ECE 440 – Spring 2019
Midterm 2

Instructor: Borja Peleato

Name:...

- Please fill in your name on the dotted line above.
- The exam is closed book and closed notes, but you are allowed to have a single sheet of paper handwritten on both sides.
- You can have a simple scientific calculator without communication capabilities, but you should not need one. You are free to leave your answers in terms of any expression that could be computed with a scientific calculator (e.g., $\sin(\pi/5)$).
- There is a formula sheet at the back of the exam. Feel free to tear it off.
- The formula sheet also specifies a lot of notations used throughout the exam. If there is a symbol you do not understand, please check the formula sheet before asking.
- You should be able to answer all the questions in the space provided, but you can use the blank pages at the back of the exam if you need additional space or scratch paper. However, make sure to include all calculations and explanations with your answer. If you need additional scratch paper, feel free to ask for it.
- You have 50 minutes to complete the exam. That should give you plenty of time to justify your answers, please do so.

Q1	/30
Q2	/40
Q3	/30
Total	/100

Question 1 (30 points) A message $m_1(t)$ is sampled with a sampling period of $T_s = 10ms$, resulting in samples distributed uniformly between -1 and 1, i.e. $m_1(nT_s) \sim U[-1,1]$ for $n = \ldots, -2, -1, 0, 1, 2, \ldots$. The transmitter uses a Pulse Amplitude Modulation (PAM): for each sample, it transmits a square pulse of duration T_s and amplitude equal to the sample value.

(a) (8 points) What is the expected (average) power of the transmitted signal?

(b) (7 points) What is the peak instantaneous power of the transmitted signal?

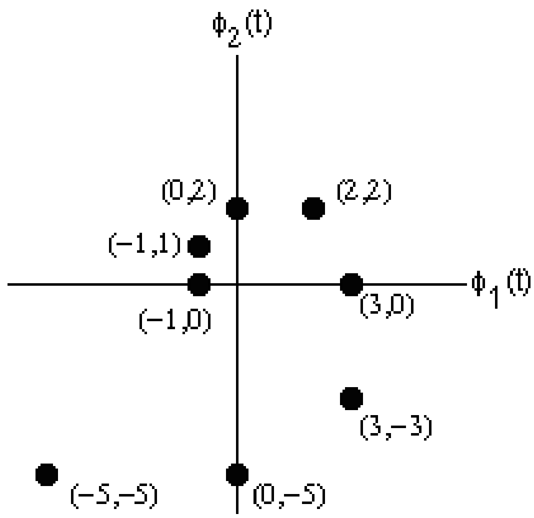
(c) (7 points) If we wanted to use Pulse Code Modulation (PCM) instead of PAM, we would need to quantize the samples. How many quantization levels would we need if we want a quantization error of $\pm 5\%$?

(d) (8 points) PCM then transmits each sample as a sequence of binary pulses. Assume that we decide to use M=64 quantization levels. What would the bandwidth of the signal be?

Question 2 (40 points)

(a) (8 points) Let $\phi_1(t) = \sqrt{2}$ and $\phi_2(t) = A\cos(2\pi 10^3 t)$, both of them defined in the interval $t \in [0, T_s]$. Find A and T_s so that they are orthonormal.

(b) (8 points) Consider the constellation shown below, where $\phi_1(t)$ and $\phi_2(t)$ are the orthonormal functions from part (a). Find the average energy per symbol.

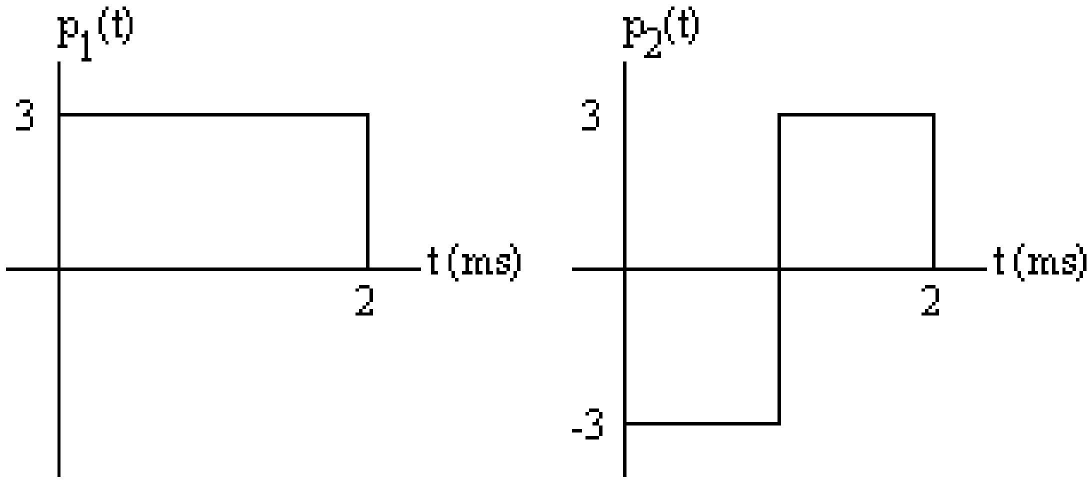


(c) (8 points) Find the probability of symbol error for the symbol at (2,2), assuming that the signal is corrupted with AWGN noise with variance $N_0/2$. Feel free to use the union bound as an approximation and to leave your answer in terms of the Q function.

