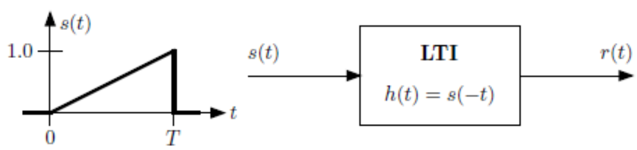
ECE 440 – Spring 2019
Homework 5

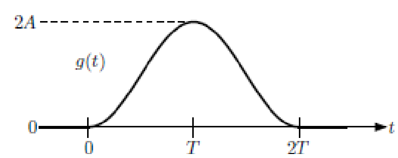
Due before class on Wednesday 03/20

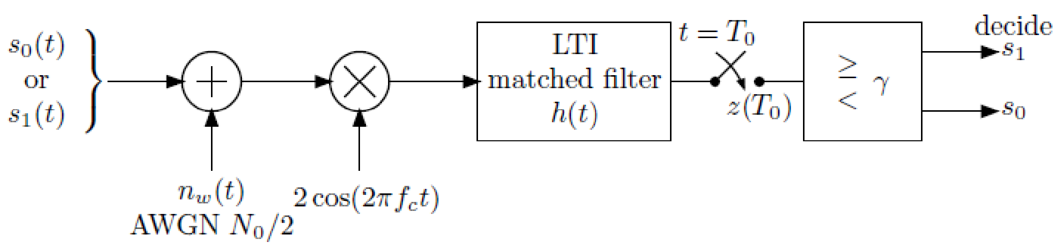
1. (Spring 2012 Final Exam) Consider the LTI system shown below where the impulse response is "matched" to the input as indicated. Find: (a) The range of t for which the output r(t) is nonzero and (b) The maximum value of the output r(t) and the time $t^\star$ where the maximum occurs.



2. (Spring 2012 Final Exam) The purpose of this problem is to step through the development of the optimal receiver for ASK with signals $s_0(t) = 0$ and $s_1(t) = g(t)\cos(2\pi f_c t)$, where $g(t)$ is the time-domain raised cosine shaped pulse:

$$g(t) = \begin{cases} A(1+\cos(\pi(t-T)/T) & 0 \le t \le 2T \\ 0 & \text{else} \end{cases} .$$





Assume that $f_c >> 1/T$ (which will suggest a certain approximation simplifying the results below).

(a) For the receiver shown above and the assumed signals $s_0(t)$ and $s_1(t)$ choose the impulse response $h(t)$ of the matched filter and specify the sampling time T_0. Note that the downconversion via multiplication by $2\cos(2\pi f_c t)$ occurs before the matched filter in this architecture.

(b) Assuming that the transmitted signal is actually $s_1(t)$ find:(i) The message-related part of $z(T_0)$, (ii) The noise-related part of $z(T_0)$ (Specify its distribution and its mean and variance) and (iii) What is the distribution of the random variable $z(T_0)$ conditioned on $s_1(t)$ being transmitted?

(c) Repeat the previous part assuming that $s_0(t)$ is transmitted. You can do this by inspection given your derivation from (b).

(d) Assuming that the prior probabilities of $s_0(t)$ and $s_1(t)$ are 1/2 choose the threshold for minimum average probability of error.

(e) Find the minimum probability of error.

3. Problem 9.10: Assume that the probabilities of sending the signals $s_0(t)$ and $s_1(t)$ are not equal, but are given by p and q=1-p, respectively. Derive an expression for the probability of error P_E that replaces $P_E = Q\left(\frac{s_{02}(T)-s_{01}(T)}{2\sigma_0}\right)$ that takes this into account. Show that the probability of error is minimized by choosing the threshold to be

$$k_{\text{opt}} = \frac{\sigma_0^2}{s_{01}(T) - s_{02}(T)} \ln(p/q) + \frac{s_{01}(T) + s_{02}(T)}{2}.$$

4. Problem 9.32 below. Doing numbers 0 through 16 is enough, no need to reach 32.

9.32 Gray encoding of decimal numbers ensures that only one bit changes when the decimal number changes by one unit. Let $b_1 b_2 b_3 \ldots b_n$ represent an ordinary binary representation of a decimal number, with b_1 being the most significant bit. Let the corresponding Gray code bits be $g_1 g_2 g_3 \ldots g_n$. Then the Gray code representation is obtained by the algorithm

$$g_1 = b_1$$
$$g_n = b_n \oplus b_{n-1}$$

where $\oplus$ denotes modulo-2 addition (i.e., $0 \oplus 0 = 0$, $0 \oplus 1 = 1, 1 \oplus 0 = 1$, and $1 \oplus 1 = 0$). Find the Gray code representation for the decimal numbers 0 through 32.