Noise and Noise Reduction

- Noise Characterization
  - Signal-to-Noise Ratio (SNR, S/N)
  - Power Spectral Density
- Sources of Noise
- Modes of Interference
- Noise Reduction
  - Filtering
  - Modulation
  - Shielding & Grounding
  - Differential Amplifier
  - Averaging

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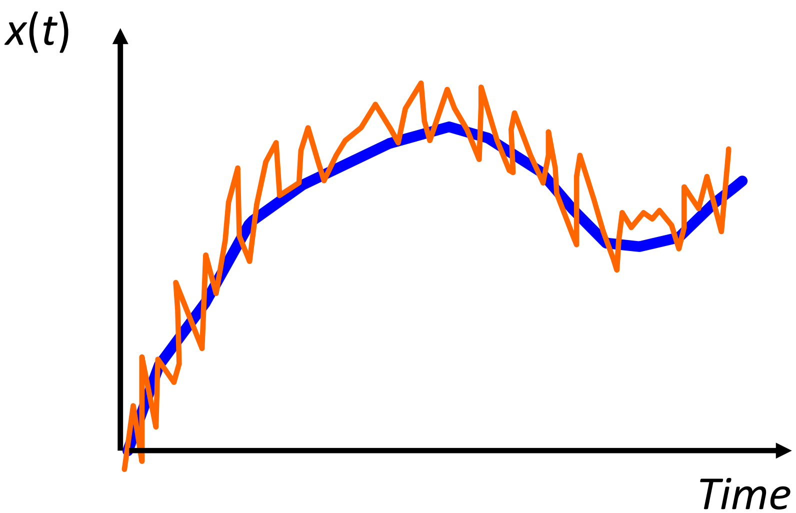
Noise

- Noise

  Noise is any unsteady component of the measurement signal that causes the instantaneous value of the signal to differ from the instrument's rendition of the "true value".



$$\underbrace{s(t)}_{\text{Actual Signal}} + \underbrace{n(t)}_{\text{Noise}} = x(t)$$

Noise Characterization

- **Average Signal Power**

Average power (in Watts) dissipated when voltage signal $V = x(t)$ is connected to a 1 Ω resistor:

$$P = V \cdot I = \frac{V^2}{R} = x^2(t) \qquad \Rightarrow \qquad \overline{P} = \overline{x^2(t)} = \frac{1}{T}\int_0^T x^2(t)\cdot dt$$

– Assume the actual signal *s*(*t*) and the noise *n*(*t*) are ***independent*** of one another:

$$\overline{x^2(t)} = \underbrace{\overline{s^2(t)}}_{\text{Signal Power}} + \underbrace{\overline{n^2(t)}}_{\text{Noise Power}}$$

– Root Mean Square (RMS) voltage of a signal is:

$$x(t)_{\text{rms}} = \sqrt{\overline{x^2(t)}}$$

Ex: *The average power of a sinusoidal signal* $x(t) = A\sin(\omega t + \phi)$ *is*:

$$\overline{x^2(t)} = \frac{1}{T}\int_0^T x^2(t)dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} A^2 \sin^2(\omega t + \phi)dt = \frac{A^2}{2} \quad \Rightarrow \quad x(t)_{rms} = \frac{A}{\sqrt{2}}$$

Noise Characterization

- **Signal-to-Noise Ratio** (SNR, S/N)

Characterizes the "noisiness" of a particular signal, $x(t) = s(t) + n(t)$:

$$SNR = S/N = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\overline{s^2(t)}}{\overline{n^2(t)}}$$

– SNR is usually expressed in decibels (dB):

$$S/N\Big|_{dB} = 10 \cdot \log_{10}\left(\frac{\text{Signal Power}}{\text{Noise Power}}\right) = 10 \cdot \log_{10}\left(\frac{\overline{s^2(t)}}{\overline{n^2(t)}}\right)$$

or

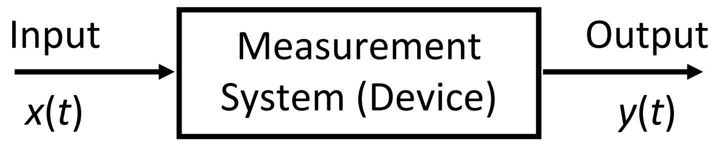
$$S/N\Big|_{dB} = 20 \cdot \log_{10}\left(\frac{\text{Signal RMS}}{\text{Noise RMS}}\right) = 20 \cdot \log_{10}\left(\frac{s(t)_{rms}}{n(t)_{rms}}\right)$$

– The *uncertainty* of a measurement is the inverse of the SNR.

