Hydraulic (Fluid) Systems

- Basic Modeling Elements
  - Resistance
  - Capacitance
  - Inertance
  - Pressure and Flow Sources
- Interconnection Relationships
  - Compatibility Law
  - Continuity Law
- Derive Input/Output Models

Variables

- q : volumetric flow rate **[m³/sec]** (current)
- V: volume **[m³]** (charge)
- p : pressure **[N/m²]** (voltage)

The analogy between the hydraulic system and the electrical system will be used often. Just as in electrical systems, the flow rate (current) is defined to be the time rate of change (derivative) of volume (charge):

$$q = \frac{d}{dt}V = \dot{V}$$

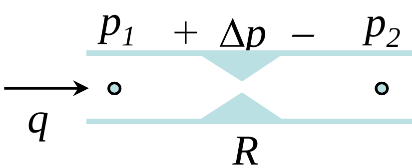
The pressure, p, used in this chapter is the *absolute pressure*. You need to be careful in determining whether the pressure is the absolute pressure or *gauge pressure*, p^*. Gauge pressure is the difference between the absolute pressure and the atmospheric pressure, i.e.

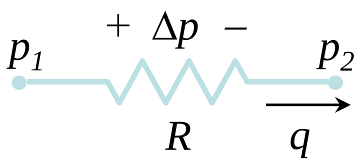
$$p^* = p - p_{atmospheric}$$

Basic Modeling Elements

- **Fluid Resistance**

  Describes any physical element with the characteristic that the pressure drop, Δp , across the element is proportional to the flow rate, q.





$$\Delta p = p_1 - p_2 = p_{12} = R \cdot q$$

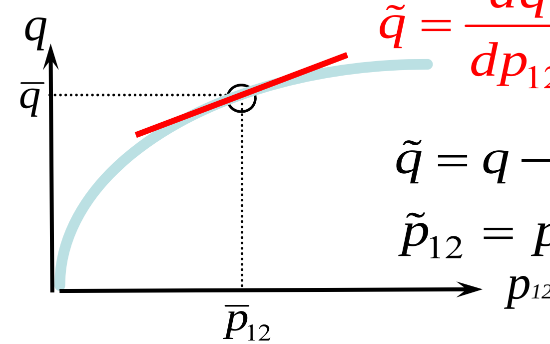
$$q = \frac{1}{R}\Delta p = \frac{1}{R} p_{12}$$

  – Orifices, valves, nozzles and friction in pipes can be modeled as fluid resistors.

Ex: The flow that goes through an orifice or a valve and the turbulent flow that goes through a pipe is related to the pressure drop by

$$q = k\sqrt{p_{12}}$$

Find the effective flow resistance of the element at certain operating point $(\bar{q}, \bar{p}_{12})$.



$$\tilde{q} = \frac{dq}{dp_{12}}\tilde{p}_{12}$$

$$\tilde{q} = q - \bar{q},$$

$$\tilde{p}_{12} = p_{12} - \bar{p}_{12}$$

$$\frac{1}{R} = \left.\frac{dq}{dp_{12}}\right|_{(\bar{q},\bar{p}_{12})} = \frac{k}{2\sqrt{\bar{p}_{12}}}$$

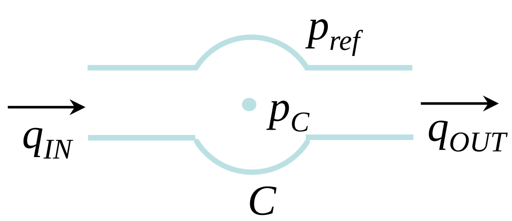
$$R = \frac{2\sqrt{\bar{p}_{12}}}{k} = \frac{2\bar{q}}{k^2}$$

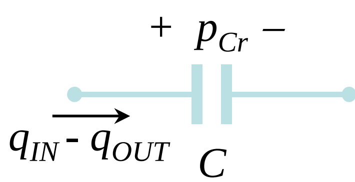
PURDUE MECHANICAL ENGINEERING

Basic Modeling Elements

- **Fluid Capacitance**

  Describes any physical element with the characteristic that the rate of change in pressure, p, in the element is proportional to the difference between the input flow rate, q_{IN}, and the output flow rate, q_{OUT}.

