ECE 440 – Spring 2018
Homework 8

Due after office hours on Wednesday, April 24

1. A 16-QAM signal with symbol duration 1ms is spread using DSSS with a binary sequence with chip duration 10μs.
   (a) How does the bandwidth of the original signal compare with that of the spread signal?
   (b) Which of the following sequences do you think would be a better chip sequence, and why?
   [1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1...] (alternating 1 and -1)
   [1 1 1 1 1 -1 -1 -1 -1 -1 1 1 1 1 1 -1 -1 -1 -1 -1...] (alternating series of 1 and series of -1)
   [1 1 -1 1 -1 -1 1 1 1 -1 1 -1 -1 1 -1 1 1 1 -1 1 1...] (random 1 and -1 with slightly different probabilities)
   (c) We want to have multiple users communicating simultaneously on the same band using CDMA. What conditions do their chip sequences have to fulfil?

2. (Problem 10.40 in 7th edition) Consider a multipath channel with a delay spread (time difference between shortest and longest path) of 5 microseconds through which we want to transmit data at 500 kbps. Design an OFDM system with a symbol period at least a factor of 10 greater than the delay spread if the modulation to be used in each subcarrier is QPSK.

3. Compare the capacity of the following channels: 1) Two parallel binary symmetric channels with BER 0.1 and 0.2, respectively, 2) Two parallel binary symmetric channels, both with BER 0.15 and 3) one binary symmetric channel with BER 0.01.

4. An error correcting code is said to be perfect if it attains the Hamming bound. There are only two families of perfect codes: Hamming and Golay. The (23,12,7) binary Golay code has 2^{12} binary codewords of length 23 and a minimum distance of 7. What does that mean in terms of error correction capability? What is the rate of this code?

5. Imagine that an information source is transmitting a DNA sequence, consisting of symbols $\mathcal{X} = \{A, C, G, T\}$ distributed according to

$$\mathcal{X} = \begin{cases} \text{A} & \textbf{with probability } p_A = 0.5 \\ \text{C} & \textbf{with probability } p_C = 0.25 \\ \text{G} & \textbf{with probability } p_G = 0.125 \\ \text{T} & \textbf{with probability } p_T = 0.125 \end{cases}$$

   (a) Find the entropy of the source $H(\mathcal{X})$ in bits (average information per symbol). This gives an asymptotic limit for how much a sequence can be compressed.
   (b) Use either Huffman or Lempel-Ziv source coding to compress the following sequence: AAACT-GAACATGAACTG. Make sure to explain your answer clearly.

(c) Assume that you have compressed the DNA sequence into a string of 500 bits. You now wish to transmit them through a channel in such a way that the receiver can tolerate a single burst of 100 consecutive bits erased. How would you do it? You know that 100 consecutive bits in your transmission will be lost, but neither the transmitter nor the receiver know when it will happen.