Noise Characterization

- **System "Noisiness"**

The "noisiness" of a system (device) is called the Noise Figure (NF), which is defined to be the ratio between the input and output SNR:



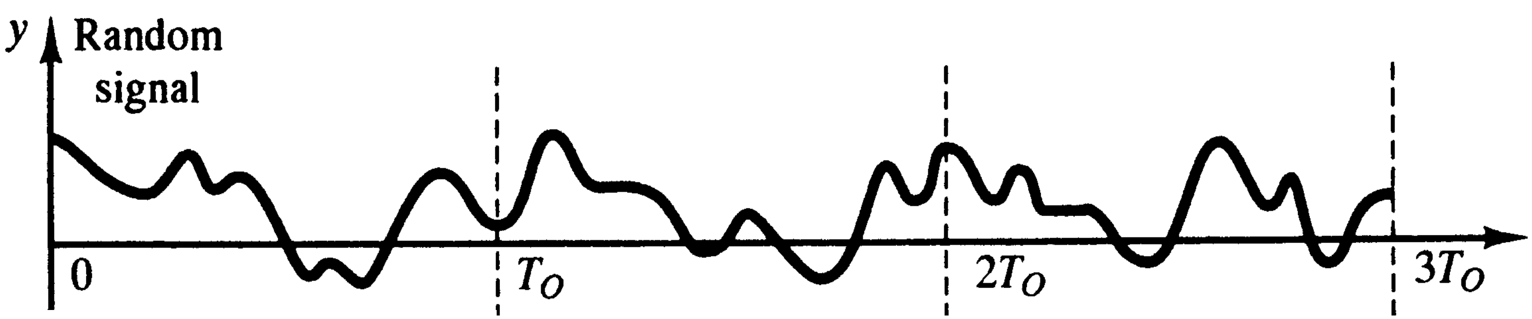
$$NF = \frac{(SNR)_{\text{INPUT}}}{(SNR)_{\text{OUTPUT}}}$$

$$\Rightarrow \quad NF\Big|_{dB} = 10 \cdot \log_{10}(NF)$$

- It is desirable to design a measurement system (device) that has a noise figure (NF) close to 1.

Noise Characterization

- **Random Noise**
  - Observe "random" signal for several observation periods:
    - Signal is different from period to period.
    - The average power for each observation period is about the same.



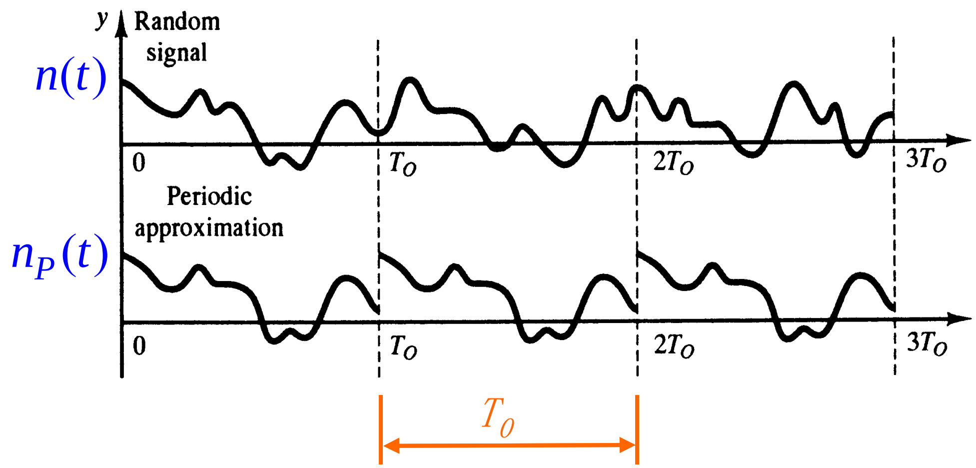
Noise Characterization

- **Random Signal**
  - Can be approximated by a periodic signal, in which the first observation period portion of the signal is repeated. The approximation is valid if:
    - The observation period is long.
    - The approximation is only used for calculating the power distribution in the random signal.

