ECE 440 – Spring 2019
Midterm 1

Instructor: Borja Peleato

Name:...

- Please fill in your name on the dotted line above.
- The exam is closed book and closed notes. You can have a simple scientific calculator without communication capabilities, but you should not need one. You are free to leave your answers in terms of any expression that could be computed with a scientific calculator (e.g., $\sin(\pi/5)$).
- There is a formula sheet at the back of the exam. Feel free to tear it off.
- The formula sheet also specifies a lot of notations used throughout the exam. If there is a symbol you do not understand, please check the formula sheet before asking.
- You should be able to answer all the questions in the space provided, but you can use the blank pages at the back of the exam if you need additional space or scratch paper. However, make sure to include all calculations and explanations with your answer. If you need additional scratch paper, feel free to ask for it.
- You have 50 minutes to complete the exam. That should give you plenty of time to justify your answers, please do so.

Q1	/10
Q2	/15
Q3	/30
Q4	/15
Q5	/15
Total	/100

Question 1 (10 points) Which of the following are correct equations for the Hilbert transform of a given signal $x(t)$? (Mark all that apply)

(a) $\hat{x}(t) = \int_{-\infty}^{\infty} \frac{x(\alpha)}{\pi(t-\alpha)} d\alpha$

(b) $\hat{x}(t) = x(t)\frac{1}{\pi t}$

(c) $\hat{x}(t) = x(t) * [(-j)sign(x(t))]$

(d) $\hat{x}(t) = \mathcal{F}^{-1}[X(f)(-j)sign(f)]$

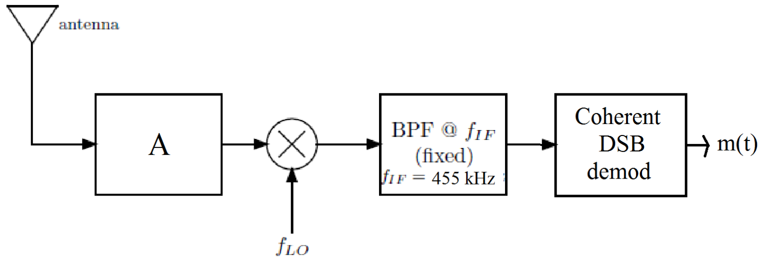
(e) $\hat{x}(t) = \mathcal{F}^{-1}[X(f) * \frac{1}{\pi f}]$

Question 2 (15 points) A signal $x(t) = 10sinc(t)$ is put through a linear time invariant (LTI) filter with impulse response $h(t) = \frac{1}{\pi t}$, resulting in an output $y(t)$.

- Find the energy of $x(t)$ (10 points)

- Find the energy of $y(t)$. You may leave your answer in terms of the result for part (a) (5 points)

Question 3 (30 points) A superheterodyne receiver with intermediate frequency $f_{\mathrm{IF}} = 455$ kHz is used to recover a double sideband modulated (DSB) signal $x_{\mathrm{DSB}} = m(t)\cos(2\pi f_c t)$ with carrier frequency $f_c = 10^6$. It is known that the message m(t) is a voice signal composed of frequencies below 5 KHz.

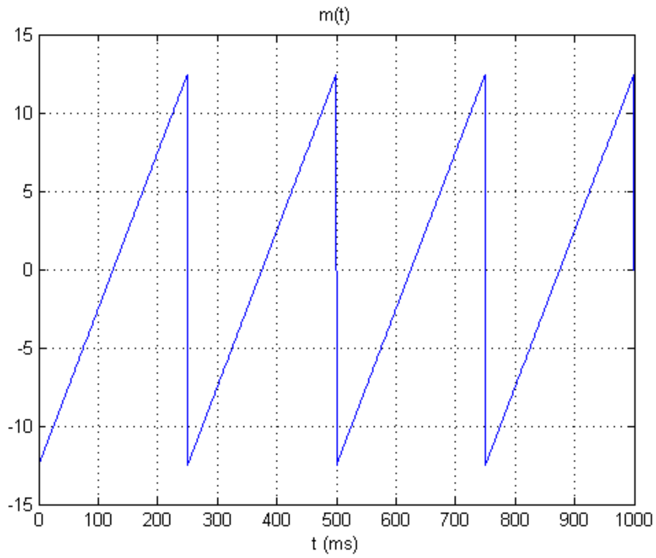


(a) **(10 points)** The block labeled A represents a linear time invariant (LTI) system. What system should that be? Sketch its frequency response. Try to pick a system which is easy to implement.

