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Inverse problems in atmospheric dispersion with randomly scattered sensors

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Abstract

We consider the problem of monitoring the concentration and dispersion of pollutants in the atmosphere using a collection of randomly scattered sensors. The sensors are capable of only indicating that the concentration has exceeded a randomly selected threshold and providing this information through a transponder-like mechanism to an airborne imaging radar. We use this information together with a standard atmospheric dispersion model to estimate the concentration as well as the time and location of the pollutant source. We use the expectation–maximization algorithm to find the maximum likelihood estimate of the quantities of interest.

Keywords: Inverse problems; Atmospheric dispersion; Gaussian plume model; EM algorithm; Pollutant detection

1. Introduction

The estimation of concentration and sources of pollutants in the atmosphere is an important problem. Currently, the techniques in use for estimating these quantities rely on taking accurate measurements at precisely known or desired locations [2,4,9]. However, in a situation when the pollutant is a toxic substance released by an exploding container or shell, it becomes hazardous and time consuming to send in people or equipment to take

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such direct measurements. In such a situation, it may however be feasible to randomly scatter inexpensive sensors on the ground by an airplane and take measurements using a receiver or radar mounted on the airplane. In this paper, we study such a scenario in which information is extracted by randomly scattered sensors.

The current methods in use for estimating the concentration of pollutants either involve running atmospheric transport models with known sources and wind velocity fields [14], or estimating the source and concentration from sparse measurements of the concentration. The second method falls in the category of inverse problems. Inverse problems in the atmosphere have mainly focused on two techniques, the back-trajectory techniques applicable to Lagrangian models [2,4,15] and inversion based on adjoint equations applicable to Eulerian models [7,12]. Both techniques rely on taking accurate measurements at desired locations.

The term "randomly scattered sensors" used above means that the locations of sensors are modeled by a spatial point process. In this paper, we will assume the sensors are scattered according to a two-dimensional homogeneous Poisson process. For this technique to be feasible, the sensors themselves must be inexpensive. Hence, it is assumed that they are simple devices which can only detect if the pollutant concentration is above a particular threshold. The sensors thus can take only rough measurements. Furthermore, we also assume that the sensors transmit their signal to a single receiver or radar mounted on an airplane which we call the "hub." The sensors are unable to communicate with each other and hence transmit asynchronously.

Apart from the concentration of the pollutant, it is also important to know the location of the pollutant source in space and time. This information may help us in quick neutralization of the source perhaps reducing the damage. If wind velocity is assumed to be constant, atmospheric dispersion of pollutants is well modeled by the Gaussian plume model [3,16–18]. We also propose using the Gaussian plume model to estimate the source location and strength. We take a series of measurements in time from the scattered sensors and construct an estimator of the location as well as the strength of the source.

2. The measurement model

The measurement model consists of the description of how we obtain data from the randomly scattered sensors. We assume that the sensors can only detect if the pollutant concentration is above a particular threshold. The threshold is randomly selected and fixed for each sensor. This randomization of threshold increases the information content of the received signal. This is so because if we have the same threshold for each sensor in a small region, we would get the same answer from each of them. However, when we select the thresholds randomly, the answers from the sensors are not the same and hence more information can be extracted from them.

We divide the region of interest into smaller regions called "cells." We further assume that the locations of the sensors are modeled by a spatial point process. In particular, we visualize a scenario in which the sensors are simply thrown out in an area, in which case, the spatial point process will be a two-dimensional homogeneous Poisson process.

The sensors are assumed to send their signals to a single hub. A sensor sends its signal if the concentration is above its threshold, otherwise it stays silent. The hub detects the power in the vector sum of the signals coming from all the active sensors in a particular cell. If the signal sent by each sensor is a constant amplitude sine wave, then the pdf of the received signal power as a function of the number of scattered sensors in a cell has been derived in Ref. [1].

We define some notation:

C	the set of all cells, and let $s \in C$ denote a particular cell,
y_s	the observed intensity of the received signal from the cell s,
N_s	number of sensors in the cell s,
N	the maximum number of sensors in the cell s,
d_s	concentration of the pollutant in the cell s,
$T_{i,s}$	the threshold, i.e., a sensor will be active if $T_{i,s} \leqslant d_s$. The cdf of $T_{i,s}$ is $F_T(t)$. Here, the subscript i represents an individual sensor in the cell s,
K_s	number of active sensors in the cell s, $K_s = \sum_{i=1}^{N_s} I_{\{T_{i,s} \leqslant d_s\}}$,
$P_N(n_s)$	the probability mass function of i.i.d. random variables N_s,
$\bar{K}_s$	$E[K_s \mid y_s, d_s]$,
$\bar{N}_s$	$E[N_s \mid y_s, d_s]$,
$\psi(y_s \mid K_s)$	the pdf of the received signal with K_s active sensors.