$n(t)$: Random Signal

$n_P(t)$: Periodic Approximation

$\Rightarrow \quad n(t) \approx n_P(t)$ w.r.t. average power



$n_P(t)$ is a periodic signal:

$$\Rightarrow \quad n_P(t) = \frac{A_0}{2} + \sum_{k=1}^{\infty} M_k \cos(k\omega_1 t - \theta_k)$$

$$\Downarrow \quad \omega_1 = \frac{2\pi}{T_0}$$

Noise Characterization

• Average Power Spectrum

$$n(t) \approx n_P(t) = \frac{A_0}{2} + \sum_{k=1}^{\infty} M_k \cos(k\omega_1 t - \theta_k)$$

– If k-th harmonic is applied across a 1 Ω resistor, the average power dissipated is:

$$W_k = \frac{1}{T_O}\int_0^{T_O} M_k^{\,2} \cos^2(\underbrace{k\omega_1}_{\omega_k} t - \theta_k)\,dt = \frac{M_k^{\,2}}{2}$$

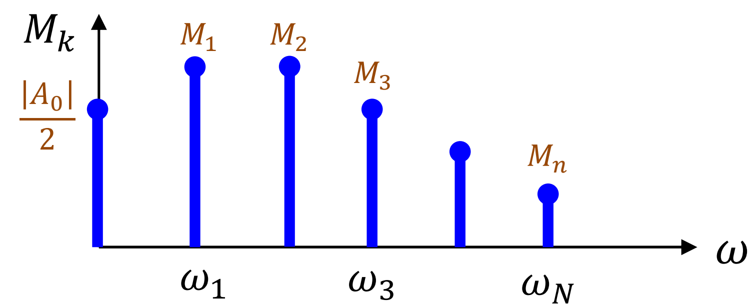
the portion of the signal power associated with frequency component w_k

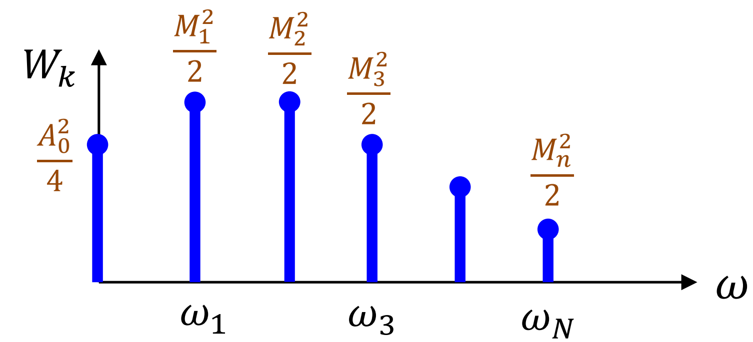
– **Power Spectrum**:

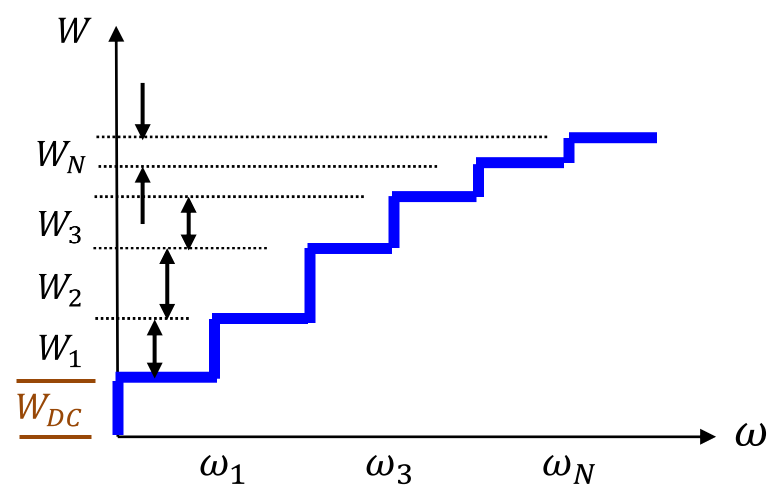
plot of W_k vs ω_k (*similar to magnitude spectrum*).

– **Cumulative Power** for the first N harmonics is:

$$W = W_{DC} + \sum_{k=1}^{N} W_k = \frac{1}{4}A_0^2 + \frac{1}{2}\sum_{k=1}^{N} M_k^2$$