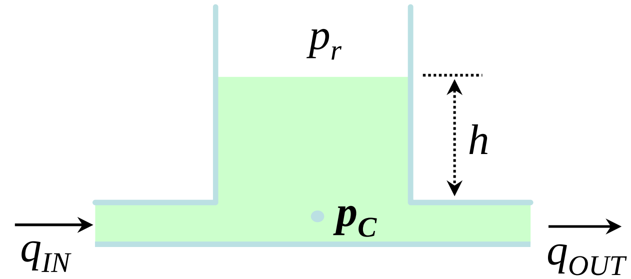




$$C\frac{d}{dt}\underbrace{\left(p_C - p_{ref}\right)}_{p_{Cr}} = C \cdot \dot{p}_{Cr} = q_{IN} - q_{OUT}$$

  – Hydraulic cylinder chambers, tanks, and accumulators are examples of fluid capacitors.

Ex: Consider an open tank with a constant cross-sectional area, A:



$p_C = p_r + \rho g h \Rightarrow p_{Cr} = \rho g h$

$q_{IN} - q_{OUT} = \frac{dV}{dt} = \frac{d(Ah)}{dt} = A\frac{dh}{dt}$

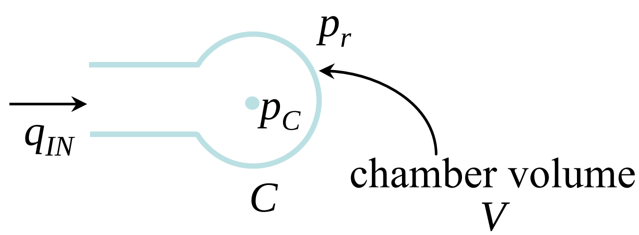
$\dot{p}_{Cr} = \rho g \dot{h}$ $\Rightarrow$ $q_{in} - q_{out} = \underbrace{A\frac{1}{\rho g}}_{C}\dot{p}_{Cr}$

$\Rightarrow$

$C = \frac{A}{\rho g}$

Fluid Capacitance Examples

Ex: Calculate the equivalent fluid capacitance for a hydraulic chamber with only an inlet port.



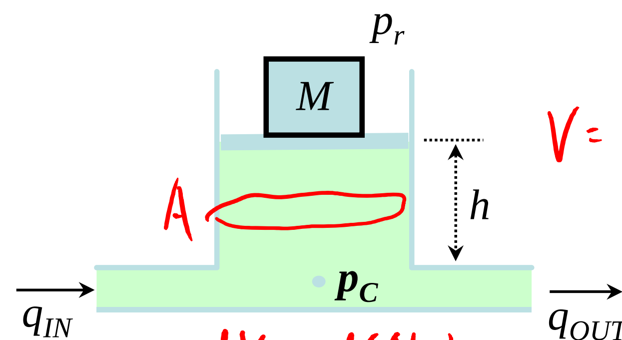
Recall the bulk modulus (β) of a fluid is defined by:

$$\beta = V\frac{dp_{Cr}}{dV} = V\frac{\left(dp_{Cr}/dt\right)}{\left(dV/dt\right)} = V\frac{\dot{p}_{Cr}}{q_{in}}$$

$$\Rightarrow$$

$$q_{IN} = \underbrace{\left(\frac{V}{\beta}\right)}_{C}\frac{d}{dt}p_{Cr}$$

Ex: Will the effective capacitance change if in the previous open tank example, a load mass M is floating on top of the tank?



$$V = Ah$$

$$q_{in} - q_{out} = \frac{dV}{dt} = \frac{d(Ah)}{dt} = A\dot{h}$$

$$P_c = P_r + \frac{Mg}{A} + \rho g h \Rightarrow P_{cr} = \frac{Mg}{A} + \rho g h$$

$$\Rightarrow \dot{p}_{cr} = \frac{d}{dt}\left(\frac{Mg}{A} + \rho g h\right) = \rho g \dot{h}$$

$$\Rightarrow q_{in} - q_{out} = \underbrace{A\frac{1}{\rho g}}_{C}\dot{p}_{cr}$$