The symbols without the subscript will denote the corresponding vector, i.e., data over all cells and time.

We now proceed to derive the pdf of the received signal y_s. The probability that a sensor is active is $\Pr(T_{i,s} \leqslant d_s) = F_T(d_s)$. Hence it can be shown that

$$\Pr(K_s = k_s \mid N_s = n_s, d_s) = \binom{n_s}{k_s} \big(1 - F_T(d_s)\big)^{n_s - k_s} \big(F_T(d_s)\big)^{k_s}. \tag{1}$$

Using this result, the conditional distribution of Y_s given d_s can be derived as

$$\begin{aligned} f(y_s \mid d_s) &= \sum_{n_s=0}^{N} \sum_{k_s=0}^{n_s} \psi(y_s \mid k_s) \Pr(K_s = k_s \mid N_s = n_s, d_s) P_N(n_s) \\ &= \sum_{n_s=0}^{N} \sum_{k_s=0}^{n_s} \psi(y_s \mid k_s) \binom{n_s}{k_s} \big(1 - F_T(d_s)\big)^{n_s - k_s} \big(F_T(d_s)\big)^{k_s} P_N(n_s). \end{aligned} \tag{2}$$

Equation (2) is then the conditional pdf of the received signal conditioned on the pollutant concentration in the cell.

3. The dispersion model

Material released into the atmosphere is transported by the wind and mixed into the surrounding air by turbulent eddies and molecular diffusion. This atmospheric transport of materials, particularly pollutants, is modeled by atmospheric dispersion models. There are

two different approaches to atmospheric dispersion modeling. The Lagrangian approach models the dispersion by computing the trajectories of a few Lagrangian particles [8,13]. The Eulerian approach on the other hand focuses on the conservation of mass equation in a fixed volume [2,4–6]. The general transport equation describing the conservation of mass in a fixed differential volume is given by

$$\frac{\partial \Gamma}{\partial t} = -\mathbf{V} \cdot \nabla \Gamma + \nabla \cdot \mathbf{K} \nabla \Gamma + S, \tag{3}$$

where Γ is the concentration of material in the atmosphere, $\Gamma = \Gamma(x, y, z, t)$, $\mathbf{V}$ is the wind velocity field, $\mathbf{K}$ is the parameter describing eddy diffusivity and molecular diffusion, $\mathbf{K} = [K_x, K_y, K_z]$, and S is the source term.

Consider the special case when $S = D\delta(x, y, z, t)$, $\mathbf{V} = [u, 0, 0]$, $K_x = ab(ut)^b/2t$, $K_y = ab(ut)^b/2t$, and $K_z = cd(ut)^d/2t$. This corresponds to an instantaneous release of a pollutant from a point source at the origin with a constant wind velocity in the x-direction. In this case, Eq. (3) has a closed form solution given by

$$\Gamma(x, y, z, t) = \frac{D}{(2\pi)^{3/2}\sigma_x(t)\sigma_y(t)\sigma_z(t)} \times \exp\left\{-\frac{1}{2}\left[\left(\frac{x - ut}{\sigma_x(t)}\right)^2 + \frac{y^2}{\sigma_y^2(t)} + \frac{z^2}{\sigma_z^2(t)}\right]\right\}, \tag{4}$$

where

$$\sigma_x^2(t) = a(ut)^b, \quad \sigma_y^2(t) = a(ut)^b, \quad \sigma_z^2(t) = c(ut)^d.$$

Equation (4) is also popularly known as the *Gaussian dispersion model* and is perhaps the most commonly used atmospheric dispersion model. The parameters of Eq. (4) are: D is the mass of material released into the atmosphere, u is the constant wind velocity, and a, b, c, and d are the constants determined by atmospheric stability. The three coefficients $\sigma_x(t)$, $\sigma_y(t)$, and $\sigma_z(t)$ are sometimes called the dispersion coefficients. They represent the standard deviations of the concentration in the downwind (x), crosswind (y) and vertical (z) planes respectively. The calculation of the dispersion coefficients and atmospheric stability has been widely discussed in Refs. [3,16,17].