Noise Characterization

- **Cumulative Power Spectrum**
  - In the limiting case when $T_O \to \infty$, $\omega_1 \to 0$, W becomes a continuous function of frequency ω :

$$W(\omega) = \lim_{T_0 \to \infty} W$$

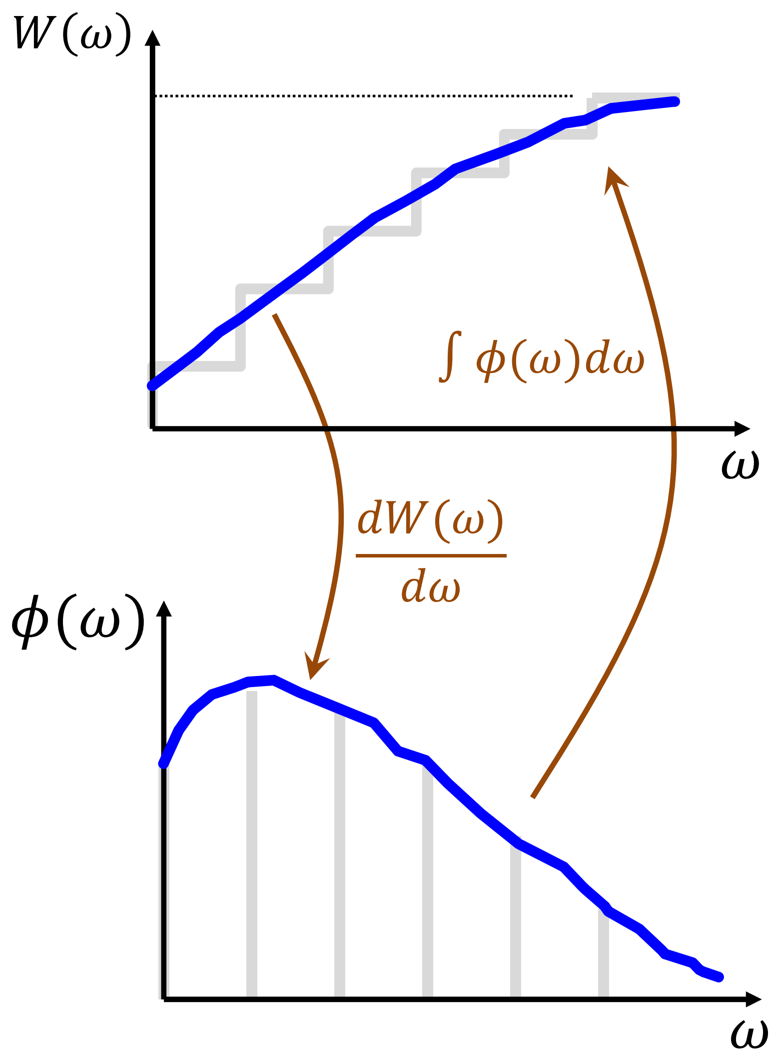
- **Power Spectral Density** $(\phi(\omega))$

The derivative of the cumulative power $W(\omega)$:

$$\phi(\omega) = \frac{d}{d\omega} W(\omega) \qquad [\text{Watt}/(\text{rad}/\text{sec})]$$

or

$$\phi(f) = \frac{d}{df} W(f) \qquad [\text{Watt}/\text{Hz}]$$



Noise Characterization

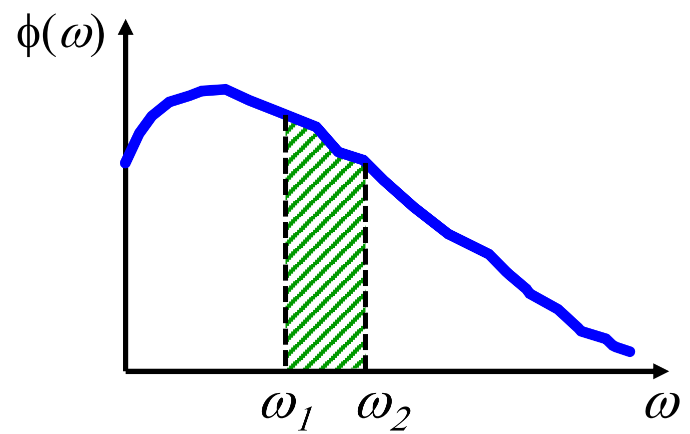
- **Power Spectral Density** ($\phi(\omega)$)
  - Signal power is a stationary quantity that can be used to quantify random signals.
  - Power Spectral Density, $\phi(\omega)$, is a quantity that is a measure of the power distribution of a random signal among all possible frequencies.

