# ECE 440 – Spring 2016
# Final Exam

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- Please fill in your name on the dotted line above.
- The exam is closed book and closed notes. You can have a simple scientific calculator without communication capabilities, but you should not need one. You are free to leave your answers in terms of any expression that could be computed with a scientific calculator (e.g., $\sin(\pi/5)$).
- There is a formula sheet at the back of the exam. Feel free to tear it off.
- You should be able to answer all the questions in the space provided, but you can use the last (blank) pages of the exam if you need additional space or scratch paper. However, make sure to include all calculations and explanations with your answer. If you need additional scratch paper, feel free to ask for it.
- You have two hours to complete the exam. That should give you plenty of time to justify your answers, please do so.

| | |
|---|---|
| Q1 | /10 |
| Q2 | /10 |
| Q3 | /30 |
| Q4 | /20 |
| Q5 | /20 |
| Q6 | /10 |
| Total | /100 |

**Question 1 (10 points)** Determine whether the following are acceptable autocorrelation functions for a real signal. If that is not the case, specify why.

(a) $R_1(\tau) = 2\cos(10\pi\tau) + \cos(30\pi\tau)$

(b) $R_2(\tau) = 4\Lambda(\tau/2)$

(c) $R_3(\tau) = 2\sin(10\pi\tau)$

(d) $R_4(\tau) = e^{-\tau}(\tau + 1)$

**Question 2 (10 points)** Explain what the **lock range** and the **steady state error** of a Phase-Locked Loop (PLL) are.

**Question 3 (30 points)** A Digital Audio Broadcasting system (DAB) uses the band 174-240 MHz to transmit multiple radio stations using frequency division multiplexing[1] and a digital modulation within each station (subchannel). Assume that you want to transmit the message $m(t) = sinc(t)$ through one of the stations. **This question has many parts, but they can be solved independently. If you cannot solve one, move on to the next.**

(a) **(5 points)** Since we are using a digital modulation, we need to sample $m(t)$. What condition does the sampling period need to fulfil in order to avoid aliasing?

(b) **(2 points)** If the samples have infinite precision, can the analog signal $m(t)$ be recovered without error from those samples? How?

(c) **(5 points)** Each sample is then quantized to have finite precision. If each sample must be known to within $\pm 1\%$ of the peak-to-peak full-scale value, how many levels should the quantizer have?

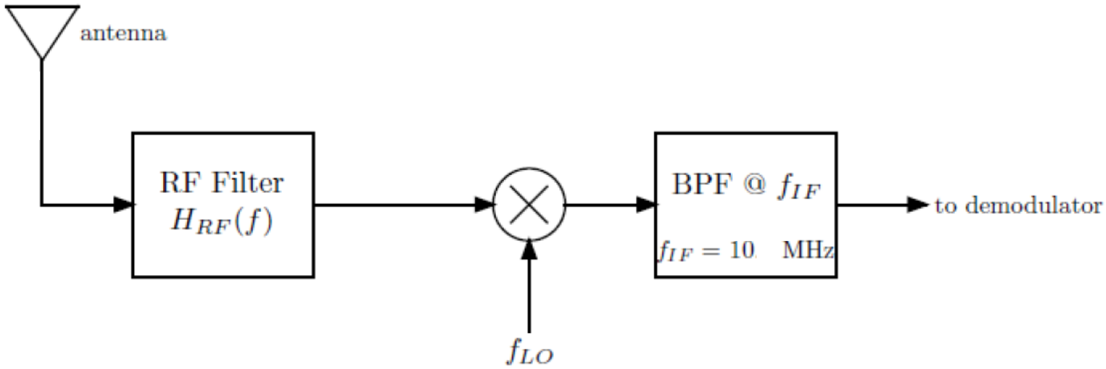
(d) **(2 points)** How many binary symbols (bits) are needed to represent each sample? Feel free to leave your answer in terms of $L$, the number of levels in the quantizer.

[1] Practical systems use a dynamic multiplexing scheme not covered in class

(e) **(3 points)** These bits are then mapped into symbols of a 4-PAM (Pulse Amplitude Modulation). How would you assign the bits to the symbols if you want to minimize the bit error rate? (Assume that they are going to be transmitted through an AWGN channel).

(f) **(3 points)** The 4-PAM baseband signal is then modulated with a sinusoidal carrier at 200 MHz. Please sketch the modulated signal as a function of time for the following sequence of 4-PAM symbols: $[1,\ -3,\ 3,\ -1]$. Assume that the symbol duration is a multiple of $\frac{1}{200}$ and make a second sketch zooming into one of the symbol transitions (for example, the transition between -3 and 3).

(g) **(5 points)** What advantages and disadvantages would 16-PAM have over 4-PAM? Explain the trade-offs between bandwidth, power, transmission rate, and probability of error.




(h) **(4 points)** In practical DAB systems, each subchannel (radio station) uses an OFDM multi-carrier modulation with 1 kHz subcarriers. Assume a bandwidth of 1 MHz for each station and a superheterodyne receiver (see figure) with an intermediate frequency $f_{IF} = 10$ MHz. Propose a local oscillator frequency $f_{LO}$ and an RF filter (either low-pass or high-pass) to receive the station centered at 200 MHz. For the RF filter, you just need to specify the type (LPF or HPF) and cut-off frequency.