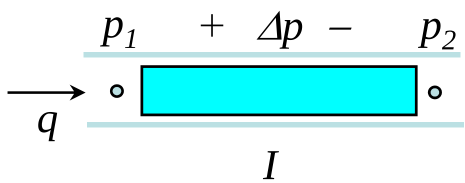
$$\Rightarrow \quad C = \frac{q_{IN} - q_{OUT}}{\dot{p}_{Cr}} = \frac{A}{\rho g}$$

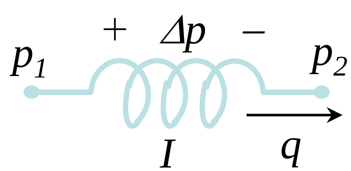
PURDUE MECHANICAL ENGINEERING

Basic Modeling Elements

- **Fluid Inertance (Inductance)**

Describes any physical element with the characteristic that the pressure drop, Δp , across the element is proportional to the rate of change (derivative) of the flow rate, q.



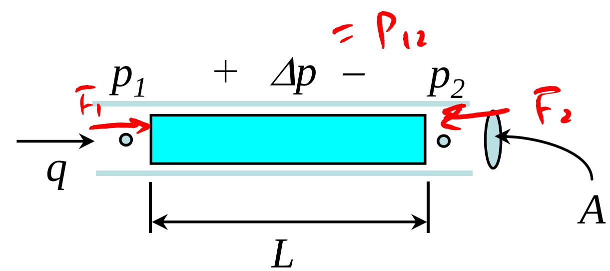


$$\Delta p = p_{12} = (p_1 - p_2) = I \frac{d}{dt} q = I \cdot \dot{q}$$

– Long pipes are examples of fluid inertance.

Q: What will happen if you suddenly shut off one end of a long tube? --- (Water Hammer effect)

Ex: *Consider a section of pipe with cross-sectional area A and length L, filled with fluid whose density is* ρ :



$$F = F_1 - F_2 = A P_1 - A P_2 = A P_{12}$$

$$m = \rho A L, \quad q = A v$$

$$F = m a = m \frac{dv}{dt} = m \frac{d}{dt}\left(\frac{q}{A}\right)$$

$$\Rightarrow A P_{12} = \rho A L \times \frac{1}{A} \frac{dq}{dt}$$

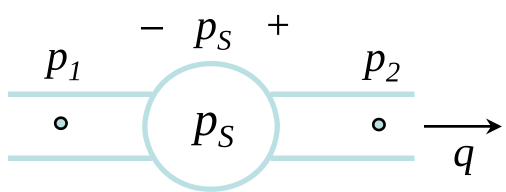
$$\Rightarrow P_{12} = \underbrace{\frac{\rho L}{A}}_{I} \dot{q}$$

$$\Rightarrow \quad I = \frac{\rho L}{A}$$

Basic Modeling Elements

- **Pressure Source (Pump)**
  - An ideal pressure source of a hydraulic system is capable of maintaining the desired pressure, regardless of the flow required for what it is driving.

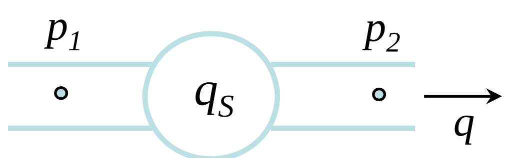
Voltage Source



$$p_{21} = p_2 - p_1 = p_S$$

- **Flow Source (Pump)**
  - An ideal flow source is capable of delivering the desired flow rate, regardless of the pressure required to drive the load.

Current Source

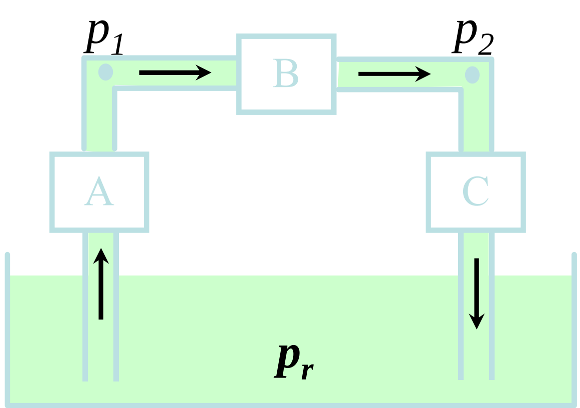


$$q = q_S$$

Interconnection Laws

- **Compatibility Law**
  - The sum of the pressure drops around a loop must be zero.
  - Similar to the Kirchhoff's voltage law.