The power of a signal due to frequency components between ω_1 and ω_2 is:

$$W_{\omega_1,\omega_2} = \int_{\omega_1}^{\omega_2} \phi(\omega) \cdot d\omega$$

The total power of a signal is the area under the $\phi(\omega)$ curve*:*

$$W_{\text{TOT}} = \int_0^{\infty} \phi(\omega) \cdot d\omega$$



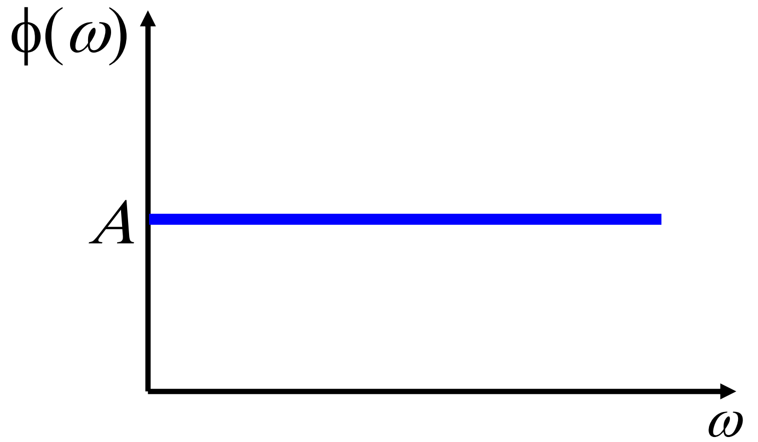
Noise Characterization

Special Random Signals

- **White Noise**:

  Uniform power spectral density

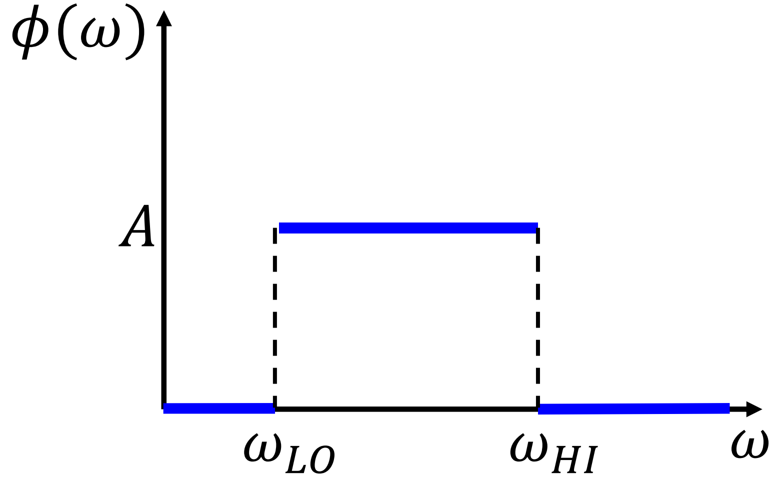
$$\phi(\omega) = \underbrace{A}_{\text{constant}} \qquad 0 \le \omega \le \infty$$



- **Band Limited Noise**:

  Uniform power spectral density across a limited frequency band

$$\phi(\omega) = \begin{cases} \underbrace{A}_{\text{constant}} & \omega_{LO} \le \omega \le \omega_{HI} \\ 0 & \omega \le \omega_{LO} \quad \text{or} \quad \omega \ge \omega_{HI} \end{cases}$$



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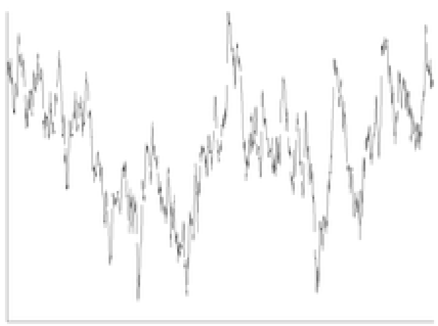
Noise Sources

- **Internal Electronics Noises**
  - Thermal (or Johnson) noise generated by the random thermal motion of charge carriers (usually electrons), inside an electrical conductor
  - Shot noise resulting from random statistical fluctuations of the electric current when the charge carriers (such as electrons) traverse a gap; The current is a flow of discrete charges, and the fluctuation in the arrivals of those charges creates shot noise
  - Flicker noise, also known as 1/f noise, with a frequency spectrum that falls off steadily into the higher frequencies, with a pink spectrum.
- **External Noises**
  - AC Interference
    - 60/120/180/240 Hz sinusoidal interference due to power lines,…
  - Communication Interference
    - Radio/TV (1-100MHz), cordless phone, cellular communication,…
  - Switching Interference
    - Switching power supplies, relays, lighting, arc welders, …
  - Mechanical/Structural Vibration
    - Normally under 20 Hz



Analog display of random fluctuations in voltage in pink noise
https://en.wikipedia.org/wiki/Noise_(electronics)

Noise Sources

- **Thermal (or Johnson) Noise**
  - Johnson noise is a type of *white noise* with uniform power spectral density
    - For current signal:

$$\phi_{i_J}(f) = \frac{4kT}{R} \qquad [\mathrm{A}^2/\mathrm{Hz}]$$

    - For voltage signal:

$$\phi_J(f) = 4kRT \qquad [\mathrm{volts}^2/\mathrm{Hz}]$$

where $k = 1.38 \times 10^{-23} \;\; [J/{}^\circ K]$ - Boltzmann's constant

$R =$ Resistance in the device $[\Omega]$

$T =$ Absolute temperature of the device $[{}^\circ K]$

  - Thermal noise amplitude roughly obeys a Gaussian distribution.
  - Can only be reduced by reducing temperature and measurement bandwidth.