(i) **(3 points)** Given that the superheterodyne receiver has an intermediate frequency of $f_{IF} = 10MHz$ and we want to tune it to a range of 174-240 MHz with 1 MHz bands, can we use a fixed (non-tunable) RF filter? Why? (Hint: The answers to part h could be helpful.)

(j) **(3 points)** Assume that the 4-PAM symbols have been transmitted over an Additive White Gaussian Noise (AWGN) Channel. Will all the symbols have the same probability of error? Would that change if we used a quadrature phase shift keying (QPSK) modulation?

**Question 4: (20 points)**

(a) **(3 points)** Draw an 8-PSK (Phase Shift Keying) constellation.

(b) **(2 points)** Draw the optimal decision regions in the figure above, assuming that all symbols are equally likely.

(c) **(3 points)** Choose two orthonormal functions for the basis of the PSK constellation.

(d) **(2 points)** Which symbols in the 8-PSK constellation have the highest probability of error? And the lowest?

(e) **(3 points)** If I send one symbol from this constellation every 20 ms, what is the bit rate?

(f) **(3 points)** The distance between the transmitter and receiver can change the channel fading (attenuation of the signal). Does a detector for 8-PSK require an estimate of the channel fading? How about a detector for 16-ASK (Amplitude Shift Keying)? Why?

(g) **(4 points)** Assume that one of the symbols is transmitted more often than the others (e.g., one symbol is transmitted with probability 0.5 and each of the others with probability 1/14). A Bayesian or MAP (Maximum a posteriori) detector would take into account those prior probabilities and adjust the decision regions accordingly. Sketch the new decision regions (no need for precise numbers, just an approximate illustration).

(h) **(5 points) Bonus:** Would the average probability of error per symbol increase or decrease respect to the equiprobable case? Why?

**Question 5 (20 points):** Imagine that an information source is transmitting a DNA sequence, consisting of symbols $\mathcal{X} = \{A, C, G, T\}$ distributed according to

$$\mathcal{X} = \begin{cases} \text{A} & \textbf{with probability} \;\; p_A = 0.5 \\ \text{C} & \textbf{with probability} \;\; p_C = 0.25 \\ \text{G} & \textbf{with probability} \;\; p_G = 0.125 \\ \text{T} & \textbf{with probability} \;\; p_T = 0.125 \end{cases}$$

(a) **(5 points)** Find the entropy of the source $H(\mathcal{X})$ in bits (average information per symbol). This gives an asymptotic limit for how much a sequence can be compressed.

(b) **(10 points)** Use either Huffman or Lempel-Ziv source coding to compress the following sequence: AAACTGAACATGAACTG. Make sure to explain your answer clearly.

(c) **(5 points)** Assume that you have compressed the DNA sequence into a string of 500 bits. You now wish to transmit them through a channel in such a way that the receiver can tolerate a single burst of 100 consecutive bits erased. How would you do it? You know that 100 consecutive bits in your transmission will be lost, but neither the transmitter nor the receiver know when it will happen.

**Question 6 (10 points):** Specify whether the following statements are true or false, and justify your answer.

(a) **(2 points)** Both Double sideband (DSB) modulation and vestigial sideband (VSB) modulation can be used to transmit a message with low frequency components.

(b) **(2 points)** For similar bandwidth and power, amplitude modulation has higher SNR than frequency modulation, which explains why AM radio reaches longer distances than FM radio.

(c) **(2 points)** Since a pulse cannot be simultaneously time- and band-limited, inter-symbol or inter-band interference (ISI) is inevitable in digital modulations.

(d) **(2 points)** Code division multiple access provides provides robustness to narrow-band noise.

(e) **(2 points)** If the symbols are equiprobable, MAP detection is equivalent to ML (Maximum Likelihood) detection.

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**Formulas and notation:**

- Trigonometric:
  - $\sin^2(x) + \cos^2(x) = 1$
  - $\sin(u + v) = \sin u \cos v + \cos u \sin v$
  - $\cos(u + v) = \cos u \cos v - \sin u \sin v$
  - $\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$
  - $\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$
  - $\sin(\alpha)\cos(\beta) = \frac{\sin(\beta+\alpha)-\sin(\beta-\alpha)}{2}$
- Functions:
  - Unit step: $u(t) = 0$ for $t < 0$, $u(t) = 1$ for $t > 0$.
  - Triangle: $\Lambda(t) = 1 - |t|$ for $|t| \leq 1$, $\Lambda(t) = 0$ otherwise
  - Square pulse: $\Pi(t) = 1$ for $|t| \leq 0.5$, $\Pi(t) = 0$ otherwise
  - $sinc(t) = \frac{sin(\pi t)}{\pi t}$
- Fourier transforms:
  - $\Pi(t) \leftrightarrow sinc(f)$
  - $\Lambda(t) \leftrightarrow sinc^2(f)$
  - $u(t) \leftrightarrow \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$
  - $\frac{1}{\pi t} \leftrightarrow -jsign(f)$
  - $\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \leftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(t - nf_s)$ where $f_s = \frac{1}{T_s}$.