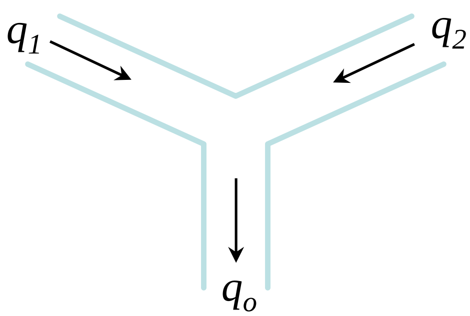
$$\sum_{\substack{\text{Closed}\\\text{Loop}}} \Delta p_j = \sum_{\substack{\text{Closed}\\\text{Loop}}} p_{ij} = 0$$



- **Continuity Law**
  - The algebraic sum of the flow rates at any junction in the loop is zero.
  - This is the consequence of the conservation of mass.
  - Similar to the Kirchhoff's current law.

$$\sum_{\substack{\text{Any}\\\text{Node}}} q_j = 0$$

$$\text{or} \quad \sum q_{IN} = \sum q_{OUT}$$



PURDUE MECHANICAL ENGINEERING

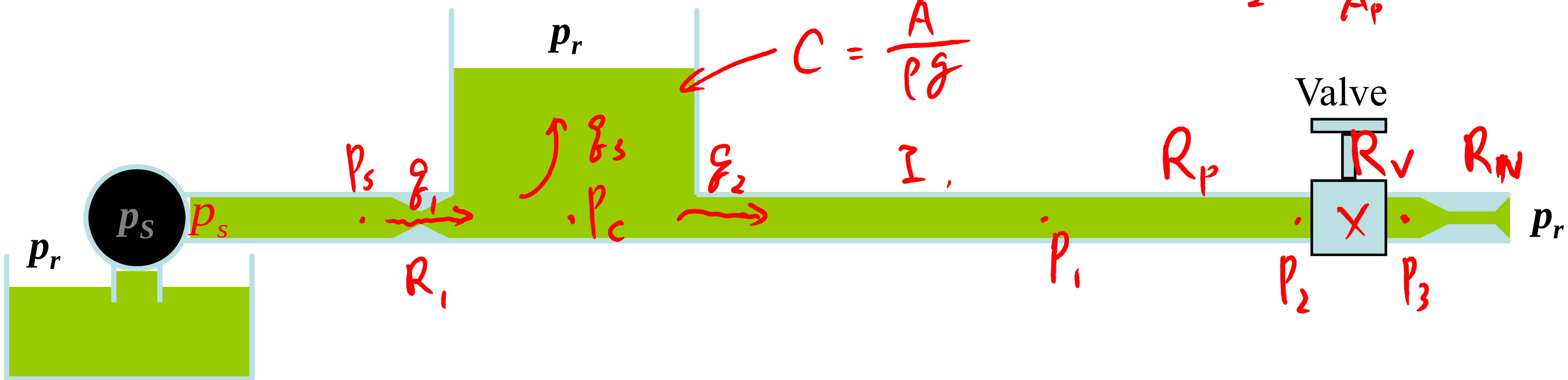
Modeling Steps

- Understand System Function and Identify Input/Output Variables
- Draw Simplified Schematics Using Basic Elements
- Develop Mathematical Model
  - Label Each Element and the Corresponding Pressures.
  - Label Each Node and the Corresponding Flow Rates.
  - Write Down the Element Equations for Each Element.
  - Apply Interconnection Laws.
  - Check to make sure that the Number of Unknown Variables equals the Number of Equations.
  - Eliminate Intermediate Variables to Obtain Standard Forms:
    - Laplace Transform
    - Block Diagrams

In Class Exercise

Derive the input/output model for the following fluid system. The pump supplies a constant pressure p_S to the system and we are interested in finding out the volumetric flow rate through the nozzle at the end of the pipe.