Noise Sources

Ex: For current flowing through a $1\ \mathrm{M\Omega}$ resistor with a temperature of $37°\mathrm{C}$, what RMS voltage results from Johnson noise between the frequencies of 10 Hz and 10 kHz?

$$\phi_J(f) = 4kRT = 4 \cdot 1.28 \times 10^{-23} \cdot 10^6 \cdot (37 + 273) \ = 1.58 \times 10^{-14} \left[\frac{\text{volts}^2}{\text{Hz}}\right]$$

$$W_J = \overline{n^2(t)} \ = \int_{f_{LO}}^{f_{HI}} \phi_J(f) df \ = \phi_J(f) \int_{f_{LO}}^{f_{HI}} df$$

$$= \phi_J(f) \cdot \Delta f \quad = 1.58 \times 10^{-14} \left[\frac{\text{volts}^2}{\text{Hz}}\right] \cdot (10^4 - 10)[\text{Hz}]$$

$$= 1.58 \times 10^{-10} [\text{V}^2]$$

$$\Rightarrow$$

$$n(t)_{\text{rms}}\Big|_{(10-10^4\text{Hz})} = \sqrt{\overline{n^2(t)}} \ = 12.6\ \mu\text{V}$$

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Noise Sources

• Shot Noise

– Shot noise is a *white noise* with a uniform power spectral density:

- For current measurement:

$$\phi_{S_I}(f) = 2I_n q \qquad [\text{amperes}^2/\text{Hz}]$$

- For voltage measurement:

$$\phi_{S_V}(f) = 2I_n q R^2 \qquad [\text{volts}^2/\text{Hz}]$$

where

q = 1.59 × 10^{-19} [C] – electron charge

R : resistance in the device [Ω]

I_n : nominal current flowing in the device [A]

Q: What is the RMS voltage due to Shot noise?

$$V_{RMS} = \sqrt{\int_0^{BW} \phi(f) \cdot df} = \sqrt{2I_n q R^2 \cdot \Delta f}$$

Δf is the frequency bandwidth of interest

Noise Sources

• Flicker Noise

Low-frequency noise, including random drift, occurs in near all electronic devices, and can show up with a variety of other effects, such as impurities in a conductive channel, generation and recombination noise in a transistor due to base current, and so on

– Power spectral density:

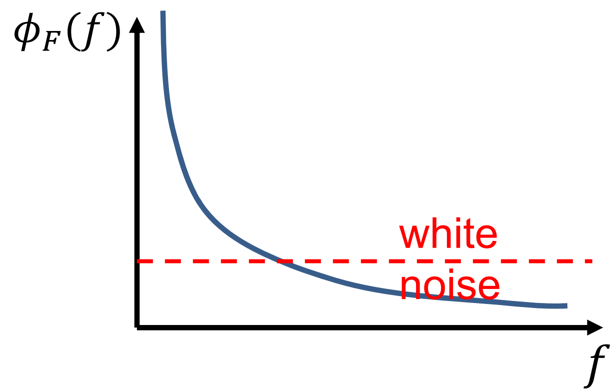
$$\phi_F(f) = \frac{C}{f^n} \quad \propto \quad \frac{1}{f} \qquad [\text{volts}^2/\text{Hz}]$$

where

C : material dependent constant

f : frequency [Hz]

$n \approx 1$



– Usually important at low frequencies, $f < 1000$ Hz

– At higher frequencies, flicker noise is overshadowed by white noise

– Avoid DC measurement when very small signals are to be measured; One solution is “chopping” DC signal into a AC signal