Equation (4) does not specify what happens when the gas reaches the ground. To resolve this problem, we assume that the gas is reflected back into the atmosphere by the ground. This can be modeled by a virtual source below the ground [3]. Further, if we now fix the origin and assume that the gas was released at (x_0, y_0, z_0, t_0), the concentration of the gas on the ground i.e., at $z = 0$ is given by the new equation

$$\Phi(x, y, t) = \Gamma(x, y, 0, t) = \frac{2D}{(2\pi)^{3/2}\sigma_x(t)\sigma_y(t)\sigma_z(t)} \times \exp\left\{-\frac{1}{2}\left[\left(\frac{x - x_0 - u(t - t_0)}{\sigma_x(t)}\right)^2 + \left(\frac{y - y_0}{\sigma_y(t)}\right)^2 + \frac{z_0^2}{\sigma_z^2(t)}\right]\right\}. \tag{5}$$

4. Estimation

We use the expectation–maximization (EM) algorithm proposed by Dempster et al. [10] to estimate the quantities of interest. The idea of the EM algorithm is to define two groups of data: "incomplete" data $y \in \mathbf{Y}$ which is also the measured or observed data and "complete" data $x \in \mathbf{X}$ where $\mathbf{Y} \subset \mathbf{X}$. Let θ be a point in parameter space Θ which parameterizes the density of the incomplete data $\mathbf{Y}$. The EM algorithm is the iterative application of two steps:

$$E\text{-step:} \quad Q(\theta, \theta^k) = E\big[\log f_X(X|\theta)|y, \theta^k\big],$$
$$M\text{-step:} \quad \theta^{k+1} = \arg\max_{\theta} Q(\theta, \theta^k),$$

where $f_X(X|\theta)$ denotes the density of $\mathbf{X}$ given θ. It can be shown, [10] that $Q(\theta, \theta^{k+1}) \geqslant Q(\theta, \theta^k) \Rightarrow f_Y(y|\theta^{k+1}) \geqslant f_Y(y|\theta^k)$. Thus, the EM algorithm converges to a local maximum of the likelihood [11].

4.1. Estimation of concentration of pollutant in a cell

To estimate the concentration of the pollutant d_s, in cell s, we use the EM algorithm with $\mathbf{Y} = Y_s$ and $\mathbf{X} = \{Y_s, K_s\}$. The joint density of Y_s, K_s, and N_s given d_s is given by

$$f(y_s, k_s, n_s|d_s) = \psi(y_s|k_s)\binom{n_s}{k_s}\big(1 - F_T(d_s)\big)^{n_s - k_s}\big(F_T(d_s)\big)^{k_s} P_N(n_s). \tag{6}$$

In the particular case when N_s is a Poisson random variable with mean μ, we can simplify Eq. (6). In this case, given d_s, K_s is a Poisson random variable with mean $F_T(d_s)\mu$. Hence we get the conditional density

$$f(y_s, k_s|d_s) = \psi(y_s|k_s)\frac{e^{-F_T(d_s)\mu}(F_T(d_s)\mu)^{k_s}}{k_s!}, \tag{7}$$

which results in the simplified Q-function given by

$$Q(d_s, d_s^k) = \bar{K}_s \log\big(\mu F_T(d_s)\big) - \mu F_T(d_s), \tag{8}$$

where the constant terms in d_s have been dropped and

$$\bar{K}_s = \sum_{k_s=0}^{\infty} k_s \frac{f(y_s, k_s|d_s^k)}{f(y_s|d_s^k)}.$$

The M-step for this Poisson case is given by $d_s^{k+1} = F_T^{-1}(\bar{K}_s/\mu)$.

4.2. Maximum a posteriori estimation

We use the maximum a posteriori (MAP) estimator to estimate the concentration of the pollutant over several cells. This is useful when we do not have a model for the dispersion of the pollutant but can impose some relationship between the concentration in adjacent cells. The MAP estimate of a parameter vector λ given the data vector y is given by

$$\tilde{\lambda} = \arg\max_{\lambda} f(\lambda|y) = \arg\max_{\lambda}\big[\log f(y|\lambda) + \log p(\lambda)\big].$$

The term $\log p(\lambda)$ incorporates a priori information in the estimate, which may represent the relationship we expect between the concentration of adjacent cells. One possible prior model for d is the generalized Gaussian–Markov random field (GGMRF) model [21,22] which has been shown to have the desirable properties of convexity and scalability. The GGMRF model is given by

$$\log p(d) = -\frac{1}{p\sigma^p} \sum_{\{s,r\}\in C} b_{s,r}|d_s - d_r|^p + \text{constant}. \tag{9}$$

The parameters control different features of the model. They are: p is the parameter which controls edge preservation, $b_{s,r}$ is the parameter controlling the magnitude of influence of the neighbors, and σ is the normalizing constant of the GGMRF.

