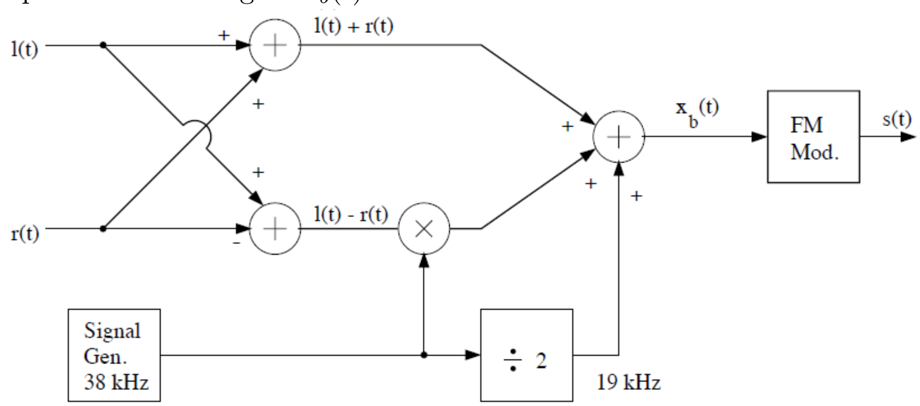
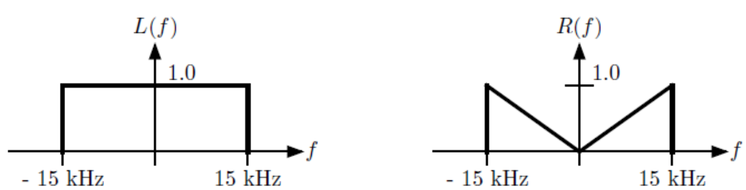
ECE 440 – Spring 2016
Exam 1

Question 1 (15 points)

(a) (10 points)The block diagram below is used as part of the transmitter for stereophonic FM broadcasting although you don't need to know anything about FM to solve this problem since we only ask about the spectrum of the signal $x_b(t)$.



Suppose that the output of the 38kHz signal generator is $2\cos(2\pi f_1 t)$ and the output of the frequency divider block is $2\cos(2\pi f_2 t)$ where $f_1 = 38$ kHz and $f_2 = 19$ kHz and that the CTFTs of the left $l(t)$ and right $r(t)$ signals are real-valued (to make the sketching simple) and given by:



Sketch the spectrum $X_b(f)$, showing the steps that you took to find it (graphically is enough).

(b) **(5 points)** If we wanted to use a digital modulation instead of FM, we would need to sample $x_b(t)$. What is the minimum number of samples we would have to take per second in order to avoid aliasing?

Question 2 (10 points): Find the energy and power for the following signals (0 and ∞ are possible answers). Tell whether they are energy signals or power signals.

- $x_1(t) = \cos(10\pi t)u(t)u(2-t)$
- $x_2(t) = \sum_{n=-\infty}^{\infty} \Lambda(t-3n)$

Question 3 (10 points): Determine whether each of the following statements is true or false, and justify your answer:

1. An envelope detector can be used to recover the message in a double sideband (DSB) modulation.

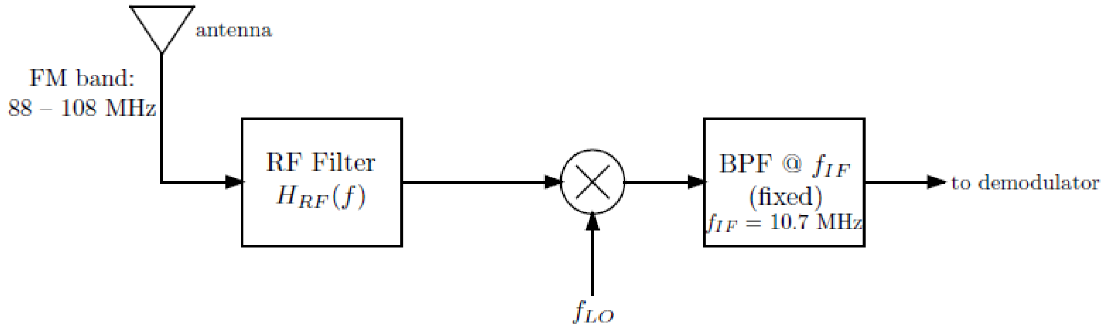
2. Single sideband (SSB) modulation should not be used when the message has low frequency and DC components, but DSB can be used.