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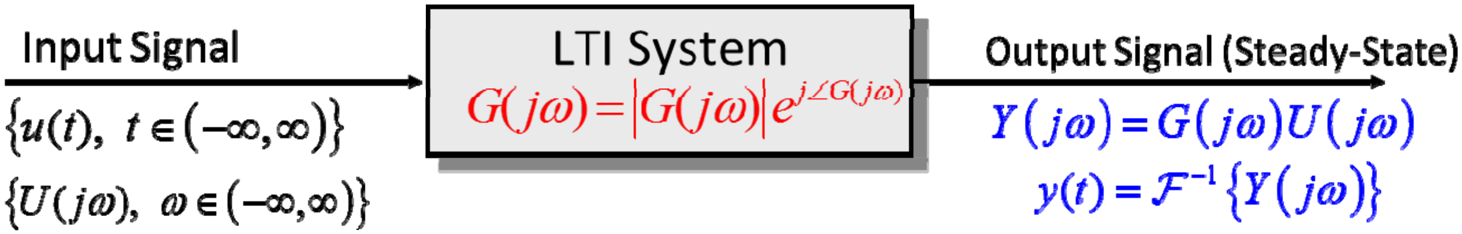
Noise Propagation

- **Noise measured at the output**:

When a device, such as an amplifier, amplifies/filters the input signal, it also amplifies/filters the noises



Let

$\phi_{N\text{-}in}(f)$ be the PSD of the input noise component with frequency unit of Hz

$\phi_{N\text{-}out}(f)$ be the PSD of the output noise component with freq. unit of Hz

then

$$\phi_{N-out}(f)=|G(j2\pi f)|^2\,\phi_{N-in}(f)$$

and

$$W_{N-\mathrm{TOT}}=\int_0^\infty \phi_{N-out}(f)\cdot df$$

Note:

$$\omega=2\pi f$$

$\forall x(t)$:

$$\underbrace{\int_{-\infty}^{\infty}|x(t)|^2dt}_{\text{energy in time-domain}}=\underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|^2d\omega}_{\text{energy in frequency-domain}}$$

$$=\underbrace{\int_{-\infty}^{\infty}|X(j2\pi f)|^2df}_{\text{energy in frequency-domain}}$$

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Example

A voltage amplifier uses transistors in its circuitry, and has a total resistance of 10 KΩ, with nominal current of 30 mA, at a temperature of 40ºC. Its frequency response function can be approximated as an ideal low-pass filter with gain of 10 and cutoff frequency of 1000 Hz.

$$\Rightarrow \quad |G(j2\pi f)| = \begin{cases} 10 & \text{for } 0 \le f \le 1000 \\ 0 & \text{otherwise} \end{cases}$$

a) Determine total noise power at the amplifier output, considering both Johnson and Shot noise.

$$R = 10^4\ [\Omega], \quad I = 30\times10^{-3}\ [A], \quad T = 40°C \ = (273+40) = 313\ [°K]$$

Johnson noise

$$\phi_J(f) = 4kRT \ = 4\cdot 1.38\times10^{-23}\cdot 10^4 \cdot 313 \ = 1.73\times10^{-16}\ [\text{V}^2/\text{Hz}]$$

Shot noise

$$\phi_S(f) = 2I_n qR^2 \ = 2\cdot 30\times10^{-3}\cdot 1.59\times10^{-19}\cdot (10^4)^2 \ = 9.54\times10^{-13}\ [\text{V}^2/\text{Hz}]$$

$$\Rightarrow$$

$$\phi_{total} = \phi_J(f) + \phi_S(f) \ = 0.00173\times10^{-13} + 9.54\times10^{-13} \ \approx 9.54\times10^{-13}\ [\text{V}^2/\text{Hz}]$$

$$W_{N-\text{TOT}} = \int_0^\infty |G(j2\pi f)|^2 \underbrace{\phi_{N-in}(f)}_{\phi_{total}}\cdot df \quad = \int_0^{1000} 10^2 \cdot 9.54\times10^{-13}\cdot df$$

$$= 9.54\times10^{-8}\ [V^2] \quad = \overline{n^2(t)}$$

Example

a) Assuming that the measured input signal is a pure sinusoid having amplitude of 0.33 V and frequency of 10 Hz, calculate the SNR of the output signal (in dB).

input signal: $x(t) = 0.33 \cdot \sin(2\pi \cdot 10 \cdot t)$

$$\Rightarrow \text{ output signal: } y(t) = \left|T(j2\pi \cdot 10)\right| \cdot 0.33 \cdot \sin(2\pi \cdot 10 \cdot t + \phi)$$

$$= 10 \cdot 0.33 \cdot \sin(2\pi \cdot 10 \cdot t + \phi)$$

Recall that signal power of a sine wave, $A\sin(\omega t + \phi)$, is $\dfrac{A^2}{2}$