In particular we choose $p = 2$ and the parameter σ^2 is estimated by its maximum likelihood estimator $\hat{\sigma}^2 = \frac{1}{|C|}\sum_{\{i,j\}\in C} b_{i,j}|x_i - x_j|^2$, where $|C|$ denotes the cardinality of the set C. The function $b_{s,r}$ is shift invariant so that $b_{i,j} = b_{i-j} = b_{j-i}$. In this paper $b_{i,j}$ is given by the 2-D array

$$\begin{bmatrix} \frac{1}{4(1+\sqrt{2})} & \frac{1}{2(2+\sqrt{2})} & \frac{1}{4(1+\sqrt{2})} \\ \frac{1}{2(2+\sqrt{2})} & 0 & \frac{1}{2(2+\sqrt{2})} \\ \frac{1}{4(1+\sqrt{2})} & \frac{1}{2(2+\sqrt{2})} & \frac{1}{4(1+\sqrt{2})} \end{bmatrix},$$

where the $b_0 = 0$ term is at the center of the array. The function $b_{s,r}$ has been normalized so that $\sum_{j\in\partial i} b_{i,j} = 1$, where ∂i denotes the neighborhood of i such that if $j \in \partial i$ then $b_{i,j} \neq 0$. Furthermore, if $b_{s,r} \neq 0$, then it is inversely proportional to the Euclidean distance between cells s and r.

For computing the MAP estimate we use the generalized EM (GEM) [19] and iterative coordinate descent (ICD) [20] algorithms. The GEM algorithm is based on the following result: $Q(d^{k+1}, d^k) > Q(d^k, d^k) \Rightarrow f(d^{k+1}|y) > f(d^k|y)$. Thus, instead of finding the global maximum, we just ensure that $Q(d^{k+1}, d^k) > Q(d^k, d^k)$. The ICD algorithm for optimization of any function of a vector of independent variables $\{x_1, x_2, \ldots, x_n\}$ works by sequentially optimizing the function with respect to each variable keeping the rest fixed.

Using Eq. (8) and the assumption that N_s is Poisson, the Q-function is given by

$$\begin{aligned} Q(d, d^k) &= \sum_{s\in C} E\big[\log f(y_s, K_s|d_s)|y_s, d_s^k\big] + \log p(d) \\ &= \sum_{s\in C} \big(\bar{K}_s \log\big(\mu F_T(d_s)\big) - \mu F_T(d_s)\big) - \frac{1}{2\sigma^2} \sum_{\{i,j\}\in C} b_{i,j}|d_i - d_j|^2, \end{aligned} \tag{10}$$

where again the constant terms in d_s have been dropped. The expectation can be computed as described in Section 4.1. For the M-step, we use the ICD algorithm and maximize $Q(d, d^k)$ sequentially with respect to one cell at a time fixing the others. In this case, the ICD algorithm satisfies all the conditions given in Ref. [23] and hence is convergent. Thus, the update of the concentration d_i in the cell i is given by

$$d_i = \arg\max_d \left(\bar{K}_i \log\big(\mu F_T(d)\big) - \mu F_T(d) - \frac{1}{2\sigma^2} \sum_{j\in\partial i} b_{i,j}|d - d_j|^2 \right). \tag{11}$$

4.3. Estimation of source location and strength

Since the pollutant disperses in the atmosphere with time, we need to take a sequence of measurements in time to estimate any temporal information about the pollutant, like finding its source, rate of spread, etc. We therefore assume that M measurements are made for each cell at times corresponding to $0, T, 2T, \ldots, (M-1)T$. We use the Gaussian Plume dispersion model given by Eq. (5) to model the spread of pollutant in the atmosphere. We assume that a, b, c, and d are known, so the parameters of the equation are $g = (D, x_0, y_0, z_0, t_0, u)$. Estimating the parameters of the equation, we can predict the source location (x_0, y_0, z_0), the time of release t_0 and the strength D of the source.

We define some more notation. If s indicates a particular cell, then let $y_{s,i} = y_s(iT)$. Let the concentration of the gas be $d_{s,i}$, which is given by Eq. (5), thus $d_{s,i} = \Phi(x_s, y_s, iT|g)$. We also define the time sequence of measurements in a cell by $\tilde{y}_s = \{y_{s,0}, y_{s,1}, y_{s,2}, \ldots, y_{s,(M-1)}\}$ and the time sequence of active sensors in a cell by $\tilde{k}_s = \{k_{s,0}, k_{s,1}, k_{s,2}, \ldots, k_{s,(M-1)}\}$, where $k_{s,i} = k_s(iT)$.

