ECE 440 – Spring 2018
Final Exam

Instructor: Borja Peleato

Name:..

- Please fill in your name on the dotted line above.
- The exam is closed book and closed notes. You can have a simple scientific calculator without communication capabilities, but you should not need one. You are free to leave your answers in terms of any expression that could be computed with a scientific calculator (e.g., $\sin(\pi/5)$).
- There is a formula sheet at the back of the exam. Feel free to tear it off.
- You should be able to answer all the questions in the space provided, but you can use the blank pages at the back of the exam if you need additional space or scratch paper. However, make sure to include all calculations and explanations with your answer. If you need additional scratch paper, feel free to ask for it.
- If you don't know how to solve part of a question but need the solution for a subsequent part, feel free to leave your answer to the latter as a function of the unknown solution.
- You have 120 minutes to complete the exam. That should give you enough time to briefly justify your answers, please do so.

Q1	/8
Q2	/12
Q3	/12
Q4	/15
Q5	/16
Q6	/10
Q7	/12
Q8	/15
Total	/100

Question 1: (8 points) Find the power of the following signals:

- $x_1(t) = 2\cos(4\pi t + 2\pi/3)$

- $x_2(t) = (-1)^{\lfloor t \rfloor}$, where $\lfloor t \rfloor$ represents rounding down to the nearest integer.

Question 2: (12 points)

(a) **(2 points)** A linear time-invariant (LTI) multipath wireless channel outputs

$$y(t) = 0.5x(t) + 0.25x(t - 0.005)$$

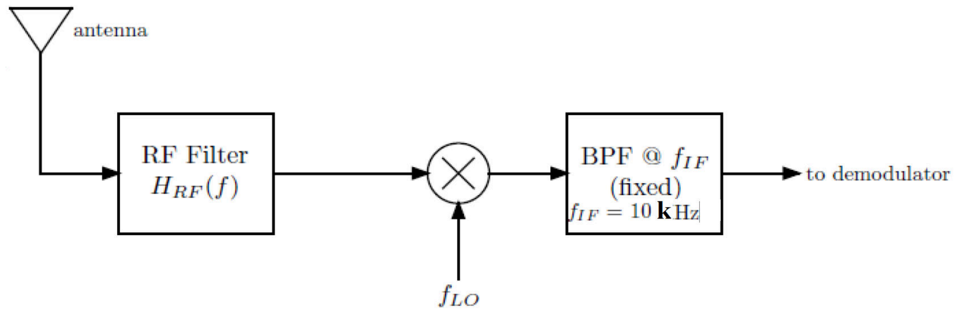
for any input $x(t)$. Find its impulse response $h(t)$.

(b) **(4 points)** Find the frequency response $H(f)$ for the channel described above. Specify the channel attenuation at $f = 0$ and $f = 100$ Hz, i.e. $|H(0)|$ and $|H(100)|$.

(c) **(3 points)** A baseband signal $x(t)$ of bandwidth 10kHz (spectrum between -10kHz and 10kHz) is transmitted through this channel. If we wish to sample the **output** $y(t)$ without aliasing, what is the minimum sampling frequency that we would need? Would the required sampling frequency be the same if we wanted to discretize the **input** $x(t)$?

(d) **(3 points)** Assume that you use a sampling **period** of 10^{-5} seconds between samples, quantize each sample into N levels, and then transmit them as 16-PSK symbols. If your processor can handle a maximum of $75,000$ **16-PSK symbols** per second, what is the finest quantization (value of N) that you can use?

Question 3: (12 points) The following figure shows a superheterodyne receiver with an intermediate frequency of 10kHz.



(a) **(3 points)** Assuming that the RF filter and local oscillator frequency are fully adjustable, is it possible to tune this receiver to receive signals centered at the following carrier frequencies? 1 kHz, 10 kHz, 100 MHz (You do not need to specify how, just whether it is possible and why)

(b) **(3 points)** Specify the RF filter and local oscillator to be used if we wish to receive a radio station centered at 100 kHz using **low-side** tuning.

(c) **(3 points)** Assuming that the RF filter is a fixed (non-tunable) ideal band-pass filter for frequencies between 100kHz and 200kHz, what is the usable band? By usable band, we mean a range of frequencies over which signals can be multiplexed in frequency and correctly received. You may assume that there are no other signals apart from the ones being multiplexed.

(d) **(3 points)** Assume, once again, that the RF filter is a fixed (non-tunable) ideal band-pass filter for frequencies between 100kHz and 200kHz. If you are free to replace the IF filter with a different one, can you increase the usable band? Specify how and how much.