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\overline{y^2(t)}}{\overline{n^2(t)}} = \frac{\dfrac{(10 \cdot 0.33)^2}{2}}{\overline{n^2(t)}} = \frac{5.445}{9.54 \times 10^{-8}} = 0.571 \times 10^8$$

$$SNR\big|_{dB} = 10 \cdot \log_{10}(SNR) = 10 \cdot \log_{10}(0.571 \times 10^8) = 77.6\ dB$$

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Noise Spectrum

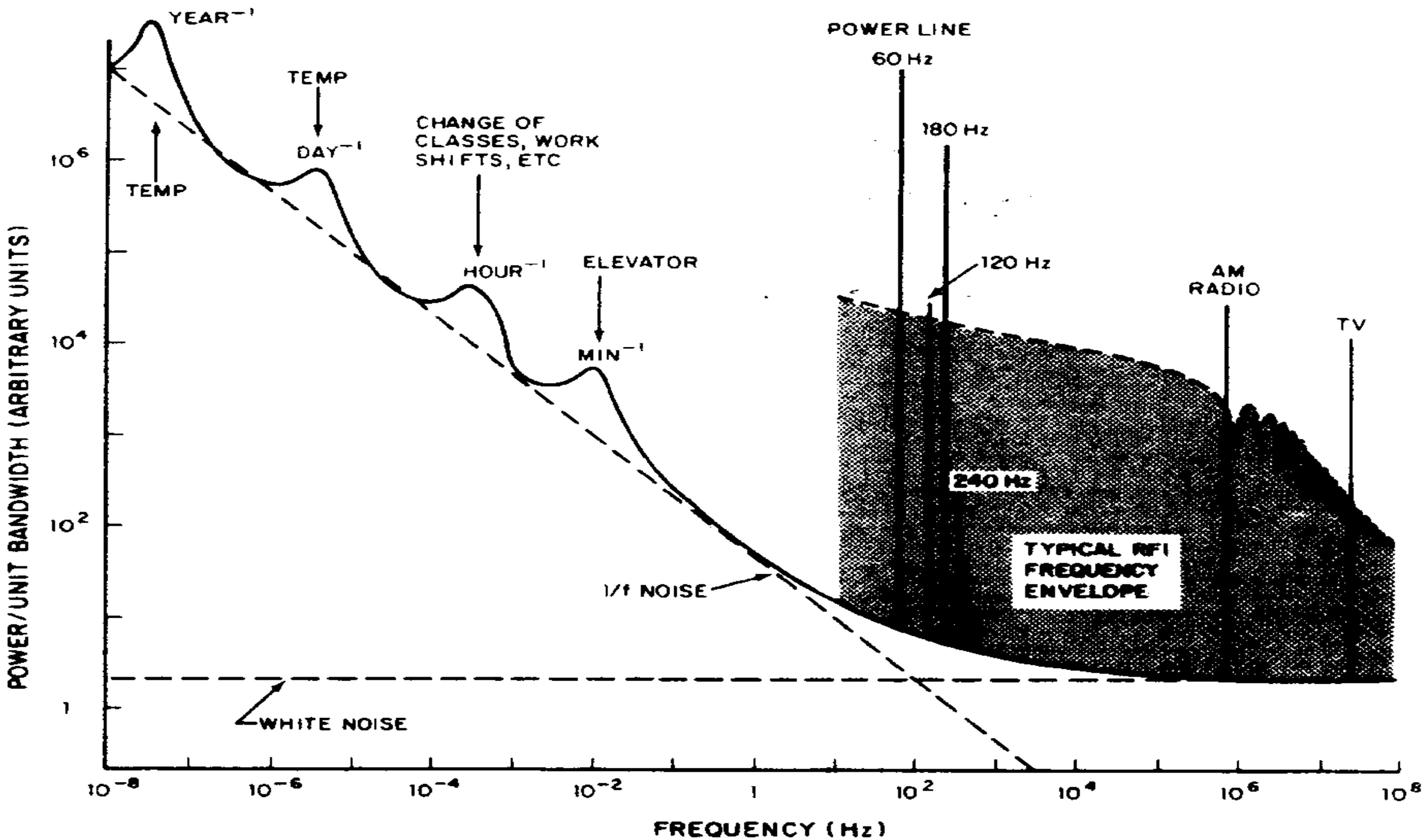

Figure 6: The Spectrum of Common Environmental Noise
(From Vol. 1 - Handbook of Measurement Science,
P. Sydenham, ed., John Wiley and Sons, 1982.)