We note that if g is known, the time series measurements for different cells $\tilde{y}_s$ are independent of each other. We assume that the hub, which we visualize to be mounted on an airplane, moves a distance greater than the wavelength of the signal independently for each look. The measurements then become conditionally independent given $\tilde{K}_s$. Thus, we have the relationships

$$f(y|g) = \prod_{s\in C} f\big(\tilde{y}_s|g\big), \qquad f\big(\tilde{y}_s|\tilde{K}_s = \tilde{k}_s\big) = \prod_{i=0}^{M-1} \psi(y_{s,i}|k_{s,i}).$$

In Appendix A we show how to derive the EM algorithm when N_s does not change with time. However, this strategy is computationally expensive and has poor convergence properties. A simplification can be made if $N_{s,i}$ is assumed to be an independent Poisson random variable with mean μ for each look. This assumption is equivalent to perturbing the cell structure such that at each look the number of sensors in each cell become independent of each other. In this case, given g, $\tilde{K}_s$ is independent for each look, i.e., $f(\tilde{k}_s|g) = \prod_{i=0}^{M-1} f(k_{s,i}|g)$. Using the assumptions above and Eq. (7) we get the conditional density

$$f(y,k|g) = \prod_{s\in C}\prod_{i=0}^{M-1} \psi(y_{s,i}|k_{s,i}) \frac{e^{-F_T(d_{s,i})\mu}(F_T(d_{s,i})\mu)^{k_{s,i}}}{k_{s,i}!} \tag{12}$$

and the simplified Q-function

$$\begin{aligned} Q(g, g^k) &= \sum_{s\in C}\sum_{i=0}^{M-1} \big(\bar{K}_{s,i} \log\big(F_T(d_{s,i})\mu\big) - F_T(d_{s,i})\mu\big) \\ &= \sum_{s\in C}\sum_{i=0}^{M-1} \big(\bar{K}_{s,i} \log\big(F_T(\Phi(x_s, y_s, iT|g))\mu\big) \\ &\quad - F_T\big(\Phi(x_s, y_s, iT|g)\big)\mu\big), \end{aligned} \tag{13}$$

where terms constant in g have been dropped. The expectation can be computed as described in Section 4.1.

For the M-step, we need to maximize $Q(g, g^k)$ with respect to g. This can be done using an iterative strategy like steepest-descent or conjugate-gradient. In this paper, we used the conjugate-gradient algorithm for the maximization. We constrained the parameter D to be nonnegative. The starting point for the maximization algorithm was selected to be $D = 1000$, $x_0 = 0$, $y_0 = 10$, $z_0 = 10$, $t_0 = -5$, and $u = 5$, but could be based on the rough estimates given by the MAP estimate described in Section 4.2.

5. Simulation results

We simulated the algorithms derived above to estimate the concentration and the source parameters. We considered the case when the signal sent by the sensor is a constant amplitude sine wave and we use a power law detector in the hub. Thus, the hub detects the power in the vector sum of the signal sent by the sensors in a particular cell. The pdf of the received signal $\psi(y_s|K_s)$, in this case, has been derived in Ref. [1]. We also assumed that the number and locations of the sensors are modeled by a two-dimensional homogeneous Poisson (μ) process. The cdf of the randomized thresholds $F_T(t)$, was assumed to be linear in the range [0, 1], i.e. $F_T(t) = t,\ t \in [0, 1]$.

We found that the expected value of the received signal could be closely approximated as a linear function of d_s as illustrated in Fig. 1. This also motivated the use of an adaptive starting point for the algorithm $d_s^{\text{start}} = y_s/\mu$. This adaptive starting point helped increase the convergence speed of the EM algorithm as illustrated in Fig. 2.

The mean value of $\mu = E[N_s]$, was found to be of critical importance in the performance of the algorithm. For the high-rate case, i.e. for a large number of sensors in a cell, the MLE was found to be well-behaved compared to the low-rate case where the MLE behaved very peculiarly. Figures 2 and 3 show the behavior of the MLE at a high-rate case

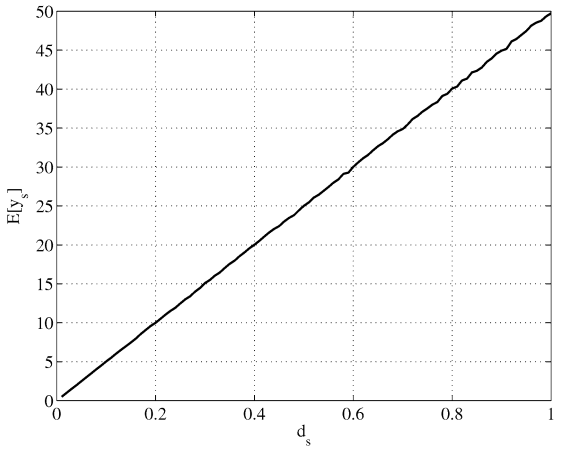

Fig. 1. The Monte Carlo simulation result of $E[y_s]$ vs d_s for $\mu = 50$ with 100,000 tries. The plot shows that $E[y_s]$ can be approximated very closely as a linear function of d_s.

