{d}_{ci} z_i > 0 \\ \gamma_{di} z_i & & else \end{array} \right. \quad (21)$$

with $\gamma_{di} > 0$ and $|\hat{d}_{ci}(0)| \le d_{ciM}$, in which d_{ciM} is a pre-set bound for $\hat{d}_{ci}(t)$. In (20), k_{is1} is a robust gain to be specified later, and k_{is2} is a nonlinear gain large enough so that the following robust performance condition is satisfied

$$z_i \left[b_i \alpha_{is2} - \tilde{b}_i \alpha_{ia2} - \tilde{d}_{ci} + \Delta_i^* - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{b}_l} \dot{\hat{b}}_l \right] \le \varepsilon_{ci} + \varepsilon_{di} \breve{d}_i^2 \quad (22)$$

where ε_{ci} and ε_{di} are positive constant design parameters that can be thought as the theoretical indexes for the attenuation level of model uncertainties.

With the virtual DIARC control function (20), the i^{th} error equation can be written as

$$\dot{z}_i = b_i z_{i+1} + b_i \alpha_{is1} + \left[b_i \alpha_{is2} - \tilde{b}_i \alpha_{ia2} - \tilde{d}_{ci} + \Delta_i^* \right. \quad (23)$$

$$\left. - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{b}_l} \dot{\hat{b}}_l - \frac{\partial \alpha_{i-1}}{\partial \hat{d}_{c(i-1)}} \dot{\hat{d}}_{c(i-1)} \right] \quad (24)$$

and the derivative of the augmented non-negative function

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 \quad (25)$$

is given by

$$\begin{array}{l} \dot{V}_i = \sum_{l=1}^{i} z_l b_l \alpha_{ls1} + \sum_{l=1}^{i} b_l z_l z_{l+1} + \sum_{l=1}^{i} z_l \left\{ b_l \alpha_{ls2} - \tilde{b}_l \alpha_{la2} - \tilde{d}_{cl} \right. \\ \qquad \left. + \Delta_l^* - \frac{\partial \alpha_{l-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - \sum_{j=1}^{l-1} \frac{\partial \alpha_{l-1}}{\partial \hat{b}_j} \dot{\hat{b}}_j - \frac{\partial \alpha_{l-1}}{\partial \hat{d}_{c(l-1)}} \dot{\hat{d}}_{c(l-1)} \right\} \end{array} \quad (26)$$

Lemma 2 can be proved via direct verifications.

Remark 1 *P1 and P3 of Lemma 1 guarantee that the parameter estimation error $\tilde{\theta}_b$ and its derivative $\dot{\hat{\theta}}_b$ are bounded with known bounds. Noting (19) and (21), $\tilde{d}_{ci}$ and Δ_i^* are bounded with known functions of states. Thus, as in [2], there exists a large enough nonlinear feedback gain k_{is2} that is a function of $\bar{x}_i$ and t only such that the robust performance condition (22) can be satisfied. Furthermore, how to choose $k_{is2}(\bar{x}_i, t)$ to satisfy (22) can be worked out in the same way as in [2]. Note also that the use of projection type adaptation law guarantees that $\hat{b}_i$ is non-zero, which makes the control law (20) free of singularity.* △

Step n

Letting $x_{n+1} = u$, then, the step n is exactly the same as the previous steps but with $z_{n+1} = 0$ if we actually choose the input u as

$$u = \alpha_n(\bar{x}_n, \hat{\theta}, \hat{\bar{b}}_n, t) \quad (27)$$

where α_n is given by equations (20) with $i = n$. The following theorem states the theoretic achievable performance of such a DIARC law:

Theorem 1 *Consider the ARC law (27) with the rate limited projection type adaptation law (5), in which τ could be any estimation function. If the gains $k_{is1}, i = 1, \ldots, n$ are sufficiently large such that the following matrix is non-negative*

$$M_s = \begin{bmatrix} b_1 k_{1s1} - \kappa_1 & -\frac{1}{2}\left(b_1 + \frac{\gamma_{d1}}{\hat{b}_1}\right) & \ldots & 0 \\ -\frac{1}{2}\left(b_1 + \frac{\gamma_{d1}}{\hat{b}_1}\right) & \times & \ddots & \vdots \\ \vdots & \ddots & \ddots & \times \\ 0 & \ldots & \times & b_n k_{ns1} - \kappa_n \end{bmatrix} \ge 0 \quad (28)$$

where $\kappa_i, i = 1, \ldots, n$, are some positive numbers, then, in general, all signals in the resulting closed loop system are bounded. In addition, the tracking errors are bounded by

$$||\bar{z}_n(t)||^2 \le e^{-\lambda_v t} ||\bar{z}_n(0)||^2 + \frac{2\varepsilon_{vmax}}{\lambda_v}\left[1 - e^{-\lambda_v t}\right] \quad (29)$$

where $\lambda_v = 2\left(\min_{i=1,\ldots,n}\{\kappa_i\}\right)$ and $\varepsilon_v(t) = \sum_{i=1}^{n}\left[\varepsilon_{ci} + \varepsilon_{di}\breve{d}_i^2(t)\right]$