(b) **(10 points)** What frequency f_{LO} should the local oscillator be tuned to?

(c) **(10 points)** The superheterodyne receiver is followed by a coherent DSB demodulator. Provide a block diagram of the demodulator, making sure to specify all frequencies and bandwidths.

Question 4 (15 points) Assume that we want to modulate the message $m(t)$ shown in the figure below using an FM modulator with an unmodulated carrier $x_c(t) = 5\cos(200\pi t)$ and frequency deviation constant of $f_d = 4$ Hz/V.



(a) (10 points) Sketch the shape of the modulated signal $x_{FM}(t)$. Your drawing does not have to be very precise, but please specify the amplitude and the instantaneous frequency at $t = 0.025$, $t = 0.125$, and $t = 0.225$. Observe that the message has zero mean and $m(0.025) = -10$, $m(0.125) = 0$, $m(0.225) = 10$.

(b) (5 points) If we wanted to sample the modulated signal $x_{FM}(t)$ without aliasing, how many samples per second would we need?

Question 6 (15 points) Specify whether the following random processes are stationary and ergodic. In all cases $A \sim U[-\pi, \pi]$. To save time, you do not need to justify your answers . Feel free to guess if you do not know.

(a) (5 points) $X(t) = 3 + A\cos(2\pi f_c t)$

(b) (5 points) $X(t) = 3 + \cos(2\pi f_c t + A)$

(c) (5 points) $X(t) = 3 + \cos(2At)$

Question 5 (15 points) Find the noise-equivalent bandwidth of a filter with frequency response $|H(f)| = min(3, \sqrt{10 - |f|})$.

Formulas and notation:

- Trigonometric:
  - $\sin^2(x) + \cos^2(x) = 1$
  - $\sin(u+v) = \sin u \cos v + \cos u \sin v$
  - $\cos(u+v) = \cos u \cos v - \sin u \sin v$
  - $\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$
  - $\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$
  - $\sin(\alpha)\cos(\beta) = \frac{\sin(\beta+\alpha)-\sin(\beta-\alpha)}{2}$
- Functions:
  - Unit step: $u(t) = 0$ for $t < 0$, $u(t) = 1$ for $t > 0$.
  - Triangle: $\Lambda(t) = 1 - |t|$ for $|t| \leq 1$, $\Lambda(t) = 0$ otherwise
  - Square pulse: $\Pi(t) = 1$ for $|t| \leq 0.5$, $\Pi(t) = 0$ otherwise
  - $sinc(t) = \frac{sin(\pi t)}{\pi t}$
  - $sign(t) = \frac{t}{|t|}$
- Fourier transforms:
  - $\Pi(t) \leftrightarrow sinc(f)$
  - $sinc(t) \leftrightarrow \Pi(f)$
  - $\Lambda(t) \leftrightarrow sinc^2(f)$
  - $u(t) \leftrightarrow \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$
  - $\frac{1}{\pi t} \leftrightarrow (-j)sign(f)$
  - $\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \leftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(t - nf_s)$ where $f_s = \frac{1}{T_s}$.
- Fourier transform theorems:
  - $ax(t) + by(t) \leftrightarrow aX(f) + bY(f)$
  - $x(t - t_0) \leftrightarrow X(f)e^{-j2\pi f t_0}$
  - $x(-t) \leftrightarrow X(-f) = X*(f)$
  - $\frac{d^n}{dt^n} \leftrightarrow (j2\pi f)^n X(f)$
  - $x(t)\cos(2\pi f_0 t) \leftrightarrow \frac{1}{2}[X(f-f_0) + X(f+f_0)]$
  - $x(t)\sin(2\pi f_0 t) \leftrightarrow \frac{1}{2j}[X(f-f_0) - X(f+f_0)]$
- Notation:
  - $X(f) = \mathcal{F}[x(t)]$ denotes the Fourier transform of a time function $x(t)$, and $\mathcal{F}^{-1}$ denotes the inverse Fourier transform
  - $\hat{x}(t)$ denotes the Hilbert transform of a function $x(t)$
  - $j = \sqrt{-1}$
  - $x(t) * y(t)$ denotes the convolution of $x(t)$ and $y(t)$