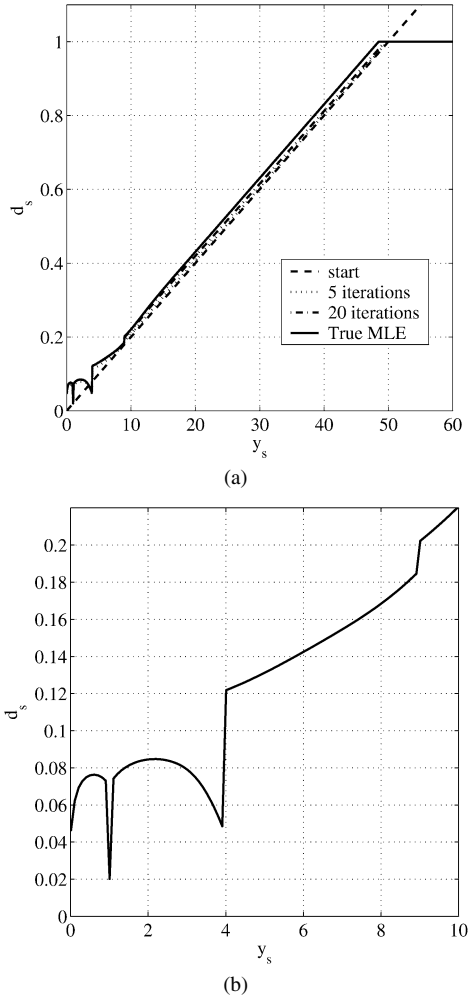

Fig. 2. Shows the behavior of the MLE and convergence of the EM algorithm for $\mu = 50$. (a) Shows that the EM algorithm converges to the MLE, (b) shows the peculiar behavior of the MLE at low values of y_s.

of $\mu = 50$ and at a low-rate case of $\mu = 1$. It is seen from Fig. 2 that for a typical high-rate case, there may be some peculiarities in the MLE for low values of y_s but for large values of y_s the MLE is well-behaved. However, as illustrated in Fig. 3, the MLE acts almost like a step-function and hence does not give a reliable estimate of d_s for the low-rate case. It was also observed that for the low rate case, increasing the noise variance just shifts the step-function.

Though it was difficult to reconstruct information in a particular cell, we found that estimating parameters over several cells was possible for a low-rate case. Figure 4 shows the result of estimating a cloud of pollutant dispersing in the atmosphere with different techniques for $\mu = 1$. The different constants were assumed to have the following values:

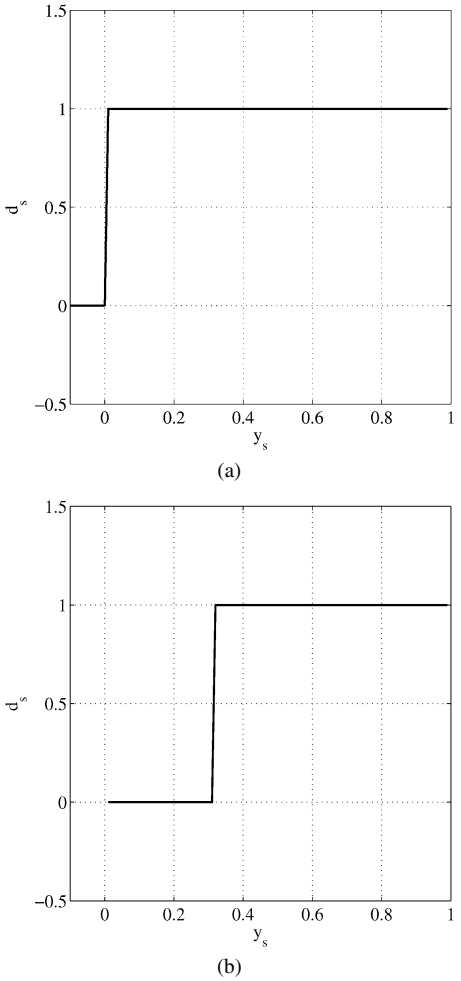

Fig. 3. Shows the peculiar behavior of the MLE in the low-rate case of $\mu = 1$. (a) Shows the MLE with $\psi(y_s|k_s)$ as described in Ref. [1], (b) shows the effect of adding noise of variance $\sigma^2 = 0.08$, to the signal.