Proof of Theorem 1: From (20), $\frac{\partial \alpha_i}{\partial \hat{d}_{ci}} = -\frac{1}{\hat{b}_i}$. Thus, from (21), $\left|\frac{\partial \alpha_i}{\partial \hat{d}_{ci}} \dot{\hat{d}}_{ci}\right| \le \frac{1}{\hat{b}_i} \gamma_{di} |z_i|$. Noting (20), the robust performance condition (22), and the fact that $z_{n+1} = 0$, from (26),

$$\dot{V}_n \le -\sum_{l=1}^{n} \kappa_l z_l^2 - |\bar{z}_n|^T M_s |\bar{z}_n| + \sum_{i=1}^{n}\left(\varepsilon_{ci} + \varepsilon_{di}\breve{d}_i^2\right) \quad (30)$$

where the matrix M_s is defined in (28). Thus, when (28) is satisfied,

$$\dot{V}_n \le -\lambda_v V_n + \varepsilon_v(t) \quad (31)$$

which leads to (29) by using the Comparison Lemma [13]. As the boundedness of $\hat{\theta}_b$ and $\dot{\hat{\theta}}_b$ is guaranteed by P1 and P3 of Lemma 1 respectively, one can follow the standard backstepping proofs to show that all the control functions (20) and state $\bar{x_n}$ are bounded for any bounded desired trajectory $x_{1d}(t)$ with bounded higher derivatives. Thus all the signals of the closed-loop system are bounded, which completes the proof of the theorem. $\diamondsuit$

3.3 Parameter Estimation Algorithm

In the above subsection, a DIARC law which can admit any estimation function τ has been constructed and a guaranteed transient and final tracking performance is achieved even in the presence of uncertain nonlinearities. Thus, the reminder of the paper is to construct suitable estimation functions τ so that an improved final tracking accuracy–asymptotic tracking or zero final tracking error in the presence of parametric uncertainties only–can be obtained with an emphasis on good parameter estimation process as well. As such, in this subsection, we assume the system is absence of uncertain nonlinearities, i.e., let $\Delta_i = 0, i = 1,...,n,$ in (1). For the same practical reasons as in [9], the original system model (1), rather than any transformed tracking error dynamics, will be used to construct specific estimation functions for better accuracy of parameter estimates in implementation as detailed as follows.

Note that, when $\Delta_i = 0$, the system dynamics (1) can be rewritten as

$$\dot{\bar{x}}_n = f_0(\bar{x}_n,u) + F^T(\bar{x}_n,u)\theta_b \tag{32}$$

where the matrix F is defined as

$$F^T(\bar{x}_n,u) = \begin{bmatrix} \varphi_1^T & x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_n^T & 0 & \dots & u \end{bmatrix} \tag{33}$$

and the vector of known functions $f_0 \in \Re^n$ is added for generality and represents the lumped effect of all known nonlinearities, which is zero for (1). Construct the following filters:

$$\begin{aligned} \dot{\Omega}^T &= A\Omega^T + F^T \\ \dot{\Omega}_0 &= A(\Omega_0 + \bar{x}_n) - f_0 \end{aligned} \tag{34}$$

where A is an exponentially stable matrix. Let $y = \bar{x}_n + \Omega_0$. From (32) and (34),

$$\begin{aligned} \dot{y} &= f_0 + F^T\theta_b + A(\Omega_0 + \bar{x}_n) - f_0 \\ &= F^T\theta_b + A(\Omega_0 + \bar{x}_n) \end{aligned} \tag{35}$$

Let $\tilde{\varepsilon} = \bar{x}_n + \Omega_0 - \Omega^T\theta_b$. As in [8], it is easy to verify that y can be written as

$$y = \Omega^T\theta_b + \tilde{\varepsilon} \tag{36}$$

where $\tilde{\varepsilon}$ exponentially decays to zero and is governed by

$$\dot{\tilde{\varepsilon}} = A\tilde{\varepsilon} \tag{37}$$

Now define the estimate of y as

$$\hat{y} = \Omega^T\hat{\theta}_b \tag{38}$$

and define the prediction error as

$$\varepsilon = \hat{y} - y = \Omega^T\hat{\theta}_b - \bar{x}_n - \Omega_0 \tag{39}$$

which is calculable. The resulting prediction error model is

$$\varepsilon = \Omega^T\tilde{\theta}_b - \tilde{\varepsilon} \tag{40}$$

Thus, one has a static model (40) that is linearly parameterized in terms of $\tilde{\theta}_b$, with an additional term $\tilde{\varepsilon}$ that exponentially decays to zero. With this static model, various estimation algorithms can be used to identify unknown parameters, of which the gradient estimation algorithm and the least squares estimation algorithm [8] are given below.

