

Markov Random Fields and Stochastic Image Models

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1 Breakdown of References in Statistical Image Modeling

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 - ii. Markov Chains [41, 42, 152, 129]
 - iii. Ising model [124, 123, 119, 127, 97, 128]
 - iv. Discrete MRF [13, 14, 156, 47, 157, 46, 16, 165, 36, 50, 48, 113, 163, 96, 49, 70, 101, 153, 54, 177, 118, 120, 23, 88, 172, 89, 37, 122, 125, 137, 164, 94, 116, 11, 39, 75, 168, 90]
 - v. MRF with Line Processes [66, 52, 173, 171, 174, 169, 167]
 - (b) Continuous Models
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 - ii. Gaussian MRF [18, 15, 84, 92, 91, 33, 111, 149, 38, 103, 144]
 - iii. Nonconvex potential functions [68, 69, 21, 79, 104, 64, 32, 140]
 - iv. Convex potential functions [17, 73, 104, 105, 151, 24, 143, 87, 25, 27, 32, 146, 26, 145, 147]
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- (a) Quadratic [161, 154, 134, 99, 100, 95, 135, 58]
 - (b) Nonconvex [136, 82, 85, 155, 19, 20]
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 - iii. Greedy optimization [156, 16, 157, 36, 50, 101, 153, 54, 122, 89]
 - iv. Multiscale optimization [22, 70, 23, 125, 94, 102, 107]
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 - v. ICM/Gauss-Seidel/ICD [24, 143, 144, 25, 27]
 - vi. Continuation methods [19, 20, 149, 140]
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 - A. Maximum likelihood [127, 62, 69, 128, 118, 105]
 - B. Coding/maximum pseudolikelihood [15, 16, 18, 67, 101]
 - C. Least squares [48, 75]
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 - B. Non-Gaussian [121, 145, 26, 130, 142, 141]
 - iii. EM based [69, 172, 173, 39, 176, 26, 174, 130, 142, 141]
 - (b) For other models
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 - ii. Order identification [2, 91, 139, 37, 175, 176]
7. Application

- (a) Texture classification [92, 33, 38, 112]
 - (b) Texture modeling [55, 91]
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 - (c) Parameter estimation [34, 29, 162, 150]
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