$T = 2$, $M = 10$, $a = b = d = 1$, and $c = 0.5$. In the first technique, we used the most general method by using a Markov random field prior on d_s with $\sigma^2 = 0.000192$ and estimating the concentration in each look. Next we used the semi-parameterized method which relied on assuming that the concentration could be modeled as a Gaussian, i.e., $d_s = h \exp\{-(s-c)^2/\gamma^2\}$ and estimating its parameters $\{h, c, \gamma^2\}$ for each look. Thus, no correlation in time was assumed in this method. This method is the special case for a single look, i.e., for $M = 1$, as described in Section 4.3. Finally we used the Gaussian plume model to estimate the parameters in the fully parameterized equation of the pollutant. It was found that the fully parameterized version using the Gaussian plume model out-performed the other methods and the result was very close to the true values. Tables 1 and 2 shows the numerical result of the estimation algorithms corresponding to Fig. 4.

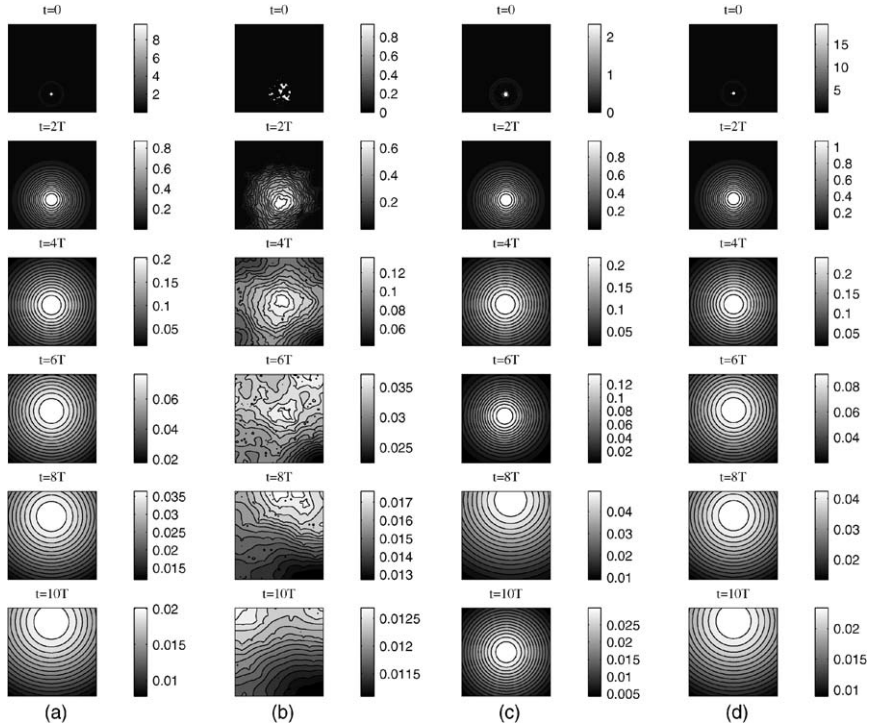

Fig. 4. The figure shows a sequence of snapshots of the pollutant dispersing in the atmosphere in an area which is divided into 64×64 cells. It compares the result of estimating the concentration of the pollutant with different methods for the low-rate case of $\mu = 1$. (a) True sequence of the pollutant dispersing in the atmosphere, (b) the estimated sequence using a Markov random field prior, (c) the estimated sequence using a semi-parameterized model, and (d) the estimated sequence using the fully parameterized Gaussian plume model.

Table 1
This table shows the result of estimating the parameters by using the fully parameterized Gaussian plume model

	D	x_0	y_0	z_0	t_0	u
True	1800	10	32	4	−2	2
Estimated	2083	10.7572	32.8335	2.6642	−2.0711	1.8507

Table 2
This table shows the result of estimating the parameters by using the semi-parameterized model

Time	True				Estimated			
t	h	x_c	y_c	γ^2	h	x_c	y_c	γ^2
0	10.3466	14	32	32	2.4995	13.793	32.3	53.233
$2T$	0.9321	22	32	288	1.0448	21.774	32.421	282.787
$4T$	0.2162	30	32	800	0.2308	30.557	31.857	839.06
$6T$	0.0803	38	32	1568	0.1436	34.43	31.456	535.795
$8T$	0.0381	46	32	2592	0.052	57.45	35.661	2500.0
$10T$	0.0210	54	32	3872	0.0317	32.364	32.605	1001.01

6. Conclusion

In this paper, we have shown that useful information can be reconstructed by using inexpensive randomly scattered sensors. In particular, we showed how the concentration of a pollutant can be estimated using sensors which only respond if the concentration has exceeded a threshold. An algorithm was also derived to find the source location and strength from these sensors using the Gaussian plume dispersion model which was shown to work in the low-rate case. We also showed that, in some circumstances, the algorithm can be simplified and the convergence rate can be increased.

