EE 641 DIGITAL IMAGE PROCESSING II
Assignment #3 - Spring 1996
Tuesday February 20, 1996

1) Let Y be a 1-D AR process with $h_n = \rho\delta_{n-1}$ and σ^2 prediction variance. Compute (σ^2_{NC}, g) the noncausal prediction variance and the noncausal prediction filter.

2) Consider the following functional

$$f(x) = ||y - x||^2 + x^t Bx$$

where y and x are vectors, and B is symmetric and positive definite matrix.
a) Compute the Gradient decent algorithm for minimizing $f(x)$.

$$x^{(k+1)} = x^{(k)} + \omega \bigtriangledown_x f(x^{(k)})$$

b) Compute the Coordinate decent algorithm for minimizing $f(x)$.
c) Compute the Steepest decent algorithm for minimizing $f(x)$. (Hint: steepest decent is the same as gradient decent with ω choosen to minimize the functional at each step.)

3) Assume that the noisy image Y is related to the noiseless image X by

$$Y = X + N$$

where N is i.i.d. Gaussian noise with mean 0 and variance $\sigma^2 = 32^2$. Furthermore, assume that X is a Gaussian MRF with noncausal prediction variance of $\sigma_x^2 = 2^2$ and a noncausal prediction filter of g_s that has the form

1/12	1/6	1/12
1/6	0	1/6
1/12	1/6	1/12

where the 0 term is at the center of the filter.
a) Use the monochrome image from the previous homework (img03.tif) as X and produce a noisy image Y. Approximate Y by truncating the pixels to the range $[0, \cdots 255]$. Print out the image Y.
b) Compute the MAP estimate of X using 20 iterations for coordinate decent optimization. Print out the optimization result for interations 1,5,10, and 20.
c) Repeat step b) for $\sigma_x^2 = 0.2^2$, and $\sigma_x^2 = 20^2$. Explain your results. For each case, print out the result after 20 iterations.