A Local Update Strategy for Iterative Reconstruction from Projections

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Abstract

We present a method for Bayesian reconstruction from projections which updates single pixel values, rather than the entire image, at each step. The technique is similar to Gauss-Seidel (GS) iteration for the solution of differential equations on finite grids. The computational cost per iteration of the GS approach is found to be approximately equal to that of gradient methods. For continuously valued images, GS is found to have significantly better convergence at modes representing high spatial frequencies. In addition, GS is well-suited to segmentation when the image is constrained to be discrete-valued [1, 2].

1 Introduction

2 Model of Physical System

In practice, reconstruction requires finite-dimensional representation of both the projection data, p, and the modeled image, f. The Radon transform equations may be written in the discrete form

$$p = \mathbf{A}f$$

where $\mathbf{A}$ is a sparse MxN matrix with A_{ji} equal to the length of the intersection of projection ray j and pixel i.

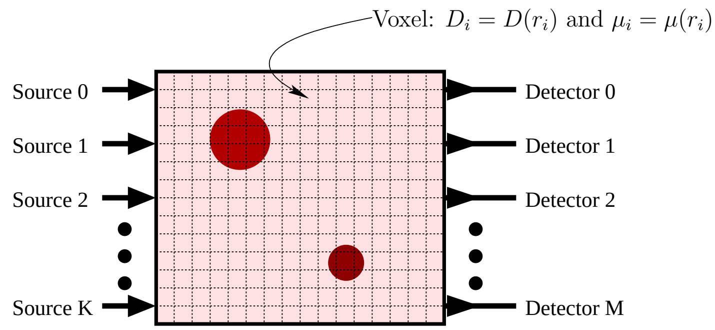


Figure 1: This is an example of a figure drawn in xfig that contains Latex equations.

3 Optimization Techniques

3.1 Gradient Ascent

We will first consider gradient ascent, an iterative scheme whose update equation may be written in the form of a standard discrete time system.

$$\begin{aligned} f^{(n+1)} &= \left[I - \alpha(\mathbf{A}^t\mathbf{D}\mathbf{A} + \gamma\mathbf{R})\right] f^{(n)} + \alpha\mathbf{A}^t\mathbf{D}\hat{p} \\ &= \left[I - \alpha(\mathbf{H} + \gamma\mathbf{R})\right] f^{(n)} + \alpha b \end{aligned} \tag{1}$$

Each iteration of (1) requires the computation of a projection, a backprojection and multiplication by the matrices $\mathbf{D}$ and $\mathbf{R}$.

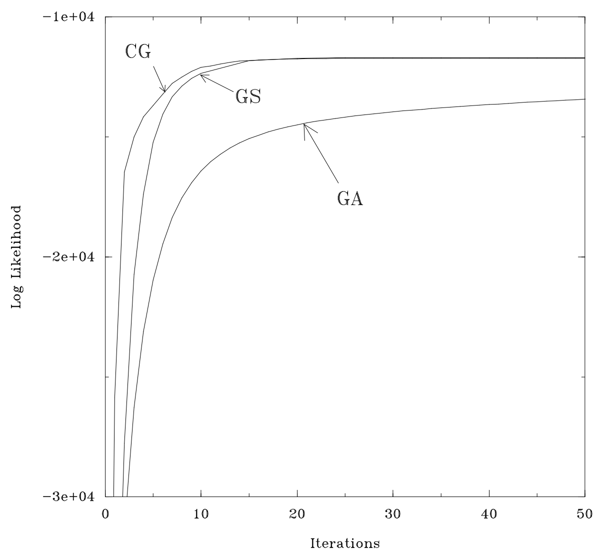
4 Experimental Results

Regularization both speeds convergence, and prevents excessive oscillation in the estimate. For the following results, we use an $\mathbf{R}$ with the form of a discrete 5-point Laplacian. Typical convergence rates are shown in Fig. 2 for MAP estimation with the same optimization methods and $\gamma = 100\text{cm}^2$. (This corresponds to a standard deviation of a pixel given its neighbors of 0.1cm^{-1}) The associated error spectrum has substantial energy at very low frequencies, plus an approximately flat spectral content across the higher frequencies. Here CG enjoys a slight advantage in convergence rate, and both CG and GS are essentially completely converged at fewer than 15 iterations. GA is much slower, as expected. Trials with larger γ yielded still faster convergence, but very similar relationships among the three techniques.

References

[1] K. Sauer and C. Bouman, "Bayesian estimation of transmission tomograms using segmentation based optimization," *IEEE Trans. on Nuclear Science*, vol. 39, pp. 1144–1152, 1992.

[2] K. Sauer and C. A. Bouman, "A local update strategy for iterative reconstruction from projections," *IEEE Trans. on Signal Processing*, vol. 41, no. 2, pp. 534–548, February 1993.



a	b
c	d

Figure 2: Convergence comparison for real projection weighting matrix D and regularization using a Gaussian prior.