ECE 440 – Spring 2018
Midterm 1

Name:..

- Please fill in your name on the dotted line above.
- The exam is closed book and closed notes. You can have a simple scientific calculator without communication capabilities, but you should not need one. You are free to leave your answers in terms of any expression that could be computed with a scientific calculator (e.g., $\sin(\pi/5)$).
- There is a formula sheet at the back of the exam. Feel free to tear it off.
- You should be able to answer all the questions in the space provided, but you can use the blank pages at the back of the exam if you need additional space or scratch paper. However, make sure to include all calculations and explanations with your answer. If you need additional scratch paper, feel free to ask for it.
- You have 50 minutes to complete the exam. That should give you plenty of time to justify your answers, please do so.

Q1	/10
Q2	/15
Q3	/15
Q4	/15
Q5	/15
Q6	/15
Q7	/15
Total	/100

Question 1 (10 points) Determine whether the following are acceptable autocorrelation functions for a real signal. If that is not the case, specify why.

(a) $R_1(\tau) = 4\Lambda(\tau/2)$

(b) $R_2(\tau) = 2\sin(10\pi\tau)$

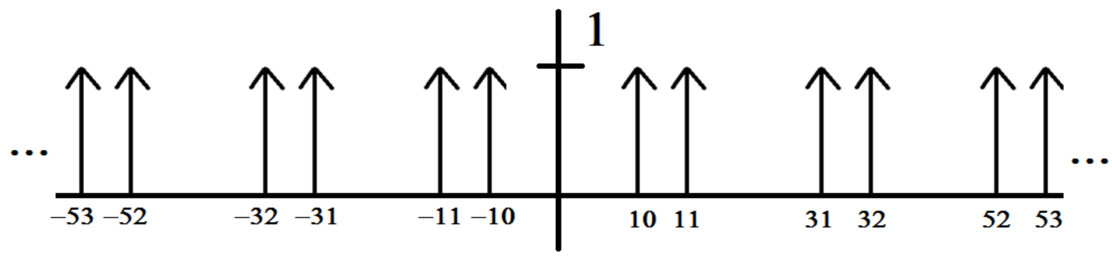
Question 2 (15 points)

(a) **(6 points)** The spectrum of an audio message is known to be 0 for frequencies above 4 kHz. What is the minimum sampling rate you would need to allow recovery without distortion?

(b) **(9 points)** If the message is restricted to the band between 3 kHz and 4 kHz, and this fact is known by both the transmitter and the receiver, can they use a lower sampling rate? If so, give an example, otherwise explain why not.

Question 3 (15 points) Sketch the time domain signal whose spectrum is

$$X(f) = \sum_{k=-\infty}^{\infty} [\delta(f - 10 + 21k) + \delta(f + 10 + 21k)]$$



Question 4 (15 points) An AM modulator has output (in volts)

$$x_c(t) = 40\cos[2\pi(200)t] + 5\cos[2\pi(180)t] + 5\cos[2\pi(220)t]$$

(Hint: You might want to look at the trigonometric formulas provided in the last page of this exam).

(a) **(4 points)** Assuming that the message had been scaled so that $-1 \leq m_n(t) \leq 1$, what is the modulation index a being used for the AM modulation?

(b) **(5 points)** What is the power efficiency of the modulation?

(c) **(4 points)** What is the average transmission power for this modulator?

(d) **(2 points)** What is the peak transmission power for this modulator? In other words, what is maximum instantaneous power of $x_c(t)$ for any t.

Question 5: (15 points) An angle modulator uses a carrier frequency of 1 kHz and produces an output $x_c(t) = 40\cos[2\pi(1200)t^2]$ (watch the squared t!)

(a) **(5 points)** Find the phase deviation

(b) **(5 points)** Find the frequency deviation.

(c) **(3 points)** What is the average transmission power?

(d) **(2 points)** What is the peak transmission power?

Question 6: (15 points) A continuous random variable has the following CUMULATIVE distribution function

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ Ax^4, & 0 < x \leq 12 \\ B, & x > 12 \end{cases}$$

(a) **(5 points)** Find the values of A and B.

(b) **(5 points)** Find the pdf $f_X(x)$.

(c) **(5 points)** Compute the probability that X takes a value between 4 and 6.

Question 7 (15 points): A receiver has four antennas that receive the same signal $x_c(t)$ corrupted with independent but identically distributed AWGN noise $n_i(t)$, $i = 1, 2, 3, 4$. That is, the first antenna receives $y_1(t) = x_c(t) + n_1(t)$, the second $y_2(t) = x_c(t) + n_2(t)$, etc. Each antenna has a signal to noise power ratio of $SNR_i = \frac{P_T}{N_0}$ where P_T is the power of the signal $x_c(t)$ and N_0 is the power (variance) of the $n_i(t)$ noise. If the receiver combines all four signals $y(t) = y_1(t) + y_2(t) + y_3(t) + y_4(t)$ what would the ratio between overall signal and noise power (SNR) be?

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Formulas and notation:

- Trigonometric:
  - $\sin^2(x) + \cos^2(x) = 1$
  - $\sin(u+v) = \sin u \cos v + \cos u \sin v$
  - $\cos(u+v) = \cos u \cos v - \sin u \sin v$
  - $\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$
  - $\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$
  - $\sin(\alpha)\cos(\beta) = \frac{\sin(\beta+\alpha)-\sin(\beta-\alpha)}{2}$
- Functions:
  - Unit step: $u(t) = 0$ for $t < 0$, $u(t) = 1$ for $t > 0$.
  - Triangle: $\Lambda(t) = 1 - |t|$ for $|t| \leq 1$, $\Lambda(t) = 0$ otherwise
  - Square pulse: $\Pi(t) = 1$ for $|t| \leq 0.5$, $\Pi(t) = 0$ otherwise
  - $sinc(t) = \frac{sin(\pi t)}{\pi t}$
- Fourier transforms:
  - $\Pi(t) \leftrightarrow sinc(f)$
  - $\Lambda(t) \leftrightarrow sinc^2(f)$
  - $u(t) \leftrightarrow \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$
  - $\frac{1}{\pi t} \leftrightarrow -jsign(f)$
  - $\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \leftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(t - nf_s)$ where $f_s = \frac{1}{T_s}$.