(d) (8 points) Sketch the signal corresponding to the symbol at (2,2). There is no need for a precise drawing, but make sure to specify its range (maximum and minimum value), time interval, and the frequency of any oscillations. Observe the atypical basis functions given in part (a).

(e) (8 points) Consider the same constellation from part (b), but with $\phi_1(t) = \sqrt{2/T_s}\cos(2\pi 10^3 t)$ and $\phi_2(t) = \sqrt{2/T_s}\sin(2\pi 10^3 t)$. In order to differentiate all the symbols in this constellation, would you need an envelope detector (capable of detecting magnitude, but not phase), a phase detector (capable of detecting phase, but not magnitude), or a coherent detector capable of detecting both amplitude and phase? Justify your answer.

Question 3 (30 points) A binary communication system uses the two pulses below to send bit values 0 and 1 through an AWGN channel.

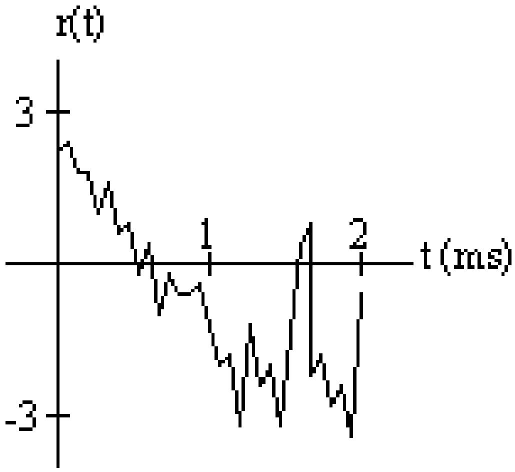


(a) (7 points) What would be the data rate (bits per second) of such a system?

(b) (8 points) Sketch the impulse response of a causal filter capable of differentiating the two pulses above with minimum BER (matched filter).

(c) (7 points) Specify the optimal time at which the output of that filter should be sampled so as to minimize BER after thresholding.

(d) (8 points) If you were to receive the signal below, which of the two pulses would a matched filter receiver estimate that had been transmitted? Justify your answer.



(e) **Bonus: (5 points)** Refine your matched filter above to minimize delay and avoid noise amplification (energy at the input must be the same as energy at the output). Your refined filter needs to be causal and yield the same BER as before. If you believe that the filter in part (a) already yields minimum latency and no amplification, feel free to answer "same".

.

Formulas and notation:

- Trigonometric:
  - $\sin^2(x) + \cos^2(x) = 1$
  - $\sin(u + v) = \sin u \cos v + \cos u \sin v$
  - $\cos(u + v) = \cos u \cos v - \sin u \sin v$
  - $\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$
  - $\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$
  - $\sin(\alpha)\cos(\beta) = \frac{\sin(\beta+\alpha)-\sin(\beta-\alpha)}{2}$
- Functions:
  - Unit step: $u(t) = 0$ for $t < 0$, $u(t) = 1$ for $t > 0$.
  - Triangle: $\Lambda(t) = 1 - |t|$ for $|t| \leq 1$, $\Lambda(t) = 0$ otherwise
  - Square pulse: $\Pi(t) = 1$ for $|t| \leq 0.5$, $\Pi(t) = 0$ otherwise
  - $sinc(t) = \frac{sin(\pi t)}{\pi t}$
  - $sign(t) = \frac{t}{|t|}$
- Fourier transforms:
  - $\Pi(t) \leftrightarrow sinc(f)$
  - $sinc(t) \leftrightarrow \Pi(f)$
  - $\Lambda(t) \leftrightarrow sinc^2(f)$
  - $u(t) \leftrightarrow \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$
  - $\frac{1}{\pi t} \leftrightarrow (-j)sign(f)$
  - $\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \leftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(t - nf_s)$ where $f_s = \frac{1}{T_s}$.
- Fourier transform theorems:
  - $ax(t) + by(t) \leftrightarrow aX(f) + bY(f)$
  - $x(t - t_0) \leftrightarrow X(f)e^{-j2\pi f t_0}$
  - $x(-t) \leftrightarrow X(-f) = X * (f)$
  - $\frac{d^n}{dt^n} \leftrightarrow (j2\pi f)^n X(f)$
  - $x(t)\cos(2\pi f_0 t) \leftrightarrow \frac{1}{2}[X(f - f_0) + X(f + f_0)]$
  - $x(t)\sin(2\pi f_0 t) \leftrightarrow \frac{1}{2j}[X(f - f_0) - X(f + f_0)]$
- Notation:
  - $X(f) = \mathcal{F}[x(t)]$ denotes the Fourier transform of a time function $x(t)$, and $\mathcal{F}^{-1}$ denotes the inverse Fourier transform
  - $\hat{x}(t)$ denotes the Hilbert transform of a function $x(t)$
  - $j = \sqrt{-1}$
  - $x(t) * y(t)$ denotes the convolution of $x(t)$ and $y(t)$