$$I = \frac{\rho L}{A_p}$$



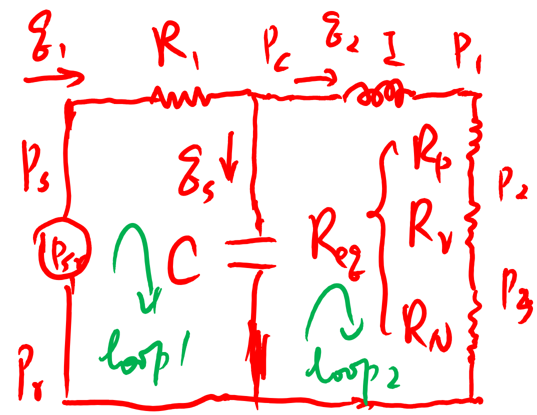
- *Label the pressures at nodes and flow rates*
- *Write down element equations:*

$$p_s - p_c = R_1 q_1 \quad (1)$$

$$q_s = C\frac{d}{dt}(p_c - p_r) = C\dot{p}_c \quad (2)$$

$$p_c - p_1 = I\dot{q}_2 \quad (3)$$

$$p_1 - p_r = R_{eq} q_2 \quad (4)$$



$$R_{eq} = R_p + R_v + R_N$$

PURDUE MECHANICAL ENGINEERING

In Class Exercise

- *Interconnection laws:*

loop 1: $P_s - P_r = P_s - P_c + P_c - P_r$

Node: $q_1 = q_s + q_2$ ⑤

loop 2: $P_c - P_r = P_c - P_1 + P_1 - P_r$

- *No. of unknowns and equations:*

P_c, q_1, q_s, P_1, q_2 5 UVs = 5 eqs. ① to ⑤

- *Eliminate intermediate variables and obtain I/O model:*

② → ① → ⑤ : $\frac{P_s - P_c}{R_1} = C\dot{P}_c + q_2 \overset{③}{\Longrightarrow} P_s - (P_1 + I\dot{q}_2) = R_1 C(\dot{P}_1 + I\ddot{q}_2) + R_1 q_2$

$\overset{④}{\Longrightarrow} P_s - (P_r + R_{eq} q_2) - I\dot{q}_2 = R_1 C(0 + R_{eq}\dot{q}_2) + R_1 C I \ddot{q}_2 + R_1 q_2$

$\Rightarrow R_1 C I \ddot{q}_2 + (I + R_1 R_{eq} C)\dot{q}_2 + (R_1 + R_{eq}) q_2 = P_s - P_r = P_{sr}$

or

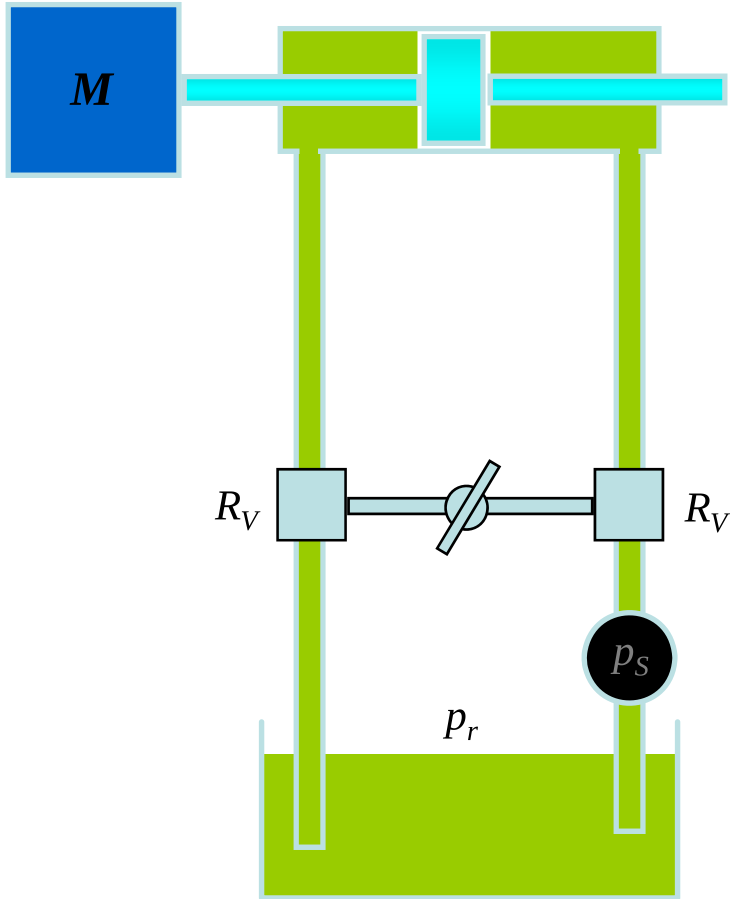
$$\ddot{q}_2 + \frac{I + R_1 R_{eq} C}{R_1 C}\dot{q}_2 + \frac{R_1 + R_{eq}}{R_1 C I} q_2 = \frac{1}{R_1 C I} P_{sr}$$