Appendix A

Our objective in this appendix is to provide an algorithm for estimating the source location and strength in the case when N_s is not assumed to be an independent Poisson random variable for each look, but instead is constant in time.

We make an assumption that if N_s is known for a particular cell s, then $\tilde{K}_s$ becomes independent for each look. This assumption is not true in the case of having a fixed threshold for each sensor but is equivalent to the dithering of threshold of the sensors for each look. We thus have the relationship $f(\tilde{k}_s|N_s = n_s, g) = \prod_{i=0}^{M-1} f(k_{s,i}|n_s, g)$. Using this assumption and Eq. (1) we get

$$f(y,k,n|g) = \prod_{s\in C}\Bigg(\prod_{i=0}^{M-1}\psi(y_{s,i}|k_{s,i})\binom{n_s}{k_{s,i}} \times \big(1-F_T(d_{s,i})\big)^{n_s-k_{s,i}}\big(F_T(d_{s,i})\big)^{k_{s,i}}\Bigg)P_N(n_s). \tag{A.1}$$

Taking the expectation of Eq. (A.1) we get the simplified Q-function

$$Q(g,g^k) = \sum_{s\in C}\sum_{i=0}^{M-1}\big(\big(\bar{N}_s - \bar{K}_{s,i}\big)\log\big(1-F_T(d_{s,i}) + \bar{K}_{s,i}\log\big(F_T(d_{s,i})\big)\big), \tag{A.2}$$

where the terms constant in g have been dropped. We note that $\bar{N}_s = E[N_s|\tilde{y}_s, g]$, $\bar{K}_{s,i} = E[K_{s,i}|\tilde{y}_s, g]$, and

$$\begin{aligned} f(\tilde{y}_s|g) &= \sum_{n_s=0}^{\infty}\sum_{k_{s,0}=0}^{n_s}\sum_{k_{s,1}=0}^{n_s}\cdots\sum_{k_{s,M-1}=0}^{n_s} f\big(\tilde{y}_s,\tilde{k}_s,n_s|g\big) \\ &= \sum_{n_s=0}^{\infty}P_N(n_s)\prod_{i=0}^{M-1}\Bigg(\sum_{k_{s,i}=0}^{n_s}\psi(y_{s,i}|k_{s,i})\binom{n_s}{k_{s,i}} \times \big(1-F_T(d_{s,i})\big)^{n_s-k_{s,i}}\big(F_T(d_{s,i})\big)^{k_{s,i}}\Bigg). \end{aligned} \tag{A.3}$$

The expectation can now be computed by using equations (A.1) and (A.3) and the equations

$$\bar{N}_s = \sum_{n_s=0}^{\infty} \sum_{k_{s,0}=0}^{n_s} \sum_{k_{s,1}=0}^{n_s} \cdots \sum_{k_{s,M-1}=0}^{n_s} n_s \frac{f(\tilde{y}_s, \tilde{k}_s, n_s|g)}{f(\tilde{y}_s|g)}$$

$$= \frac{1}{f(\tilde{y}_s|g)} \sum_{n_s=0}^{\infty} n_s P_N(n_s) \prod_{i=0}^{M-1} \left(\sum_{k_{s,i}=0}^{n_s} \psi(y_{s,i}|k_{s,i}) \binom{n_s}{k_{s,i}} \times \left(1 - F_T(d_{s,i})\right)^{n_s - k_{s,i}} \left(F_T(d_{s,i})\right)^{k_{s,i}} \right),$$

$$\bar{K}_{s,i} = \sum_{n_s=0}^{\infty} \sum_{k_{s,0}=0}^{n_s} \sum_{k_{s,1}=0}^{n_s} \cdots \sum_{k_{s,M-1}=0}^{n_s} k_{s,i} \frac{f(\tilde{y}_s, \tilde{k}_s, n_s|g)}{f(\tilde{y}_s|g)}$$

$$= \frac{1}{f(\tilde{y}_s|g)} \sum_{n_s=0}^{\infty} P_N(n_s) \prod_{i=0}^{M-1} \left(\sum_{k_{s,i}=0}^{n_s} k_{s,i} \psi(y_{s,i}|k_{s,i}) \binom{n_s}{k_{s,i}} \times \left(1 - F_T(d_{s,i})\right)^{n_s - k_{s,i}} \left(F_T(d_{s,i})\right)^{k_{s,i}} \right).$$

The maximization in the M-step for the Q-function can be done through an iterative maximization strategy such as the steepest-descent or conjugate-gradient algorithm.

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