Distributed Signal Decorrelation in WSNs Using the Sparse Matrix Transform (SMT)

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Distributed Anomaly Detection

Anomaly detection is
- **Important:** Central to Detection theory
- **Ubiquitous:** Many applications in security-related areas
  - Remote Sensing, Surveillance, Network Intrusion Detection, etc...

### Wireless Network of Cameras
- Collectively monitor the environment
- Each outputs image (*vector*) from own viewpoint

**Goal:** detect anomalies based on *joint* measurements from *all cameras*

**Decorrelation requires:**
- $O(p^2)$ computation/communication
- $N \gg p$ samples to design transform

**Big problems**
Approach: Sparse Matrix Transform (SMT)

- Allows covariance to be estimated when $n<<p$
  - Imposes sparsity constraint in non-linear manifold
  - Maintains full rank of covariance estimate

- Results in a fast decorrelating transformation
  - Computation of transform is $O(p)$
  - Generalization of FFT and orthonormal wavelet transform

- **Problem:** Requires lots of communications between sensors

**In this paper:** The Vector SMT

- Improvement on original SMT
- Suitable for implementation in a network of sensors
  - Distributed *in network* implementation
  - Restrict communication between pairs of sensors
Covariance Estimation Framework

• **Data:** We observe \( n \) independent \( N(0,R) \) vectors, each of dimension \( p \).

\[
Y = [y_1, \cdots, y_n]
\]

• **Sample Covariance:**

\[
S = \frac{1}{n} YY^t
\]

• **Model:** Covariance can be represented by

\[
R = E[S] = E\Lambda E^t \quad E - \text{eigen transform} \\
\Lambda - \text{eigenvalues}
\]

• **Maximum Likelihood (ML) Estimate:**

\[
\hat{E} = \arg \min_{E \in \Omega_K} \left\{ \| \text{diag}(E^t SE) \| \right\} \\
\hat{\Lambda} = \text{diag}(\hat{E}^t S\hat{E})
\]

Unconstrained minimization \( \longrightarrow \) PCA of \( S \)

**Big Idea:** Constrain \( \Omega_K \) \( \longrightarrow \) SMT of order \( K \)
The Sparse Matrix Transform (SMT)

- An SMT is a product of Givens rotations
  \[ E = \prod_{k=1}^{K} E_k = E_1 \cdots E_K \], where \( E_k = \cos \theta_k - \sin \theta_k \) and \( K = r \cdot p \), where \( r \) is typically a small constant

- SMT is a generalization of the FFT

- SMT is also a generalization of the orthonormal (paraunitary) wavelet transform
Design of SMT using Cost Optimization

- Cost optimization problem:
- Greedy optimization algorithm:
  - Decorrelating transform, $K = rp \Rightarrow O(p)$ computation
The Vector SMT

Decorrelates $2h$-dimensional vector, $\begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}$

Sensor Node 1
$h$-dimensional output

Sensor Node 2
$h$-dimensional output

Sensor Node 3
$h$-dimensional output

Sensor Node L
$h$-dimensional output

Correlated aggregated vector

Decorrelated aggregated vector
Vector SMT Design in Data Domain

\[ \text{maximize } \Delta \text{Likelihood} \]

\[ p = hL \]

\[ n \text{ random vectors} \]

\[ T_1 \]

\[ T_2 \]

\[ x^{(L)} \]

\[ x^{(1)} \]

\[ x^{(2)} \]

\[ x^{(3)} \]

\[ \ldots \]

\[ x^{(L)} \]

\[ x'^{(1)} \]

\[ x'^{(2)} \]

\[ x'^{(3)} \]

\[ \ldots \]

\[ x''^{(2)} \]

\[ x^{(L)} \]

\[ x''^{(L)} \]
Anomaly Detection with the Vector SMT

- Vector SMT: models parent distribution of typical data
- Anomaly Detection: significance test against parent distribution
- Gaussian with covariance $R$
  
  - Measure of anomalousness: $x^T R^{-1} x$

**Metric for detection accuracy:**

Volume of ellipsoid: $x^T R^{-1} x \leq \eta^2$

- Proxy for missed detection rate
  - Minimum volume desired

Computed by:

$$V(R, \eta) = \frac{\pi^{p/2}}{\Gamma\left(1 + \frac{p}{2}\right)} \eta^p \sqrt{|R|}$$

$\eta$: Controls probability of false alarm
Simulations Setup

We consider two scenarios:
1 – Assume sensor measurements are independent
2 – Assume sensor measurements are correlated – use vector SMT for decorrelation

10-dimensional vectors (h=10)
Monitoring a Moving Sphere

“Typical” trajectory

“Anomalous” trajectory

Side view

Top view
Moving Sphere Anomaly Detection

ROC analysis

Log-Volume

Ellipsoid Log-Volume

- Independent Processing
- Joint Processing
Eigen-Images

Under Independent sensor measurements assumption

Under correlated sensor measurements assumption
Relative Sensitivity of the Two Detectors

Relative sensitivity given by the ratio:

\[
\frac{x^t R_2^{-1} x}{x^t R_1^{-1} x}
\]

Covariance matrices:

- \( R_1 \) - sensor independence assumption
- \( R_2 \) - correlated sensor measurements assumption

Generalized Eigen-decomposition:

Transform \( H \):

\[
HH^t = R_1
\]

\[
H \tilde{\Lambda}_2 H^t = R_2
\]

The ratio becomes a weighted sum of independent components:

\[
\frac{x^t R_2^{-1} x}{x^t R_1^{-1} x} \quad \tilde{x} = H^t x \quad \sum_{k=1}^{p} \frac{1}{[\tilde{\Lambda}_2]_{kk}} \tilde{x}_k^2
\]

Relative sensitivity of generalized coordinate \( k \).
Generalized Eigenvalues/Eigen-Images

Dimensions with largest relative sensitivity
**Goal:** Monitor the clouds of 30 spheres scene using 14 cameras and decide whether
(1) it is a typical configuration (hollow cloud)
(2) OR it is an anomalous configuration (dense cloud)
Cloud Sample – Camera Views 1-14

Sample with typical configuration: Hollow cloud

Sample with anomalous configuration: Dense cloud
Sphere Cloud - Detection

ROC analysis

Ellipsoid Log-Volume

- Independent Processing
- Joint Processing
Conclusions and Future Work

• Vector SMT framework
  – Based on the SMT
  – Decorrelates vector measurements across multiple sensors in a WSN
  – One pair of sensors per iteration

• Simulation results suggest
  – Great potential for use in distributed monitoring applications
  – Multi-view detection of visual anomalies

• Future
  – Analysis of communication costs
  – Comparison with other methods
Thank You