Q: Can you draw an equivalent electrical circuit of this hydraulic system ?

PURDUE MECHANICAL ENGINEERING

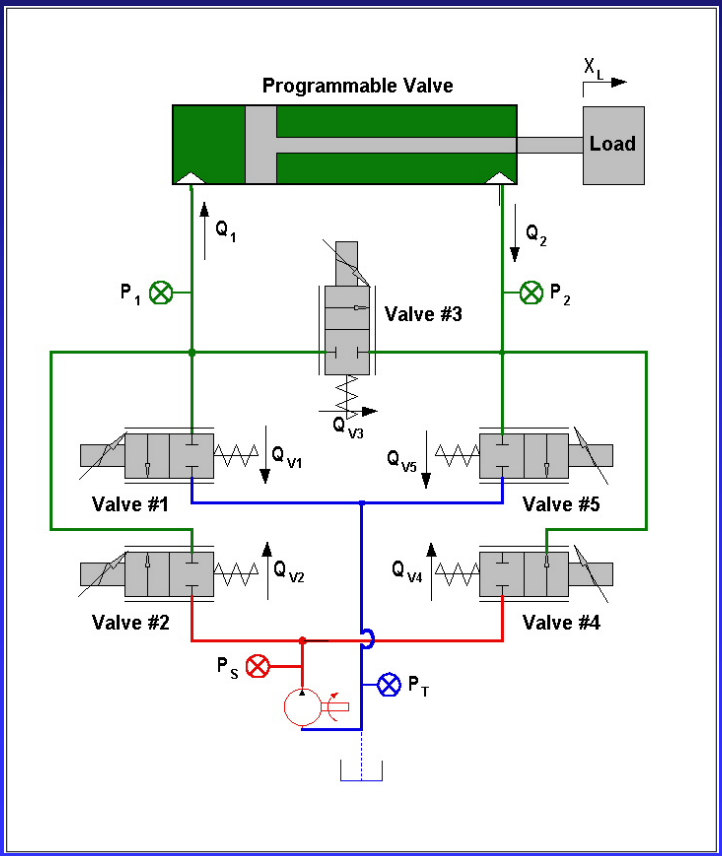
Motion Control of Hydraulic Cylinders

Hydraulic actuation is attractive for applications when large power is needed while maintaining a reasonable weight. Not counting the weight of the pump and reservoir, hydraulic actuation has the edge in power-to-weight ratio compared with other cost effective actuation sources. Earth moving applications (wheel loaders, excavators, mining equipment, ...) are typical examples where hydraulic actuators are used extensively. A typical motion application involves a hydraulic cylinder connected to certain mechanical linkages (inertia load). The motion of the cylinder is regulated via a valve that is used to regulate the flow rate to the cylinder. It is well known that such system chatters during sudden stop and start. Can you analyze the cause and propose solutions?