Question 4: (15 points) An angle modulator uses a carrier frequency of 1000 Hz and is used to transmit a message with bandwidth W.

(a) **(3 points)** In a strict sense, what will the bandwidth of the output be?

(b) **(3 points)** Briefly explain Carson's rule.

(c) **(4 points)** Determine both the phase and frequency deviation if the modulator output is

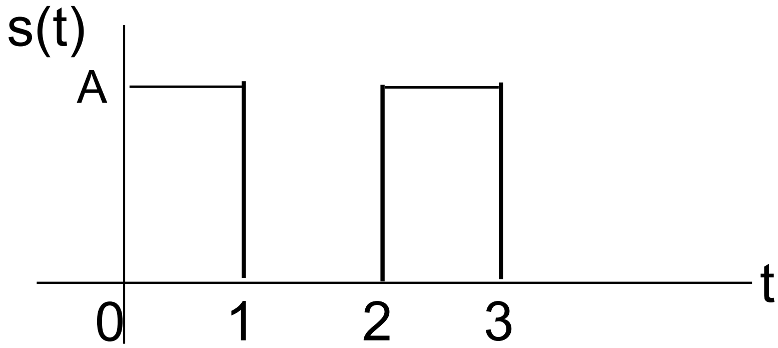
$$x_c(t) = \cos(2\pi 600t)$$

(Hint: this exact problem was in one of the homeworks).

(d) **(2 points)** Again, if the modulator output is $x_c(t) = \cos(2\pi 600t)$, what is the message being transmitted? Give the answer both for the case of phase modulation (PM) and frequency modulation (FM). Feel free to include an unknown scaling factor in your answer, to account for the deviation constants.

(e) **(3 points)** Again, if the modulator output is $x_c(t) = \cos(2\pi 600t)$, does this signal agree with Carson's rule? Why or why not?

Question 5: (16 points) The received signal in a binary communication system that employs antipodal signals is $r(t) = \pm s(t) + n(t)$, where $s(t)$ is shown below and $n(t)$ is AWGN with power spectral density $N_0/2$ W/Hz.



(a) **(4 points)** Sketch the impulse response of the filter matched to $s(t)$.

(b) **(4 points)** Determine the optimal sampling time for the filter output

(c) **(4 points)** Determine the variance of the noise at the output of the matched filter at $t = 3$

(d) **(4 points)** Determine the probability of error as a function of A and N_0

Question 6: (10 points) A 16-QAM signal with symbol duration 1ms is spread using DSSS with a binary sequence with chip duration 10μs.

(a) **(5 points)** How does the bandwidth of the original signal compare with that of the spread signal? An approximate numerical answer is required, not just greater or smaller.

(b) **(3 points)** Which of the following sequences do you think would be a better chip sequence, and why?

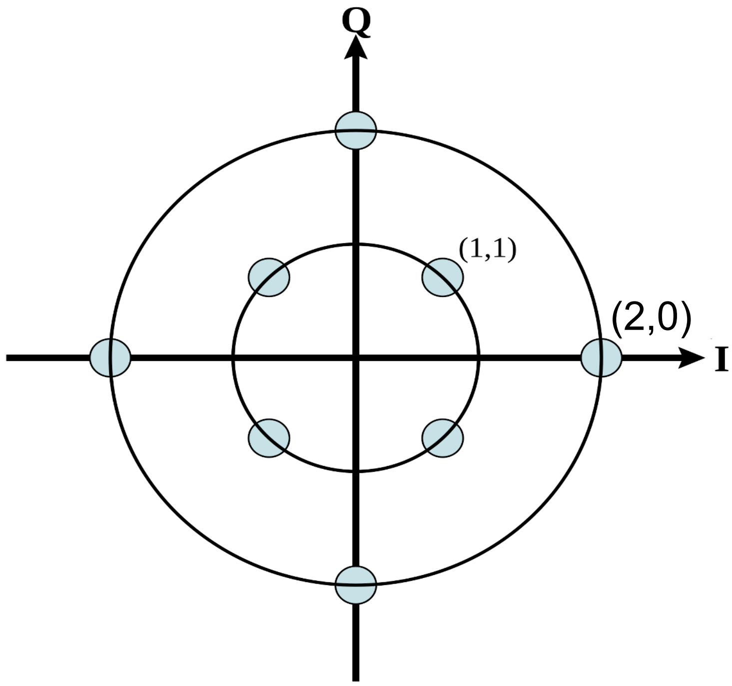
[1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1...] (alternating 1 and -1)

[1 1 1 1 1 -1 -1 -1 -1 -1 1 1 1 1 1 -1 -1 -1 -1 -1...] (alternating series of 1 and series of -1)

[1 1 -1 1 -1 -1 1 1 1 -1 1 -1 -1 1 -1 1 1 1 -1 1 1...] (random 1 and -1 with slightly different probabilities)

(c) **(2 points)** We want to have multiple users communicating simultaneously on the same band using CDMA. What conditions do their chip sequences have to fulfil?