3.3.1 Gradient Estimator: With the gradient type estimation algorithm, the resulting adaptation law is given by (5), in which Γ can be chosen as a constant positive diagonal matrix, i.e., $\Gamma = diag[\gamma_1,...,\gamma_p]$, and τ is defined as

$$\tau = -\frac{\Omega\varepsilon}{1+\nu\|\Omega\|_F^2}, \quad \nu \geq 0 \tag{41}$$

where by allowing $\nu = 0$, we encompass unnormalized adaptation function, and $\|\Omega\|_F$ represents the Frobenius norm of Ω, given by $\|\Omega\|_F^2 = tr\{\Omega^T\Omega\}$, in which $tr\{\bullet\}$ is the trace operation.

3.3.2 Least Squares Estimator: When the least squares type estimation algorithm with co-variance resetting [14] and exponential forgetting [15] is used, the resulting adaptation law is given by (5), in which $\Gamma(t)$ is updated by

$$\dot{\Gamma} = \alpha\Gamma - \Gamma\frac{\Omega\Omega^T}{1+\nu tr\{\Omega^T\Gamma\Omega\}}\Gamma, \ \ \Gamma(0) > 0, \ \ \Gamma(t_r^+) = \rho_0 I \tag{42}$$

where $\nu \geq 0$ and $\nu = 0$ leads to the unnormalized algorithm, and τ is defined as

$$\tau = -\frac{\Omega\varepsilon}{1+\nu tr\{\Omega^T\Gamma\Omega\}} \tag{43}$$

In (42), α is the forgetting factor, t_r is the covariance resetting time, i.e., the time when $\lambda_{min}(\Gamma(t)) = \rho_1$ where ρ_1 is a pre-set lower limit for $\Gamma(t)$ satisfying $0 < \rho_1 < \rho_0$. In practice, the above least square estimator may lead to estimator windup (i.e., $\lambda_{max}(P(t)) \longrightarrow \infty$) when the regressor is not persistently exciting. To prevent this estimator windup and take into account the effect of the rate-limited adaptation law (5), (42) is modified to

$$\dot{\Gamma} = \begin{cases} \alpha\Gamma - \Gamma\frac{\Omega\Omega^T}{1+\nu tr\{\Omega^T\Gamma\Omega\}}\Gamma, & \text{if} \quad \begin{array}{l}\lambda_{max}(P(t)) \leq \rho_M \text{ and} \\ \|Proj_{\hat{\theta}_b}(\Gamma\tau)\| \leq \dot{\theta}_M\end{array} \\ 0 & \quad\quad \text{otherwise} \end{cases} \tag{44}$$

where ρ_M is the pre-set upper bound for $\|P(t)\|$ with $\rho_M > \rho_0$. With these practical modifications, $\rho_1 I \leq \Gamma(t) \leq \rho_M I, \ \forall t$. To prove that asymptotic output tracking can be achieved, the following lemma which summarizes the properties of the estimators is needed [9]:

Lemma 3 *When the rate-limited projection type adaptation law (5) with either the gradient estimator (41) or the least squares estimator (43) is used with the prediction error calculated from (39), the following results hold:*

$$\varepsilon \in L_2[0,\infty) \cap L_\infty[0,\infty) \tag{45}$$

$$\dot{\hat{\theta}}_b \in L_2[0,\infty) \cap L_\infty[0,\infty) \tag{46}$$

Theorem 2 *In the presence of parametric uncertainties only, i.e.,* $\Delta_i = 0$, $i = 1, \ldots, n$, *by using the control law (27) and the rate limited projection type adaptation law (5) with either the gradient type estimation function (41) or the least squares type estimation function (43), if the following persistent excitation condition is satisfied:*

$$\int_t^{t+T} \Omega\Omega^T d\tau \geq \kappa_p I_p, \quad \text{for some } \kappa_p > 0 \text{ and } T > 0 \tag{47}$$

then, the physical parameter estimate $\hat{\theta}_b$ *converge to their true values (i.e.,* $\tilde{\theta}_b \to 0$ *as* $t \to \infty$*), and, in addition to the robust performance results stated in Theorem 1, an improved final tracking performance–asymptotic tracking–is also achieved, i.e.,* $\bar{z}_n \to 0$ *as* $t \to \infty$*.* △