Coordinated Motion Control of Hydraulic Arm

Automated Modeling and Energy Saving Adaptive Robust Control of Electro-Hydraulic Systems with Programmable Valves



Novel Programmable Valves

Energy-Saving Control Experiments

Motion Control of Hydraulic Cylinders

Let's look at a simplified problem:

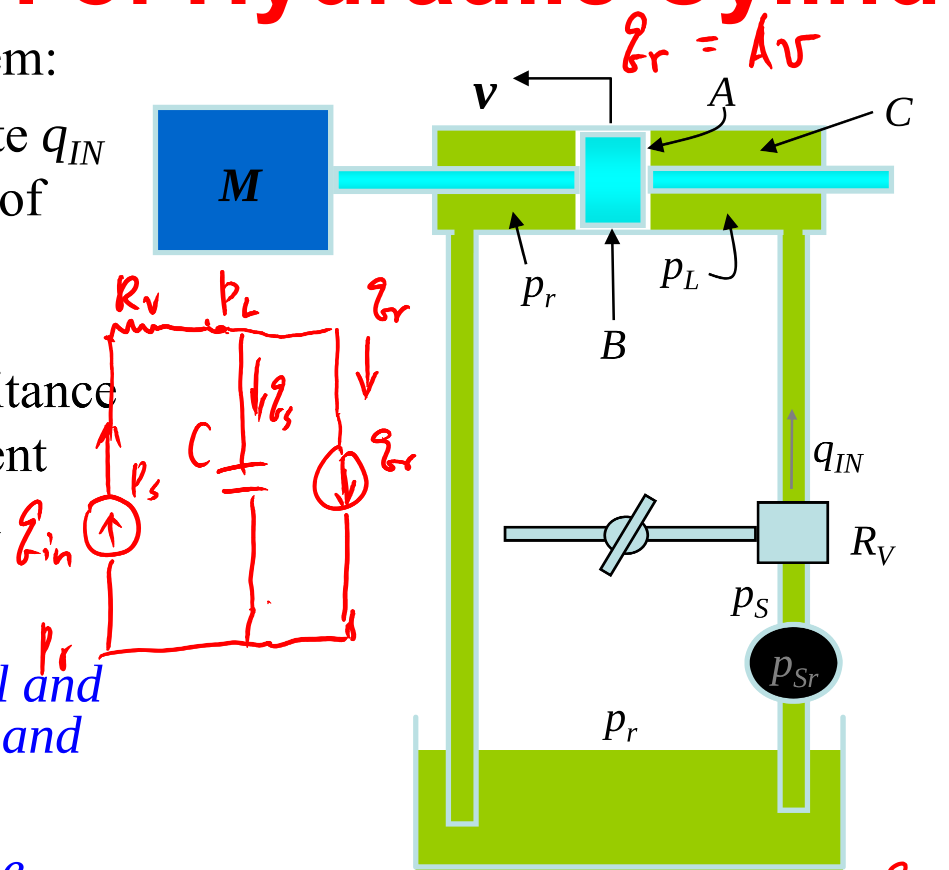
The input is the input flow rate q_{IN} and the output is the velocity of the mass, V.

A: Cylinder bore area

C: Cylinder chamber capacitance

B: Viscous friction coefficient between piston head and cylinder wall.

- *Derive the input/output model and transfer function between q_{IN} and V.*
- *Draw the block diagram of the system.*
- *Can this model explain the vibration problem noticed in practice when one suddenly closes the valve?*



$$q_s + q_r = q_{in}$$

$$q_s = C\frac{d}{dt}(p_L - p_r) = C\dot{p}_L$$

$$\Rightarrow C\dot{p}_L = q_{in} - q_r$$

Motion Control of Hydraulic Cylinders

Element equations and interconnection equations:

Hydraulic system	Hydraulic-Mechanical	Mechanical system
$q_s = C\frac{d}{dt}p_{Lr} = C\dot{p}_{Lr}$	$F = A(p_2 - p_r)$	$M\dot{V} = F - BV$
$q_{in} = q_s + q_r$	$= A p_{Lr}$	
	$q_r = AV$	

Take Laplace transforms:

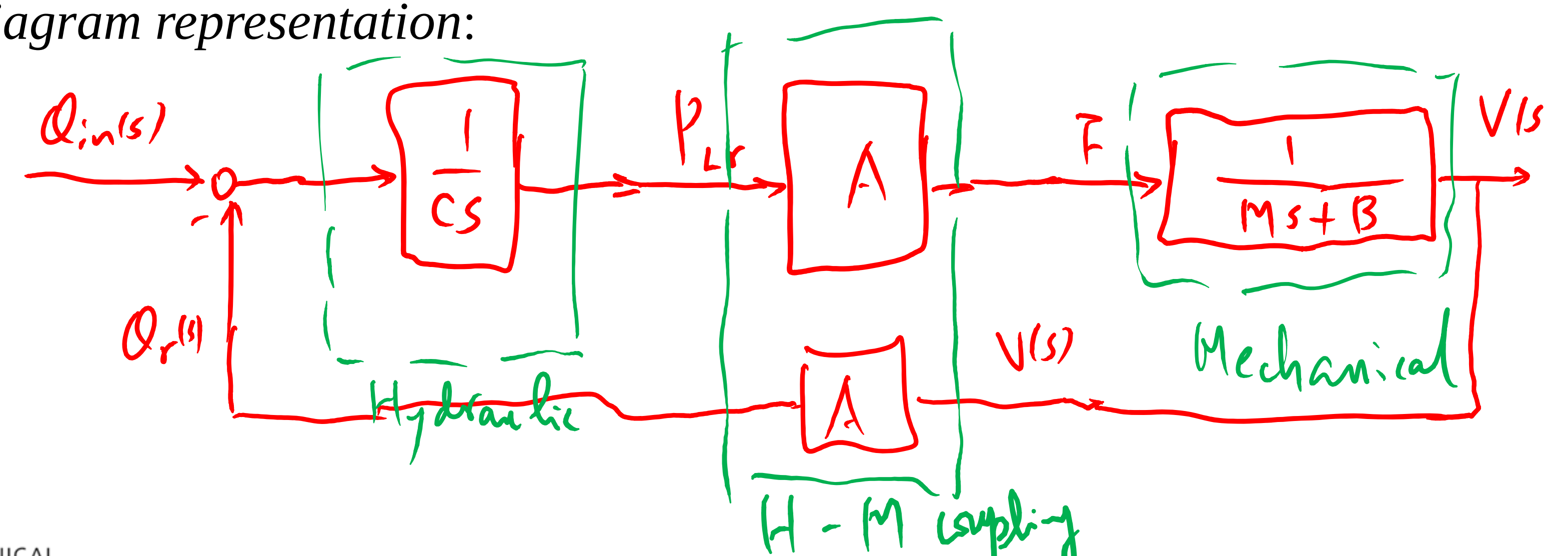
$$Cs\,P_{Lr}(s) = Q_{in}(s) - Q_r(s)$$

$$\Rightarrow P_{Lr}(s) = \frac{1}{Cs}\left[Q_{in}(s) - Q_r(s)\right]$$

$$(Ms + B)\,V(s) = F$$

$$\Rightarrow V(s) = \frac{1}{Ms + B}F$$

Block diagram representation:



PURDUE MECHANICAL ENGINEERING

Motion Control of Hydraulic Cylinders

Transfer function between q_{IN} and V:

$$G(s) = \frac{V(s)}{Q_{in}(s)} = \frac{A}{MCs^2 + \beta Cs + A^2}$$

$$= \frac{\frac{A}{MC}}{s^2 + \frac{\beta}{M}s + \frac{A^2}{MC}}, \quad \frac{A}{MC} = K\omega_n^2, \quad \frac{\beta}{M} = 2\zeta\omega_n, \quad \frac{A^2}{MC} = \omega_n^2$$

Natural Frequency

$$\omega_n = \sqrt{\frac{A^2}{MC}} = \frac{A}{\sqrt{MC}}$$

Damping Ratio

$$\zeta = \frac{\frac{B}{M}}{2\omega_n} = \frac{1}{2}\frac{\sqrt{C}}{\sqrt{M}}\frac{B}{A}$$

Steady State Gain

$$K = \frac{\frac{A}{MC}}{\omega_n^2} = \frac{1}{A}$$

How would the velocity response look like if one suddenly opens the valve to reach constant input flow rate Q for some time T and suddenly closes the valve to stop the flow?

In reality,

large M, Small C, Reasonable B

⇒ reasonal value for ω_n

Very small ζ