ECE 440 – Spring 2016
Exam 2

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Name:..

Question 1 (20 points) A continuous speech signal with bandwidth 4 kHz is quantized and transmitted using a Pulse Coded Modulation (PCM) system.

(a) **(5 points)** If each data sample at the receiving end of the system must be known to within $\pm 0.25\%$ of the peak-to-peak full-scale value, how many binary symbols (bits) must each transmitted digital word contain?

(b) **(5 points)** Assuming that the sampling frequency is 8kHz, to avoid aliasing, what would the transmission speed be in bits per second?

(c) **(10 points)** How would the power, bandwidth and speed requirements change if we were to use Pulse Amplitude Modulation (PAM) instead of PCM to encode each sample? Quantization noise should still be $\pm 0.25\%$ of the peak-to-peak value.

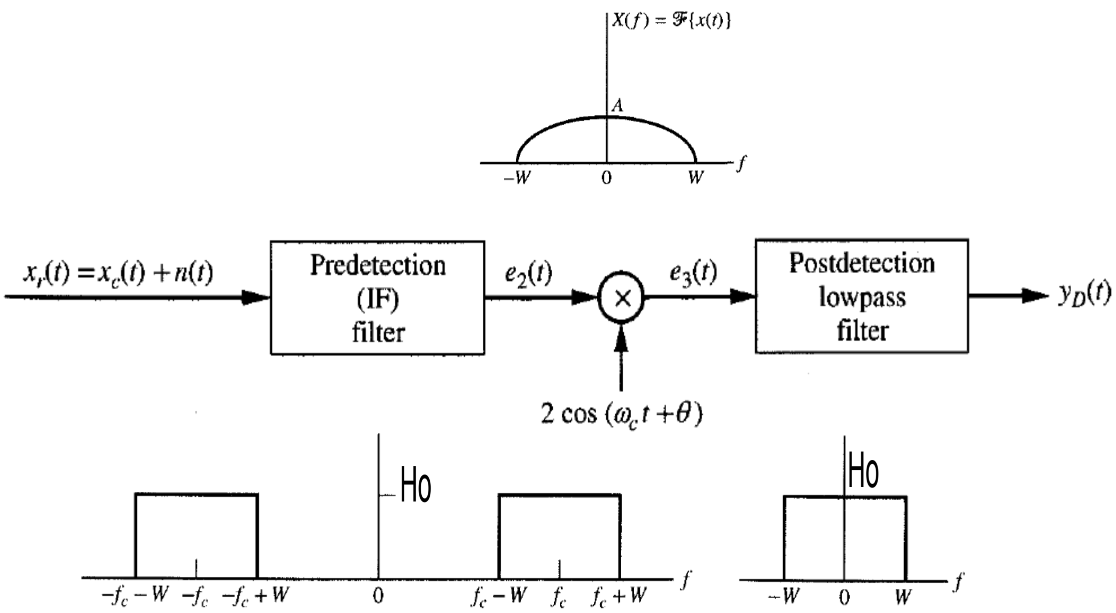
Question 2 (15 points): An FDM system is capable of transmitting a baseband signal having a bandwidth of 100 kHz. One channel is input to the system without modulation ($f = 0$). Assume that all messages have a lowpass spectrum with a bandwidth of 2 kHz (i.e. spectrum goes from -2 kHz to 2 kHz) and that the guardband between channels is 1 KHz.

(a) **(5 points)** How many channels can be multiplexed together to form the baseband?

(b) **(5 points)** What modulation should be used for the messages to achieve that number? (i.e., to maximize bandwidth efficiency)

(c) **(5 points)** What modulation would you use if the messages have a DC component? Why? (feel free to sacrifice part of the guardband)

Question 3 (15 points): A message $x(t)$ with bandwidth W and power A^2 is transmitted through an AWGN channel using double sideband (DSB) modulation. The noise is assumed to have a power spectral density of $N_0/2$ and the signal is received using the coherent demodulator below.