Proof of Theorem 2: When $\Delta_i = 0$, from (17), $\breve{\Delta}_i = 0, \forall i$. From Theorem 1 and Lemma 3, it is easy to check that $\bar{z}_n, \bar{x}_n, \tilde{\theta}_b, \hat{\theta}_b, W, Q, \dot{\hat{\theta}}_b, \Omega \in L_\infty[0,\infty)$. From (24) and (39), it is clear that $\dot{\bar{z}}_n \in L_\infty[0,\infty)$ and $\dot{\varepsilon} \in L_\infty[0,\infty)$, which indicates that $\bar{z}_n$ and ε are uniformly continuous. As $\varepsilon \in L_2$ from Lemma 3, by Barbalat's Lemma, $\varepsilon \to 0$ as $t \to \infty$. Thus, from (37) and (40), $\Omega^T\tilde{\theta}_b \to 0$, and from (41) and (43), $\tau \to 0$ as $t \to \infty$. From (5), $\dot{\hat{\theta}}_b \to 0$ as $t \to \infty$. Following the standard technique in adaptive control [12], it is easy to show that the PE condition (47) guarantees the exponential convergence of parameter estimates. So $\tilde{\theta}_b \to 0$ as $t \to \infty$ and $\tilde{\theta}_b \in L_2[0,\infty)$.

Noting (19), from (26), it is easy to verify that

$$\begin{aligned}\dot{V}_n \leq & -\sum_{l=1}^n b_l k_{ls1} z_l^2 + \sum_{l=1}^{n-1} b_l |z_l||z_{l+1}| + \sum_{l=1}^n |z_l||\zeta_i| - \sum_{l=1}^n z_l \hat{d}_{cl} \\ & + \sum_{l=2}^n |z_l| \frac{\gamma_{dl-1}}{\hat{b}_{l-1}} |z_{l-1}| \leq -\sum_{l=1}^n \kappa_l z_l^2 - |\bar{z}_n|^T M_s |\bar{z}_n| + \sum_{l=1}^n |z_l||\zeta_i| \\ & - \sum_{l=1}^n z_l \hat{d}_{cl} \leq -\sum_{l=1}^n \kappa_l z_l^2 + \sum_{l=1}^n |z_l||\zeta_i| - \sum_{l=1}^n z_l \hat{d}_{cl}\end{aligned} \tag{48}$$

where

$$\zeta_i = -\tilde{b}_l \alpha_{la} - \phi_l^T \tilde{\theta} + \sum_{j=1}^{l-1} \frac{\partial \alpha_{l-1}}{\partial x_j} \tilde{b}_j x_{j+1} - \frac{\partial \alpha_{l-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - \sum_{j=1}^{l-1} \frac{\partial \alpha_{l-1}}{\partial \hat{b}_j} \dot{\hat{b}}_j \tag{49}$$

Choose a positive definite function

$$V_{an} = V_n + \sum_{i=1}^n \frac{1}{2\gamma_{di}} \tilde{d}_{ci}^2 \tag{50}$$

From (21) and (48),

$$\begin{aligned}\dot{V}_{an} & \leq -\sum_{l=1}^n \kappa_l z_l^2 + + \sum_{l=1}^n \hat{d}_{cl}[-z_l + \frac{1}{\gamma_{di}} \dot{\hat{d}}_{ci}] \quad (51) \\ & \leq -\sum_{l=1}^n \kappa_l z_l^2 + \sum_{l=1}^n |z_l||\zeta_i| \quad (52)\end{aligned}$$

where the last term of (51) is less than zero due to the property $P2$ (6) of the projection mapping used (or via direct verification). As ζ_i defined by (49) is linear w.r.t. to the parameter estimation errors $\tilde{\theta}_b$ and their derivatives $\dot{\hat{\theta}}_b$ with all coefficients being uniformly bounded by Theorem 1, the fact that $\tilde{\theta}_b \in L_2[0,\infty)$ and $\dot{\hat{\theta}}_b \in L_2[0,\infty)$ implies that $\zeta_i \in L_2[0,\infty)$. Therefore, from (52), $\bar{z}_n \in L_2$. As $\bar{z}_n$ is uniformly continuous, by Barbalat's lemma, $\bar{z}_n \to 0$ as $t \to \infty$, i.e., asymptotic output tracking is achieved, which leads to Theorem 2. ♢

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