Question 7: (12 points) An amplitude and phase-shift keying (APSK) has the constellation below, where the basis functions for the horizontal and vertical axis are $\phi_1(t) = \sqrt{2/T}\cos(2\pi f_0 t)$ and $\phi_2(t) = \sqrt{2/T}\sin(2\pi f_0 t)$ for $0 < t < T$, respectively.



(a) **(5 points)** Assuming that all symbols are equally likely to be transmitted, what is the average transmit power of this modulation?

(b) **(2 points)** Would all the symbols in the constellation suffer the same probability of error in an AWGN channel? Otherwise, which ones would have lowest probability of error? Why?

(c) **(3 points)** If the demodulator were to receive the signal $x_r(t) = 1.5\sqrt{2/T}\cos(2\pi f_0 t) + 1.5\sqrt{2/T}\sin(2\pi f_0 t)$, what symbol would it return? Re-draw the constellation and mark your answer graphically.

(d) **(2 points)** If a channel is able to transmit this constellation without error, what is the capacity of that channel?

Question 8: (15 points) Real world questions:

(a) **(3 points)** What type of multiplexing do radio stations use?

(b) **(3 points)** If we can design codes with arbitrary error correction capability, why don't communication systems use codes that can correct an INFINITE number of errors?

(c) **(3 points)** You are trying to transmit an FM signal through an AWGN channel, but the receive SNR is too low (below the threshold). Propose a simple parameter adjustment that could increase SNR.

(d) **(3 points)** If you were asked to transmit a play by Shakespeare to a friend on the other side of a BSC(0.1) channel (zeros and ones go in, zeros and ones come out, 10% of the time they are different). How would you do it? Be as specific as possible, with the steps you would take, but there is no need to compute numerical parameters.

(e) **(3 points)** If the play above has 10^6 characters, the entropy of the English language is 2.6 bits per letter, and we can transmit 1000 bits per second into the BSC(0.1) channel, how long do you estimate it would take you to transmit the whole play **without errors**?

(Hint: $H(0.1) = -0.1\log_2(0.1) - 0.9\log_2(0.9) \simeq 0.47$)

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Formulas and notation:

- Trigonometric:
  - $\sin^2(x) + \cos^2(x) = 1$
  - $\sin(u + v) = \sin u \cos v + \cos u \sin v$
  - $\cos(u + v) = \cos u \cos v - \sin u \sin v$
  - $\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$
  - $\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$
  - $\sin(\alpha)\cos(\beta) = \frac{\sin(\beta+\alpha)-\sin(\beta-\alpha)}{2}$
- Functions:
  - Unit step: $u(t) = 0$ for $t < 0$, $u(t) = 1$ for $t > 0$.
  - Triangle: $\Lambda(t) = 1 - |t|$ for $|t| \leq 1$, $\Lambda(t) = 0$ otherwise
  - Square pulse: $\Pi(t) = 1$ for $|t| \leq 0.5$, $\Pi(t) = 0$ otherwise
  - $sinc(t) = \frac{sin(\pi t)}{\pi t}$
- Fourier transforms:
  - $\Pi(t) \leftrightarrow sinc(f)$
  - $\Lambda(t) \leftrightarrow sinc^2(f)$
  - $u(t) \leftrightarrow \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$
  - $\frac{1}{\pi t} \leftrightarrow -jsign(f)$
  - $\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \leftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(t - nf_s)$ where $f_s = \frac{1}{T_s}$.
- Fourier transform theorems:
  - $ax(t) + by(t) \leftrightarrow aX(f) + bY(f)$
  - $x(t - t_0) \leftrightarrow X(f)e^{-j2\pi f t_0}$
  - $x(-t) \leftrightarrow X(-f) = X*(f)$
  - $\frac{d^n}{dt^n} \leftrightarrow (j2\pi f)^n X(f)$
  - $x(t)\cos(2\pi f_0 t) \leftrightarrow \frac{1}{2}[X(f - f_0) + X(f + f_0)]$
  - $x(t)\sin(2\pi f_0 t) \leftrightarrow \frac{1}{2j}[X(f - f_0) - X(f + f_0)]$