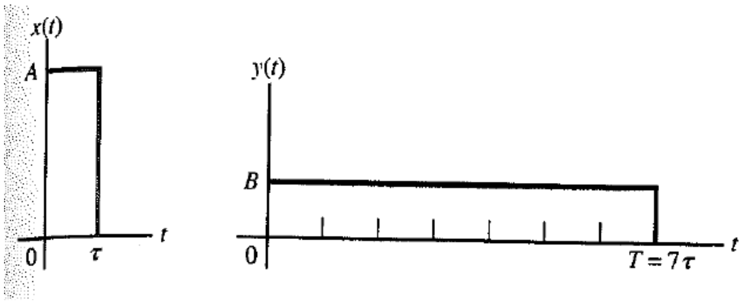


(a) **(4 points)** What is the (pre-detection) SNR for signal $e_2(t)$ at the output of the IF filter?

(b) **(4 points)** What is the (post-detection) SNR for signal $y_D(t)$ at the output of the demodulator?

(c) **(3 points)** How does the gain of the filters affect the post-detection SNR? How does the amplitude of the local oscillator at the demodulator affect the post-detection SNR?

(d) **(4 points)** Is the post-detection low pass filter necessary? How would the SNR change without it?



Question 4: (15 points)

(a) **(5 points)** Sketch or give an equation for the optimum (matched) filter impulse response to detect the two pulses above. Make sure that it is causal.

(b) **(5 points)** Relate the constants A and B so that both pulses give the same SNR at the matched filter output.

(d) **(5 points)** If peak transmitted power is a consideration, which waveform is preferable?

Question 5: (20 points)

(a) **(5 points)** Draw a 16-QAM (Quadrature amplitude modulation) constellation and assign bits to each symbol so as to minimize the bit error rate under AWGN noise.

(b) **(3 points)** If I send one symbol from this constellation every 20 ms, what is the bit rate?

(c) **(9 points)** If the average energy per symbol (assuming equally probable) is constrained to be $P_{ave} = 10J$, what is the probability of error for each symbol? Feel free to leave your answers in terms of the noise variance σ_0^2 and to approximate the probability using the union bound.

(d) **(3 points)** How does the average probability of error per bit compare with the probability of error per symbol? An exact equation is required, not just which one is greater.

(e) **(5 points)** What advantages does 16-QAM have over 16-PAM (Pulse amplitude modulation) and 16-PSK (Phase shift keying)?

Question 6: (10 points) In order to build a 2 dimensional modulation, we need pairs of orthogonal functions (they can then be scaled to become orthonormal). Specify conditions on the parameter A so that the following pairs of functions are orthogonal in the interval [0,T]. If it is not possible or they are orthogonal regardless of A, say so.

(a) **(2 points)** $\cos(2\pi At)$ and $\sin(2\pi At)$.

(b) **(2 points)** $\cos(2\pi f_c t)$ and $\cos(2\pi(f_c + A)t)$.

(c) **(2 points)** $\Pi(At)$ and $-\Pi(t/A)$ (square pulses, see definition in formula page).

(d) **(2 points)** $\Pi(\frac{t}{T} - 0.5)$ and $\Pi(\frac{t}{T} - 0.5) * sign(t - A)$.

(e) **(2 points)** $\Lambda(\frac{4t}{T} - 1)$ and $\Lambda(\frac{4t}{T} - A)$.

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Formulas and notation:

- Trigonometric:
  - $\sin^2(x) + \cos^2(x) = 1$
  - $\sin(u + v) = \sin u \cos v + \cos u \sin v$
  - $\cos(u + v) = \cos u \cos v - \sin u \sin v$
  - $\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$
  - $\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$
  - $\sin(\alpha)\cos(\beta) = \frac{\sin(\beta+\alpha)-\sin(\beta-\alpha)}{2}$
- Functions:
  - Unit step: $u(t) = 0$ for $t < 0$, $u(t) = 1$ for $t > 0$.
  - Triangle: $\Lambda(t) = 1 - |t|$ for $|t| \leq 1$, $\Lambda(t) = 0$ otherwise
  - Square pulse: $\Pi(t) = 1$ for $|t| \leq 0.5$, $\Pi(t) = 0$ otherwise
  - $sinc(t) = \frac{sin(\pi t)}{\pi t}$
- Fourier transforms:
  - $\Pi(t) \leftrightarrow sinc(f)$
  - $\Lambda(t) \leftrightarrow sinc^2(f)$
  - $u(t) \leftrightarrow \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$
  - $\frac{1}{\pi t} \leftrightarrow -jsign(f)$
  - $\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \leftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(t - nf_s)$ where $f_s = \frac{1}{T_s}$.