3. Amplitude modulation (AM) is efficient in terms of transmitted power, because the delta that appears in the PSD of the modulated signal is very narrow and has negligible power.

4. Vestigial sideband (VSB) modulation requires scaling the message to ensure that its maximum value is below 1.

5. AM is more efficient in terms of bandwidth than DSB.

Question 4: (15 points) A superheterodyne receiver uses an IF frequency of 10.7 MHz. We wish to tune the receiver over the entire FM band which includes center frequencies between $88 \le f_c \le 108$ MHz with 200 kHz spacing. The superheterodyne architecture employed is:



The IF filter is an ideal bandpass filter with passband of width 200 kHz centered at $f_{IF} = 10.7$ MHz.

(a) **(5 points)** Assuming that highside tuning is used (i.e., the local oscillator frequency is higher than any desired frequency in the FM band) find the equation that shows how to chose f_{LO}, i.e., find

$$f_{LO}(f_c) \text{ for } 88 \text{ MHz } \le f_c \le 108 \text{ MHz}.$$

(b) **(5 points)** What are the corresponding image frequencies?

(c) **(5 points)** What type of RF filter is needed to reject the image frequencies (e.g., lowpass, highpass, bandpass, etc.). Can you use a fixed RF filter over the entire tuning range? Explain.

Question 5: (10 points) A transmitter uses a carrier frequency of $f_c = 1000$ Hz so that the unmodulated carrier is $A_c \cos(2\pi f_c t)$. Determine the phase AND frequency deviation if the transmitter output is $x_c(t) = \cos(2400\pi t^2)$.

Question 6: (10 points) A sinusoidal message $m(t) = A\sin(2\pi f_m t)$ is transmitted using Phase Modulation (PM) with deviation constant k_p.

(a) **(3 points)** What is the bandwidth of the transmitted signal, if we want to capture 100% of the transmitted power?

(b) **(3 points)** In order to improve the SNR at the demodulator, we decide to pass the received signal through a band pass filter. What bandwidth should this filter have if we want to capture 98% of the signal power?

(c) **(4 points)** Later, we decide to use Frequency modulation (FM) with frequency deviation constant f_d instead of PM. What blocks would you need to add to the modulator?

Question 7: (15 points) Two random variables X and Y have joint pdf $f_{XY} = Ae^{-|x|-2|y|}$ for $-\infty < x, y < \infty$.

(a) **(5 points)** Is it possible to determine whether they are they independent? If it is, please do so. Otherwise explain why it cannot be determined.

(b) **(5 points)** Is it possible to find the pdf of X? If it is, please do so. Otherwise explain why it cannot be found.

(c) **(5 points)** Find the value of A.

Question 8: (15 points) An ideal finite-time integrator is characterized by the input-output relationship $Y(t) = \int_{t-T}^{t} X(\alpha)d\alpha$.

(a) **(3 points)** Find its impulse response $h(t)$.

(b) **(3 points)** Find and sketch its frequency response $H(f)$.

(c) **(3 points)** If the input X(t) is white noise with (two-sided) power spectral density $\frac{N_0}{2}$, find the power spectral density of the output $Y(t)$.

(d) **(3 points)** Find the autocorrelation function of the output.

(e) **(3 points)** What is the equivalent noise bandwidth of the integrator?

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Formulas and notation:

- Trigonometric:
  - $\cos(2x) = 2\cos^2(u) - 1$
- Functions:
  - Unit step: $u(t) = 0$ for $t < 0$, $u(t) = 1$ for $t > 0$.
  - Triangle: $\Lambda(t) = 1 - |t|$ for $|t| \leq 1$, $\Lambda(t) = 0$ otherwise
  - Square pulse: $\Pi(t) = 1$ for $|t| \leq 0.5$, $\Pi(t) = 0$ otherwise
  - $sinc(t) = \frac{sin(\pi t)}{\pi t}$
- Fourier transforms:
  - $\Pi(t) \leftrightarrow sinc(f)$
  - $\Lambda(t) \leftrightarrow sinc^2(f)$
  - $u(t) \leftrightarrow \frac{1}{j2\pi f} + \frac{\delta(f)}{2}$
  - $\frac{1}{\pi t} \leftrightarrow -jsign(f)$
  - $\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \leftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(t - nf_s)$ where $f_s = \frac{1}{T_s}$.